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$Z(4430)$ and a new paradigm for spin interactions in tetraquarks

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Following the recent confirmation of the $Z^+(4430)$ resonance with $J^{PG} = 1^{++}$, we have re-examined the model of S- and P-wave tetraquarks. We propose a "type-II" diquark-antidiquark model which shows to be very effective at producing a simple and comprehensive picture of the $J^{PG} = 1^{++}$ and 1^{--} sectors of the recently discovered charged tetraquarks and of the observed Y resonances. The model is still faced with the unresolved difficulty of explaining why some states seem to have incomplete isospin multiplets.

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I. INTRODUCTION

The recent observation by LHCb [1] of the $Z(4430)$ charged tetraquark supports the earlier Belle [2,3] indication, later cast into doubt by *BABAR* [4]. This confirmation is decisive for the understanding of the complex system of charmonium-like exotic resonances, the so-called X, Y, Z states, discovered in recent years and the subject of diverse and conflicting theoretical interpretations.

The idea that charmed meson molecules might be formed in high-energy reactions, proposed originally in Ref. [5], has been invoked several times in the context of X, Y, Z spectroscopy with special reference to loosely bound molecules whose prototypical example is the $X(3872)$, which is supposed to be a $D^0\bar{D}^{*0}$ molecule with a binding energy $\mathcal{E} \approx 0$ and a width $\Gamma \lesssim 1$ MeV. Such an idea appears to be at odds with the large prompt production cross sections observed at CDF [6] and CMS [7], as is confirmed by the calculations done with hadronization algorithms [8]. Final-state interactions within the $D^0\bar{D}^{*0}$ pair are often invoked as effective in coalescing the pair into a barely bound state, even if the components are initially recoiling with high relative momenta [9]. The limits of such an approach were further discussed in Ref. [10].

The molecular picture has also been proposed to explain the nature of the $Z(4430)$ resonance. In this case, however, the loosely bound mechanism does not work as there are no open charm thresholds with $J^{PG} = 1^{++}$ quantum numbers at that mass. In Ref. [11] it was suggested that the $Z(4430)$ might be a $D^*(2010)\bar{D}_1(2420)$ S-wave bound state, but this has $J^P = 0^-$, 1^-, 2^- which is not consistent with the recent observations strongly suggesting $J^P = 1^+$. For the molecular picture see also Refs. [12] and [13]. Other theoretical interpretations include $\Lambda_c\bar{\Sigma}_c$ baryonium [14], the cusp effect [15], and D_s radial excitation [16], as well as sum rules calculations based on the $D^*\bar{D}_1$ molecule [17,18]. All these speculations envisage effects due to the residual, short-range, forces generated by colorless meson exchange between color-neutral objects.

Here we shall follow the tetraquark interpretation of states made by *colored components*, diquarks and antidiquarks, bound by the long-range color forces [19]. Hidden beauty tetraquark states have been considered in Ref. [20].

A diquark is made by a $[cq]$ pair in a color-antisymmetric state, with c the charm and q a light, u or d, quark. This picture supports the existence of bound states that have higher orbital angular momentum and/or are radially excited and is consistent with production at Tevatron and LHC with cross sections similar to the ones of normal charmonia.

In a 2007 paper [21], after the observation of the charged $Z(4430)$ resonance by Belle, and judging from the considerably higher mass with respect to the lowest-lying tetraquark, we proposed this resonance to be the first radial excitation of a $1S$ state. The $Z(4430) - X(3872)$ mass difference is indeed very similar to the $\psi(2S) - \psi(1S)$ mass difference and in line with the observed decay

$$Z^+(4430) \to \pi^+ + \psi(2S). \tag{1}$$

We also noted, "A crucial consequence of a $Z(4430)$ charged particle is that a charged state decaying into $J/\psi + \pi^\pm$ or $\eta_c + \rho^\pm$ should be found around 3880 MeV".

Now that the $Z_c(3900)$ has been seen by BES III [22] and Belle [23] with the decay

$$Z^+(3900) \to \pi^+ + \psi(1S) \tag{2}$$

and a neutral partner suggested by CLEO [24], and with the further observation of $Z(4020)$ by the BES III Collaboration [25], [26], the tetraquark picture looks more attractive and constrained as compared to some years ago [27].

