

Towards an explanation of transverse single-spin asymmetries in proton-proton collisions: The role of fragmentation in collinear factorization

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We study the transverse single-spin asymmetry for single-hadron production in proton-proton collisions within the framework of collinear twist-3 factorization in quantum chromodynamics. By taking into account the contribution due to parton fragmentation, we obtain a very good description of all high transverse-momentum data for neutral and charged pion production from the Relativistic Heavy Ion Collider. Our study may provide the crucial step toward a final solution to the long-standing problem of what causes transverse single-spin asymmetries in hadronic collisions within quantum chromodynamics. We show for the first time that it is possible to simultaneously describe spin/azimuthal asymmetries in proton-proton collisions, semi-inclusive deep-inelastic scattering, and electron-positron annihilation by using collinear twist-3 factorization in the first process along with transverse-momentum-dependent functions extracted from the latter two reactions.

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I. INTRODUCTION

The field of transverse single-spin asymmetries (SSAs) in hard semi-inclusive processes began some four decades ago with the observation of the large transverse polarization (up to about 30%) of neutral Λ hyperons in the process $pBe \rightarrow \Lambda^\uparrow X$ at FermiLab [1]. People noticed early on that the naïve collinear parton model cannot generate such large effects [2]. It was then pointed out that SSAs for single-particle production in hadronic collisions are genuine twist-3 observables for which, in particular, collinear three-parton correlations have to be taken into account in order to have a proper description within quantum chromodynamics (QCD) [3]. This formalism later on was worked out in more detail and also successfully applied to SSAs in processes like hadron production in proton-proton collisions, $p^\uparrow p \rightarrow hX$ (see, e.g., Refs. [4–10]). Here we focus on SSAs in such reactions, which were extensively investigated in fixed target and in collider experiments.

Let us now look at the generic structure of the spin-dependent cross section for $A(P, \vec{S}_\perp) + B(P') \rightarrow C(P_h) + X$, where the 4-momenta and polarizations of the incoming protons A , B and outgoing hadron C are specified. In twist-3 collinear QCD factorization, one has

$$\begin{aligned} d\sigma(\vec{S}_\perp) = & H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{C/c(2)} \\ & + H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{C/c(2)} \\ & + H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}, \end{aligned} \quad (1)$$

with $f_{a/A(t)}$ ($f_{b/B(t)}$) indicating the distribution function associated with parton a (b) in proton A (B), while $D_{C/c(t)}$ represents the FF associated with hadron C in parton c . The

twist of the functions is denoted by t . The hard factors corresponding to each term are given by H , H' , and H'' , and the symbol \otimes represents convolutions in the appropriate momentum fractions. In Eq. (1), a sum over partonic channels and parton flavors in each channel is understood.

The first term in (1) has already been studied in quite some detail in the literature [5,7–12]. It contains both quark-gluon-quark correlations and trigluon correlations in the polarized proton, where for the former one needs to distinguish between contributions from so-called soft gluon poles (SGPs) and soft fermion poles (SFPs). The second term in (1), arising from twist-3 effects in the unpolarized proton, was shown to be small [13]. Only recently, a complete analytical result was obtained for the third term in (1), which describes the twist-3 contribution due to parton fragmentation [14].

For quite some time, many in the community believed that the first term in (1) dominates the transverse SSA in $p^\uparrow p \rightarrow hX$ (typically denoted by A_N) for the production of light hadrons (see, e.g., Refs. [5,7,10]), where the SGP contribution is generally considered the most important part. Note that the SGP contribution to A_N is determined by the Qiu-Sterman function T_F [4,5], which can be related to the transverse-momentum-dependent (TMD) Sivvers parton density f_{1T}^\perp [15,16]. For a given quark flavor q , these entities satisfy [17]

$$T_F^q(x, x) = - \int d^2\vec{p}_\perp \frac{\vec{p}_\perp^2}{M} f_{1T}^{\perp q}(x, \vec{p}_\perp^2)|_{\text{SIDIS}}, \quad (2)$$

where M is the nucleon mass. Because of the relation in (2), one can extract T_F from data on either A_N or on the Sivvers transverse SSA in semi-inclusive deep-inelastic scattering

