Standard model with new order parameters at finite temperature

Makoto Sakamoto[1,*] and Kazunori Takenaga[2,†]
[1]*Department of Physics, Kobe University, Rokkodai Nada, Kobe 657-8501, Japan*
[2]*Faculty of Health Science, Kumamoto Health Science University, Izumi-machi, Kita-ku, Kumamoto 861-5598, Japan*
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We consider the finite temperature effective potential of the standard model at the one-loop level in four dimensions by taking account of two kinds of order parameters, the Higgs vacuum expectation value and the zero modes of gauge fields for the Euclidean time direction. We study the vacuum structure of the model, focusing on the existence of the new phase, where the zero modes, that is, the new order parameters, develop nontrivial vacuum expectation values except for the center of the gauge group. We find that under certain conditions there appears no new phase at finite temperature.

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I. INTRODUCTION

Quantum field theory at finite temperature [1] provides useful tools to study phase transition in high-energy physics. The effective potential at finite temperature actually plays an important role for studying the scenario of the electroweak baryogenesis [2] and the deconfinement phase of QCD [3] in weak coupling regime. Quantum field theory at finite temperature has been used in various contexts [4].

The imaginary time formulation of quantum field theory at finite temperature is familiar, and in this formulation, the Euclidean time direction is compactified on a circle S_τ^1 whose length of the circumference is the inverse temperature T^{-1}. When one considers gauge theory on such a space, it is well known that zero modes of component gauge fields for the S_τ^1 direction cannot be gauged away and become dynamical degrees of freedom [3] so that they can develop vacuum expectation values [5]. We can determine the vacuum expectation values by minimizing the effective potential for the zero modes. One should notice that such zero modes must be taken into account as long as they are the dynamical degrees of freedom.

In the context of higher dimensional gauge theory at finite temperature, zero modes of component gauge fields for the S_τ direction should be taken into account in addition to possible zero modes of component gauge fields corresponding to topological spatial extra dimensions. Gauge symmetry breaking through the zero modes has been discussed in [6,7]. The high-temperature phase transition of the standard model through the dynamics of the zero modes of the $SU(2)_L, U(1)_Y$ gauge fields for the S_τ^1 direction has been studied in [8], where the Higgs potential has been ignored. It has been found that there appear metastable states by studying the effective potential for the zero modes.

One may think that the zero mode of the $SU(2)_L$ gauge fields for the S_τ direction takes the value at the center of the $SU(2)_L$ gauge group like QCD in weak coupling regime at finite temperature [3]. This is, however, not so trivial because the models contain the Higgs potential, and the vacuum expectation value of the Higgs field may influence the location of the minimum for the zero mode in the effective potential. This is actually the case in physics with extra dimensions [6].

In this paper, we investigate the phase structure of the standard model in four dimensions at finite temperature by studying the effective potential at the one-loop level. In doing it, we correctly take the zero modes of the $SU(3)_c, SU(2)_L, U(1)_Y$ gauge fields for the S_τ^1 direction into account in addition to the usual order parameter, the vacuum expectation value of the Higgs field. It is expected that there appear new phases in which the zero modes of the $SU(2)_L, U(1)_Y$ gauge fields, that is, new order parameters in the model, take nontrivial values except for the center of the gauge group. If this is the case, the zero modes give a source for the gauge symmetry breaking. We focus on seeking whether such a new phase appears or not.

One encounters the situation that has never been seen before due to the new order parameters. The parametrization of the vacuum expectation value of the Higgs field changes, contrary to the usual case. The electromagnetic component in the Higgs field, which is usually gauged away by using the $SU(2)_L \times U(1)_Y$ degrees of freedom, remains even after using the gauge degrees of freedom because of the new order parameters. As a result, the number of the order parameters increases in the model. We follow the standard prescription to calculate the effective potential. We expand fields around the vacuum expectation values and take up to quadratic terms. The increased order parameters complicate the quadratic terms, which contain the couplings that break the electromagnetic $U(1)$, denoted

*dragon@kobe-u.ac.jp
†takenaga@kumamoto-hsu.ac.jp

by $U(1)_{\rm em}$, invariance. This makes it difficult to obtain the effective potential in an analytic way.

We impose plausible conditions among the order parameters in order to study the effective potential as analytically as possible. Even under these conditions we can still investigate a possibility of new phases in which new order parameters take nontrivial values. Our analyses tell us that there is no new phase in the standard model in four dimensions. It may be essential for the result that the boundary condition of fermions for the S^1_τ direction is fixed by the Fermi statistics. This is quite different from the case of the physics with spatial extra dimensions.

This paper is organized as follows. In Sec. II we introduce the order parameters of the model and discuss the minimum of the tree-level potential for latter convenience. We obtain the effective potential at the one-loop level under certain conditions among the order parameters and study the phase structure by minimizing the effective potential, focusing on the new phase in Sec. III. Conclusions and discussions are devoted to Sec. IV. In the Appendix, we present the detail of the calculations in the presence of the new order parameters.

II. ORDER PARAMETERS

The imaginary time formulation of quantum field theory at finite temperature is to consider the theory on $S^1_\tau \times M^3$, where the Euclidean time direction τ is compactified on the S^1_τ whose circumference is the inverse temperature T^{-1}. The M^3 is the three-dimensional flat space whose coordinate is denoted by $x^i (i = 1,2,3)$.

We consider the standard model in four dimensions at finite temperature. As discussed in the literature [3,5], the zero modes of the Euclidean time components of the gauge fields, which cannot be gauged away, become the dynamical variable to parametrize the vacuum of the theory. They are order parameters of the theory. The vacuum expectation values are determined by minimizing the effective potential for the order parameters.

In the present case, the order parameters we have to take into account are

$$\langle A_\tau\rangle, \qquad \langle B_\tau\rangle, \qquad \langle G_\tau\rangle, \qquad \langle\Phi\rangle, \tag{1}$$

where $A_\tau (B_\tau, G_\tau)$ is the Euclidean time component of the $SU(2)_L [U(1)_Y, SU(3)_c]$ gauge field and Φ is the Higgs field.

Let us discuss the parametrization of the vacuum expectation value (1) in the electroweak sector. By using the $SU(2)_L \times U(1)_Y$ degrees of freedom, we can parametrize the vacuum expectation values as

$$\frac{g}{T}\langle A_\tau\rangle = 2\pi \mathrm{diag.}(\varphi, -\varphi), \qquad \frac{g_Y}{T}\langle B_\tau\rangle = 2\pi\theta,$$
$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} p \\ v\end{pmatrix}. \tag{2}$$

Here g, g_Y are the $SU(2)_L, U(1)_Y$ gauge couplings, respectively. Let us note that φ (also θ) is physically identical to $\varphi + l (l \in \mathbb{Z})$. The φ, θ, v, p are real parameters. One can choose another parametrization, but equivalent to (2), given by

$$\frac{g}{T}\langle A_\tau\rangle = 2\pi\begin{pmatrix} \varphi_3 & \varphi_1 \\ \varphi_1 & -\varphi_3\end{pmatrix}, \qquad \frac{g_Y}{T}\langle B_\tau\rangle = 2\pi\theta,$$
$$\langle\Phi\rangle = \frac{1}{\sqrt{2}} p\begin{pmatrix} 0 \\ v'\end{pmatrix}. \tag{3}$$

The parametrizations (2) and (3) are mutually related by the transformations

$$\langle\Phi'\rangle = V\langle\Phi\rangle, \qquad \langle A_\tau{}'\rangle = V\langle A_\tau\rangle V^\dagger,$$

where V is defined by

$$V = \frac{1}{\sqrt{p^2+v^2}}\begin{pmatrix} v & -p \\ p & v\end{pmatrix} \qquad \text{with}$$
$$VV^\dagger = V^\dagger V = 1, \qquad \det V = 1. \tag{4}$$

One easily finds that

$$\varphi_1 \equiv \frac{2vp}{v^2+p^2}, \qquad \varphi_3 \equiv \frac{v^2-p^2}{v^2+p^2}\varphi, \qquad v' = \sqrt{v^2+p^2}. \tag{5}$$

We employ parametrization (2) in the paper. Let us note that in the vacuum expectation value of the Higgs field there remains the component p that can break the electromagnetic $U(1)$, denoted by $U(1)_{\rm em}$, invariance contrary to the usual case where $\langle A_\tau\rangle$ and $\langle B_\tau\rangle$ are not taken into account.

In the $SU(3)_c$ sector, one can parametrize $\langle G_\tau\rangle$ as

$$\frac{g_s}{T}\langle G_\tau\rangle = 2\pi \mathrm{diag.}(\omega_1, \omega_2, \omega_3) \qquad \text{with} \qquad \sum_{r=1}^{3}\omega_r = 0. \tag{6}$$

g_s is the $SU(3)_c$ gauge-coupling constant. $\omega_r (r = 1,2,3)$ is physically identical to $\omega_r + l (l \in \mathbb{Z})$.

Let us discuss the potential at the tree level. The Higgs potential is given by

$$V_{\rm H} = -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2. \tag{7}$$

In the background of (2), the potential at the tree level is given by the Higgs potential (7) and the contribution from the Higgs kinetic term, which yields the third term below,

$$\begin{aligned} V_{\rm H}^{\rm tree} &= -\mu^2|\langle\Phi\rangle|^2 + \frac{\lambda}{2}(|\langle\Phi\rangle|^2)^2 \\ &\quad + \langle\Phi\rangle^\dagger\left(g\langle A_\tau\rangle + \frac{g_Y}{2}\langle B_\tau\rangle\right)^2\langle\Phi\rangle \\ &= -\frac{\mu^2}{2}(p^2+v^2) + \frac{\lambda}{8}(p^2+v^2)^2 + \frac{(2\pi T)^2}{2} \\ &\quad \times\left\{\left(\varphi+\frac{\theta}{2}\right)^2 p^2 + \left(\varphi-\frac{\theta}{2}\right)^2 v^2\right\}. \end{aligned} \tag{8}$$

There are three extreme points:

$$(\mathrm{I}):\ p=0, \qquad v=\sqrt{\frac{2\mu^2}{\lambda}}, \qquad \varphi=\theta=0, \tag{9}$$

$$(\mathrm{II}):\ p=v=0, \qquad \theta=\frac{2\mu}{2\pi T}, \qquad \varphi=0, \tag{10}$$

$$(\mathrm{III}):\ p=v=0, \qquad \theta=0, \qquad \varphi=\frac{\mu}{2\pi T}. \tag{11}$$

It is easy to show that (I) is the vacuum configuration and that (II) and (III) are the saddle point configurations. The configuration (I) is the usual vacuum in the standard model.

For latter convenience, let us minimize the potential under the assumption $p=0$,

$$V_{\rm H}^{\rm tree}|_{p=0} = -\frac{\mu^2}{2}v^2 + \frac{\lambda}{8}v^4 + \frac{(2\pi T)^2}{2}\left(\varphi-\frac{\theta}{2}\right)^2 v^2. \tag{12}$$

There are two extreme points:

$$(\mathrm{I}):\ v=\sqrt{\frac{2\mu^2}{\lambda}}, \qquad \varphi-\frac{\theta}{2}=0, \tag{13}$$

$$(\mathrm{II}):\ v=0, \qquad \left(\varphi-\frac{\theta}{2}\right)^2=\left(\frac{\mu}{2\pi T}\right)^2. \tag{14}$$

It is easy to show that configuration (II) is a saddle point and that the vacuum configuration is given by (I). Let us note that as long as the second equation in Eq. (13) is satisfied arbitrary configurations for φ, θ are allowed.

III. ONE-LOOP EFFECTIVE POTENTIAL

The one-loop effective potential is obtained by the standard prescription. To this end, one needs to expand the fields around the vacuum expectation values (2) and takes quadratic terms with respect to fluctuations. The calculation is straightforward, but a little bit tedious because of the new order parameters. Namely in the present case, as discussed in the previous section, there remains the component p in the vacuum expectation values of the Higgs field that can break the $U(1)_{\rm em}$ invariance, and accordingly, this results in couplings that do not conserve the $U(1)_{\rm em}$ charge. This never happened in the past calculations of the standard model at finite temperature. This is entirely due to the new order parameters, that is, the vacuum expectation values for A_τ. We present the details of the calculations in the Appendix.

The quadratic terms are given by Eqs. (A23), (A28), (A35), and (A44) in the Appendix. One needs to find the eigenvalues for the matrices $M^2_{\rm gauge}, M^2_{\rm scalar}, M^2_{\rm ghost}, M_{\rm quark}$, and $M_{\rm lepton}$ and has to sum up all of the Matsubara mode labeled by the integer n whose dependence in the eigenvalues is quite nontrivial in the present case. It may be also difficult to carry out the summation with respect to n though we obtain the eigenvalues. The matrices are too complex to calculate the effective potential as analytically as possible because of the increased order parameters.

We are very much interested in the possibility of whether the new order parameters, namely, φ, θ, take the nontrivial values or not. Taking account of the fact that at the tree level there is no vacuum that breaks the $U(1)_{\rm em}$ invariance, it is likely that perturbative corrections do not induce the vacuum that breaks the $U(1)_{\rm em}$ invariance. Therefore, it may be natural to assume $p=0$. Since the tree-level potential has the global minimum (13) under the assumption $p=0$ as shown in Sec. II, let us impose an ansatz, which is given by

$$p=0, \qquad \varphi-\frac{\theta}{2}=0. \tag{15}$$

This drastically simplifies the quadratic terms with the dependence on the new order parameters φ, θ, and we are able to perform the analytic calculations for obtaining the effective potential at the one-loop level by the standard prescription. The details of the calculations under the ansatz are also given in the Appendix.