With only the $X(3872)$ at hand, the couplings characterizing spin interactions of the different flavors were deduced in Ref. [19] from the spectrum of mesons and baryons under rather uncontrolled hypotheses, such as the one-gluon exchange approximation and the equality of $|\psi(0)|^2$, the overlap probability of quarks and antiquarks in mesons or baryons and in the tetraquark (see Ref. [28]).

We introduce in this paper a "type-II" model, based on a simple, new ansatz of spin-spin couplings, whereby the cq interaction inside the diquark is assumed to dominate over all other possible pairings. A value of this coupling, $\kappa_{qc} \approx 67$ MeV, larger than the one deduced in Ref. [19] from baryon masses, explains the near degeneracy of $X(3872)$ with $Z(3900)$ as well as the $Z(4020)-Z(3900)$ mass difference. Predictions for the other S-wave tetraquarks with $J^P = 0^+, 2^+$ are provided.

In the "type-II" diquark model, we propose that diquarks more closely resemble compact bosonic building blocks. Indeed we are neglecting spin-spin interactions between different diquarks as we suppose that the size of the entire tetraquark is consistently larger than the size of its building blocks. As for the color force, the diquark-antidiquark pair is described as a bound state of two "point-like" color sources—the same configuration of a quark-antiquark system. For this reason we make the hypothesis that the spacings in radial excitations could closely resemble those observed in standard P-wave charmonia, as indicated by the $Z(4430) - Z(3990)$ mass difference.

In parallel with Z states, there have been extensive experimental investigations on the Y states, pioneered by the *BABAR* discovery of the $Y(4260)$ resonance in e^+e^- annihilation with initial-state radiation [29]. A considerable number of $J^{PC} = 1^{--}$ states has been observed by *BABAR* and Belle, although several are not confirmed. A nonexhaustive survey (see also Refs. [30] and [31]) includes $Y(4008)$ and $Y(4260)$ decaying into J/ψ [32], [33], $Y(4360)$ and $Y(4660)$, decaying into $\psi(2S)$ [34], [35], and $Y(4630)$ [36], where it is not clear if the last two are the same particle or if they are different.[1] BES III has recently studied decay channels with $h_c(1P)$, which give access to Y states dominated by $c\bar{c}$ configurations with $S_{c\bar{c}} = 0$, with the possible indication of one narrow state, $Y(4220)$, and a second wider one, $Y(4290)$ (see Refs. [31,38]).

Negative-parity states have to be P-wave excitations since the basic diquark-antidiquark relative parity is positive. We note in this paper that four Y states with $L = 1$ are expected, separated by fine and hyperfine mass differences due to spin-orbit and spin-spin couplings, and one with $L = 3$.

Tentatively, we identify $Y(4360)$ and $Y(4660)$ with the $n = 2$ radial excitations of $Y(4008)$ and $Y(4260)$, on the basis of their decay into $\psi(2S)$ and of mass differences very similar to the mass differences of the radial excitations of $\chi_{(c,b)J}(2P) - \chi_{(c,b)J}(1P)$ charmonia.

The selection rule corresponding to $S_{c\bar{c}}$ conservation leads us to identify $Y(4008)$, $Y(4260)$ and $Y(4630)$ with the $n = 1$, $L = 1$ states with a dominant component $s_{c\bar{c}} = 1$. The scheme may also accommodate one of the two possible states with a dominant decay into h_c, indicating a dominant $s_{c\bar{c}} = 0$ component, namely either $Y(4220)$ or $Y(4290)$, but not both. An experimental clarification of the real situation in the $h_c 2\pi$ channel is needed for further progress.

The $Y(4260)$ is assigned the same quark spin structure of the $X(3872)$, making an electromagnetic, $E1$, transition possible,

$$Y(4260) \to \gamma + X(3872), \tag{3}$$

a decay observed by BES III [39]. We discuss the selection rules of similar $E1$ transitions of the other Y states, which could provide an effective tool to determine the internal spin structure of Y and X states.

In this paper we do not explore the case of exotic hadrons with hidden charm and strangeness. Tetraquark $[cs][\bar{c}\,\bar{s}]$ states were considered e.g. in Ref. [40] where it was suggested to study the decay channels into $J/\psi\phi$ and $D_s^{(*)} D_s^{(*)}$. Successively, the first tetraquark candidate decaying into $J/\psi\phi$ was observed by CDF [41], the $Y(4140)$, recently confirmed by CMS [42]. Its mass and quantum numbers fall within the spectrum predicted in Ref. [40].[2] However, the LHCb collaboration reported a negative result in Ref. [43] and similar results from Belle can be found in Ref. [44].

