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Light composite scalar in eight-flavor QCD on the lattice

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We present the first observation of a flavor-singlet scalar meson as light as the pion in $N_f = 8$ QCD on the lattice, using the highly improved staggered quark action. Such a light scalar meson can be regarded as a composite Higgs with mass 125 GeV. In accord with our previous lattice results showing that the theory exhibits walking behavior, the light scalar may be a technidilaton, a pseudo-Nambu-Goldstone boson of the approximate scale symmetry in walking technicolor.

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Recently, a Higgs boson with mass around 125 GeV has been discovered at the Large Hadron Collider (LHC) [1,2]. While the current LHC data show good agreement with the standard model Higgs boson, there exists a possibility that the Higgs boson is a composite particle in an underlying strongly coupled gauge theory. A typical example is the walking technicolor theory, featuring approximate scale invariance and a large anomalous dimension, $\gamma_m \approx 1$ [3] (see also similar works [4–6]). Such a theory predicts a light composite Higgs, a "technidilaton" [3], emerging as a pseudo-Nambu-Goldstone (NG) boson of the spontaneously broken approximate scale symmetry. It was shown [7,8] that the technidilaton is phenomenologically consistent with the current LHC data.

Thus, the most urgent theoretical task to test walking technicolor theories would be to check whether or not such a light flavor-singlet scalar bound state exists from first-principle calculations with lattice gauge theory. Since the composite Higgs should be associated with the electroweak symmetry breaking, it must be predominantly a bound state of technifermions carrying electroweak charges, but not of technigluons having no electroweak charges (up to some mixings between them). Thus we look for a light flavor-singlet scalar meson in the correlator of fermionic operators on the lattice.

One of the most popular candidates for walking technicolor theories is QCD with a large number of (massless) flavors (N_f) in the fundamental representation. For the past few years, we have studied the SU(3) gauge theory with $N_f = 4$, 8, 12, and 16, in a common lattice setup [9–11]. (For reviews of lattice studies in search of candidates for walking technicolor theories, see [12–15].)

In $N_f = 12$ QCD we actually observed [11,16] a flavor-singlet scalar meson (σ) lighter than the "pion" having the quantum numbers corresponding to the NG pion (π) in the broken phase. (Recently a light flavor-singlet scalar meson consistent with ours was also observed by another group [17] using a different lattice action.)

We found [9] that $N_f = 12$ QCD is consistent with a conformal theory. If it is a conformal theory, it should have no bound states ("unparticle") in the exact chiral limit, and hence a light bound state can only be formed in the presence of a fermion mass m_f which explicitly (not spontaneously) breaks the scale/chiral/electroweak symmetry.

Hence such a light scalar meson in $N_f = 12$ QCD would not be a composite Higgs associated with the spontaneous symmetry breaking. However, its presence strongly suggests that a walking theory would have a similar light scalar meson as a composite Higgs associated with the spontaneous scale/chiral/electroweak symmetry breaking, since in the walking theory the gauge coupling is similar to that of a conformal theory with the role of the explicit breaking mass m_f replaced by the dynamically generated fermion mass, m_D, arising from the spontaneous chiral symmetry breaking.

In this paper we indeed observe such a light flavor-singlet scalar fermionic bound state σ as light as π in $N_f = 8$ QCD, which we found [10] is a candidate for walking technicolor, with the spontaneous breaking of chiral symmetry and large anomalous dimension near unity. Thus it can be a candidate for the composite Higgs (technidilaton) with a 125 GeV mass. The preliminary results of this work were already reported in Ref. [18].