Let us quote relevant results from the Appendix where we give notations and present details of calculations. The quadratic terms for the gauge sector are given by

$$\mathcal{L}_{\rm gauge}^{\rm EW(2)}|_{\rm ansatz} = W_i^-\bar{D}^{W^\pm}\delta_{ij}W_j^+ + \frac{1}{2}(\bar{A}_i^3, \bar{B}_i) \times\begin{pmatrix}\bar{D}^{A^3} & \frac{gg_Y}{4}v^2 \\ \frac{gg_Y}{4}v^2 & \bar{D}^B\end{pmatrix}\delta_{ij}\begin{pmatrix}\bar{A}_j^3 \\ \bar{B}_j\end{pmatrix}, \tag{16}$$

where

$$\bar{D}^{W^\pm} = \partial_l^2 + (\partial_\tau - i(2\pi T)(2\varphi))^2 - \frac{g^2}{4}v^2,$$

$$\bar{D}^{A^3} = \partial_l^2 + \partial_\tau^2 - \frac{g^2}{4}v^2, \qquad \bar{D}^B = \partial_l^2 + \partial_\tau^2 - \frac{g_Y^2}{4}v^2. \tag{17}$$

As we can see, the second term in Eq. (16) can be diagonalized by the usual rotation defined by Eq. (A55) in the Appendix. The eigenvalues are given by Eq. (A56) in the Appendix. The new order parameter φ appears in the $\bar{D}^{W^\pm}$. Then the one-loop contributions from the gauge sector are given by

$$V^{\text{one-loop}}_{\text{gauge}} = 6\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2(n-2\varphi)^2+\frac{g^2}{4}v^2\right] + 3\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2n^2+\frac{g^2+g_Y^2}{4}v^2\right] + 3\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2n^2\right], \quad (18)$$

where we have defined

$$\int_k \equiv iT\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}. \quad (19)$$

The integer n stands for the Matsubara mode.

The quadratic terms from the scalar sector under the ansatz, including the Euclidean time components of the gauge fields, $\bar{A}^a_\tau, \bar{B}_\tau$ are given by

$$\mathcal{L}^{(2)}_{\text{scalar}}|_{\text{ansatz}} = \frac{1}{2}(\bar{A}^1_\tau,\bar{A}^2_\tau)\begin{pmatrix}\bar{A} & a\\ -a & \bar{A}\end{pmatrix}\begin{pmatrix}\bar{A}^1_\tau\\ \bar{A}^2_\tau\end{pmatrix} + \frac{1}{2}(g^1,g^2)\times\begin{pmatrix}\bar{B} & \bar{g}\\ -\bar{g} & \bar{C}\end{pmatrix}\begin{pmatrix}g^1\\ g^2\end{pmatrix} + \frac{1}{2}(\bar{A}^3_\tau,\bar{B}_\tau)\begin{pmatrix}\bar{D} & \bar{l}\\ \bar{l} & \bar{E}\end{pmatrix}\times\begin{pmatrix}\bar{A}^3_\tau\\ \bar{B}_\tau\end{pmatrix} + \frac{1}{2}h\bar{F}h + \frac{1}{2}G^0\bar{G}G^0, \quad (20)$$

where

$$\bar{A} = \partial_i^2+\partial_\tau^2-(2\pi T)^2(2\varphi)^2-\frac{g^2}{4}v^2,$$
$$\bar{B} = \partial_i^2+\partial_\tau^2-(2\pi T)^2(2\varphi)^2+\mu^2-\frac{\lambda}{2}v^2-\frac{g^2}{4}v^2 = \bar{C},$$
$$\bar{D} = \partial_i^2+\partial_\tau^2-\frac{g^2}{4}v^2, \qquad \bar{E} = \partial_i^2+\partial_\tau^2-\frac{g_Y^2}{4}v^2,$$
$$\bar{F} = \partial_i^2+\partial_\tau^2+\mu^2-\frac{3}{2}\lambda v^2,$$
$$\bar{G} = \partial_i^2+\partial_\tau^2+\mu^2-\frac{\lambda v^2}{2}-\frac{g^2+g_Y^2}{4}v^2,$$
$$a = -2(2\pi T)(2\varphi)\partial_\tau, \quad \bar{g} = -2(2\pi T)(2\varphi)\partial_\tau, \quad \bar{l} = \frac{gg_Y}{4}v^2. \quad (21)$$

The first and second terms in Eq. (20) are automatically diagonalized by the original complex base,

$$W^\pm_\tau = \frac{1}{\sqrt{2}}(\bar{A}^1_\tau \mp i\bar{A}^2_\tau), \qquad G^\pm = \frac{1}{\sqrt{2}}(g^1 \mp ig^2). \quad (22)$$

The third term is diagonalized by the usual rotation by Eq. (A55), as before. The eigenvalues are given by Eq. (A61). Then the contributions from the scalar sector are

$$V^{\text{one-loop}}_{\text{scalar}} = 2\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2(n-2\varphi)^2+\frac{g^2}{4}v^2\right] + 1\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2n^2+\frac{g^2+g_Y^2}{4}v^2\right] + 1\times\frac{1}{2i}\int_k \ln[k_i^2+(2\pi T)^2n^2] + 2\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2(n-2\varphi)^2-\mu^2+\frac{\lambda}{2}v^2+\frac{g^2}{4}v^2\right] + 1\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2n^2-\mu^2+\frac{\lambda}{2}v^2+\frac{g^2+g_Y^2}{4}v^2\right] + 1\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2n^2-\mu^2+\frac{3\lambda}{2}v^2\right]. \quad (23)$$

The quadratic terms from the ghost sector are given by

$$\mathcal{L}^{(2)}_{\text{ghost}}|_{\text{ansatz}} = -i\bar{C}^+\bar{D}^{W^\pm}C^- - i\bar{C}^-\bar{D}^{W^\pm}C^+ - i(\bar{C}^3,\bar{C})\times\begin{pmatrix}\bar{D}^{A^3} & \frac{gg_Y}{4}v^2\\ \frac{gg_Y}{4}v^2 & \bar{D}^B\end{pmatrix}\begin{pmatrix}C^3\\ C\end{pmatrix}, \quad (24)$$

where $\bar{D}^{W^\pm}, \bar{D}^{A^3}, \bar{D}^B$ are the same as the ones given in Eq. (17). Then the contributions to the effective potential are

$$V^{\text{one-loop}}_{\text{ghost}} = -4\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2(n-2\varphi)^2+\frac{g^2}{4}v^2\right] - 2\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2n^2+\frac{g^2+g_Y^2}{4}v^2\right] - 2\times\frac{1}{2i}\int_k \ln\left[k_i^2+(2\pi T)^2n^2\right]. \quad (25)$$

We observe from Eqs. (18) and (25) that the on-shell degrees of freedom for the gauge fields are extracted by the ghost fields. Let us note that the ghost fields obey the periodic boundary conditions [9].

The $SU(3)_c$ gauge contribution to the effective potential is given from Eq. (A51) in the Appendix by

$$V^{\text{one-loop}}_{SU(3)_c} = (4-2)\sum_{r,q=1}^{3}\frac{1}{2i}\int_k \ln[k_i^2 + (2\pi T)^2(n+\omega_r-\omega_q)^2]. \tag{26}$$

We consider only the third generation for fermions. Our results do not change even if we introduce the first and second generations with the mixings among the generations, and we simply assume that the neutrino is massless. Then the quadratic terms from the fermions are given by

$$\mathcal{L}^{(2)}_{\text{fermion}}|_{\text{ansatz}} = (\bar{t}_L, \bar{t}_R)\begin{pmatrix} -i\bar{D}_{t_L} & \frac{f_t}{\sqrt{2}}v \\ \frac{f_t}{\sqrt{2}}v & -i\bar{D}_{t_R}\end{pmatrix}\begin{pmatrix} t_L \\ t_R\end{pmatrix} + (\bar{b}_L, \bar{b}_R)\begin{pmatrix} -i\bar{D}_{b_L} & \frac{f_b}{\sqrt{2}}v \\ \frac{f_b}{\sqrt{2}}v & -i\bar{D}_{b_R}\end{pmatrix}\begin{pmatrix} b_L \\ b_R\end{pmatrix} + (\bar{\tau}_L, \bar{\tau}_R)\begin{pmatrix} -i\bar{D}_{\tau_L} & \frac{f_\tau}{\sqrt{2}}v \\ \frac{f_\tau}{\sqrt{2}}v & -i\bar{D}_{\tau_R}\end{pmatrix}\begin{pmatrix} \tau_L \\ \tau_R\end{pmatrix} - i\bar{\nu}_L\bar{D}_{\nu_L}\nu_L, \tag{27}$$

where

$$\begin{aligned} \bar{D}_{t_L} &\equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\omega_r + \frac{4}{3}\varphi\right)\right) + \gamma_i\partial_i = \bar{D}_{t_R}, \\ \bar{D}_{b_L} &\equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\omega_r - \frac{2}{3}\varphi\right)\right) + \gamma_i\partial_i = \bar{D}_{b_R}, \\ \bar{D}_{\tau_L} &\equiv \gamma_\tau\left(\partial_\tau + i(2\pi T)2\varphi\right) + \gamma_i\partial_i = \bar{D}_{\tau_R}, \\ \bar{D}_{\nu_L} &\equiv \gamma_\tau\partial_\tau + \gamma_i\partial_i. \end{aligned} \tag{28}$$

The contributions to the effective potential are given by the determinants of the two-by-two matrices in Eq. (27) and by taking the logarithm of them, which is given by

$$\begin{aligned} V^{\text{one-loop}}_{\text{fermion}} &= (-1)2^2\times\frac{1}{2i}\sum_{r=1}^{3}\int_k \ln\left[k_i^2 + (2\pi T)^2\left(n+\frac{1}{2}-\omega_r-\frac{4}{3}\varphi\right)^2 + \frac{f_t^2}{2}v^2\right] \\ &+ (-1)2^2\times\frac{1}{2i}\sum_{r=1}^{3}\int_k \ln\left[k_i^2 + (2\pi T)^2\left(n+\frac{1}{2}-\omega_r+\frac{2}{3}\varphi\right)^2 + \frac{f_b^2}{2}v^2\right] \\ &+ (-1)2^2\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2\left(n+\frac{1}{2}+2\varphi\right)^2 + \frac{f_\tau^2}{2}v^2\right] \\ &+ (-1)\frac{2^2}{2}\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2\left(n+\frac{1}{2}\right)^2\right]. \end{aligned} \tag{29}$$

Let us note that the half integer in the Matsubara mode n comes from the antiperiodic boundary condition of fermion for the S^1_τ direction. Collecting all of the contributions obtained above, we obtain the effective potential at the one-loop level under the ansatz (15), as given by Eq. (A70) in the Appendix.

The typical expression we have to evaluate is

$$V_{\text{basics}} = (-1)^F N_{\text{deg.}}\left(\frac{1}{2i}\right)\int_k \ln\left(k^2 + (2\pi T)^2 \times (n+\varphi)^2 + m(v)^2\right), \tag{30}$$

where F takes 1(0) for fermions (bosons) and $N_{\text{deg.}}$ counts the degrees of freedom. Following the standard prescription, V_{basics} consists of the zero-temperature part and the finite-temperature part,

$$V_{\text{basics}} = V^{T=0}_{\text{basics}} + V^{T\neq 0}_{\text{basics}}, \tag{31}$$

where

$$V^{T=0}_{\text{basics}} = -(-1)^{F+1}N_{\text{deg}}\frac{m(v)^4}{4(4\pi)^2}\left(\ln\left(\frac{m(v)^2}{M^2}\right) - \frac{3}{2}\right), \tag{32}$$

$$V^{T\neq 0}_{\rm basics} = (-1)^{F+1} N_{\rm deg} \frac{2}{(2\pi)^2} T^4 \sum_{m=1}^{\infty} \frac{1}{m^4} \cos(2\pi m\varphi) \times \left(\frac{m(v)^2}{T^2} m^2\right) K_2\left(\frac{m(v)}{T} m\right). \qquad (33)$$

Let us note that the Matsubara mode n is now replaced by the "winding" mode m through the Poisson's resummation formula,

$$\sqrt{4\pi t}T \sum_{n=-\infty}^{\infty} e^{-t(2\pi T)^2(n+\varphi)^2} = \sum_{m=-\infty}^{\infty} e^{-(\frac{m^2}{4T^2}\frac{1}{t} + 2\pi i m\varphi)}. \qquad (34)$$

Here we have employed the $\overline{\rm MS}$ scheme for the zero-temperature part of the effective potential, which comes from the $m = 0$ mode and M is a certain mass scale. $K_2(z)$ is the modified Bessel function defined by

$$\int_0^{\infty} dt t^{-\nu-1} e^{-At-\frac{B}{t}} = 2\left(\frac{A}{B}\right)^{\frac{\nu}{2}} K_2(2\sqrt{AB}). \qquad (35)$$

Equipped with Eqs. (32) and (33), we finally obtain the effective potential at the one-loop level, as given by Eqs. (A73) and (A74) in the Appendix. Let us note that the finite-temperature part (33) becomes the same as the one obtained by Dolan and Jackiw [1] for $\varphi = 0$ and $m(v)/T \ll 1$ by expanding the modified Bessel function in polynomial [10].

We are very much interested in the new phase, in which the new order parameter φ takes the nontrivial value except for the center of gauge group. The new order parameters φ and ω_r appear only in the finite temperature parts of the contributions from the $W^{\pm}, G^{\pm}, t, b, \tau, G_{\mu}$, which is given from Eq. (A74) by

$$\begin{aligned} V^{T\neq 0}_{\varphi,\omega_r-{\rm dep.}} = &-4\frac{2}{(2\pi)^2} T^4 \sum_{m=1}^{\infty} \frac{1}{m^4} \cos[2\pi m(2\varphi)] \left(\frac{m_W(v)^2}{T^2} m^2\right) K_2\left(\frac{m_W(v)}{T} m\right) \\ &- 2\frac{2}{(2\pi)^2} T^4 \sum_{m=1}^{\infty} \frac{1}{m^4} \cos[2\pi m(2\varphi)] \left(\frac{m_{G^{\pm}}(v)^2}{T^2} m^2\right) K_2\left(\frac{m_{G^{\pm}}(v)}{T} m\right) \\ &+ 4\frac{2}{(2\pi)^2} T^4 \sum_{r=1}^{3}\sum_{m=1}^{\infty} \frac{(-1)^m}{m^4} \cos\left[2\pi m\left(\omega_r + \frac{4}{3}\varphi\right)\right] \left(\frac{m_t(v)^2}{T^2} m^2\right) K_2\left(\frac{m_t(v)}{T} m\right) \\ &+ 4\frac{2}{(2\pi)^2} T^4 \sum_{r=1}^{3}\sum_{m=1}^{\infty} \frac{(-1)^m}{m^4} \cos\left[2\pi m\left(\omega_r - \frac{2}{3}\varphi\right)\right] \left(\frac{m_b(v)^2}{T^2} m^2\right) K_2\left(\frac{m_b(v)}{T} m\right) \\ &+ 4\frac{2}{(2\pi)^2} T^4 \sum_{m=1}^{\infty} \frac{(-1)^m}{m^4} \cos[2\pi m(2\varphi)] \left(\frac{m_\tau(v)^2}{T^2} m^2\right) K_2\left(\frac{m_\tau(v)}{T} m\right) \\ &- 2\frac{2}{(2\pi)^2} T^4 \sum_{r,q=1}^{3}\sum_{m=1}^{\infty} \frac{2}{m^4} \cos[2\pi m(\omega_r - \omega_q)]. \end{aligned} \qquad (36)$$

The notation $m_i(v) (i = W, G^{\pm}, t, b, \tau)$ is defined by Eq. (A75) in the Appendix.