In conclusion, we find that the tetraquark scheme with the new ansatz of spin-spin couplings is able to reproduce the main features of the spectrum of the observed X, Y, Z particles. This makes even more puzzling the remarkable lack of evidence for a neutral state close to the $X(3872)$ and for its charged counterpart. Further experimental and theoretical investigations are needed, to get to a satisfactory picture of the exotic charmonia.

II. S-WAVE TETRAQUARKS

S-wave tetraquarks have positive parity and have been classified in Ref. [19] according to the diquark and

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[1] The latter option was proposed in Ref. [37].

[2] Especially for a second peak—observed by CDF at $Y(4274)$ in the same decay mode—which, however, is not (yet) confirmed by other experiments.

antidiquark spin, $s_{qc} = s$ and $s_{\bar{q}\bar{c}} = \bar{s}$, respectively, and the resulting angular momentum, J. For each value of these quantum numbers we have four charge states, with isospin $I = 1, 0$. Neutral states have a definite charge conjugation, $C = \pm 1$, while we can assign a G-parity, $G = C(-1)^I = -C$ to the charged states, which is conserved in their decays.

In the $|s, \bar{s}\rangle_J$ basis we have the following states:

$$\mathrm{J^P} = 0^+, \quad C = +, \quad X_0 = |0,0\rangle_0, \quad X_0' = |1,1\rangle_0, \tag{4}$$

$$\mathrm{J^P} = 1^+, \quad C = +, \quad X_1 = \frac{1}{\sqrt{2}}(|1,0\rangle_1 + |0,1\rangle_1), \tag{5}$$

$$\mathrm{J^P} = 1^+, \qquad G = +,$$
$$Z = \frac{1}{\sqrt{2}}(|1,0\rangle_1 - |0,1\rangle_1), \qquad Z' = |1,1\rangle_1, \tag{6}$$

$$\mathrm{J^P} = 2^+, \qquad C = +, \qquad X_2 = |1,1\rangle_2. \tag{7}$$

We use the symbol Z when the charged states have been identified and give the corresponding G-parity, and use the symbol X for all other cases, reporting the C conjugation of the neutral state.

We identify $X_1 = X(3872)$, while the physical $Z(3900)$ and $Z(4020)$ are identified with the linear combinations of Z and Z' which diagonalize the spin-spin Hamiltonian.

There are neutral X states quoted in Refs. [30,45], which could be identified with X_0' and X_2, notably $X(3915)$ and $X(3940)$, as we discuss below.

There are indications of neutral counterparts of the Z states [24], but charged counterparts for the X states have not been detected so far.

Searches by *BABAR* [46] and Belle [47] exclude the pure $I = 1$ assignment of $X(3872)$; however, a mixed $I = 1$ and $I = 0$ seems still possible. The possibility of a very broad $I = 1$ state was considered in Ref. [48].

It is convenient to put into evidence the heavy quark spin, by introducing the basis where the spins of each quark-antiquark pair are diagonal, which we denote by $|s_{q\bar{q}}, s_{c\bar{c}}\rangle_J$. Charge conjugation is given by[3]

$$C = (-1)^{s_{q\bar{q}} + s_{c\bar{c}}}, \tag{8}$$

so that states with $C = +1$ (alternatively, $C = -1$ and $G = +1$), have to have equal (unequal) quark-antiquark spins.

It is not difficult to see that[4]

$$X_0 = \frac{1}{2}|0_{q\bar{q}}, 0_{c\bar{c}}\rangle_0 + \frac{\sqrt{3}}{2}|1_{q\bar{q}}, 1_{c\bar{c}}\rangle_0, \tag{11}$$

$$X_0' = \frac{\sqrt{3}}{2}|0_{q\bar{q}}, 0_{c\bar{c}}\rangle_0 - \frac{1}{2}|1_{q\bar{q}}, 1_{c\bar{c}}\rangle_0, \tag{12}$$

$$X_1 = |1_{q\bar{q}}, 1_{c\bar{c}}\rangle_1, \tag{13}$$

$$Z = \frac{1}{\sqrt{2}}(|1_{q\bar{q}}, 0_{c\bar{c}}\rangle_1 - |0_{q\bar{q}}, 1_{c\bar{c}}\rangle_1), \tag{14}$$

$$Z' = \frac{1}{\sqrt{2}}(|1_{q\bar{q}}, 0_{c\bar{c}}\rangle_1 + |0_{q\bar{q}}, 1_{c\bar{c}}\rangle_1), \tag{15}$$

$$X_2 = |1_{q\bar{q}}, 1_{c\bar{c}}\rangle_2. \tag{16}$$

A tentative mass spectrum for the S-wave tetraquarks was derived in Ref. [19], based on an extrapolation of the spin-spin interactions in conventional S-wave mesons and baryons.