We carry out simulations of SU(3) gauge theory with eight fundamental fermions using two degenerate staggered fermion species with bare fermion mass m_f, where each species has four fermion degrees of freedom, called tastes. We use a tree-level Symanzik gauge action and the highly improved staggered quark (HISQ) [19] action without the tadpole improvement or the mass correction in the Naik term [20]. The flavor symmetry breaking of this action is highly suppressed in QCD [20]. It is also true in our $N_f = 8$

TABLE I. Simulation parameters for $N_f = 8$ QCD at $\beta = 3.8$. $N_{\rm cf}(N_{\rm st})$ is the total number of gauge configurations (Markov chain streams). The second error of m_σ is a systematic error coming from the fit range. The values for m_π and F_π are from Ref. [10], but the ones with (*) have been updated.

m_f	$L^3 \times T$	$N_{\rm cf}[N_{\rm st}]$	m_σ	m_π	F_π
0.015	$36^3 \times 48$	3200[2]	$0.155(21)(^{0}_{41})$	$0.1861(4)^*$	$0.0503(2)^*$
0.02	$36^3 \times 48$	5000[1]	$0.190(17)(^{39}_{0})$	$0.2205(3)^*$	$0.0585(1)^*$
0.02	$30^3 \times 40$	8000[1]	$0.201(21)(^{0}_{60})$	0.2227(9)	0.0578(2)
0.03	$30^3 \times 40$	16500[1]	$0.282(27)(^{24}_{0})$	$0.2812(2)^*$	$0.07140(9)^*$
0.03	$24^3 \times 32$	36000[2]	$0.276(15)(^{6}_{0})$	0.2832(14)	0.0715(4)
0.04	$30^3 \times 40$	12900[3]	$0.365(43)(^{17}_{0})$	$0.3349(3)^*$	$0.0826(1)^*$
0.04	$24^3 \times 32$	50000[2]	$0.322(19)(^{8}_{0})$	0.3353(7)	0.0823(2)
0.04	$18^3 \times 24$	9000[1]	$0.228(30)(^{0}_{16})$	0.3421(29)	0.0823(5)
0.06	$24^3 \times 32$	18000[1]	$0.46(7)(^{12}_{0})$	0.4295(6)	0.1012(3)
0.06	$18^3 \times 24$	9000[1]	$0.386(77)(^{12}_{0})$	0.4317(15)	0.0999(5)

QCD simulations, where the breaking is almost negligible in the meson masses [10]. At the same bare coupling $\beta \equiv 6/g^2 = 3.8$ as in our previous work [10], we calculate the mass of the flavor-singlet scalar (m_σ) at five fermion masses, $m_f = 0.015$, 0.02, 0.03, 0.04, and 0.06, to investigate the m_f dependence of m_σ. We use four volumes of spatial extent $L = 18$, 24, 30, and 36, with fixed aspect ratio $T/L = 4/3$, to check for finite size effects on m_σ. All the simulation parameters are tabulated in Table I. In this paper all dimensionful quantities are expressed in lattice units.

We generate between 6400 and 100,000 trajectories depending on the simulation parameters with the standard hybrid Monte Carlo algorithm using the MILC code version 7 [21] with some modifications to suit our needs, such as Hasenbusch mass preconditioning [22] to reduce the computational cost. For the thermalization we discard more than 2000 trajectories. In some parameters we make several Markov chain streams to collect thermalized configurations more efficiently. The total numbers of configurations and Markov chain streams are tabulated in Table I. For the measurement of the flavor-singlet scalar mass we use interpolating operators of the fermionic bilinear with the appropriate quantum numbers, $J^{PC} = 0^{++}$. In this measurement we use the MILC code [21] and exploit GPGPU acceleration thanks to the QUDA library [23]. The measurements are performed every 2 trajectories. The vacuum-subtracted disconnected correlator has large statistical noise; however, it is essential to obtain m_σ. For the noise reduction, as in the $N_f = 12$ QCD calculation [11], we utilize a method [24] based on the axial Ward-Takahashi identity [25], which has been employed in the literature [24–27]. We use 64 random sources spread in spacetime and color spaces for this noise-reduction method. The statistical errors are estimated by the jackknife method, with a bin size of 200 trajectories to eliminate autocorrelation sufficiently.

