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Generalized second law of thermodynamics in scalar-tensor gravity

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Within the context of scalar-tensor gravity, we explore the generalized second law (GSL) of gravitational thermodynamics. We extend the action of ordinary scalar-tensor gravity theory to the case in which there is a nonminimal coupling between the scalar field and the matter field (as a chameleon field). Then we derive the field equations governing the gravity and the scalar field. For a Friedmann-Robertson-Walker universe filled only with ordinary matter, we obtain the modified Friedmann equations as well as the evolution equation of the scalar field. Furthermore, we assume the boundary of the Universe to be enclosed by the dynamical apparent horizon that is in thermal equilibrium with the Hawking temperature. We obtain a general expression for the GSL of thermodynamics in the scalar-tensor gravity model. For some viable scalar-tensor models, we first obtain the evolutionary behaviors of the matter density, the scale factor, the Hubble parameter, the scalar field, and the deceleration parameter, as well as the effective equation of state (EoS) parameter. We conclude that in most of the models, the deceleration parameter approaches a de Sitter regime at late times, as expected. Also, the effective EoS parameter acts like the ΛCDM model at late times. Finally, we examine the validity of the GSL for the selected models.

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I. INTRODUCTION

During the last decade, observational cosmology has entered an era of unprecedented precision. Measurements of the cosmic microwave background ([1,2]), the Hubble constant (H_0) [3], the luminosity and distance at high redshift with the supernovae Ia [4], and baryon acoustic oscillations surveys [5] suggest that our Universe is currently undergoing a phase of accelerated expansion. The proposals that have been put forth to explain these interesting discoveries can basically be classified into two categories. One is to assume the cosmic speeding-up might be caused within general relativity (GR) by a mysterious cosmic fluid with negative pressure, which is usually called dark energy (DE). However, the nature of DE is still unknown and the problem of DE is one of the hardest and unresolved problems in modern theoretical physics (see [6,7] and references therein).

Alternatively, the acceleration could be due to purely gravitational effects, named modified gravity, i.e., one may consider modifying the current gravitational theory to produce an effective DE. One such modification is referred to as $f(R)$-gravity, in which the Einstein-Hilbert action in GR is generalized from the Ricci scalar R to an arbitrary function of the Ricci scalar (for a good review, see [8] and references therein). There are also some other classes of modified gravities containing $f(\mathcal{G})$ [9], $f(R,\mathcal{G})$ [10] and $f(T)$ [11], which are considered as gravitational alternatives for DE. Here, $\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet invariant term. Also, $R_{\mu\nu\rho\sigma}$ and $R_{\mu\nu}$ are the Riemann and Ricci tensors, respectively, and T is the torsion scalar. The modified gravity can unify the early-time inflation with late-time acceleration without resorting to the DE [8]. Moreover, modified gravity may serve as dark matter [12].

In the context of modified gravity, there is also a large class of models called scalar-tensor theories [13,14], which take into account the effects due to the nonminimal coupling term $F(\phi)R$ between a scalar field, ϕ, and a Ricci scalar curvature. In scalar-tensor theories, if the evolution of matter perturbations $\delta_{\rm m} = \delta\rho_{\rm m}/\rho_{\rm m}$ is known observationally, together with the Hubble parameter $H(z)$, one can even determine the function $F(\phi)$ together with the potential $V(\phi)$ of the scalar field [15]. Scalar-tensor theories also contain a class of models called chameleon gravity [16–18] in which there is a nonminimal coupling between the scalar field and the matter field. Historically, one of the first scalar-tensor theories is the Brans-Dicke theory of gravity, which has been motivated from Mach's principle. This is achieved in Brans-Dicke theory by making the effective gravitational coupling strength $G_{\rm eff} \sim \phi^{-1}$ depend on the space-time position and being governed by distant matter sources. Modern interest in Brans-Dicke and scalar-tensor theories is motivated by the fact that they are obtained as low-energy limits of string theories. It was shown that metric and Palatini (but not metric-affine)

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modified gravities can be reduced to scalar-tensor theories [19].