Let us minimize $V^{T\neq 0}_{\varphi,\omega_r-{\rm dep.}}$ with respect to φ, ω_r. It has been well known that the $SU(3)_c$ gauge sector, the last line in Eq. (36), is minimized at

$$\omega_r = \frac{k}{3}\ (k = 0, 1, 2)\ ({\rm mod} 1). \qquad (37)$$

One sees from the Polyakov loop defined by

$$W_p^{SU(3)_c} = \mathcal{P}\exp\left(i g_s \int_0^{\frac{1}{T}} d\tau \langle G_\tau\rangle\right)\Bigg|_{\omega_r = \frac{k}{3}} = e^{i2\pi\frac{k}{3}} \mathbf{1}_{3\times 3} \qquad (38)$$

that it is the center of the $SU(3)_c$ gauge group.

The typical structure of the potential (36) is given by the following two types of the functions:

$$\begin{aligned} f(x,\tilde{z}) &= -\sum_{m=1}^{\infty} \frac{1}{m^4} \cos[2\pi m x](\tilde{z}m)^2 K_2(\tilde{z}m), \\ g(x,\tilde{z}) &= +\sum_{m=1}^{\infty} \frac{(-1)^m}{m^4} \cos[2\pi m x](\tilde{z}m)^2 K_2(\tilde{z}m). \end{aligned} \qquad (39)$$

We numerically depict $f(x,\tilde{z})$ and $g(x,\tilde{z})$ for positive values of $\tilde{z}$ in Figs. 1 and 2. We find that both of the functions are minimized at $x = 0$ for $\tilde{z} > 0$. This result is understood by noting that the $m = 1$ mode dominates the functions $f(x,\tilde{z})$ and $g(x,\tilde{z})$ to yield $x = 0$ as the minimum configuration. It is also confirmed numerically that the $m = 1$ mode dominates the two functions. Then the configurations that minimize $V^{T\neq 0}_{\varphi,\omega_r-{\rm dep.}}$ are given by

$$\omega_r + \frac{4}{3}\varphi = 0\ (r = 1,2,3), \qquad \omega_r - \frac{2}{3}\varphi = 0\ (r = 1,2,3), \qquad 2\varphi = 0\ \ ({\rm mod} 1) \qquad (40)$$

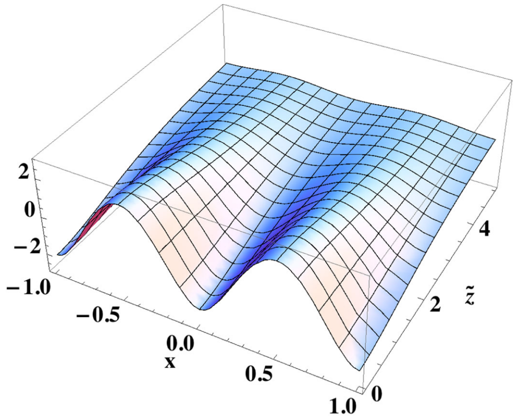

FIG. 1 (color online). The behavior of $f(x, \tilde{z})$.

and (37). The first and the second equations in Eq. (40) are a result of the top and bottom contributions, and the third one is resulted by $W^{\pm}, G^{\pm}$, and tau contributions. We immediately find that $V^{T\neq 0}_{\varphi,\omega_r-\mathrm{dep.}}$ is minimized at $\varphi = \tilde{k}/2(\tilde{k}=0,1)$ and $\omega_r = k/3(k=0,1,2)$ (mod 1) for nonzero temperature. This also implies $\theta = 0$ (mod 1) at the vacuum from the second equation in the ansatz (15). We conclude that the center of $SU(3)_c$ and $SU(2)_L$ is the vacuum configuration at finite temperature.

There is no new phase in which the new order parameters take nontrivial values other than the center of the gauge group. It is crucial that the boundary condition of fermions for the Euclidean time direction must be antiperiodic due to the Fermi statistics. This is essential for φ to take the center of $SU(2)_L$ at the minimum of the effective potential. This is a remarkable difference when we consider the boundary conditions of the fields for extra dimensions, which is *a priori* unknown.

Let us mention the studies of [8], where the very high-temperature behavior of the standard model has been studied by the effective potential for the zero modes of A_τ, B_τ, and G_τ with the assumption of $p = v = 0$. We can reproduce the results of [8] by taking $p = v = 0$ in the quadratic terms (A25), (A32), (A37), (A46), and (A47).

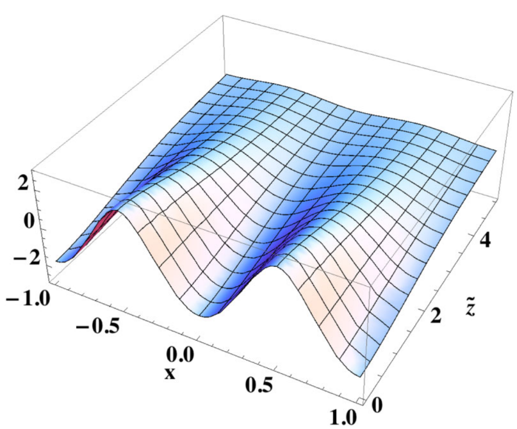

FIG. 2 (color online). The behavior of $g(x, \tilde{z})$.

IV. CONCLUSIONS AND DISCUSSIONS

We have taken account of the new order parameters arising from the zero mode of the Euclidean time components of the gauge fields for studying the effective potential of the standard model at finite temperature in four dimensions. Because of the increased number of the order parameters, there remains the component that can break the electromagnetic $U(1)$, denoted by $U(1)_{\mathrm{em}}$, invariance in the parametrization of the vacuum expectation values of the Higgs field.

The existence of such a parameter complicates the quadratic terms of the fluctuating fields by which one obtains the effective potential at the one-loop level. We have imposed the ansatz, which preserves the $U(1)_{\mathrm{em}}$ invariance, in order to study the effective potential as analytically as possible. Then we have obtained the analytic expression for the effective potential and study the vacuum structure, namely, the possibility of whether the new order parameters take nontrivial values except for the center of the gauge group or not.

We find that the new order parameters do not take the nontrivial values. It is important that the fermion obey the antiperiodic boundary condition for the Euclidean time direction due to the Fermi statistics. Thanks to this fact, the new order parameter φ always takes zero at the vacuum for finite temperature. It has been pointed out in [6] that the nontrivial phase exists if there is a cross term between v and φ in the tree-level potential. One may think that the absence of the cross term in Eq. (8) due to the ansatz is the reason that no new phases exist. Such a cross term, however, is not important for the existence of the new phase. The boundary condition for the compactified direction is essential. In the case of [6], the boundary condition can be taken to be periodic even for fermions because of the spatial compactified direction, which contrary to the Euclidean time direction is free from the Fermi statistics. This makes it possible that the new order parameter φ can take a nontrivial value except for the center of the gauge group.

Some comments are in order. Originally there are four order parameters, v, p, φ, θ. Even though it may be unlikely for p, φ, θ to have nontrivial values at the vacuum, one should calculate the effective potential at the one-loop level by using the matrices in Eqs. (A25), (A32), (A37), (A46), and (A47) without imposing any conditions among the order parameters. To this end, one needs to develop the technique to sum up all of the Matsubara modes n that have the complicated dependence in the eigenvalues of the matrices. Concerning this point, we mention the limit $g, g_Y \to 0$. Even in the limit, the dependence of the quadratic terms on φ, θ is survived in the simplified form. Nevertheless, it is still difficult to perform the analytic calculation for obtaining the effective potential at the one-loop level.

It may be true that the order parameters arising from the Euclidean components of the gauge fields do not develop nontrivial values except for the center of the gauge group. If one considers two Higgs doublets models, including the minimal supersymmetric standard model, for example, the number of order parameters in the models is larger than the usual case because of the additional zero modes of the Euclidean components of the gauge fields. Then there may be a possibility of a new source for CP violation at finite temperature in dependence on the structure of the Higgs potential.[1]

We stress that as long as the zero mode of the Euclidean time component of the gauge field becomes the dynamical variable at finite temperature field theory, it is important and natural to take into account the effective potential at finite temperature in addition to the usual order parameter such as the Higgs field. We are now studying the electroweak models such as the two Higgs doublet models, including the minimal supersymmetric standard model, at finite temperature by taking account all of the order parameters [12]. We are interested in whether the order parameters take the nontrivial values or not. This will be reported elsewhere in the future.

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APPENDIX

In this Appendix we present the details of notations and calculations, some of which are used in the text.

1. Quadratic terms

In the imaginary time formulation of finite temperature field theory, the Euclidean time τ is defined by the Wick rotation

$$\tau = it. \tag{A1}$$

Here we use $\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ as the Minkowski metric. Accordingly, the Euclidean time component of the gauge field is related with the Minkowski component of the gauge field by

$$A_\tau = -iA_0, \qquad B_\tau = -iB_0, \qquad G_\tau = -iG_0. \tag{A2}$$

[1] A spontaneous CP violation at finite temperature has been reported in [11].

The Euclidean time direction is compactified on S^1_τ whose circumference is the inverse temperature T^{-1}. Bosons (Fermions) must obey the (anti) periodic boundary conditions because of quantum statistics. Then the Euclidean component of the momentum $k_\tau = -ik_0$ is discretized as

$$k_\tau = \begin{cases} \omega_n^B = 2\pi T n & \text{for bosons,} \\ \omega_n^F = 2\pi T(n+\frac{1}{2}) & \text{for fermions,} \end{cases} \tag{A3}$$

where n denotes the Matsubara mode, $n = 0, \pm 1, \pm 2, \ldots$.

In order to obtain the effective potential at the one-loop level, we expand the fields around the vacuum expectation values as defined by Eq. (2) in the text and take up to the quadratic terms with respect to the fluctuations.

Let us start with the gauge sector of the standard model,

$$\mathcal{L}^{\mathrm{EW}}_{\mathrm{gauge}} = -\frac{1}{2}\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{A4}$$

where $F_{\mu\nu}, B_{\mu\nu}$ are the $SU(2)_L, U(1)_Y$ field strengths, respectively. We consider the $SU(3)_c$ gauge sector later. The quadratic terms from the gauge kinetic terms are obtained by

$$\begin{aligned}\mathcal{L}^{\mathrm{EW}(2)}_{\mathrm{gauge}} = &-\frac{1}{2}\left(\partial_i\bar{A}^a_j\partial_i\bar{A}^a_j - \partial_i\bar{A}^a_j\partial_j\bar{A}^a_i\right) \\ &-\frac{1}{2}(D^{SU(2)}_\tau\bar{A}_i)^a(D^{SU(2)}_\tau\bar{A}_i)^a - \frac{1}{2}\partial_i\bar{A}^a_\tau\partial_i\bar{A}^a_\tau \\ &-\frac{1}{2}\left(\partial_i\bar{B}_j\partial_i\bar{B}_j - \partial_j\bar{B}_i\partial_i\bar{B}_j\right) - \frac{1}{2}\partial_i\bar{B}_\tau\partial_i\bar{B}_\tau \\ &-\frac{1}{2}\partial_\tau\bar{B}_i\partial_\tau\bar{B}_i + (D^{SU(2)}_\tau\bar{A}_i)^a\partial_i\bar{A}^a_\tau + \partial_i\bar{B}_\tau\partial_\tau\bar{B}_i,\end{aligned} \tag{A5}$$

where we have defined

$$D^{SU(2)}_\tau\bar{A}_i \equiv \partial_\tau\bar{A}_i - ig[\langle A_\tau\rangle, \bar{A}_i] \quad (i = 1,2,3). \tag{A6}$$

$a(=1,2,3)$ is the $SU(2)_L$ index, and the i, j, l stand for the space component. The bar on the field denotes the fluctuation. The Higgs kinetic term and potential are

$$\mathcal{L}_{\mathrm{Higgs}} = (D_\mu\Phi)^\dagger D^\mu\Phi - V_{\mathrm{H}}, \tag{A7}$$

where

$$\begin{aligned} D_\mu\Phi &= \partial_\mu\Phi - igA_\mu\Phi - i\frac{g_Y}{2}B_\mu\Phi, \\ V_{\mathrm{H}} &= -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2. \end{aligned} \tag{A8}$$