(SIDIS) $A_{\text{SIDIS}}^{\text{Siv}}$. It therefore came as a major surprise when an attempt failed to simultaneously explain both A_N and $A_{\text{SIDIS}}^{\text{Siv}}$ [11]. The striking result pointed out in Ref. [11] was that the two extractions for T_F differ in sign. This ‘‘sign-mismatch’’ puzzle could not be resolved by more flexible parameterizations of f_{1T}^{\perp} [18]. Also trigluon correlations are unlikely to fix this issue [12], while SFPs may play some role [9].

At this point, one may start to question the dominance of the first term in (1). In fact, data on the transverse SSA in inclusive DIS [19,20] seem to support this point of view, i.e., that the first term in (1) is not the main cause of A_N [21]. Therefore, in the present work, we study the potential role of the twist-3 fragmentation part of (1). After fixing the SGP contribution to A_N through the Siverson function extracted from data on $A_{\text{SIDIS}}^{\text{Siv}}$ [22,23] and the relation in (2), we obtain a very good fit to all high transverse-momentum forward-region pion data for A_N from the Relativistic Heavy Ion Collider (RHIC). As explained below in more detail, our analysis shows for the first time that one can simultaneously describe A_N using collinear factorization, $A_{\text{SIDIS}}^{\text{Siv}}$, the Collins transverse SSA $A_{\text{SIDIS}}^{\text{Col}}$ in SIDIS, and $A_{e^+e^-}^{\cos(2\phi)}$ that represents a particular azimuthal asymmetry in electron-positron annihilation into two hadrons, $e^+e^- \rightarrow h_1 h_2 X$ [24].

II. FRAGMENTATION CONTRIBUTION TO A_N

The fragmentation contribution to the cross section in (1) reads [14]

$$\begin{aligned} & \frac{P_h^0 d\sigma(\vec{S}_{\perp})}{d^3\vec{P}_h} \\ &= -\frac{2\alpha_s^2 M_h}{S} \epsilon_{\perp, \alpha\beta} S_{\perp}^{\alpha} P_{h\perp}^{\beta} \sum_i \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^3} \\ & \times \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x' S + T/z} \frac{1}{-x'\hat{t} - x\hat{u}} h_1^a(x) f_1^b(x') \\ & \times \left\{ \left[\hat{H}^{C/c}(z) - z \frac{d\hat{H}^{C/c}(z)}{dz} \right] S_{\hat{H}}^i + \frac{1}{z} H^{C/c}(z) S_{\hat{H}}^i \right. \\ & \left. + 2z^2 \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{C/c, \mathfrak{S}}(z, z_1) \frac{1}{\xi} S_{\hat{H}_{FU}}^i \right\}, \quad (3) \end{aligned}$$

where i denotes the channel, $x = -x'(U/z)/(x'S + T/z)$, $x'_{\min} = -(T/z)/(U/z + S)$, $z_{\min} = -(T + U)/S$, and $\xi = (1 - z/z_1)$. Here we used the Mandelstam variables $S = (P + P')^2$, $T = (P - P_h)^2$, and $U = (P' - P_h)^2$, which on the partonic level give $\hat{s} = xx'S$, $\hat{t} = xT/z$, and $\hat{u} = x'U/z$. Oftentimes one also uses $x_F = 2P_{hz}/\sqrt{S}$, where P_{hz} is the longitudinal momentum of the hadron, as well as the pseudo-rapidity $\eta = -\ln \tan(\theta/2)$, where θ is the scattering angle. The variables x_F , η are further related by $x_F = 2P_{h\perp} \sinh(\eta)/\sqrt{S}$, where $P_{h\perp}$ is the transverse

momentum of the hadron. The nonperturbative parts in (3) are the transversity distribution h_1 , the unpolarized parton density f_1 , and the three (twist-3) fragmentation functions (FFs) \hat{H} , H , and $\hat{H}_{FU}^{\mathfrak{S}}$, with the last one parameterizing the imaginary part of a three-parton correlator. The definition of those functions and the results for the hard scattering coefficients S^i can be found in [14]. (An alternative notation of the relevant FFs is given in Ref. [25], where twist-3 effects in SIDIS were computed.) We note that the so-called derivative term in (3), associated with $d\hat{H}/dz$, was first obtained in [26].