Thermodynamics of the accelerating Universe driven by the DE or dark gravity (due to the modified gravity effect) is one of the interesting issues in modern cosmology. In the context of black hole thermodynamics, Jacobson using the first law of thermodynamics on the local Rindler horizons and assuming the Bekenstein-Hawking entropy-area relation $S_{\rm BH} = A/(4G)$, where A is the area of the horizon and G is Newton's constant, was able to derive the Einstein equations [20]. The study on the connection between gravity and thermodynamics has been extended to the cosmological context. It was pointed out that the Friedmann equation in the Einstein gravity can be obtained using the first law of thermodynamics (the Clausius relation) $-{\rm d}E = T_A {\rm d}S_A$ on the apparent horizon $\tilde{r}_{\rm A}$ with the Hawking temperature $T_{\rm A} = 1/(2\pi\tilde{r}_{\rm A})$ and Bekenstein-Hawking entropy $S_A = \frac{A}{4G}$ [21]. The relation between gravity and thermodynamics has been further disclosed in extended gravitational theories, including the $f(\mathcal{G})$ theory [21], scalar-tensor gravity and $f(R)$-gravity [22], Lovelock theory [23], and braneworld scenarios (such as DGP, RSI, and RSII) [24].

Note that the entropy-area relation $S_A = \frac{A}{4G}$ familiar from GR is still valid in the other modified gravity theories provided that Newton's constant, G, is replaced by a suitable effective gravitational coupling strength, $G_{\rm eff}$. For instance, the effective Newton's constant in $f(R)$-gravity and $f(T)$-gravity are given by $G_{\rm eff} = G/f'(R)$ [25] and $G_{\rm eff} = G/f'(T)$ [26], respectively, where prime denotes a derivative with respect to the Ricci R and torsion T scalars. In scalar-tensor gravity, the geometric entropy is also given by $S_A = \frac{A}{4G_{\rm eff}}$ [19,27] with $G_{\rm eff} = G/F(\phi)$ [19,27].

In addition to the first law of thermodynamics, the generalized second law (GSL) of gravitational thermodynamics, which states that entropy of the fluid inside the horizon plus the geometric entropy does not decrease with time, has been studied extensively in the literature [28]–[37]. The GSL of thermodynamics, like the first law, is a universal principle governing the Universe. Here, our aim is to investigate the GSL of thermodynamics in the framework of scalar-tensor gravity. As one of the most important theoretical touchstones to examine whether scalar-tensor gravity can be an alternative gravitational theory to GR, we explore the GSL of thermodynamics in scalar-tensor gravity, and derive the condition for the GSL to be satisfied. The paper is organized as follows. In Sec. II we investigate the scalar-tensor gravity and extend it to the case in which there is a nonminimal coupling between the scalar field and the matter field (as the chameleon field). In Sec. III, we explore the GSL of thermodynamics on the dynamical apparent horizon of a Friedmann-Robertson-Walker (FRW) universe filled with the ordinary matter that is in thermal equilibrium with the Hawking temperature. In Secs. IV–VIII, we examine the validity of the GSL for some viable scalar-tensor gravity models containing Brans-Dicke gravity, Brans-Dicke gravity with a self interacting potential, chameleon gravity, chameleonic generalized Brans-Dicke gravity, and chameleonic Brans-Dicke gravity with a self interacting potential. Section IX is devoted to conclusions.