Since we employ the same fermion bilinear operator as in $N_f = 12$ QCD [11], in this paper we describe it briefly. We use the local fermionic bilinear operator of the $(\mathbf{1} \otimes \mathbf{1})$ staggered spin-taste structure defined as

$$\mathcal{O}_S(t) = \sum_{i=1}^{2} \sum_{\vec{x}} \bar{\chi}_i(\vec{x}, t) \chi_i(\vec{x}, t), \tag{1}$$

where the index i runs through different staggered fermion species. The correlator of the operator is given by the connected $C(t)$ and also vacuum-subtracted disconnected $D(t)$ correlators, $\langle \mathcal{O}_S(t) \mathcal{O}_S{}^\dagger(0) \rangle = 2D(t) - C(t)$, where the factor in front of $D(t)$ comes from the number of species. Due to the staggered fermion symmetry, at large time, the correlator has two contributions from σ and also its parity partner, which is a flavor-nonsinglet (taste-nonsinglet but species-singlet) pseudoscalar ($\pi_{\overline{\rm SC}}$)

$$2D(t) - C(t) = A_\sigma(t) + (-1)^t A_{\pi_{\overline{\rm SC}}}(t), \tag{2}$$

where $A_H(t) = A_H(e^{-m_H t} + e^{-m_H(T-t)})$, with m_H being the mass of state H. Since $-C(t)$ can be regarded as a flavor-nonsinglet scalar correlator, it should have contributions from the nonsinglet scalar (a_0), and its staggered parity partner, which is another flavor-nonsinglet (taste-nonsinglet and species-nonsinglet) pseudoscalar ($\pi_{\rm SC}$). When t is large, we can write

$$-C(t) = A_{a_0}(t) + (-1)^t A_{\pi_{\rm SC}}(t). \tag{3}$$

From Eqs. (2) and (3), at large time $2D(t)$ can be written as

$$2D(t) = A_\sigma(t) - A_{a_0}(t) + (-1)^t (A_{\pi_{\rm SC}}(t) - A_{\pi_{\overline{\rm SC}}}(t)). \tag{4}$$

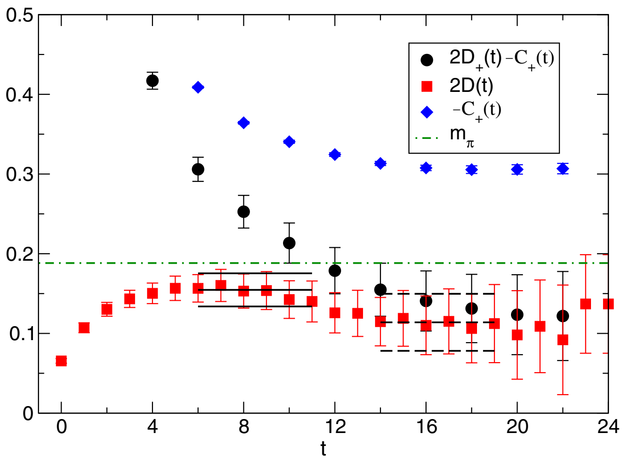

FIG. 1 (color online). Effective scalar mass m_σ from correlators in Eq. (2), with the projection explained in the text, and in Eq. (4) for $L = 36$ and $m_f = 0.015$. The solid and dashed lines highlight the fit results for m_σ with statistical error band. The dashed-dotted line represents m_π. Effective mass of the projected connected correlator in Eq. (3) is also plotted.