The quadratic terms of the Higgs sector are given by

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm Higgs} = &-(\bar{D}_\tau\bar{\Phi})^\dagger\bar{D}_\tau\bar{\Phi} - (\partial_i\bar{\Phi})^\dagger\partial_i\bar{\Phi}\\
&-(\bar{D}_\tau\bar{\Phi})^\dagger\left(-ig\bar{A}_\tau\langle\Phi\rangle - i\frac{g_Y}{2}\bar{B}_\tau\langle\Phi\rangle\right) - \left(ig\langle\Phi\rangle^\dagger\bar{A}_\tau + i\frac{g_Y}{2}\langle\Phi\rangle^\dagger\bar{B}_\tau\right)\bar{D}_\tau\bar{\Phi}\\
&-\partial_i\bar{\Phi}^\dagger\left(-ig\bar{A}_i\langle\Phi\rangle - i\frac{g_Y}{2}\bar{B}_i\langle\Phi\rangle\right) - \left(ig\langle\Phi\rangle^\dagger\bar{A}_i + i\frac{g_Y}{2}\langle\Phi\rangle^\dagger\bar{B}_i\right)\partial_i\bar{\Phi}\\
&-g^2\left(\langle\Phi\rangle^\dagger\langle A_\tau\rangle\bar{A}_\tau\bar{\Phi} + \bar{\Phi}^\dagger\bar{A}_\tau\langle A_\tau\rangle\langle\Phi\rangle + \langle\Phi\rangle^\dagger\bar{A}_\tau\bar{A}_\tau\langle\Phi\rangle\right)\\
&-\frac{gg_Y}{2}\left(\langle\Phi\rangle^\dagger\langle A_\tau\rangle\bar{B}_\tau\bar{\Phi} + \bar{\Phi}^\dagger\bar{B}_\tau\langle A_\tau\rangle\langle\Phi\rangle + \langle\Phi\rangle^\dagger\bar{A}_\tau\bar{B}_\tau\langle\Phi\rangle\right)\\
&-\frac{gg_Y}{2}\left(\langle\Phi\rangle^\dagger\langle B_\tau\rangle\bar{A}_\tau\bar{\Phi} + \bar{\Phi}^\dagger\bar{A}_\tau\langle B_\tau\rangle\langle\Phi\rangle + \langle\Phi\rangle^\dagger\bar{B}_\tau\bar{A}_\tau\langle\Phi\rangle\right)\\
&-\left(\frac{g_Y}{2}\right)^2\left(\langle\Phi\rangle^\dagger\langle B_\tau\rangle\bar{B}_\tau\bar{\Phi} + \bar{\Phi}^\dagger\bar{B}_\tau\langle B_\tau\rangle\langle\Phi\rangle + \langle\Phi\rangle^\dagger\bar{B}_\tau\bar{B}_\tau\langle\Phi\rangle\right)\\
&-g^2\langle\Phi\rangle^\dagger\bar{A}_i\bar{A}_i\langle\Phi\rangle - gg_Y\langle\Phi\rangle^\dagger\bar{B}_i\bar{A}_i\langle\Phi\rangle - \left(\frac{g_Y}{2}\right)^2\langle\Phi\rangle^\dagger\bar{B}_i\bar{B}_i\langle\Phi\rangle\\
&-\mu^2\bar{\Phi}^\dagger\bar{\Phi} + \frac{\lambda}{2}(2|\langle\Phi\rangle|^2|\bar{\Phi}|^2 + 2(\langle\Phi\rangle^\dagger\bar{\Phi})(\bar{\Phi}^\dagger\langle\Phi\rangle) + (\langle\Phi\rangle^\dagger\bar{\Phi})^2 + (\bar{\Phi}^\dagger\langle\Phi\rangle)^2).\end{aligned} \tag{A9}$$

Here we have defined

$$\bar{D}_\tau\bar{\Phi} \equiv \partial_\tau\bar{\Phi} - ig\langle A_\tau\rangle\bar{\Phi} - i\frac{g_Y}{2}\langle B_\tau\rangle\bar{\Phi}. \tag{A10}$$

Now let us introduce the gauge fixing and the ghosts,

$$\mathcal{L}_{\rm gf+FP} = \mathcal{L}^{SU(2)_L}_{\rm gf+FP} + \mathcal{L}^{U(1)_Y}_{\rm gf+FP} = (-i)\delta_B(\bar{C}^aF^a) + (-i)\delta_B(\bar{C}F), \tag{A11}$$

where δ_B denotes the BRS transformations [13]. Let us note that $\bar{C}^a$ and $\bar{C}$ are antighost fields. The gauge fixing functions are chosen to be

$$F^a \equiv -\partial_i\bar{A}^a_i - \alpha_1\left[(D^{SU(2)}_\tau\bar{A}_\tau)^a - ig\left(\bar{\Phi}^\dagger\frac{\tau^a}{2}\langle\Phi\rangle - \langle\Phi\rangle^\dagger\frac{\tau^a}{2}\bar{\Phi}\right)\right] + \frac{\alpha_1}{2}b^a, \tag{A12}$$

$$F \equiv -\partial_i\bar{B}_i - \alpha_2\left[\partial_\tau\bar{B}_\tau - i\frac{g_Y}{2}(\bar{\Phi}^\dagger\langle\Phi\rangle - \langle\Phi\rangle^\dagger\bar{\Phi})\right] + \frac{\alpha_2}{2}b, \tag{A13}$$

where α_1 and α_2 are the gauge parameters. Hereafter we take $\alpha_1 = \alpha_2 \equiv \xi$ for simplicity. After operating the BRS transformations and performing the integration of b^a and b fields, the quadratic terms from the $SU(2)_L$ part are given by

$$\begin{aligned}\mathcal{L}^{SU(2)_L\ (2)}_{\rm gf} = (-i)\delta_B(C^aF^a)|_{\rm quadratic} = &-\frac{1}{2\xi}\partial_i\bar{A}^a_i\partial_j\bar{A}^a_j - (D^{SU(2)}_\tau\bar{A}_\tau)^a\partial_i\bar{A}^a_i\\
&+ig\left[\partial_i\bar{A}^a_i\left(\bar{\Phi}^\dagger\frac{\tau^a}{2}\langle\Phi\rangle - \langle\Phi\rangle^\dagger\frac{\tau^a}{2}\bar{\Phi}\right)\right] - \frac{\xi}{2}(D^{SU(2)}_\tau\bar{A}_\tau)^a(D^{SU(2)}_\tau\bar{A}_\tau)^a\\
&+i\xi g\left[(D^{SU(2)}_\tau\bar{A}_\tau)^a\left(\bar{\Phi}^\dagger\frac{\tau^a}{2}\langle\Phi\rangle - \langle\Phi\rangle^\dagger\frac{\tau^a}{2}\bar{\Phi}\right)\right] + \frac{\xi g^2}{2}\left(\bar{\Phi}^\dagger\frac{\tau^a}{2}\langle\Phi\rangle - \langle\Phi\rangle^\dagger\frac{\tau^a}{2}\bar{\Phi}\right)^2\\
&-i\bar{C}^a\partial_i^2C^a - i\xi\bar{C}^a(D^{SU(2)}_\tau D^{SU(2)}_\tau C)^a\\
&+ig\xi\bar{C}^a\left(\frac{g}{2}\langle\Phi\rangle^\dagger\langle\Phi\rangle C^a + \frac{g_Y}{2}C\langle\Phi\rangle^\dagger\tau^a\langle\Phi\rangle\right).\end{aligned} \tag{A14}$$

Likewise we obtain the quadratic terms of the $U(1)_Y$ part,

$$\mathcal{L}_{\rm gf}^{U(1)_Y\ (2)} = (-i)\delta_B(CF)|_{\rm quadratic} = -\frac{1}{2\xi}\partial_i\bar{B}_i\partial_j\bar{B}_j - \partial_i\bar{B}_i\partial_\tau\bar{B}_\tau + \frac{ig_Y}{2}\left[\partial_i\bar{B}_i\left(\bar{\Phi}^\dagger\langle\Phi\rangle - \langle\Phi\rangle^\dagger\bar{\Phi}\right)\right] - \frac{\xi}{2}\partial_\tau\bar{B}_\tau\partial_\tau\bar{B}_\tau + i\frac{\xi g_Y}{2}\left[\partial_\tau\bar{B}_\tau\left(\bar{\Phi}^\dagger\langle\Phi\rangle - \langle\Phi\rangle^\dagger\bar{\Phi}\right)\right] + \frac{\xi g_Y^2}{8}\left(\bar{\Phi}^\dagger\langle\Phi\rangle - \langle\Phi\rangle^\dagger\bar{\Phi}\right)^2 - i\bar{C}\partial_i^2 C - i\xi\bar{C}\partial_\tau^2 C + i\xi g_Y C\langle\Phi\rangle^\dagger\left(g\frac{\tau^a}{2}C^a + \frac{g_Y}{2}C\right)\langle\Phi\rangle. \tag{A15}$$

The first term in the third line of Eq. (A5) [the second term in the third line of Eq. (A5)] is canceled by the second term in the second line of Eq. (A14) [the second term in the second line of Eq. (A15)] after the partial integration. The third line of Eq. (A9) is canceled by the first term in the third line of Eq. (A14) and the third term in the second line of Eq. (A15) after the partial integration.

By noting that

$$(\bar{D}_\tau\bar{\Phi})^\dagger\bar{A}_\tau\langle\Phi\rangle = -\bar{\Phi}^\dagger(D_\tau^{SU(2)}\bar{A}_\tau)\langle\Phi\rangle + ig\bar{\Phi}^\dagger\bar{A}_\tau\langle A_\tau\rangle\langle\Phi\rangle + i\frac{g_Y}{2}\bar{\Phi}^\dagger\langle B_\tau\rangle\bar{A}_\tau\langle\Phi\rangle, \tag{A16}$$

$$(\bar{D}_\tau\bar{\Phi})^\dagger\bar{B}_\tau\langle\Phi\rangle = -\bar{\Phi}^\dagger\partial_\tau\bar{B}_\tau\langle\Phi\rangle + ig\bar{\Phi}^\dagger\langle A_\tau\rangle\bar{B}_\tau\langle\Phi\rangle + i\frac{g_Y}{2}\bar{\Phi}^\dagger\langle B_\tau\rangle\bar{B}_\tau\langle\Phi\rangle, \tag{A17}$$

where the partial integration has been performed, the second line of Eq. (A9) is recast as

$$\begin{aligned} &-(\bar{D}_\tau\bar{\Phi})^\dagger\left(-ig\bar{A}_\tau\langle\Phi\rangle - i\frac{g_Y}{2}\bar{B}_\tau\langle\Phi\rangle\right) - \left(ig\langle\Phi\rangle^\dagger\bar{A}_\tau + i\frac{g_Y}{2}\langle\Phi\rangle^\dagger\bar{B}_\tau\right)\bar{D}_\tau\bar{\Phi} \\ &= ig\Big[-\bar{\Phi}^\dagger(D_\tau^{SU(2)}\bar{A}_\tau)\langle\Phi\rangle + ig\bar{\Phi}^\dagger\bar{A}_\tau\langle A_\tau\rangle\langle\Phi\rangle + i\frac{g_Y}{2}\bar{\Phi}^\dagger\langle B_\tau\rangle\bar{A}_\tau\langle\Phi\rangle \\ &\quad + \langle\Phi\rangle^\dagger(D_\tau^{SU(2)}\bar{A}_\tau)\bar{\Phi} + ig\langle\Phi\rangle^\dagger\langle A_\tau\rangle\bar{A}_\tau\bar{\Phi} + i\frac{g_Y}{2}\langle\Phi\rangle^\dagger\bar{A}_\tau\langle B_\tau\rangle\bar{\Phi}\Big] \\ &\quad + i\frac{g_Y}{2}\Big[-\bar{\Phi}^\dagger\partial_\tau\bar{B}_\tau\langle\Phi\rangle + ig\bar{\Phi}^\dagger\langle A_\tau\rangle\bar{B}_\tau\langle\Phi\rangle + i\frac{g_Y}{2}\bar{\Phi}^\dagger\langle B_\tau\rangle\bar{B}_\tau\langle\Phi\rangle \\ &\quad + \langle\Phi\rangle^\dagger\partial_\tau\bar{B}_\tau\bar{\Phi} + ig\langle\Phi\rangle^\dagger\bar{B}_\tau\langle A_\tau\rangle\bar{\Phi} + i\frac{g_Y}{2}\langle\Phi\rangle^\dagger\bar{B}_\tau\langle B_\tau\rangle\bar{\Phi}\Big]. \end{aligned} \tag{A18}$$

The first terms in the second and the third line of Eq. (A18), together with the first term in the fourth line of Eq. (A14), are summarized into the compact form given by the first term in Eq. (A19) below. Likewise, the first terms in the fourth and the fifth lines of Eq. (A18), together with the first term in the third line of Eq. (A15), are summarized into the compact form given by the second term in Eq. (A19) below. The compact form is given by

$$ig(\xi-1)\left(\bar{\Phi}^\dagger(D_\tau^{SU(2)}\bar{A}_\tau)\langle\Phi\rangle - \langle\Phi\rangle^\dagger(D_\tau^{SU(2)}\bar{A}_\tau)\bar{\Phi}\right) + \frac{ig_Y}{2}(\xi-1)\left(\partial_\tau\bar{B}_\tau\bar{\Phi}^\dagger\langle\Phi\rangle - \langle\Phi\rangle^\dagger\bar{\Phi}\partial_\tau\bar{B}_\tau\right), \tag{A19}$$

which vanishes for the Feynman gauge $\xi = 1$.

The quadratic part of the Lagrangian for the gauge fields $\bar{A}_i^a$ and $\bar{B}_i$ is given by

$$\begin{aligned} \mathcal{L}_{\rm gauge}^{(2)} &= \frac{1}{2}\bar{A}_i^a\left(\delta_{ij}\partial_l^2 - \left(1-\frac{1}{\xi}\right)\partial_i\partial_j\right)\bar{A}_j^a - \frac{1}{2}(D_\tau^{SU(2)}\bar{A}_i)^a(D_\tau^{SU(2)}\bar{A}_i)^a \\ &\quad + \frac{1}{2}\bar{B}_i\left(\delta_{ij}\partial_l^2 - \left(1-\frac{1}{\xi}\right)\partial_i\partial_j\right)\bar{B}_j - \frac{1}{2}\partial_\tau\bar{B}_i\partial_\tau\bar{B}_i \\ &\quad - g^2\langle\Phi\rangle^\dagger\bar{A}_i\bar{A}_i\langle\Phi\rangle - gg_Y\langle\Phi\rangle^\dagger\bar{A}_i\bar{B}_i\langle\Phi\rangle - \left(\frac{g_Y}{2}\right)^2\langle\Phi\rangle^\dagger\bar{B}_i\bar{B}_i\langle\Phi\rangle. \end{aligned} \tag{A20}$$

The quadratic part for the ghost fields C^a, $\bar{C}^a$, C, and $\bar{C}$ is given by

$$\mathcal{L}^{(2)}_{\rm ghost} = -i\bar{C}^a\partial_i^2 C^a - i\xi\bar{C}^a(D_\tau^{SU(2)}D_\tau^{SU(2)}C)^a - i\bar{C}\partial_i^2 C - i\xi\bar{C}\partial_\tau^2 C + i\xi g\bar{C}^a\left(\frac{g}{2}|\langle\Phi\rangle|^2 C^a + \frac{g_Y}{2}C\langle\Phi\rangle^\dagger\tau^a\langle\Phi\rangle\right) + i\xi g_Y\bar{C}\langle\Phi\rangle^\dagger\left(\frac{g}{2}\tau^a C^a + \frac{g_Y}{2}C\right)\langle\Phi\rangle. \quad \text{(A21)}$$

Let us comment on the gauge fixing. We have chosen the gauge fixing in order to cancel all of the unwanted mixing terms with derivatives between the gauge and Higgs fields. We found that this can be done by the gauge choices of (A12) and (A13) with $\alpha_1 = \alpha_2 = \xi = 1$, as seen above. One might think that our results are gauge dependent. This is not, however, the case. The gauge invariance of the effective potential at minima (also maxima) has been proved in [14]. Furthermore, the vacuum expectation values of gauge fields are also gauge invariant because they correspond to the Wilson (Polyakov) line phases for the Euclidean time direction, which are manifestly gauge-invariant quantities.