The resulting couplings turn out to be dominated by the $q\bar{q}$ coupling, pairing the spin of particles in different diquarks, with a much weaker coupling of the $[qc]$ pair in the same diquark. To a first approximation, the Hamiltonian for this case can simply be taken as

$$H \approx 2\kappa_{q\bar{q}} \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}} = \kappa_{q\bar{q}} s_{q\bar{q}}(s_{q\bar{q}} + 1). \tag{17}$$

Mass eigenstates are diagonal in the basis where the $q\bar{q}$ and the $c\bar{c}$ have definite spin, where the states with $s_{q\bar{q}} = 1$ are heavier. One finds

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[3] The formula holds for states with $L = 0$, L being the relative orbital angular momentum of the diquark-antidiquark pair; for general L the formula is $C = (-1)^{L + s_{q\bar{q}} + s_{c\bar{c}}}$.

[4] A spin-zero diquark in the color antitriplet channel is defined by $[q_1 q_2]_i = \epsilon_{ijk}(\bar{q}_1^j)_c \gamma_5 q_2^k$, which indeed, apart from a $(-i)$ phase factor, corresponds to the bispinor expression $\epsilon_{ijk}(q_1^j)^T \sigma^2 q_2^k$. Therefore, as far as the spin is concerned, the tetraquark state can be described by $(q_1^T \sigma^2 q_2)(\bar{q}_3^T \sigma^2 \bar{q}_4)$. Using appropriate normalizations, we can define (using e.g. $q_1 = c$, $q_2 = q$, $\bar{q}_3 = \bar{q}$, $q_4 = \bar{c}$)

$$|0,0\rangle_0 \equiv \frac{1}{2}\sigma^2 \otimes \sigma^2, \tag{9}$$

$$|1,1\rangle_0 \equiv \frac{1}{2\sqrt{3}}\sigma^2\sigma^i \otimes \sigma^2\sigma^i, \tag{10}$$

where the i index is summed in the latter and the normalization comes from the request $1/12\sum_i \mathrm{Tr}((\sigma^i)^T\sigma^i)^2 = 1$. Next we use the completeness relation $1/2\boldsymbol{\sigma}_{ad} \cdot \boldsymbol{\sigma}_{cb} + 1/2\delta_{ad}\delta_{cb} = \delta_{ab}\delta_{cd}$ which immediately allows to sort out the spin of $c\bar{c}$ and $q\bar{q}$ observing that $(c_s\sigma^2_{sa}\delta_{ab}q_b)(\bar{q}_r\sigma^2_{rc}\delta_{cd}\bar{c}_d)$ contains $\delta_{ab}\delta_{cd}$. Indeed, substituting the completeness relation leads to Eq. (11). Equation (12) can be obtained by taking Eq. (10), where we have $(c_s\sigma^2_{sa}\sigma^i_{ab}q_b)\times(\bar{q}_r\sigma^2_{rc}\sigma^i_{cd}\bar{c}_d)$, and using the relation $3/2\delta_{ad}\delta_{cb} - 1/2\boldsymbol{\sigma}_{ad} \cdot \boldsymbol{\sigma}_{cb} = \boldsymbol{\sigma}_{ab} \cdot \boldsymbol{\sigma}_{cd}$.

$$X(3872) \approx X_1 = |1_{q\bar{q}}, 1_{c\bar{c}}\rangle_1, Z(3900) \approx |1_{q\bar{q}}, 0_{c\bar{c}}\rangle_1, \qquad (18)$$

but the other Z would have to be lighter than 3900 MeV [49], in contradiction with the BES III finding [25].

In addition, $Z(3900)$ would be made essentially by $s_{c\bar{c}} = 0$ [28], which is at variance with heavy spin conservation and its observed decay into J/ψ.