The function \hat{H} is related to the TMD Collins function H_1^{\perp} [27] according to [14,26]

$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_{\perp} \frac{\vec{k}_{\perp}^2}{2M_h^2} H_1^{\perp h/q}(z, z^2\vec{k}_{\perp}^2). \quad (4)$$

This relation can be considered the fragmentation counterpart of Eq. (2). Exploiting the universality of the Collins function [28], one can simultaneously extract H_1^{\perp} and h_1 from data on $A_{\text{SIDIS}}^{\text{Col}}$ [29,30] and data on $A_{e^+e^-}^{\cos(2\phi)}$ [31,32] (see [33] and references therein). Below we utilize such information for H_1^{\perp} and h_1 when describing A_N . The FFs in (3) are related via [14]

$$H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3 \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q, \mathfrak{S}}(z, z_1), \quad (5)$$

implying that in the collinear twist-3 framework one has two independent FFs. It is important to realize that this is different from the so-called TMD approach for A_N , where only H_1^{\perp} enters the fragmentation piece [34].

III. PHENOMENOLOGY OF A_N FOR PION PRODUCTION

We consider A_N for $p^{\uparrow} p \rightarrow \pi X$ in the forward region of the polarized proton, which has been studied by the STAR [35–37], BRAHMS [38,39] and PHENIX [40] Collaborations at RHIC. We mainly focus on data taken at $\sqrt{S} = 200$ GeV for which typically $P_{h\perp} > 1$ GeV. Throughout we use the GRV98 unpolarized parton distributions [41] and the DSS unpolarized FFs [42]. Note that the GRV98 parton distributions were also used in Refs. [22,23,33] for extracting the Siverson function and the transversity, which we take as input in our calculation. The SGP contribution to (1) is computed by fixing T_F through Eq. (2) with two different inputs for the Siverson function—SV1: f_{1T}^{\perp} from Ref. [22], obtained from SIDIS data on $A_{\text{SIDIS}}^{\text{Siv}}$ [43,44]; and SV2: f_{1T}^{\perp} from Ref. [23], ‘‘constructed’’ such that, in the TMD approach, the contribution of the Siverson effect to A_N is maximized while maintaining a good description of $A_{\text{SIDIS}}^{\text{Siv}}$. These two inputs

for f_{1T}^\perp are mainly distinct by their quite different large- x behavior. To compute the contribution in (3), we take h_1 and H_1^\perp [which fixes \hat{H} through (4)] from [33]. For favored fragmentation into π^+ , we make for $\hat{H}_{FU}^{\mathbb{S}}$ the ansatz

$$\frac{\hat{H}_{FU}^{\pi^+/(u,\bar{d}),\mathbb{S}}(z, z_1)}{D^{\pi^+/(u,\bar{d})}(z)D^{\pi^+/(u,\bar{d})}(z/z_1)} = \frac{N_{\text{fav}}}{2I_{\text{fav}}J_{\text{fav}}} z^{\alpha'_{\text{fav}}(z/z_1)} (1-z)^{\beta_{\text{fav}}} (1-z/z_1)^{\beta'_{\text{fav}}}, \quad (6)$$

with the parameters N_{fav} , α'_{fav} , β_{fav} , β'_{fav} , and the unpolarized FF D . Note that the allowed range for z and z/z_1 is $[0, 1]$ [45] and that our ansatz satisfies the constraint $\hat{H}_{FU}(z, z) = 0$ [45,46]. With the use of DSS FFs [42], the factor I_{fav} reads $I_{\text{fav}} \equiv I_{u+\bar{u}} - I_{\bar{u}}$ where I_i ($i = u + \bar{u}, \bar{u}$) is defined as