The quadratic part for the scalar fields $\bar{A}^a_\tau, \bar{B}_\tau$, and $\bar{\Phi}$ is given by

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm scalar} =& -\frac{1}{2}\partial_i\bar{A}^a_\tau\partial_i\bar{A}^a_\tau - \frac{\xi}{2}(D_\tau^{SU(2)}\bar{A}_\tau)^a(D_\tau^{SU(2)}\bar{A}_\tau)^a \\ & -\frac{1}{2}\partial_i\bar{B}_\tau\partial_i\bar{B}_\tau - \frac{\xi}{2}\partial_\tau\bar{B}_\tau\partial_\tau\bar{B}_\tau - \partial_i\bar{\Phi}^\dagger\partial_i\bar{\Phi} - (\bar{D}_\tau\bar{\Phi})^\dagger\bar{D}_\tau\bar{\Phi} \\ & - g^2(2\langle\Phi\rangle^\dagger\langle A_\tau\rangle\bar{A}_\tau\bar{\Phi} + 2\bar{\Phi}^\dagger\bar{A}_\tau\langle A_\tau\rangle\langle\Phi\rangle + \langle\Phi\rangle^\dagger\bar{A}_\tau\bar{A}_\tau\langle\Phi\rangle) \\ & -\frac{gg_Y}{2}(2\bar{\Phi}^\dagger\langle B_\tau\rangle\bar{A}_\tau\langle\Phi\rangle + 2\langle\Phi\rangle^\dagger\bar{A}_\tau\langle B_\tau\rangle\bar{\Phi} + 2\bar{\Phi}^\dagger\langle A_\tau\rangle\bar{B}_\tau\langle\Phi\rangle \\ & + 2\langle\Phi\rangle^\dagger\bar{B}_\tau\langle A_\tau\rangle\bar{\Phi} + 2\langle\Phi\rangle^\dagger\bar{A}_\tau\bar{B}_\tau\langle\Phi\rangle) \\ & -\left(\frac{g_Y}{2}\right)^2(2\bar{\Phi}^\dagger\langle B_\tau\rangle\bar{B}_\tau\langle\Phi\rangle + 2\langle\Phi\rangle^\dagger\bar{B}_\tau\langle B_\tau\rangle\bar{\Phi} + \langle\Phi\rangle^\dagger\bar{B}_\tau\bar{B}_\tau\langle\Phi\rangle) \\ & + \mu^2\bar{\Phi}^\dagger\bar{\Phi} - \frac{\lambda}{2}(2|\langle\Phi\rangle|^2|\bar{\Phi}|^2 + 2(\langle\Phi\rangle^\dagger\bar{\Phi})(\bar{\Phi}^\dagger\langle\Phi\rangle) + (\langle\Phi\rangle^\dagger\bar{\Phi})^2 + (\bar{\Phi}^\dagger\langle\Phi\rangle)^2) \\ & + \xi\frac{g^2}{2}\left(\frac{1}{4}(\bar{\Phi}^\dagger\langle\Phi\rangle)^2 + \frac{1}{4}(\langle\Phi\rangle^\dagger\bar{\Phi})^2 + \frac{1}{2}(\bar{\Phi}^\dagger\langle\Phi\rangle)(\langle\Phi\rangle^\dagger\bar{\Phi}) - |\langle\Phi\rangle|^2|\bar{\Phi}|^2\right) \\ & + \xi\frac{g_Y^2}{8}((\bar{\Phi}^\dagger\langle\Phi\rangle)^2 + (\langle\Phi\rangle^\dagger\bar{\Phi})^2 - 2(\bar{\Phi}^\dagger\langle\Phi\rangle)(\langle\Phi\rangle^\dagger\bar{\Phi})) \\ & + ig(\xi-1)(\bar{\Phi}^\dagger(D_\tau^{SU(2)}\bar{A}_\tau)\langle\Phi\rangle - \langle\Phi\rangle^\dagger(D_\tau^{SU(2)}\bar{A}_\tau)\bar{\Phi}) \\ & + \frac{ig_Y}{2}(\xi-1)(\partial_\tau\bar{B}_\tau\bar{\Phi}^\dagger\langle\Phi\rangle - \langle\Phi\rangle^\dagger\bar{\Phi}\partial_\tau\bar{B}_\tau). \end{aligned} \quad \text{(A22)}$$

The last two lines in Eq. (A22) vanish when we take the Feynman gauge $\xi = 1$.

Let us put the parametrization (2) into Eqs. (A20)–(A22). We obtain for the gauge sector $\mathcal{L}^{(2)}_{\rm gauge}$ that

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm gauge} =& W_i^-\left[\delta_{ij}\partial_l^2 - \left(1-\frac{1}{\xi}\right)\partial_i\partial_j + \left((\partial_\tau - i(2\pi T)(2\varphi))^2 - \frac{g^2}{4}(v^2+p^2)\right)\delta_{ij}\right]W_j^+ \\ & + \frac{1}{2}\bar{A}^3_i\left[\delta_{ij}\partial_l^2 - \left(1-\frac{1}{\xi}\right)\partial_i\partial_j + \left(\partial_\tau^2 - \frac{g^2}{4}(v^2+p^2)\right)\delta_{ij}\right]\bar{A}^3_j \\ & + \frac{1}{2}\bar{B}_i\left[\delta_{ij}\partial_l^2 - \left(1-\frac{1}{\xi}\right)\partial_i\partial_j + \left(\partial_\tau^2 - \frac{g_Y^2}{4}(v^2+p^2)\right)\delta_{ij}\right]\bar{B}_j \\ & - \frac{gg_Y}{4}(p^2 - v^2)\bar{A}^3_i\bar{B}_i - \frac{\sqrt{2}}{4}gg_Y pv(W_i^+ + W_i^-)\bar{B}_i \end{aligned} \quad \text{(A23)}$$

$$= (W_i^-, \bar{A}^3_i, \bar{B}_i)M^2_{\rm gauge}\begin{pmatrix} W_j^+ \\ \bar{A}^3_j \\ \bar{B}_j \end{pmatrix}, \quad \text{(A24)}$$

where

$$M^2_{\rm gauge} \equiv \begin{pmatrix} D^{W^\pm}_{ij} & 0 & -\frac{\sqrt{2}}{4} g g_Y p v \delta_{ij} \\ 0 & \frac{1}{2} D^{A^3}_{ij} & -\frac{1}{2}\frac{1}{4} g g_Y (p^2 - v^2)\delta_{ij} \\ -\frac{\sqrt{2}}{4} g g_Y p v \delta_{ij} & -\frac{1}{2}\frac{1}{4} g g_Y (p^2 - v^2)\delta_{ij} & \frac{1}{2} D^B_{ij} \end{pmatrix}. \tag{A25}$$

Here we have defined

$$W^\pm_\mu \equiv \frac{1}{\sqrt{2}}(\bar{A}^1_\mu \mp i \bar{A}^2_\mu) \quad (\mu = \tau, 1, 2, 3) \tag{A26}$$

and

$$\begin{aligned} D^{W^\pm}_{ij} &= \delta_{ij}\partial_l^2 - \left(1 - \frac{1}{\xi}\right)\partial_i\partial_j + \left((\partial_\tau - i(2\pi T)(2\varphi))^2 - \frac{g^2}{4}(v^2 + p^2)\right)\delta_{ij}, \\ D^{A^3}_{ij} &= \delta_{ij}\partial_l^2 - \left(1 - \frac{1}{\xi}\right)\partial_i\partial_j + \left(\partial_\tau^2 - \frac{g^2}{4}(v^2 + p^2)\right)\delta_{ij}, \\ D^{B}_{ij} &= \delta_{ij}\partial_l^2 - \left(1 - \frac{1}{\xi}\right)\partial_i\partial_j + \left(\partial_\tau^2 - \frac{g_Y^2}{4}(v^2 + p^2)\right)\delta_{ij}. \end{aligned} \tag{A27}$$

One observes that there is a coupling between $\bar{W}_i$ and $\bar{B}_j$ that breaks the $U(1)_{\rm em}$ invariance due to the vacuum expectation value p. This also happens in the scalar, ghost, and fermion sectors discussed below. The scalar sector is given by

$$\begin{aligned} \mathcal{L}^{(2)}_{\rm scalar} = & W^-_\tau\left[\partial_i^2 + \xi(\partial_\tau - i(2\pi T)(2\varphi))^2 - \frac{g^2}{4}(p^2 + v^2)\right]W^+_\tau \\ & + \frac{1}{2}\bar{A}^3_\tau\left[\partial_i^2 + \xi\partial_\tau^2 - \frac{g^2}{4}(v^2 + p^2)\right]\bar{A}^3_\tau + \frac{1}{2}\bar{B}_\tau\left[\partial_i^2 + \xi\partial_\tau^2 - \frac{g_Y^2}{4}(v^2 + p^2)\right]\bar{B}_\tau \\ & + \frac{1}{2}h\left[\partial_i^2 + \left(\partial_\tau - i(2\pi T)\left(-\varphi + \frac{\theta}{2}\right)\right)^2 + \mu^2 - \lambda\left(\frac{3}{2}v^2 + \frac{p^2}{2}\right) - \frac{\xi}{4}g^2p^2\right]h \\ & + \frac{1}{2}G^0\left[\partial_i^2 + \left(\partial_\tau - i(2\pi T)\left(-\varphi + \frac{\theta}{2}\right)\right)^2 + \mu^2 - \lambda\left(\frac{v^2}{2} + \frac{p^2}{2}\right) - \xi\frac{g^2 + g_Y^2}{4}v^2 - \xi\frac{g^2}{4}p^2\right]G^0 \\ & + G^-\left[\partial_i^2 + \left(\partial_\tau - i(2\pi T)\left(\varphi + \frac{\theta}{2}\right)\right)^2 + \mu^2 - \lambda\left(\frac{v^2}{2} + p^2\right) - \xi\frac{g^2}{4}v^2 - \xi\frac{g^2 + g_Y^2}{8}p^2\right]G^+ \\ & - \frac{\lambda}{2}\left[\frac{p^2}{2}(G^{+2} + G^{-2}) + \sqrt{2}pv(G^+ + G^-)h\right] + \xi\frac{g^2 + g_Y^2}{16}p^2(G^{+2} + G^{-2}) \\ & - g(2\pi T)\left[\frac{p}{\sqrt{2}}\left(\varphi + \frac{\theta}{2}\right)\{(G^+ + G^-)\bar{A}^3_\tau + (W^+_\tau + W^-_\tau)h\right. \\ & \left. + i(W^+_\tau - W^-_\tau)G^0\} - v\left(\varphi - \frac{\theta}{2}\right)(W^-_\tau G^+ + W^+_\tau G^- - \bar{A}^3_\tau h)\right] \\ & - g_Y(2\pi T)\left[\frac{p}{\sqrt{2}}\left(\varphi + \frac{\theta}{2}\right)(G^+ + G^-)\bar{B}_\tau - v\left(\varphi - \frac{\theta}{2}\right)\bar{B}_\tau h\right] \\ & - g g_Y\left[\frac{1}{4}(p^2 - v^2)\bar{A}^3_\tau\bar{B}_\tau + \frac{\sqrt{2}}{4}pv(W^+_\tau + W^-_\tau)\bar{B}_\tau\right] + \xi\frac{\sqrt{2}}{8}g^2pv(G^+ + G^-)h + \xi\frac{\sqrt{2}}{8}g_Y^2pv(G^+ - G^-)iG^0 \\ & + ig(\xi - 1)\frac{1}{2\sqrt{2}}[p(G^- - G^+)\partial_\tau\bar{A}^3_\tau + \sqrt{2}v(G^-(\partial_\tau - i(2\pi T)(2\varphi))W^+_\tau - G^+(\partial_\tau - i(2\pi T)(-2\varphi))W^-_\tau) \\ & + p((\partial_\tau - i(2\pi T)(-2\varphi))W^-_\tau - (\partial_\tau - i(2\pi T)(2\varphi))W^+_\tau)h - ip((\partial_\tau - i(2\pi T)(-2\varphi))W^-_\tau \\ & + (\partial_\tau - i(2\pi T)(2\varphi))W^+_\tau)G^0 + i\sqrt{2}vG^0\partial_\tau\bar{A}^3_\tau] + ig_Y(\xi - 1)\frac{1}{2\sqrt{2}}[p(G^- - G^+)\partial_\tau\bar{B}_\tau - \sqrt{2}ivG^0\partial_\tau\bar{B}_\tau]. \end{aligned} \tag{A28}$$

Here we have defined

$$\bar{\Phi} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h + iG^0) \end{pmatrix}.$$