III. SPIN INTERACTIONS IN TETRAQUARKS: A NEW ANSATZ

Rather than trying to enforce our prejudices derived from conventional mesons and baryons, the strength of spin interactions in tetraquarks should be derived from the observed masses of tetraquark candidates. Remarkably, there is a simple approximate ansatz replacing Eq. (17) which reproduces the correct spectrum. This consists in taking the dominant spin interactions to be the ones *within each diquark*,

$$H \approx 2\kappa_{qc}(\boldsymbol{s}_q \cdot \boldsymbol{s}_c + \boldsymbol{s}_{\bar{q}} \cdot \boldsymbol{s}_{\bar{c}}) = \kappa_{qc}[s(s+1) + \bar{s}(\bar{s}+1) - 3]. \qquad (19)$$

In this approximation, the mass eigenvectors coincide with the states given in Eqs. (4)–(7). We are led to identify

$$X(3872) = X_1, \quad Z(3900) \approx Z, \quad Z(4020) \approx Z' \qquad (20)$$

with the mass ordering

$$M(X_1) \approx M(Z), \qquad M(Z') - M(Z) \approx 2\kappa_{qc}. \qquad (21)$$

A value of

$$\kappa_{qc} = 67 \text{ MeV} \qquad (22)$$

reproduces the two mass differences within less than 20 MeV.

The value in Eq. (22) is considerably larger than $(\kappa_{qc})_{\bar{3}} = 22$ MeV obtained from the $\Sigma_c - \Lambda_c$ mass difference [19] and may indicate that diquarks in tetraquarks are more compact than diquarks in baryons.

Considering the other states, in the same approximation we find

$$M(X_2) \approx M(X'_0) \approx 4000 \text{ MeV}, \qquad (23)$$

$$M(X_0) \approx 3770 \text{ MeV}. \qquad (24)$$

We may wish to identify the first two states with the $X(3940)$ and $X(3916)$, respectively. There is no X state yet identified at masses below the $X(3872)$ [45].

In this scheme $Z(4430)$ is the first radial excitation of the $Z(3900)$, with a mass difference $Z(4430) - Z(3900) = 593$ MeV, very close to $\psi(2S) - \psi(1S) = 589$ MeV.

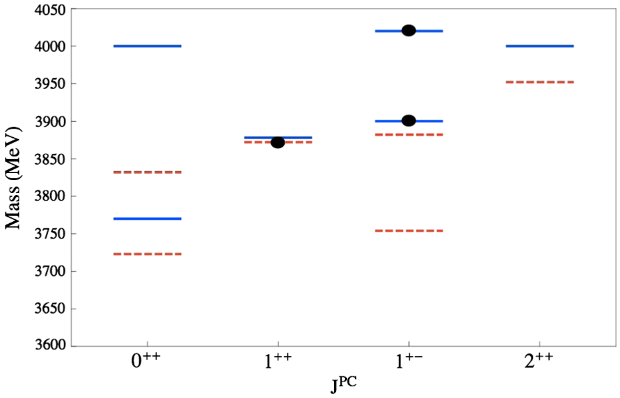

FIG. 1 (color online). The dashed (red) levels correspond to the color-spin Hamiltonian introduced in Ref. [19]. The solid (blue) levels correspond to the approximation used here [Eq. (19)]. Black disks represent the $X(3872)$, $Z(3900)$ and $Z(4020)$ masses. The C quantum number is the charge-conjugation eigenvalue of the neutral component of the multiplet.

For comparison, we report in Fig. 1 the spectra computed with the old ansatz [19] and with the new one.

Finally, we note that the states representing $Z(3900)$ and $Z(4020)$ now have both components of $s_{c\bar{c}} = 1, 0$ so as to be at least not in contradiction with the observed decays [25],

$$Z(4020) \to \pi + h_c(1^1P_1)(\text{seen}), \qquad (25)$$

$$Z(3900) \to \pi + h_c(1^1P_1)(\text{seen})(?). \qquad (26)$$

IV. *Y* STATES

We have previously interpreted the $Y(4260)$ as a P-wave tetraquark, with the composition $[cs][\bar{c}\,\bar{s}]$ [50] due to its dominant decay with $f_0(980)$ production. It was later realized [28,51,52], that instanton-induced effects may mix the f_0 with $q\bar{q}$ states and produce the dominant $f_0 \to 2\pi$ decay. The same mechanisms allow $q\bar{q} \to f_0$ and a composition for $Y(4260)$ similar to the one of the S-wave X and Z states we are considering.