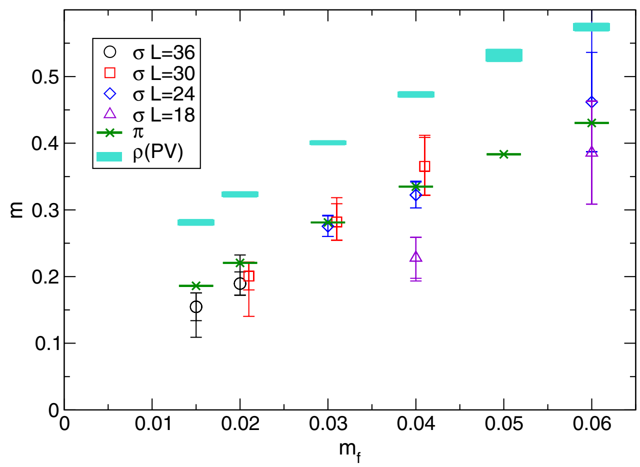

FIG. 2 (color online). Mass of the flavor-singlet scalar m_σ compared to the mass of NG pion m_π as a function of the fermion mass m_f. The outer error represents the statistical and systematic uncertainties added in quadrature, while the inner error is only statistical. Square symbols are slightly shifted for clarity. Mass of vector meson with 1 standard deviation is expressed by full boxes.

If the flavor symmetry is exact, all the flavor-nonsinglet pseudoscalars, $\pi_{\overline{\mathrm{SC}}}$, π_{SC}, and also the NG π, are degenerate. Furthermore, in the flavor symmetric limit, their amplitudes in Eq. (4) also coincide, so that $A_{\pi_{\mathrm{SC}}}(t) = A_{\pi_{\overline{\mathrm{SC}}}}(t)$ in this limit.

After applying the positive parity projection, $C_+(t) = 2C(t) + C(t+1) + C(t-1)$ at even t to minimize $A_{\pi_{\overline{\mathrm{SC}}}}(t)$ in Eq. (2), we evaluate the effective mass of the projected correlator $2D_+(t) - C_+(t)$. Figure 1 shows that the effective mass at large t is almost equal to m_π, although the error is large. We also plot the effective mass of $2D(t)$ without the projection, which does not have an oscillating behavior. This means that the flavor symmetry breaking between $A_{\pi_{\mathrm{SC}}}(t)$ and $A_{\pi_{\overline{\mathrm{SC}}}}(t)$ in Eq. (4) is small. The effective mass plateau of $2D(t)$ is statistically consistent with the one of $2D_+(t) - C_+(t)$ in the large time region. Note that the effective mass of $-C_+(t)$ is always larger than the one of $2D(t)$ in our simulations, as shown in Fig. 1. Since the plateau of $2D(t)$ appears at earlier time with smaller error than the one of $2D_+(t) - C_+(t)$, we choose $2D(t)$ to extract m_σ in all the parameters. The earlier plateau suggests that the contribution of a_0 tends to cancel with that from excited flavor-singlet scalar states in $2D(t)$. It should be noted that, because of the small m_σ, comparable to m_π, the exponential damping of $D(t)$ is slow, and this helps prevent a rapid degradation of the signal-to-noise ratio.

We fit $2D(t)$ in the region $t = 6 - 11$ by a single cosh form assuming only σ propagating in this region to obtain m_σ for all the parameters. The fit result on $L = 36$ at $m_f = 0.015$ is shown in Fig. 1. In this parameter it is possible to fit $2D(t)$ with a longer fit range, while in some parameters the effective mass of $2D(t)$ in the large time region is unstable with large error in the current statistics. Thus, we choose this fixed fit range in all the parameters. In order to estimate the systematic error coming from the fixed fit range, we carry out another fit in a region at larger t than the fixed one, with the same number of data points. An example of this fit is shown in Fig. 1. We quote the difference between the two central values as the systematic error.

The values of m_σ and also m_π for all the parameters are summarized in Table I. Figure 2 presents m_σ as a function of m_f together with m_π. These are our main results. The data on the largest two volumes at each m_f, except for $m_f = 0.015$, agree with each other, and suggest that finite size effects are negligible in our statistics. We find a clear signal that σ is as light as π for all the fermion masses we simulate. This property is distinctly different from the one in usual QCD, where m_σ is clearly larger than m_π [28,29], while it is similar to the one in $N_f = 12$ QCD observed in our previous study [11]. Thus, this might be regarded as a reflection of the approximate scale symmetry in this theory, no matter whether the main scale symmetry breaking in the far infrared comes from m_f or m_D, as we noted before. The figure also shows that our simulation region is far from the heavy-fermion limit, because the vector meson mass obtained from the $(\gamma_i\gamma_4 \otimes \xi_i\xi_4)$ operator, denoted by ρ(PV), is clearly larger than m_π.