In terms of the real fields defined by Eq. (A26) and

$$G^{\pm} \equiv \frac{1}{\sqrt{2}}(g^1 \mp ig^2), \tag{A29}$$

Eq. (A28) becomes

$$\begin{aligned}
\mathcal{L}^{(2)}_{\text{scalar}} &= \frac{1}{2}\bar{A}^1_\tau\left[\partial_i^2 + \xi(\partial_\tau^2 - (2\pi T)^2(2\varphi)^2) - \frac{g^2}{4}(p^2+v^2)\right]\bar{A}^1_\tau \\
&+ \frac{1}{2}\bar{A}^2_\tau\left[\partial_i^2 + \xi(\partial_\tau^2 - (2\pi T)^2(2\varphi)^2) - \frac{g^2}{4}(p^2+v^2)\right]\bar{A}^2_\tau - 2\pi T(2\varphi)(\bar{A}^1_\tau\partial_\tau\bar{A}^2_\tau - \bar{A}^2_\tau\partial_\tau\bar{A}^1_\tau) \\
&+ \frac{1}{2}\bar{A}^3_\tau\left[\partial_i^2 + \xi\partial_\tau^2 - \frac{g^2}{4}(v^2+p^2)\right]\bar{A}^3_\tau + \frac{1}{2}\bar{B}_\tau\left[\partial_i^2 + \xi\partial_\tau^2 - \frac{g_Y^2}{4}(v^2+p^2)\right]\bar{B}_\tau \\
&+ \frac{1}{2}h\left[\partial_i^2 + \partial_\tau^2 - (2\pi T)^2\left(-\varphi + \frac{\theta}{2}\right)^2 + \mu^2 - \lambda\left(\frac{3}{2}v^2 + \frac{p^2}{2}\right) - \frac{\xi}{4}g^2p^2\right]h \\
&+ \frac{1}{2}G^0\left[\partial_i^2 + \partial_\tau^2 - (2\pi T)^2\left(-\varphi + \frac{\theta}{2}\right)^2 + \mu^2 - \lambda\left(\frac{v^2}{2} + \frac{p^2}{2}\right) - \xi\frac{g^2+g_Y^2}{4}v^2 - \xi\frac{g^2}{4}p^2\right]G^0 \\
&+ \frac{1}{2}g^1\left[\partial_i^2 + \partial_\tau^2 - (2\pi T)^2\left(\varphi + \frac{\theta}{2}\right)^2 + \mu^2 - \frac{\lambda}{2}v^2 - \frac{3}{2}\lambda p^2 - \xi\frac{g^2}{4}v^2\right]g^1 \\
&+ \frac{1}{2}g^2\left[\partial_i^2 + \partial_\tau^2 - (2\pi T)^2\left(\varphi + \frac{\theta}{2}\right)^2 + \mu^2 - \frac{\lambda}{2}v^2 - \frac{\lambda}{2}p^2 - \xi\frac{g^2}{4}v^2 - \xi\frac{g^2+g_Y^2}{4}p^2\right]g^2 \\
&- 2\pi T\left(\varphi + \frac{\theta}{2}\right)(g^1\partial_\tau g^2 - g^2\partial_\tau g^1) - 2\pi T\left(\varphi - \frac{\theta}{2}\right)(h\partial_\tau G^0 - G^0\partial_\tau h) \\
&- g(2\pi T)\left[p\left(\varphi + \frac{\theta}{2}\right)(g^1\bar{A}^3_\tau + \bar{A}^1_\tau h + \bar{A}^2_\tau G^0) - v\left(\varphi - \frac{\theta}{2}\right)(\bar{A}^1_\tau g^1 + \bar{A}^2_\tau g^2 - \bar{A}^3_\tau h)\right] \\
&- g_Y(2\pi T)\left[p\left(\varphi + \frac{\theta}{2}\right)g^1\bar{B}_\tau - v\left(\varphi - \frac{\theta}{2}\right)\bar{B}_\tau h\right] \\
&- gg_Y\left[\frac{1}{4}(p^2 - v^2)\bar{A}^3_\tau\bar{B}_\tau + \frac{1}{2}pv\bar{A}^1_\tau\bar{B}_\tau\right] + \xi\frac{g^2}{4}pvg^1h + \xi\frac{g_Y^2}{4}pvg^2G^0 - \lambda pvhg^1 \\
&- \frac{g}{2}(\xi - 1)[(pg^2 + vG^0)\partial_\tau\bar{A}^3_\tau - v(g^1\partial_\tau\bar{A}^2_\tau - g^2\partial_\tau\bar{A}^1_\tau) + p(h\partial_\tau\bar{A}^2_\tau - G^0\partial_\tau\bar{A}^1_\tau) \\
&- (2\pi T)(2\varphi)v(g^1\bar{A}^1_\tau + g^2\bar{A}^2_\tau) + (2\pi T)(2\varphi)p(h\bar{A}^1_\tau + G^0\bar{A}^2_\tau)] \\
&- \frac{g_Y}{2}(\xi - 1)(pg^2 - vG^0)\partial_\tau\bar{B}_\tau
\end{aligned} \tag{A30}$$

$$= \frac{1}{2}(\bar{A}^1_\tau, \bar{A}^2_\tau, g^1, g^2, \bar{A}^3_\tau, \bar{B}_\tau, h, G^0)M^2_{\text{scalar}}\begin{pmatrix} \bar{A}^1_\tau \\ \bar{A}^2_\tau \\ g^1 \\ g^2 \\ \bar{A}^3_\tau \\ \bar{B}_\tau \\ h \\ G^0 \end{pmatrix}, \tag{A31}$$

where M^2_{scalar} is defined by

$$M^2_{\text{scalar}} = \begin{pmatrix}
A & a & b & -\bar{c} & 0 & c & d & \bar{d} \\
-a & A & \bar{c} & b & 0 & 0 & -\bar{d} & d \\
b & -\bar{c} & B & g & h & i & j & 0 \\
\bar{c} & b & -g & C & \bar{d} & \bar{b} & 0 & k \\
0 & 0 & h & -\bar{d} & D & l & m & -\bar{c} \\
c & 0 & i & -\bar{b} & l & E & n & \bar{e} \\
d & \bar{d} & j & 0 & m & n & F & \tilde{g} \\
-\bar{d} & d & 0 & k & \bar{c} & -\bar{e} & -\tilde{g} & G
\end{pmatrix}. \tag{A32}$$

The components in M^2_{scalar} are given as

$$
\begin{aligned}
&A = \partial_i^2 + \xi(\partial_\tau^2 - (2\pi T)^2(2\varphi)^2) - \frac{g^2}{4}(p^2 + v^2),\\
&B = \partial_i^2 + \partial_\tau^2 - (2\pi T)^2\left(\varphi + \frac{\theta}{2}\right)^2 + \mu^2 - \frac{\lambda}{2}v^2 - \frac{3}{2}\lambda p^2 - \xi\frac{g^2}{4}v^2,\\
&C = \partial_i^2 + \partial_\tau^2 - (2\pi T)^2\left(\varphi + \frac{\theta}{2}\right)^2 + \mu^2 - \frac{\lambda}{2}v^2 - \frac{\lambda}{2}p^2 - \xi\frac{g^2}{4}v^2 - \frac{\xi}{4}(g^2 + g_Y^2)p^2,\\
&D = \partial_i^2 + \xi\partial_\tau^2 - \frac{g^2}{4}(p^2 + v^2),\\
&E = \partial_i^2 + \xi\partial_\tau^2 - \frac{g_Y^2}{4}(p^2 + v^2),\\
&F = \partial_i^2 + \partial_\tau^2 - (2\pi T)^2\left(-\varphi + \frac{\theta}{2}\right)^2 + \mu^2 - \lambda\left(\frac{3}{2}v^2 + \frac{p^2}{2}\right) - \frac{\xi}{4}g^2p^2,\\
&G = \partial_i^2 + \partial_\tau^2 - (2\pi T)^2\left(-\varphi + \frac{\theta}{2}\right)^2 + \mu^2 - \lambda\left(\frac{v^2}{2} + \frac{p^2}{2}\right) - \xi\frac{g^2 + g_Y^2}{4}v^2 - \xi\frac{g^2}{4}p^2,\\
&a = -2(2\pi T)(2\varphi)\partial_\tau, \qquad b = gv(2\pi T)\left(\varphi - \frac{\theta}{2}\right) - \eta_2(2\pi T)(2\varphi)v,\\
&c = -\frac{gg_Y}{2}pv, \qquad d = -gp(2\pi T)\left(\varphi + \frac{\theta}{2}\right) + \eta_2(2\pi T)(2\varphi)p, \qquad g = -2(2\pi T)\left(\varphi + \frac{\theta}{2}\right)\partial_\tau,\\
&\tilde{g} = -2(2\pi T)\left(\varphi - \frac{\theta}{2}\right)\partial_\tau, \qquad h = -gp(2\pi T)\left(\varphi + \frac{\theta}{2}\right), \qquad i = -g_Y p(2\pi T)\left(\varphi + \frac{\theta}{2}\right),\\
&j = \xi\frac{g^2}{4}pv - \lambda pv, \qquad k = \xi\frac{g_Y^2}{4}pv, \qquad l = -\frac{gg_Y}{4}(p^2 - v^2), \qquad m = -gv(2\pi T)\left(\varphi - \frac{\theta}{2}\right),\\
&n = g_Y v(2\pi T)\left(\varphi - \frac{\theta}{2}\right), \qquad \bar{c} = \eta_2 v\partial_\tau, \qquad \bar{d} = \eta_2 p\partial_\tau, \qquad \bar{b} = \eta_1 p\partial_\tau, \qquad \bar{e} = \eta_1 v\partial_\tau,
\end{aligned}
\tag{A33}
$$

where we have defined $\eta_2 \equiv -g(\xi - 1)/2, \eta_1 \equiv -g_Y(\xi - 1)/2$, which vanishes if we take the Feynman gauge $\xi = 1$.

Let us proceed to the ghost sector. As usual, it is convenient to introduce

$$
C^\pm \equiv \frac{1}{\sqrt{2}}(C^1 \mp iC^2), \qquad \bar{C}^\pm \equiv \frac{1}{\sqrt{2}}(\bar{C}^1 \mp iC^2). \tag{A34}
$$

Then the ghost sector is given by

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\rm ghost} = {}& i\bar{C}^+\left(-\partial_i^2 - \xi(\partial_\tau - i(2\pi T)(2\varphi))^2 + \xi\frac{g^2}{4}(p^2 + v^2)\right)C^-\\
&+ i\bar{C}^-\left(-\partial_i^2 - \xi(\partial_\tau - i(2\pi T)(2\varphi))^2 + \xi\frac{g^2}{4}(p^2 + v^2)\right)C^+\\
&+ i\bar{C}^3\left(-\partial_i^2 - \xi\partial_\tau^2 + \xi\frac{g^2}{4}(p^2 + v^2)\right)C^3\\
&+ i\bar{C}\left(-\partial_i^2 - \xi\partial_\tau^2 + \xi\frac{g_Y^2}{4}(p^2 + v^2)\right)C\\
&- i\xi\frac{gg_Y}{4}(p^2 - v^2)C\bar{C}^3 + i\xi\frac{gg_Y}{4}(p^2 - v^2)\bar{C}C^3\\
&- i\xi\frac{\sqrt{2}}{4}gg_Y pv[C(\bar{C}^+ + \bar{C}^-) - \bar{C}(C^+ + C^-)]
\end{aligned}
\tag{A35}
$$

$$
= i(\bar{C}^+, \bar{C}^-, \bar{C}^3, \bar{C})M^2_{\rm ghost}\begin{pmatrix} C^- \\ C^+ \\ C^3 \\ C \end{pmatrix}, \tag{A36}
$$

where $M^2_{\rm ghost}$ is given by

$$
M^2_{\rm ghost} = \begin{pmatrix}
-D^{W^\pm} & 0 & 0 & \xi\frac{\sqrt{2}}{4}gg_Y pv\\
0 & -D^{W^\pm} & 0 & \xi\frac{\sqrt{2}}{4}gg_Y pv\\
0 & 0 & -D^{A^3} & \xi\frac{gg_Y}{4}(p^2 - v^2)\\
\xi\frac{\sqrt{2}}{4}gg_Y pv & \xi\frac{\sqrt{2}}{4}gg_Y pv & \xi\frac{gg_Y}{4}(p^2 - v^2) & -D^B
\end{pmatrix}. \tag{A37}
$$

Here we have defined

$$D^{W^\pm} = \partial_i^2 + \xi\left[(\partial_\tau - i(2\pi T)(2\varphi))^2 - \frac{g^2}{4}(p^2+v^2)\right],$$
$$D^{A^3} = \partial_i^2 + \xi\left(\partial_\tau^2 - \frac{g^2}{4}(p^2+v^2)\right),$$
$$D^{B} = \partial_i^2 + \xi\left(\partial_\tau^2 - \frac{g_Y^2}{4}(p^2+v^2)\right). \tag{A38}$$

These are the same as (A27) with $\xi = 1$ aside from the factor δ_{ij}.

Let us introduce the fermions of the third generation,

$$Q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \quad t_R,\, b_R, \qquad l_L = \begin{pmatrix} \nu_L^\tau \\ \tau_L \end{pmatrix}, \quad \tau_R. \tag{A39}$$

Since our results do not change even if we introduce the first and second generations with the mixings among the generations, we consider only the third generation. We simply assume that the neutrino is massless. The Lagrangian for the fermions is

$$\mathcal{L}_{\rm fermion} = \bar{Q}_L i\gamma^\mu D_\mu^Q Q_L + \bar{t}_R i\gamma^\mu D_\mu^{t_R} t_R + \bar{b}_R i\gamma^\mu D_\mu^{b_R} b_R + f_t(\bar{t}_R\tilde{\Phi}^\dagger Q_L + \bar{Q}_L\tilde{\Phi} t_R) + f_b(\bar{b}_R\Phi^\dagger Q_L + \bar{Q}_L\Phi b_R) + \bar{l}_L i\gamma^\mu D_\mu^l l_L + \bar{\tau}_R i\gamma^\mu D_\mu^{\tau_R}\tau_R + f_\tau(\bar{\tau}_R\Phi^\dagger l_L + \bar{l}_L\Phi\tau_R), \tag{A40}$$

where the covariant derivatives are given by

$$D_\mu^Q = \nabla_\mu - igA_\mu - i\frac{g_Y}{2}\frac{1}{3}B_\mu, \quad D_\mu^{t_R} = \nabla_\mu - i\frac{g_Y}{2}\frac{4}{3}B_\mu,$$
$$D_\mu^{b_R} = \nabla_\mu - i\frac{g_Y}{2}\left(-\frac{2}{3}\right)B_\mu, \quad D_\mu^{l} = \partial_\mu - igA_\mu - i\frac{g_Y}{2}(-1)B_\mu,$$
$$D_\mu^{\tau_R} = \partial_\mu - i\frac{g_Y}{2}(-2)B_\mu \tag{A41}$$

and $f_{t,b,\tau}$ stands for the Yukawa couplings for the top, bottom, and tau. Here ∇_μ stands for the covariant derivative for the $SU(3)_c$, which is defined by

$$\nabla_\mu \equiv \partial_\mu - ig_s G_\mu. \tag{A42}$$

The Euclidean component of the gamma matrices is defined by $\gamma_\tau \equiv i\gamma_0$ with

$$\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}(\mu, \nu = \tau, 1, 2, 3). \tag{A43}$$