Tetraquark states with $J^{PC}(Y) = 1^{--}$ can be obtained with odd values of the orbital angular momentum $L = 1, 3$ and diquark and antidiquark spins $s, \bar{s} = 0, 1$. We set ourselves in the basis used previously, Eqs. (4)–(7), using the notation

$$|s, \bar{s}; S, L\rangle_{J=1} \qquad (27)$$

for the state with total spin $\boldsymbol{S} = \boldsymbol{s} + \bar{\boldsymbol{s}}$ and total angular momentum $J = 1$.

$L = 1$: We may combine the spin structures in Eqs. (4)–(7) with $L = 1$ to obtain $J = 1$. However, under charge conjugation, the orbital momentum produces a factor -1, so we have to keep only spin states classified with $C = +1$. Thus we get the four states

$$Y_1 = |0,0;0,1\rangle_1, \tag{28}$$

$$Y_2 = \frac{1}{\sqrt{2}}(|1,0;1,1\rangle_1 + |0,1;1,1\rangle_1, \tag{29}$$

$$Y_3 = |1,1;0,1\rangle_1, \tag{30}$$

$$Y_4 = |1,1;2,1\rangle_1. \tag{31}$$

Y states can be listed in order of increasing mass. The orderings Y_1, Y_2, Y_3, Y_4 and Y_1, Y_3, Y_2, Y_4 correspond to two different particle assignments. We shall see that inverting Y_3 with Y_2 has little impact on the choice of the phenomenological parameters in the Hamiltonian.

Comparing with the spin structure of the states in Eqs. (9)–(14), we get the following composition relative to the $c\bar{c}$ spin:

$$Y_1\colon\ P(s_{c\bar{c}} = 1)\colon\ P(s_{c\bar{c}} = 0) = 3\colon 1, \tag{32}$$

$$Y_2\colon P(s_{c\bar{c}} = 1) = 1, \tag{33}$$

$$Y_3\colon\ P(s_{c\bar{c}} = 1)\colon\ P(s_{c\bar{c}} = 0) = 1\colon 3, \tag{34}$$

$$Y_4\colon P(s_{c\bar{c}} = 1) = 1. \tag{35}$$

Only Y_3 is expected to decay preferably into the $s_{c\bar{c}} = 0$ state, $h_c(1P)$.

$L = 3$: There is only one possibility, namely

$$Y_5 = |1,1;2,3\rangle_1 \tag{36}$$

with $S_{c\bar{c}} = 1$.

Tentative particle assignments: As stated in the Introduction, there are indications for more than four states in the region of the $Y(4260)$. Tentatively, we propose the following.

(i) We leave aside the $L = 3$ state, which is expected to occur at much higher energy (see below).

(ii) We interpret the $Y(4360)$ and $Y(4660)$ as radial excitations of $Y(4008)$ and $Y(4260)$, respectively, on the basis of their decay into $\psi(2S)$, analogous to the decay of $Z(4430)$. The relative mass differences of 350 and 400 MeV are in the range of the mass differences for $L = 1$ charmonia and bottomonia,[5] e.g. $\chi_{bJ}(2P) - \chi_{bJ}(1P) \approx 360$ MeV whereas $\chi_{cJ}(2P) - \chi_{cJ}(1P) \approx 437$ MeV.[6]

(iii) We identfy $Y_{1,2,4}$ with $Y(4008)$, $Y(4260)$ and $Y(4630)$ (decaying into $\Lambda\bar{\Lambda}$).

(iv) We identify Y_3 with either $Y(4290)$ (the broad structure in the h_c channel), or $Y(4220)$ (the narrow structure).

Spin-orbit and spin-spin interactions: In the spirit of a first exploration, to confront with the data we leave aside possible tensor interactions, although we are aware that they play a role in the splitting of P-wave charmonia.

We add to the Hamiltonian of S-wave tetraquarks an orbital term proportional to $\boldsymbol{L}^2$ and a spin-orbit interaction proportional to $\boldsymbol{L}\cdot\boldsymbol{S}$. The restriction of the spin-spin couplings to the interaction within the same diquark, as discussed before, is more than justified here, due to the angular-momentum barrier, and we leave open the possibility that the coupling may take a different value from the S-wave case.