Although the accuracy of our data is not enough to make a clear conclusion for a chiral extrapolation, we shall report some results below. While in the previous paper [10] we found that the data for m_π and F_π, the π decay constant at each m_f, are consistent with chiral perturbation theory (ChPT) in the region $m_f \leq 0.04$, the updated data [30], tabulated in Table I, show consistency with ChPT in a somewhat smaller region $m_f \leq 0.03$. Thus, we shall use the lightest three data with the smallest error at each m_f, i.e., the two data on $L = 36$ and the lightest data on $L = 24$, in the following analyses.

The validity of ChPT is intact even when the light σ comparable with π is involved in the chirally broken phase: the systematic power counting rule as a generalization of ChPT including σ as a dilaton was established in Ref. [31] ["dilaton ChPT" (DChPT)] including computation of the chiral log effects. At the leading order we have $m_\pi^2 = 4m_f\langle\bar{\psi}\psi\rangle/F^2$ (the Gell-Mann—Oakes—Renner relation) and

$$m_\sigma^2 = d_0 + d_1 m_\pi^2, \tag{5}$$

where $d_0 = m_\sigma^2|_{m_f=0}$ and $d_1 = (3-\gamma_m)(1+\gamma_m)/4 \cdot (N_f F^2)/F_\sigma^2$, with γ_m being the mass anomalous dimension in the walking region, and F and F_σ being the decay constants of π and σ, respectively, in the chiral limit. ($F/\sqrt{2}$ corresponds to 93 MeV for the usual QCD π.) In the following fit, we ignore higher order terms including chiral log. We plot m_σ^2 as a function of m_π^2 in Fig. 3. The extrapolation to the chiral limit based on Eq. (5) gives a reasonable $\chi^2/\text{d.o.f.} = 0.27$, with a tiny value in the chiral limit, $d_0 = -0.019(13)(^{3}_{20})$ where the first and second errors are statistical and systematic, respectively. It agrees with zero with 1.4 standard deviation and shows a consistency with the NG nature of σ. The fit without the lightest point (with single volume) gives a consistent result, showing that finite size effects are not statistically relevant. Although errors are large at this moment, it is very encouraging for obtaining a light technidilaton to be identified with a composite Higgs with

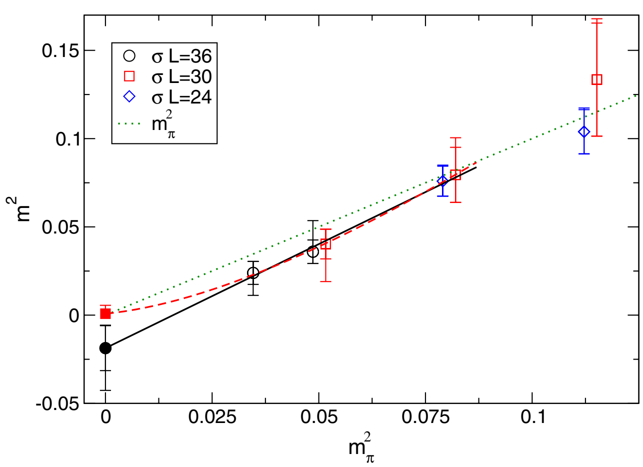

FIG. 3 (color online). Mass squared of the flavor-singlet scalar m_σ^2 as a function of m_π^2. The outer error represents the statistical and systematic uncertainties added in quadrature, while the inner error is only statistical. Open square symbols are slightly shifted for clarity. The result of a chiral extrapolation by the DChPT fit in Eq. (5) is plotted by the solid line and full circle. The linear fit result, $m_\sigma = c_0 + c_1 m_f$, is also plotted by the dashed curve and full square. The dotted line denotes $m_\sigma^2 = m_\pi^2$.