Then the quadratic terms from the fermions are

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm fermion} = & -i\bar{Q}_L\left(\gamma_\tau\left(\partial_\tau - ig_s\langle G_\tau\rangle - ig\langle A_\tau\rangle - i\frac{g_Y}{6}\langle B_\tau\rangle\right) + \gamma_i\partial_i\right)Q_L \\ & - i\bar{t}_R\left(\gamma_\tau\left(\partial_\tau - ig_s\langle G_\tau\rangle - i\frac{2g_Y}{3}\langle B_\tau\rangle\right) + \gamma_i\partial_i\right)t_R + f_t(\bar{t}_R\langle\tilde{\Phi}\rangle^\dagger Q_L + \bar{Q}_L\langle\tilde{\Phi}\rangle t_R) \\ & - i\bar{b}_R\left(\gamma_\tau\left(\partial_\tau - ig_s\langle G_\tau\rangle + i\frac{g_Y}{3}\langle B_\tau\rangle\right) + \gamma_i\partial_i\right)b_R + f_b(\bar{b}_R\langle\Phi\rangle^\dagger Q_L + \bar{Q}_L\langle\Phi\rangle b_R) \\ & - i\bar{l}_L\left(\gamma_\tau\left(\partial_\tau - ig\langle A_\tau\rangle + i\frac{g_Y}{2}\langle B_\tau\rangle\right) + \gamma_i\partial_i\right)l_L \\ & - i\bar{\tau}_R\left(\gamma_\tau\left(\partial_\tau + ig_Y\langle B_\tau\rangle\right) + \gamma_i\partial_i\right)\tau_R + f_\tau(\bar{\tau}_R\langle\Phi\rangle^\dagger l_L + \bar{l}_L\langle\Phi\rangle\tau_R).\end{aligned} \tag{A44}$$

By substituting the parametrizations for the vacuum expectation values (2) and (6) in Eq. (A44), we obtain that

$$\mathcal{L}_{\rm fermion} = (\bar{t}_L, \bar{t}_R, \bar{b}_L, \bar{b}_R)M_{\rm quark}\begin{pmatrix} t_L \\ t_R \\ b_L \\ b_R \end{pmatrix} + (\bar{\nu}_L, \bar{\tau}_L, \bar{\tau}_R,)M_{\rm lepton}\begin{pmatrix} \nu_L \\ \tau_L \\ \tau_R \end{pmatrix}, \tag{A45}$$

where we have defined

$$M_{\rm quark} = \begin{pmatrix} -iD_{t_L} & \frac{f_t}{\sqrt{2}}v & 0 & \frac{f_b}{\sqrt{2}}p \\ \frac{f_t}{\sqrt{2}}v & -iD_{t_R} & \frac{-f_t}{\sqrt{2}}p & 0 \\ 0 & \frac{-f_t}{\sqrt{2}}p & -iD_{b_L} & \frac{f_b}{\sqrt{2}}v \\ \frac{f_b}{\sqrt{2}}p & 0 & \frac{f_b}{\sqrt{2}}v & -iD_{b_R} \end{pmatrix}, \tag{A46}$$

$$M_{\rm lepton} = \begin{pmatrix} -iD_{\nu_L} & 0 & \frac{f_\tau}{\sqrt{2}}p \\ 0 & -iD_{\tau_L} & \frac{f_\tau}{\sqrt{2}}v \\ \frac{f_\tau}{\sqrt{2}}p & \frac{f_\tau}{\sqrt{2}}v & -iD_{\tau_R} \end{pmatrix}. \tag{A47}$$

The diagonal elements are given by

$$D_{t_L} \equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\omega_r + \varphi + \frac{\theta}{6}\right)\right) + \gamma_i\partial_i, \qquad D_{t_R} \equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\omega_r + \frac{2}{3}\theta\right)\right) + \gamma_i\partial_i,$$
$$D_{b_L} \equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\omega_r - \varphi + \frac{\theta}{6}\right)\right) + \gamma_i\partial_i, \qquad D_{b_R} \equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\omega_r - \frac{\theta}{3}\right)\right) + \gamma_i\partial_i,$$
$$D_{\tau_L} \equiv \gamma_\tau\left(\partial_\tau + i(2\pi T)\left(\varphi + \frac{\theta}{2}\right)\right) + \gamma_i\partial_i, \qquad D_{\tau_R} \equiv \gamma_\tau(\partial_\tau + i(2\pi T)\theta) + \gamma_i\partial_i,$$
$$D_{\nu_L} \equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\varphi - \frac{\theta}{2}\right)\right) + \gamma_i\partial_i. \qquad \text{(A48)}$$

Let us finally consider the $SU(3)_c$ gauge sector whose Lagrangian, including the gauge fixing and the ghost, is given by

$$\mathcal{L}_{SU(3)_c} = -\frac{1}{2}\,\mathrm{tr}(G_{\mu\nu}G^{\mu\nu}) - i\delta_B(\bar{C}_s^\alpha F_s^\alpha), \qquad \text{(A49)}$$

where $\alpha(= 1 \sim 8)$ is the $SU(3)_c$ color index and the gauge-fixing function F_s^α is chosen to be

$$F_s^\alpha \equiv -\partial_i\bar{G}_i^\alpha - \xi_s(D_\tau^{SU(3)_c}\bar{G}_\tau)^\alpha + \frac{\xi_s}{2}b_s^\alpha, \qquad \text{(A50)}$$

as usual. The calculations are straightforward and go in parallel with the $SU(2)_L$ case except that there is no scalar field like the Higgs field in this sector. Expanding the gauge field G_μ around the background $\langle G_\tau\rangle$ and taking the quadratic terms with respect to the fluctuations, one obtains that

$$\begin{aligned}\mathcal{L}^{(2)}_{SU(3)_c} &= \frac{1}{2}\bar{G}_i^\alpha\left(\delta_{ij}\partial_l^2 - \left(1-\frac{1}{\xi}\right)\partial_i\partial_j\right)\bar{G}_j^\alpha \\ &\quad -\frac{1}{2}(D_\tau^{SU(3)_c}\bar{G}_i)^\alpha(D_\tau^{SU(3)_c}\bar{G}_i)^\alpha \\ &\quad -\frac{1}{2}\partial_i\bar{G}_\tau^\alpha\partial_i\bar{G}_\tau^\alpha - \frac{\xi_s}{2}(D_\tau^{SU(3)_c}\bar{G}_\tau)^\alpha(D_\tau^{SU(3)_c}\bar{G}_\tau)^\alpha \\ &\quad - i\bar{C}_s^\alpha\partial_i^2C_s^\alpha - i\xi_s\bar{C}_s^\alpha(D_\tau^{SU(3)_c}D_\tau^{SU(3)_c}C_s)^\alpha, \end{aligned} \qquad \text{(A51)}$$

where the covariant derivative in Eqs. (A50) and (A51) is defined by

$$D_\tau^{SU(3)_c}\bar{G}_\mu \equiv \partial_\tau\bar{G}_\mu - ig_s[\langle G_\tau\rangle, \bar{G}_\mu] \quad (\mu = \tau, 1, 2, 3). \qquad \text{(A52)}$$

2. Quadratic terms under the ansatz

We have obtained the quadratic terms. It is, however, difficult to sum up all of the Matsubara modes n because of the complex dependence on n in the matrices (A25), (A32), (A37), (A46), and (A47).[2] As explained in the text, it may be natural to impose the ansatz (15) in order to study the effective potential at the one-loop level as analytically as possible. Under the ansatz with the Feynman gauge $\xi = 1$ and $\xi_s = 1$, the matrices become so simple that we can diagonalize them and sum up all of the Matsubara modes.

The quadratic terms under the ansatz for the gauge sector is simplified as

$$(W_i^-, \bar{A}_i^3, \bar{B}_i)M^2_{\mathrm{gauge}}|_{\mathrm{ansatz}}\begin{pmatrix} W_j^+ \\ \bar{A}_j^3 \\ \bar{B}_i \end{pmatrix} = W_i^-\bar{D}^{W^\pm}\delta_{ij}W_j^+ + \frac{1}{2}(\bar{A}_i^3, \bar{B}_i)\begin{pmatrix} \bar{D}^{A^3} & \frac{1}{4}gg_Yv^2 \\ \frac{1}{4}gg_Yv^2 & \bar{D}^B \end{pmatrix}\delta_{ij}\begin{pmatrix} \bar{A}_j^3 \\ \bar{B}_j \end{pmatrix}, \qquad \text{(A53)}$$

where

$$\bar{D}^{W^\pm} = \partial_i^2 + (\partial_\tau - i(2\pi T)(2\varphi))^2 - \frac{g^2}{4}v^2,$$
$$\bar{D}^{A^3} = \partial_i^2 + \partial_\tau^2 - \frac{g^2}{4}v^2, \qquad \bar{D}^B = \partial_i^2 + \partial_\tau^2 - \frac{g_Y^2}{4}v^2. \qquad \text{(A54)}$$

Diagonalization of the $\bar{A}_i^3$ and $\bar{B}_i$ sector can be done by the usual rotation,

$$\begin{pmatrix} Z_i \\ A_i^\gamma \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix}\begin{pmatrix} \bar{A}_i^3 \\ \bar{B}_i \end{pmatrix} \quad \text{with} \quad \times\begin{cases} c_w \equiv \cos\theta_w \equiv \frac{g}{\sqrt{g^2+g_Y^2}}, \\ s_w \equiv \sin\theta_w \equiv \frac{g_Y}{\sqrt{g^2+g_Y^2}} \end{cases}. \qquad \text{(A55)}$$

Then the eigenvalues in the momentum space are given by

$$-k_i^2 - (2\pi T)^2(n-2\varphi)^2 - \frac{g^2}{4}v^2 \cdots W_i^\pm,$$
$$-k_i^2 - (2\pi T)^2n^2 - \frac{g^2+g_Y^2}{4}v^2 \cdots Z_i,$$
$$-k_i^2 - (2\pi T)^2n^2 \cdots A_i^\gamma. \qquad \text{(A56)}$$

The quadratic terms for the scalar sector are

[2] The derivative ∂_τ is replaced by $i(2\pi T)n(i(2\pi T)(n+\frac{1}{2}))$ in the momentum space for bosons (fermions).

$$\frac{1}{2}(\bar{A}^1_\tau,\bar{A}^2_\tau,g^1,g^2,\bar{A}^3_\tau,\bar{B}_\tau,h,G^0)M^2_{\rm scalar}|_{\rm ansatz}\begin{pmatrix}\bar{A}^1_\tau\\ \bar{A}^2_\tau\\ g^1\\ g^2\\ \bar{A}^3_\tau\\ \bar{B}_\tau\\ h\\ G^0\end{pmatrix} \tag{A57}$$

$$=\frac{1}{2}(\bar{A}^1_\tau,\bar{A}^2_\tau)\begin{pmatrix}\bar{A} & a\\ -a & \bar{A}\end{pmatrix}\begin{pmatrix}\bar{A}^1_\tau\\ \bar{A}^2_\tau\end{pmatrix}+\frac{1}{2}(g^1,g^2)\begin{pmatrix}\bar{B} & \bar{g}\\ -\bar{g} & \bar{C}\end{pmatrix}\begin{pmatrix}g^1\\ g^2\end{pmatrix}$$
$$+\frac{1}{2}(\bar{A}^3_\tau,\bar{B}_\tau)\begin{pmatrix}\bar{D} & \bar{l}\\ \bar{l} & \bar{E}\end{pmatrix}\begin{pmatrix}\bar{A}^3_\tau\\ \bar{B}_\tau\end{pmatrix}+\frac{1}{2}h\bar{F}h+\frac{1}{2}G^0\bar{G}G^0, \tag{A58}$$

where

$$\bar{A}=\partial_i^2+\partial_\tau^2-(2\pi T)^2(2\varphi)^2-\frac{g^2}{4}v^2,$$
$$\bar{B}=\partial_i^2+\partial_\tau^2-(2\pi T)^2(2\varphi)^2+\mu^2-\frac{\lambda}{2}v^2-\frac{g^2}{4}v^2=\bar{C},$$
$$\bar{D}=\partial_i^2+\partial_\tau^2-\frac{g^2}{4}v^2,\quad \bar{E}=\partial_i^2+\partial_\tau^2-\frac{g_Y^2}{4}v^2,$$
$$\bar{F}=\partial_i^2+\partial_\tau^2+\mu^2-\frac{3}{2}\lambda v^2,\ \ \bar{G}=\partial_i^2+\partial_\tau^2+\mu^2-\frac{\lambda v^2}{2}-\frac{g^2+g_Y^2}{4}v^2,$$
$$a=-2(2\pi T)(2\varphi)\partial_\tau,\quad \bar{g}=-2(2\pi T)(2\varphi)\partial_\tau,\quad \bar{l}=\frac{gg_Y}{4}v^2. \tag{A59}$$

The $\bar{A}^{1,2}_\tau$ and $g^{1,2}$ sectors are diagonalized by the original base defined in Eqs. (A26) and (A29). The $\bar{A}^3_\tau$ and $\bar{B}_\tau$ sector is diagonalized by the rotation matrix given by Eq. (A55),

$$\begin{pmatrix}Z_\tau\\ A^\gamma_\tau\end{pmatrix}=\begin{pmatrix}c_w & -s_w\\ s_w & c_w\end{pmatrix}\begin{pmatrix}\bar{A}^3_\tau\\ \bar{B}_\tau\end{pmatrix}. \tag{A60}$$

Then the eigenvalues in the momentum space are

$$-k_i^2-(2\pi T)^2(n-2\varphi)^2-\frac{g^2}{4}v^2\cdots W^\pm_\tau,$$
$$-k_i^2-(2\pi T)^2n^2-\frac{g^2+g_Y^2}{4}v^2\cdots Z_\tau,$$
$$-k_i^2-(2\pi T)^2n^2\cdots A^\gamma_\tau,$$
$$-k_i^2-(2\pi T)^2(n-2\varphi)^2+\mu^2-\frac{\lambda}{2}v^2-\frac{g^2}{4}v^2\cdots G^\pm,$$
$$-k_i^2-(2\pi T)^2n^2+\mu^2-\frac{\lambda}{2}v^2-\frac{g^2+g_Y^2}{4}v^2\cdots G^0,$$
$$-k_i^2-(2\pi T)^2n^2+\mu^2-\frac{3\lambda}{2}v^2\cdots h. \tag{A61}$$

Let us note that in the above calculations the terms proportional to $-\varphi+\theta/2$ or p vanish and $\varphi+\theta/2$ becomes 2φ due to the ansatz.