$$I_i = \frac{N_i(K_{1,\text{fav}} + \gamma_i K_{2,\text{fav}})}{B[2 + \alpha_i, \beta_i + 1] + \gamma_i B[2 + \alpha_i, \beta_i + \delta_i + 1]},$$

with $K_{1,\text{fav}} = B[\alpha'_{\text{fav}} + \alpha_i + 1, \beta'_{\text{fav}} + \beta_i]$,

$$K_{2,\text{fav}} = B[\alpha'_{\text{fav}} + \alpha_i + 1, \beta'_{\text{fav}} + \beta_i + \delta_i], \quad (7)$$

and $B[a, b]$ the Euler β function. The parameters N_i , α_i , β_i , γ_i , and δ_i come from D FFs at the initial scale and are given in Table III of [42]. Note that $D^{\pi^+/u}$ in Ref. [42] differs from $D^{\pi^+/\bar{d}}$. J_{fav} in (6) is similarly defined as $J_{\text{fav}} \equiv J_{u+\bar{u}} - J_{\bar{u}}$, where J_i ($i = u + \bar{u}, \bar{u}$) follows from I_i through $\alpha'_{\text{fav}} \rightarrow (\alpha_{\text{fav}} + 4)$, $\beta'_{\text{fav}} \rightarrow (\beta_{\text{fav}} + 1)$. The factor $1/(2I_{\text{fav}}J_{\text{fav}})$ in (6) is convenient and implies $\int_0^1 dz z H_{(3)}^{\pi^+/u}(z) = N_{\text{fav}}$ at the initial scale, where $H_{(3)}$ represents the entire second term on the right-hand side of (5). For the disfavored FFs $\hat{H}_{FU}^{\pi^+/(d,\bar{u}),\mathbb{S}}$ we make an ansatz in full analogy to (6), introducing the additional parameters N_{dis} , α'_{dis} , β_{dis} , β'_{dis} . (I_{dis} and J_{dis} are calculated using $D^{\pi^+/d} = D^{\pi^+/\bar{u}}$ from [42].) The π^- FFs are then fixed through charge conjugation, and the π^0 FFs are given by the average of the FFs for π^+ and π^- . The FFs $H^{\pi/q}$ are computed by means of (5). All parton correlation functions are evaluated at the scale $P_{h\perp}$ with leading-order evolution of the collinear functions.

Using the MINUIT package, we fit the fragmentation contribution to data for $A_N^{\pi^0}$ [35–37] and $A_N^{\pi^\pm}$ [38]. To facilitate the fit, we only keep seven parameters in $\hat{H}_{FU}^{\pi^+/q,\mathbb{S}}$ free. We also allow the β -parameters $\beta'_u = \beta'_d$ of the transversity to vary within the error range given in [33]. All integrations are done using the Gauss-Legendre method with 250 steps.

For the SV1 input, the result of our eight-parameter fit is shown in Table I. Note that the values for $\beta'_{\text{fav}} = \beta'_{\text{dis}}$ and β_{fav} are at their lower limits, which we introduce to guarantee a finite integration upon z_1 in (3) and a proper behavior of A_N at large x_F , respectively. For the SV2 input,

TABLE I. Fit parameters for SV1 input.

$\chi^2/\text{d.o.f.} = 1.03$	
$N_{\text{fav}} = -0.0338$	$N_{\text{dis}} = 0.216$
$\alpha'_{\text{fav}} = \alpha'_{\text{dis}} = -0.198$	$\beta_{\text{fav}} = 0.0$
$\beta'_{\text{fav}} = \beta'_{\text{dis}} = -0.180$	$\alpha_{\text{dis}} = \alpha'_{\text{dis}} = 3.99$
$\beta_{\text{dis}} = 3.34$	$\beta'_u = \beta'_d = 1.10$

the values of the fit parameters are similar, with an equally successful fit ($\chi^2/\text{d.o.f.} = 1.10$).