II. SCALAR-TENSOR GRAVITY

In the Jordan frame, general action of the scalar-tensor gravity can be written as

$$I = \int {\rm d}^4x\sqrt{-g}\bigg[\frac{1}{2k^2}(F(\phi)R - Z(\phi)g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - 2U(\phi)) + E(\phi)L_{\rm m}\bigg], \tag{1}$$

where $k^2 = 8\pi G$. Also g, R, ϕ, and $L_{\rm m}$ are the determinant of the metric $g_{\mu\nu}$, the Ricci scalar curvature, the scalar field, and the matter Lagrangian, respectively. Also $F(\phi)$, $Z(\phi)$, and $E(\phi)$ are arbitrary dimensionless functions, and $U(\phi)$ is the scalar field potential. Note that in action (1), the terms $F(\phi)R$ and $E(\phi)L_{\rm m}$ show that the scalar field ϕ is non-minimally coupled to the scalar curvature [as the Brans-Dicke field with $F(\phi) = \phi$] and the matter Lagrangian (as the chameleon field), respectively. In the absence of the chameleon field, i.e., $E(\phi) = 1$, Eq. (1) reduces to the ordinary action of the scalar-tensor gravity theory [38].

Taking variations of the action (1) with respect to $g_{\mu\nu}$ and ϕ leads to the corresponding field equations in scalar-tensor gravity as

$$F(\phi)G_{\mu\nu} = k^2T^{\rm m}_{\mu\nu}E(\phi) + Z(\phi)\bigg[\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial_\alpha\phi)^2\bigg] + \nabla_\mu\partial_\nu F(\phi) - g_{\mu\nu}\Box F(\phi) - g_{\mu\nu}U(\phi), \tag{2}$$

$$2Z(\phi)\Box\phi = 2U_{,\phi} - F_{,\phi}R - Z_{,\phi}(\partial_\alpha\phi)^2 - \frac{k^2}{2}g^{\mu\nu}E_{,\phi}T^{\rm m}_{\mu\nu}, \tag{3}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, and $T^{\rm m}_{\mu\nu}$ is the energy-momentum tensor of the matter fields. Also, ∇_μ is the covariant derivative associated with $g_{\mu\nu}$ and the subscript ϕ denotes a derivative with respect to the scalar field ϕ (i.e., $F_{,\phi} = {\rm d}F/{\rm d}\phi$). We assume that $T^{\rm m}_{\mu\nu}$ has the form of the energy-momentum tensor of a perfect fluid,

$$T^{\rm m}_{\mu\nu} = p_{\rm m}g_{\mu\nu} + (p_{\rm m} + \rho_{\rm m})U_\mu U_\nu. \tag{4}$$

Now we consider a spatially nonflat universe described by the FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right), \quad (5)$$

where $K = 0, 1, -1$ represents a flat, closed, and open universe, respectively. Substituting the FRW metric (5) into the field equations (2) yields the Friedmann equations in scalar-tensor gravity as

$$3F(\phi)\left(H^2 + \frac{K}{a^2}\right) = k^2\rho_m E(\phi) + \frac{Z(\phi)}{2}\dot{\phi}^2 - 3H\dot{F} + U(\phi), \quad (6)$$

$$-2F(\phi)\left(\dot{H} - \frac{K}{a^2}\right) = k^2(\rho_m + p_m)E(\phi) + Z(\phi)\dot{\phi}^2 + \ddot{F} - H\dot{F}. \quad (7)$$

Also, Eq. (3) for the FRW metric (5) gives the equation governing the evolution of the scalar field as

$$2Z(\phi)(\ddot{\phi} + 3H\dot{\phi}) = RF_{,\phi} - Z_{,\phi}\dot{\phi}^2 - 2U_{,\phi} - \frac{k^2}{2}E_{,\phi}(\rho_m - 3p_m), \quad (8)$$

where

$$R = 6\left(\dot{H} + 2H^2 + \frac{K}{a^2}\right), \quad (9)$$

and $H = \dot{a}/a$ is the Hubble parameter. Here, the dot denotes a derivative with respect to the cosmic time t. Note that in the absence of the chameleon field, i.e., $E(\phi) = 1$, Eqs. (6), (7), and (8) are same as those obtained for the ordinary scalar-tensor gravity [38].