The quadratic terms for the ghost sector are

$$i(\bar{C}^+,\bar{C}^-,\bar{C}^3,\bar{C})M^2_{\rm ghost}|_{\rm ansatz}\begin{pmatrix}C^-\\ C^+\\ C^3\\ C\end{pmatrix}$$
$$=-i\bar{C}^+\bar{D}_{W^\pm}C^- - i\bar{C}^-\bar{D}_{W^\pm}C^+ - i(\bar{C}^3,\bar{C})$$
$$\times\begin{pmatrix}\bar{D}^{A^3} & \frac{gg_Y}{4}v^2\\ \frac{gg_Y}{4}v^2 & \bar{D}^B\end{pmatrix}\begin{pmatrix}C^3\\ C\end{pmatrix}, \tag{A62}$$

where

$$\bar{D}^{W^\pm}=\partial_i^2+(\partial_\tau-i(2\pi T)(2\varphi))^2-\frac{g^2}{4}v^2,$$
$$\bar{D}^{A^3}=\partial_i^2+\partial_\tau^2-\frac{g^2}{4}v^2,\qquad \bar{D}^B=\partial_i^2+\partial_\tau^2-\frac{g_Y^2}{4}v^2. \tag{A63}$$

By introducing the new bases by the rotation matrix (A55),

$$\begin{pmatrix}\bar{C}_Z\\ \bar{C}_\gamma\end{pmatrix}=\begin{pmatrix}c_w & -s_w\\ s_w & c_w\end{pmatrix}\begin{pmatrix}\bar{C}^3\\ \bar{C}\end{pmatrix},$$
$$\begin{pmatrix}C_Z\\ C_\gamma\end{pmatrix}=\begin{pmatrix}c_w & -s_w\\ s_w & c_w\end{pmatrix}\begin{pmatrix}C^3\\ C\end{pmatrix}, \tag{A64}$$

the ghost sector is diagonalized. The eigenvalues are given, in the momentum space, by

$$-k_i^2-(2\pi T)^2(n-2\varphi)^2-\frac{g^2}{4}v^2\cdots\bar{C}^\pm,C^\pm,$$
$$-k_i^2-(2\pi T)^2n^2-\frac{g^2+g_Y^2}{4}v^2\cdots\bar{C}_Z,C_Z,$$
$$-k_i^2-(2\pi T)^2n^2\cdots\bar{C}_\gamma,C_\gamma. \tag{A65}$$

Let us finally consider the fermion sector whose matrices under the ansatz are given by

$$(\bar{t}_L,\bar{t}_R,\bar{b}_L,\bar{b}_R)M_{\rm quark}|_{\rm ansatz}\begin{pmatrix}t_L\\ t_R\\ b_L\\ b_R\end{pmatrix}$$
$$+(\bar{\nu}_L,\bar{\tau}_L,\bar{\tau}_R,)M_{\rm lepton}|_{\rm ansatz}\begin{pmatrix}\nu_L\\ \tau_L\\ \tau_R\end{pmatrix} \tag{A66}$$

$$
\begin{aligned}
&= (\bar{t}_L, \bar{t}_R)\begin{pmatrix} -i\bar{D}_{t_L} & \frac{f_t}{\sqrt{2}}v \\ \frac{f_t}{\sqrt{2}}v & -i\bar{D}_{t_R}\end{pmatrix}\begin{pmatrix} t_L \\ t_R\end{pmatrix} \\
&\quad + (\bar{b}_L, \bar{b}_R)\begin{pmatrix} -i\bar{D}_{b_L} & \frac{f_b}{\sqrt{2}}v \\ \frac{f_b}{\sqrt{2}}v & -i\bar{D}_{b_R}\end{pmatrix}\begin{pmatrix} b_L \\ b_R\end{pmatrix} \\
&\quad + (\bar{\tau}_L, \bar{\tau}_R)\begin{pmatrix} -i\bar{D}_{\tau_L} & \frac{f_\tau}{\sqrt{2}}v \\ \frac{f_\tau}{\sqrt{2}}v & -i\bar{D}_{\tau_R}\end{pmatrix}\begin{pmatrix} \tau_L \\ \tau_R\end{pmatrix} - i\bar{\nu}_L\bar{D}_{\nu_L}\nu_L,
\end{aligned}
\tag{A67}
$$

where

$$
\begin{aligned}
\bar{D}_{t_L} &\equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\omega_r + \frac{4}{3}\varphi\right)\right) + \gamma_i\partial_i = \bar{D}_{t_R}, \\
\bar{D}_{b_L} &\equiv \gamma_\tau\left(\partial_\tau - i(2\pi T)\left(\omega_r - \frac{2}{3}\varphi\right)\right) + \gamma_i\partial_i = \bar{D}_{b_R}, \\
\bar{D}_{\tau_L} &\equiv \gamma_\tau\left(\partial_\tau + i(2\pi T)2\varphi\right) + \gamma_i\partial_i = \bar{D}_{\tau_R}, \\
\bar{D}_{\nu_L} &\equiv \gamma_\tau\partial_\tau + \gamma_i\partial_i.
\end{aligned}
\tag{A68}
$$

Let us note that thanks to the ansatz the diagonal components of each matrix for the top, bottom, and tau sectors become identical. The eigenvalues in the momentum space are

$$
\begin{aligned}
&k_i^2 + (2\pi T)^2\left(n + \frac{1}{2} - \omega_r - \frac{4}{3}\varphi\right)^2 + \frac{f_t^2}{2}v^2 \cdots t_L, t_R, \\
&k_i^2 + (2\pi T)^2\left(n + \frac{1}{2} - \omega_r + \frac{2}{3}\varphi\right)^2 + \frac{f_b^2}{2}v^2 \cdots b_L, b_R, \\
&k_i^2 + (2\pi T)^2\left(n + \frac{1}{2} + 2\varphi\right)^2 + \frac{f_\tau^2}{2}v^2 \cdots \tau_L, \tau_R, \\
&k_i^2 + (2\pi T)^2\left(n + \frac{1}{2}\right)^2 \cdots \nu_L.
\end{aligned}
\tag{A69}
$$

The half integer in the Matsubara mode n is due to the Fermi statistics for fermions. Since the quarks have the color degrees of freedom, the eigenvalues also depend on the order parameter ω_r of the vacuum expectation value $\langle G_\tau\rangle$.

Taking account of the eigenvalues obtained above, the one-loop contributions to the effective potential are given by

$$
\begin{aligned}
V^{\text{one-loop}} &= (6+2-4)\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2(n-2\varphi)^2 + \frac{g^2}{4}v^2\right] \\
&\quad + (3+1-2)\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2 n^2 + \frac{g^2+g_Y^2}{4}v^2\right] \\
&\quad + (3+1-2)\times\frac{1}{2i}\int_k \ln[k_i^2 + (2\pi T)^2 n^2] \\
&\quad + (4-2)\sum_{r,q=1}^{3}\frac{1}{2i}\int_k \ln[k_i^2 + (2\pi T)^2(n+\omega_r-\omega_q)^2] \\
&\quad + 2\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2(n-2\varphi)^2 - \mu^2 + \frac{\lambda}{2}v^2 + \frac{g^2}{4}v^2\right] \\
&\quad + 1\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2 n^2 - \mu^2 + \frac{\lambda}{2}v^2 + \frac{g^2+g_Y^2}{4}v^2\right] \\
&\quad + 1\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2 n^2 - \mu^2 + \frac{3\lambda}{2}v^2\right] \\
&\quad + (-1)2^2\sum_{r=1}^{3}\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2\left(n+\frac{1}{2}-\omega_r-\frac{4}{3}\varphi\right)^2 + \frac{f_t^2}{2}v^2\right] \\
&\quad + (-1)2^2\sum_{r=1}^{3}\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2\left(n+\frac{1}{2}-\omega_r+\frac{2}{3}\varphi\right)^2 + \frac{f_b^2}{2}v^2\right] \\
&\quad + (-1)2^2\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2\left(n+\frac{1}{2}+2\varphi\right)^2 + \frac{f_\tau^2}{2}v^2\right] \\
&\quad + (-1)\frac{2^2}{2}\times\frac{1}{2i}\int_k \ln\left[k_i^2 + (2\pi T)^2\left(n+\frac{1}{2}\right)^2\right],
\end{aligned}
\tag{A70}
$$

where we have defined

$$\int_k \equiv iT \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3}. \tag{A71}$$

The first to fourth lines come from $W^{\pm}_{i,\tau}, Z_{i,\tau}, A^{\gamma}_{i,\tau}$, and $G^{\alpha}_{i,\tau}$, respectively, together with the ghost fields $C^{\pm}, \bar{C}^{\pm}, C_Z, \bar{C}_Z, C_\gamma, \bar{C}_\gamma, C^{\alpha}_s, \bar{C}^{\alpha}_s$. The fields $G^{\pm}, G^0$, and h contribute to the fifth, sixth, and seventh lines, respectively.

The last four lines are the fermion contributions. In addition to the usual order parameter v, the one-loop contributions depend on the new order parameters φ and ω_r. As discussed in the text, the effective potential at the one-loop level consists of the zero and finite temperature part,

$$V^{\text{one-loop}} = V^{T=0} + V^{T\neq 0}, \tag{A72}$$

where

$$\begin{aligned} V^{T=0} = &-\frac{\mu^2}{2}v^2 + \frac{\lambda}{8}v^4 \\ &+ \frac{4}{4(4\pi)^2} m_W(v)^4\left(\ln\frac{m_W(v)^2}{M^2} - \frac{3}{2}\right) + \frac{2}{4(4\pi)^2} m_Z(v)^4\left(\ln\frac{m_Z(v)^2}{M^2} - \frac{3}{2}\right) \\ &+ \frac{2}{4(4\pi)^2} m_{G^\pm}(v)^4\left(\ln\frac{m_{G^\pm}(v)^2}{M^2} - \frac{3}{2}\right) + \frac{1}{4(4\pi)^2} m_{G^0}(v)^4\left(\ln\frac{m_{G^0}(v)^2}{M^2} - \frac{3}{2}\right) \\ &+ \frac{1}{4(4\pi)^2} m_h(v)^4\left(\ln\frac{m_h(v)^2}{M^2} - \frac{3}{2}\right) - \frac{12}{4(4\pi)^2} m_t(v)^4\left(\ln\frac{m_t(v)^2}{M^2} - \frac{3}{2}\right) \\ &- \frac{12}{4(4\pi)^2} m_b(v)^4\left(\ln\frac{m_b(v)^2}{M^2} - \frac{3}{2}\right) - \frac{4}{4(4\pi)^2} m_\tau(v)^4\left(\ln\frac{m_\tau(v)^2}{M^2} - \frac{3}{2}\right) \end{aligned} \tag{A73}$$

and

$$\begin{aligned} V^{T\neq 0} = &-4\frac{2}{(2\pi)^2}T^4\sum_{m=1}^{\infty}\frac{1}{m^4}\cos[2\pi m(2\varphi)]\left(\frac{m_W(v)^2}{T^2}m^2\right)K_2\left(\frac{m_W(v)}{T}m\right) \\ &-2\frac{2}{(2\pi)^2}T^4\sum_{m=1}^{\infty}\frac{1}{m^4}\left(\frac{m_Z(v)^2}{T^2}m^2\right)K_2\left(\frac{m_Z(v)}{T}m\right) \\ &-2\frac{2}{(2\pi)^2}T^4\sum_{m=1}^{\infty}\frac{1}{m^4}\cos[2\pi m(2\varphi)]\left(\frac{m_{G^\pm}(v)^2}{T^2}m^2\right)K_2\left(\frac{m_{G^\pm}(v)}{T}m\right) \\ &-1\frac{2}{(2\pi)^2}T^4\sum_{m=1}^{\infty}\frac{1}{m^4}\left(\frac{m_{G^0}(v)^2}{T^2}m^2\right)K_2\left(\frac{m_{G^0}(v)}{T}m\right) \\ &-1\frac{2}{(2\pi)^2}T^4\sum_{m=1}^{\infty}\frac{1}{m^4}\left(\frac{m_h(v)^2}{T^2}m^2\right)K_2\left(\frac{m_h(v)}{T}m\right) \\ &+4\frac{2}{(2\pi)^2}T^4\sum_{r=1}^{3}\sum_{m=1}^{\infty}\frac{(-1)^m}{m^4}\cos\left[2\pi m\left(\omega_r + \frac{4}{3}\varphi\right)\right]\left(\frac{m_t(v)^2}{T^2}m^2\right)K_2\left(\frac{m_t(v)}{T}m\right) \\ &+4\frac{2}{(2\pi)^2}T^4\sum_{r=1}^{3}\sum_{m=1}^{\infty}\frac{(-1)^m}{m^4}\cos\left[2\pi m\left(\omega_r - \frac{2}{3}\varphi\right)\right]\left(\frac{m_b(v)^2}{T^2}m^2\right)K_2\left(\frac{m_b(v)}{T}m\right) \\ &+4\frac{2}{(2\pi)^2}T^4\sum_{m=1}^{\infty}\frac{(-1)^m}{m^4}\cos[2\pi m(2\varphi)]\left(\frac{m_\tau(v)^2}{T^2}m^2\right)K_2\left(\frac{m_\tau(v)}{T}m\right) \\ &-2\frac{2}{(2\pi)^2}T^4\sum_{r,q=1}^{3}\sum_{m=1}^{\infty}\frac{2}{m^4}\cos[2\pi m(\omega_r - \omega_q)]. \end{aligned} \tag{A74}$$

The last line in Eq. (A74) comes from the $SU(3)_c$ gauge sector. The new order parameters φ and ω_r enter into the finite temperature part of the one-loop effective potential. Here we have defined the notations,

$$m_W(v)^2 = \frac{g^2}{4}v^2, \qquad m_Z(v)^2 = \frac{g^2+g_Y^2}{4}v^2, \qquad m_h(v)^2 = -\mu^2 + \frac{3\lambda}{2}v^2,$$
$$m_{G^\pm}(v)^2 = -\mu^2 + \frac{\lambda}{2}v^2 + m_W(v)^2, \qquad m_{G^0}(v)^2 = -\mu^2 + \frac{\lambda}{2}v^2 + m_Z(v)^2,$$
$$m_t(v)^2 = \frac{f_t^2}{2}v^2, \qquad m_b(v)^2 = \frac{f_b^2}{2}v^2, \qquad m_\tau(v)^2 = \frac{f_\tau^2}{2}v^2. \tag{A75}$$

$K_2(z)$ is the modified Bessel function defined in Eq. (35).

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