The very good description of A_N is also reflected by Fig. 1. We emphasize that such a positive outcome is nontrivial if one keeps in mind the constraint in (5) and the need to simultaneously fit data for $A_N^{\pi^0}$ and $A_N^{\pi^\pm}$. Results for the FFs $H^{\pi^+/q}$ and $\tilde{H}_{FU}^{\pi^+/q} \equiv \int_z^\infty \frac{dz_1}{z_1} \frac{1}{z-z_1} \hat{H}_{FU}^{\pi^+/q,\mathbb{S}}(z, z_1)$ are displayed in Fig. 2. In either case, the favored and disfavored FFs have opposite signs. This is like for H_1^\perp where such reversed signs are actually ‘‘preferred’’ by the Schäfer-Teryaev (ST) sum rule $\sum_h \sum_{S_h} \times \int_0^1 dz z M_h \hat{H}^{h/q}(z) = 0$ [47]. Note that the ST sum rule, in combination with (5), implies a constraint on a certain linear combination of $H^{h/q}$ and (an integral of) $\hat{H}_{FU}^{h/q,\mathbb{S}}$. In view of that, reversed signs between favored and disfavored FFs like in Fig. 2 are actually beneficial. Also depicted in

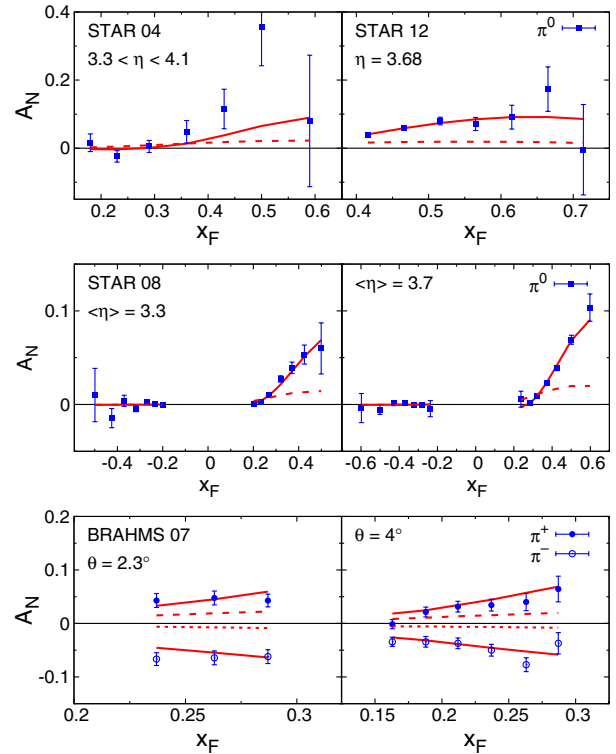


FIG. 1 (color online). Fit results for $A_N^{\pi^0}$ (data from [35–37]) and $A_N^{\pi^\pm}$ (data from [38]) for the SV1 input. The dashed line (dotted line in the case of π^-) means $\hat{H}_{FU}^{\mathbb{S}}$ switched off.

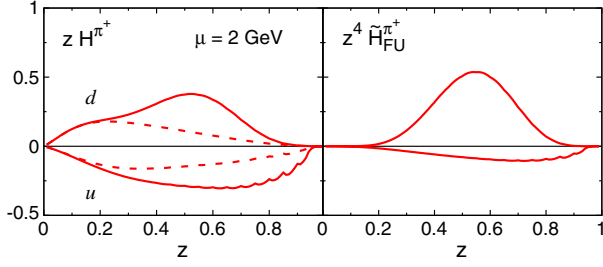


FIG. 2 (color online). Results for the FFs H^{π^+}/q and $\tilde{H}_{FU}^{\pi^+}/q$ (defined in the text) for the SV1 input. Also shown is H^{π^+}/q without the contribution from $\hat{H}_{FU}^{\pi^+}$ (dashed line).