The Friedmann equations (6) and (7) can be rewritten in the standard form as

$$H^2 + \frac{K}{a^2} = \frac{k^2}{3}\rho_{\rm eff}, \quad (10)$$

$$\dot{H} - \frac{K}{a^2} = -\frac{k^2}{2}(\rho_{\rm eff} + p_{\rm eff}), \quad (11)$$

where $\rho_{\rm eff}$ and $p_{\rm eff}$ are the effective (total) energy density and pressure defined as

$$\rho_{\rm eff} = \frac{1}{F(\phi)}\left(\rho_m E(\phi) + \frac{\rho_\phi}{k^2}\right), \quad (12)$$

$$p_{\rm eff} = \frac{1}{F(\phi)}\left(p_m E(\phi) + \frac{p_\phi}{k^2}\right). \quad (13)$$

Here, ρ_ϕ and p_ϕ are the energy density and pressure due to the scalar field contribution defined as

$$\rho_\phi = \frac{Z(\phi)}{2}\dot{\phi}^2 - 3H\dot{F} + U(\phi), \quad (14)$$

$$p_\phi = \frac{Z(\phi)}{2}\dot{\phi}^2 + \ddot{F} + 2H\dot{F} - U(\phi). \quad (15)$$

Note that the scalar field contributions ρ_ϕ and p_ϕ in scalar-tensor gravity can justify the observed acceleration of the Universe without resorting to the DE. For a special case, $F(\phi) = E(\phi) = 1$, from Eqs. (14) and (15) we have $\rho_\phi = \frac{Z(\phi)}{2}\dot{\phi}^2 + U(\phi)$ and $p_\phi = \frac{Z(\phi)}{2}\dot{\phi}^2 - U(\phi)$; then Eqs. (10) and (11) transform to the usual Friedmann equations in the Einstein gravity.

The energy conservation laws in scalar-tensor gravity can be obtained as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -\frac{3}{4}(\rho_m + p_m)\frac{\dot{E}(\phi)}{E(\phi)}, \quad (16)$$

$$\dot{\rho}_{\rm eff} + 3H(\rho_{\rm eff} + p_{\rm eff}) = 0. \quad (17)$$

Also, ρ_ϕ and p_ϕ satisfy the following energy equation:

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = k^2\left[\rho_{\rm eff}\dot{F}(\phi) - \frac{1}{4}\dot{E}(\phi)(\rho_m - 3p_m)\right]. \quad (18)$$

Note that the set of equations containing the Friedmann equations (6) and (7), the evolution equation of the scalar field (8), and the continuity equation governing the matter field (16) are not independent of each other. Taking the time derivative of Eq. (6) and using Eqs. (8) and (16), one can get the second Friedmann equation (7). In the next sections, we take the set of Eqs. (7), (8), and (16), which can uniquely determine the dynamics of the Universe.

III. GSL IN SCALAR-TENSOR GRAVITY

Here, in the context of scalar-tensor gravity theory, we explore the GSL of gravitational thermodynamics on the dynamical apparent horizon of a FRW universe filled only with ordinary matter that is in thermal equilibrium with the Hawking temperature. The GSL states that the sum of the entropy of fluid filling the Universe along with the entropy of the cosmological horizon must be an increasing (or nondecreasing) function of time [21].

For a spatially nonflat FRW universe, the dynamical apparent horizon takes the form [39]

$$\tilde{r}_A = \left(H^2 + \frac{K}{a^2}\right)^{-1/2}, \quad (19)$$

which, in the case of a flat universe ($K = 0$), reduces to the Hubble horizon, i.e., $\tilde{r}_A = H^{-1}$. On the apparent horizon, the associated Hawking temperature is defined as [21]

$$T_{\rm A} = \frac{1}{2\pi\tilde{r}_{\rm A}}\left(1 - \frac{\dot{\tilde{r}}_{\rm A}}{2H\tilde{r}_{\rm A}}\right), \tag{20}$$

where the condition $\frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} < 1$ is necessary due to having a positive temperature. Cai *et al.* [40], using the tunneling approach, proved that there is indeed a Hawking radiation with a temperature (20), for a locally defined apparent horizon of a FRW universe with any spatial curvature. They also pointed out that an observer with the Kodoma vector inside the apparent horizon can measure the Hawking temperature (20).