We write

$$M = M_{00} + B_c \frac{\boldsymbol{L}^2}{2} - 2a\boldsymbol{L}\cdot\boldsymbol{S} + 2\kappa'_{qc}[(\boldsymbol{s}_q \cdot \boldsymbol{s}_c) + (\boldsymbol{s}_{\bar{q}} \cdot \boldsymbol{s}_{\bar{c}})]. \tag{37}$$

Signs are chosen so that, for B_c, a, κ' positive, energy increases for increasing $\boldsymbol{L}^2$ and $\boldsymbol{S}^2$. With obvious manipulations, we obtain

$$M = M_{00} + B_c \frac{L(L+1)}{2} + a[L(L+1) + S(S+1) - 2] + \kappa'_{qc}[s(s+1) + \bar{s}(\bar{s}+1) - 3], \tag{38}$$

namely

$$M = M_0 + \left(\frac{B_c}{2} + a\right)L(L+1) + aS(S+1) + \kappa'_{qc}[s(s+1) + \bar{s}(\bar{s}+1)], \tag{39}$$

where

$$M_0 = M_{00} - 2a - 3\kappa'_{qc}. \tag{40}$$

We then find

$$\begin{aligned} M_1 &= M_0 + 2\left(\frac{B_c}{2} + a\right) = c, \\ M_4 - M_3 &= 6a, \\ M_3 - M_2 &= 2\kappa'_{qc} - 2a, \\ M_2 - M_1 &= 2\kappa'_{qc} + 2a. \end{aligned} \tag{41}$$

With four masses and three parameters, we find the relation

$$M_2 = \frac{3M_1 + M_3 + 2M_4}{6}. \tag{42}$$

The above formulas require[7] $M_2 > M_1$ and $M_4 > M_3$; however, the sign of the mass difference $M_3 - M_2$ can

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[5] Mass differences between ground states and radial excited states have been recently analyzed in Ref. [53] for tetraquark states and in Ref. [54] for charmonia and bottomonia.

[6] No data is available for $\chi_{c1}(2P)$.

[7] So, at variance with the assignment made in Ref. [50], $Y(4260)$ cannot be identified with Y_1.

take either sign, as it is determined by the difference of two constants which are *a priori* of a similar size.

We keep fixed the assignment

$$Y(4260) = Y_2 = \frac{1}{\sqrt{2}}(|1,0;1,1\rangle_1 + |0,1;1,1\rangle_1) \qquad (43)$$

and consider separately the two cases[8] for Y_3.

$Y_3 = Y(4290)$: The mass relation (42) is well satisfied by the nominal masses of the Y states, giving

$$(M_2)_{\rm th} = 4262\ \text{MeV}. \qquad (44)$$

We may use the first three equations in Eq. (41) to obtain the value of the parameters. Using the nominal Y masses corresponding to the assignment $Y_1 = Y(4008)$, $Y_2 = Y(4260)$, $Y_3 = Y(4290)$ and $Y_4 = Y(4630)$ we find (in MeV)

$$a = 56, \qquad \kappa'_{qc} = 71. \qquad (45)$$

$Y_3 = Y(4220)$: The mass relation (42) gives

$$(M_2)_{\rm th} = 4251\ \text{MeV} \qquad (46)$$

and the value of the parameters are

$$a = 73, \qquad \kappa'_{qc} = 53. \qquad (47)$$

In either case the value found for κ'_{qc} is close to the value in Eq. (22), supporting the difference between diquarks in tetraquarks and diquarks in baryons.

Either structure in the $h_c + 2\pi$ channel can be accommodated in the scheme, but not both. An experimental clarification is needed for further progress.

The orbital excitation energy: In the new scheme, the spin structure of $Y(4260)$ and $X(3872)$ and their spin interactions are exactly the same and we may obtain the energy of the orbital excitation directly from their mass difference. Starting from Eq. (38), neglecting the difference between κ_{qc} and κ'_{qc} and using Eq. (40) we obtain

$$\begin{aligned} M(Y_2) &= M(X) + B_c + 2a, \text{i.e.} \\ B_c &= 278\ \text{MeV}. \end{aligned} \qquad (48)$$

This value compares well with values found in normal hadrons; see the discussion in Ref. [50].