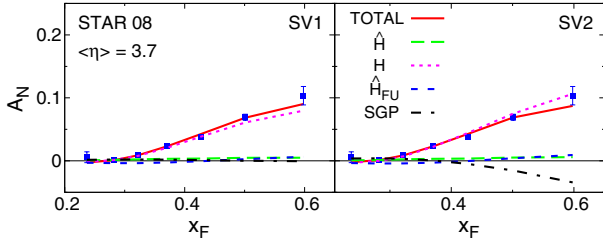


FIG. 3 (color online). Individual contributions to $A_N^{\pi^0}$ (data from [36]) for SV1 and SV2 inputs.

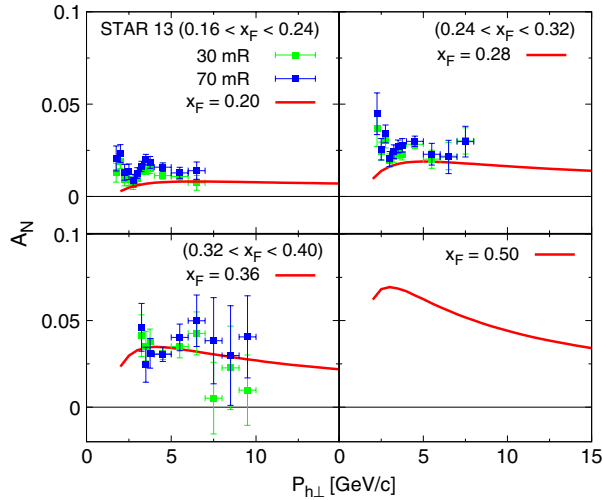


FIG. 4 (color online). A_N as function of $P_{h\perp}$ for SV1 input at $\sqrt{S} = 500$ GeV (data from [48]).

Fig. 2 is H^{π^+}/q when $\hat{H}_{FU}^{\pi^+}/q$ is switched off. As shown in Fig. 1, in such a scenario, i.e., by turning off the three-parton FF, one cannot describe the data for A_N . According to Fig. 3, the \hat{H} term (including its derivative) in (3) contributes only very little to A_N . Also, the SGP pole term

is small, except for the SV2 input at large x_F , where its contribution is opposite to the data. Clearly, A_N is governed by the H term in (3). This result can mainly be traced back to the hard scattering coefficients: e.g., for the dominant $qg \rightarrow qg$ channel, one has $S_H \propto 1/\hat{t}^3$, but $S_{\hat{H}} \propto 1/\hat{t}^2$ [14] in the forward region where \hat{t} is small. Finally, Fig. 4 shows the $P_{h\perp}$ dependence of A_N for $\sqrt{S} = 500$ GeV. Preliminary data from STAR, extending to almost $P_{h\perp} = 10$ GeV, show that A_N is rather flat [48]. The twist-3 calculation agrees with that trend, and also the magnitude of A_N is in line with the data. Note that the data of Ref. [48] were not included in our fit and that only statistical errors are shown in Fig. 4 [48].

IV. CONCLUSIONS

Collinear twist-3 QCD factorization can be considered the most natural and rigorous approach to the transverse SSA A_N in $p^\uparrow p \rightarrow hX$. However, the sign-mismatch issue of the Siverson effect had put this framework into question [11]. Here we have demonstrated for the first time that, despite the sign-mismatch problem, twist-3 factorization actually can describe high-energy RHIC data for A_N^π very well if one takes the fragmentation contribution into account. We reemphasize that this result is nontrivial. Since in the twist-3 approach part of A_N can be fixed by spin/azimuthal asymmetries in SIDIS and in $e^+e^- \rightarrow h_1 h_2 X$, we have shown that at present a simultaneous description of all of those observables is possible. We repeat that the fragmentation contribution in twist-3 factorization goes beyond the pure Collins effect. Independent information on the FFs H^{π^+}/q , $\hat{H}_{FU}^{\pi^+}/q$ from other sources is needed before one can ultimately claim that the intriguing data on A_N^π are fully understood. However, the fact that $\hat{H}_{FU}^{\pi^+}/q$ gives a reasonable contribution to (the numerically dominant) H^{π^+}/q (see Fig. 2) allows one to be optimistic in this regard.

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