mass 125 GeV, with the value very close to $F/\sqrt{2} \simeq 123$ GeV of the one-family model with 4 weak doublets, i.e., $N_f = 8$. The value of F from our data is estimated as $F = 0.0202(13)(^{54}_{67})$, which is updated from the previous paper [10] using more statistics and new smaller m_f data. (If this scalar is to be identified with a composite Higgs, we expect $d_0 \sim F^2/2 \sim 0.0002$.)

From the value of d_1, we can read F_σ, because the factor $(3-\gamma_m)(1+\gamma_m)/4$ is close to unity when we use $\gamma_m = 0.6$–1.0 [10]. The value of F_σ is important to make a prediction of the couplings of the Higgs boson from the walking technicolor theory. The obtained slope is $d_1 = 1.18(24)(^{35}_{7})$. From d_1 we estimate F_σ as $F_\sigma \sim \sqrt{N_f}F$, in curious coincidence with the holographic estimate [7] and the linear sigma model. Note that the property $d_1 \sim 1$ is another feature different from usual QCD, where a much larger slope was observed for $m_\pi > 670$ MeV [28].

With our statistics we can also fit the data with an empirical form, $m_\sigma = c_0 + c_1 m_f$, consistent with Eq. (5) up to higher order corrections, where we obtain $c_0 = 0.029(39)(^{8}_{72})$ and the ratio $m_\sigma/(F/\sqrt{2}) = 2.0(2.7)(^{8}_{5.1})$. The fit result is plotted in Fig. 3 as a function of m_π^2 using a quadratic m_f fit result for m_π^2. Several other fits, such as a linear m_π^2 fit of m_σ^2/F_π^2, are carried out, and they give reasonably consistent ratios with the one from c_0. All the fit results suggest a possibility to reproduce the Higgs boson mass within the large errors.

Note that due to the sizable error the σ spectrum could also be consistent with the hyperscaling for the conformal theory. Different, more precisely measurable quantities are required to study if the theory is conformal or near conformal [10,30].

We found that $N_f = 8$ QCD behaves consistently with a walking theory in the previous study [10]. If our σ is a candidate for the composite Higgs, m_σ should be nonzero in the chiral limit, and hence become larger than m_π at m_f smaller than the ones used in the current work. Note that it is predicted in Ref. [31] that the chiral log effect of π loops makes the m_π^2 dependence of m_σ^2 milder. Therefore, observing $m_\sigma > m_\pi$ is an important future direction and is necessary to determine a precise value of m_σ in the chiral limit, though it requires more accurate data with a much smaller fermion mass. Furthermore, in such a small m_f region, decay of σ to two pions should be taken into account to extract m_σ using a variational method, while σ in this work cannot decay due to the heavy fermion mass where $m_\sigma < 2m_\pi$. To check the consistency of the ground state mass, it is also important to calculate m_σ from gluonic operators as in our $N_f = 12$ QCD study [11,16,32].

In summary, using the same calculation techniques as in the study of $N_f = 12$ QCD [11], we have observed clear signals of a flavor-singlet scalar as light as the pion in $N_f = 8$ QCD, which was shown to be a candidate for walking technicolor [10]. Our simple chiral extrapolations suggest the possibility of the existence of a very light flavor-singlet scalar to be identified with a composite Higgs, which may be the technidilaton, with mass 125 GeV, although the errors on the extrapolated values are large.

Obviously, an important future direction is to obtain a more precise value of m_σ in the chiral limit to clarify whether this theory can really reproduce the Higgs boson mass of 125 GeV, and is really a candidate for a theory beyond the standard model. To do this, we should observe $m_\sigma > m_\pi$ discussed above, which could be regarded as another signal of walking behavior.

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