Finally, a large separation between Y_5 and the states Y_{1-4} is implied,

$$M_5 = M_2 + 5B_c + 14a \sim 6420\ \text{MeV}. \qquad (49)$$

The radiative decay of $Y(4260)$: The identification $Y(4260) = Y_2$ is reinforced by the observation [39] of a conspicuous radiative decay mode of $Y(4260)$,

$$Y(4260) \to \gamma + X(3872). \qquad (50)$$

The identical spin structure implied in the tetraquark model for the two states suggests that this mode is an unsuppressed $E1$ transition, with $\Delta L = 1$ and $\Delta S = 0$, similar to the observed transitions of the charmonium χ states.

The decay rate of Eq. (50) could provide a first estimate of the radius of the tetraquark.

A comparison of the spin structures in Eqs. (4)–(7) and Eqs. (28)–(35) provides selection rules for $E1$ transitions between Y and X states that should allow a better identification of the levels, e.g. whether $Y(4660)$ is or is not a radial excitation of a lower, P-wave tetraquark. With the assignments we made, we expect

$$Y_4 = Y(4630) \to \gamma + X_2(J^{PC} = 2^{++}) = \gamma + X(3940)(??), \qquad (51)$$

$$\begin{aligned} Y_3 = Y(4290/4220) &\to \gamma + X'_0(J^{PC} = 0^{++}) \\ &= \gamma + X(3916)(??), \end{aligned} \qquad (52)$$

$$\begin{aligned} Y_2 = Y(4260) &\to \gamma + X_1(J^{PC} = 1^{++}) \\ &= \gamma + X(3872)(\text{seen}), \end{aligned} \qquad (53)$$

$$\begin{aligned} Y_1 = Y(4008) &\to \gamma + X_0(J^{PC} = 0^{++}) \\ &= \gamma + X(3770\,??)(??). \end{aligned} \qquad (54)$$

V. CONCLUSIONS AND OUTLOOK

The confirmation of the $Z(4430)$, whose existence has been controversial up to very recently, reinforces the evidence that hidden-charm tetraquarks exist, as was first predicted in Ref. [19]. Here we chose to use again a diquark-antidiquark representation of the tetraquark but with a new assumption on spin-spin couplings: diquark building blocks are more compact than what was thought before and spin-spin forces outside diquark shells are suppressed.

This implies a simplified spin-spin interaction Hamiltonian with respect to that postulated in Ref. [19]. The new ansatz allows a good description of the 1^+ sector, $Z(3900)$, $Z'(4020)$ and $X(3872)$, as described in Fig. 1.

A consistent description of the Y particles can also be achieved. The $Y(4360)$ and $Y(4660)$ are identified as the first radial excitations of $Y(4008)$ and $Y(4260)$, respectively. The $L = 1$ tetraquarks with predominantly $s_{c\bar{c}} = 1$ are identified with $Y(4008)$, $Y(4260)$ and $Y(4630)$, which decay dominantly in $J/\Psi + 2\pi$, while the fourth state, with

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[8] We thank the referee for suggesting that we consider the two cases on a similar ground.

dominantly $s_{c\bar{c}} = 0$ component, is identified with either $Y(4290)$ or $Y(4220)$. Only one state is admitted but, at the moment, the two alternatives cannot be distinguished on theoretical grounds.

Finally, the diquark spin structure associated with the $Y(4260)$ accounts for the radiative transition into $X(3872)$, observed by BES III, as a dominant E_1 transition.

The "type-II" diquark-antidiquark model presented here does not yet explain why charged partners of the X and Y states have not been observed. As for the persisting lack of experimental confirmation of two neutral, almost degenerate, X particles at 3872 MeV, required by the diquark-antidiquark model to account for the strong isospin-violating pattern observed in X decays [19], we believe that this might be due to the sensibility of present experimental analyses. On the other hand this model provides a natural explanation of the quantum numbers of most of the X, Y, Z resonances observed, together with a very reasonable description of their decay rates.

There are several experimental hints that point to a unified description of X, Y, Z resonances. Two striking ones are i) the mass difference between the $Z(4430)$ and $Z(3900)$, the former decaying into $\psi(2S)\pi$ and the latter into $J/\psi\pi$, and ii) the observed radiative transitions between the $Y(4260)$ and the $X(3872)$.

After the recent discoveries of the $Z_c(3900)$, $Z_c(4020)$ and especially $Z(4430)$, we think that the tetraquark option is back with renovated strength and, despite the obvious limitations of the diquark-antidiquark description, we have to observe that it has a descriptive power regarding the X, Y, Z resonance physics which other explanations cannot provide.

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