The entropy of the matter inside the horizon satisfies the Gibbs equation [28],

$$T_{\rm A}{\rm d}S_{\rm m} = {\rm d}E_{\rm m} + p_{\rm m}{\rm d}V, \tag{21}$$

where $E_{\rm m} = \rho_{\rm m}V$ and $V = \frac{4\pi}{3}\tilde{r}_{\rm A}^3$ is the volume of the dynamical apparent horizon $\tilde{r}_{\rm A}$ containing the matter. Here, we have assumed the local equilibrium hypothesis to hold. This requires that the temperature $T_{\rm m}$ of the matter content inside the apparent horizon should be in equilibrium with the temperature $T_{\rm A}$ associated with the apparent horizon, so we have $T_{\rm m} = T_{\rm A}$. We notice that the assumption of the thermal equilibrium in the cosmological setting is ideal because the temperature of the matter field at the present time differs much from that of the horizon $T_{{\rm A}_0} \sim 10^{-22}$ K. Hence, the systems must interact for some length of time before they can attain thermal equilibrium. Although in this case the local equilibrium hypothesis may no longer hold [41], Karami and Ghaffari [42] showed that the contribution of the heat flow between the horizon and the fluid in the GSL in nonequilibrium thermodynamics is very small, $O(10^{-7})$. Therefore, the equilibrium thermodynamics are still preserved. In general, when we consider the thermal equilibrium state of the Universe, the temperature of the Universe is associated with the apparent horizon.

Taking the time derivative of Eq. (21) and using (16) gets the equation governing the evolution of the matter entropy as

$$T_{\rm A}\dot{S}_{\rm m} = 4\pi\tilde{r}_{\rm A}^3(\rho_{\rm m} + p_{\rm m})\left(\frac{\dot{\tilde{r}}_{\rm A}}{\tilde{r}_{\rm A}} - H - \frac{1}{4}\frac{\dot{E}}{E}\right). \tag{22}$$

Using Eq. (7), this can be rewritten as

$$T_{\rm A}\dot{S}_{\rm m} = \frac{-\tilde{r}_{\rm A}^2}{2GE(\phi)}\left(\dot{\tilde{r}}_{\rm A} - H\tilde{r}_{\rm A} - \frac{1}{4}\frac{\dot{E}}{E}\tilde{r}_{\rm A}\right) \times\left[\left(2\dot{H} - \frac{2K}{a^2} - H\frac{\rm d}{{\rm d}t} + \frac{{\rm d}^2}{{\rm d}t^2}\right)F(\phi) + Z(\phi)\dot{\phi}^2\right]. \tag{23}$$

The geometric entropy in the scalar-tensor gravity is given by [27]

$$S_{\rm A} = \frac{AF(\phi)}{4G}, \tag{24}$$

where ${\rm A} = 4\pi\tilde{r}_{\rm A}^2$ is the area of the apparent horizon. Taking the time derivative of Eq. (24) and using (20) yields the evolution of the horizon entropy as

$$T_{\rm A}\dot{S}_{\rm A} = \frac{\tilde{r}_{\rm A}}{4GH}\left(2H - \frac{\dot{\tilde{r}}_{\rm A}}{\tilde{r}_{\rm A}}\right)\left(\frac{2\dot{\tilde{r}}_{\rm A}}{\tilde{r}_{\rm A}} + \frac{\rm d}{{\rm d}t}\right)F(\phi). \tag{25}$$

Now, according to the GSL of gravitational thermodynamics, we can consider the entropy of the Universe as the sum of the entropy of the matter inside the horizon, and the horizon entropy. Adding Eqs. (23) and (25) and using the auxiliary relation

$$\dot{\tilde{r}}_{\rm A} = H\tilde{r}_{\rm A}^3\left(\frac{K}{a^2} - \dot{H}\right), \tag{26}$$

the GSL in scalar-tensor gravity reads

$$T_{\rm A}\dot{S}_{\rm tot} = \frac{1}{4G}\left(H^2 + \frac{K}{a^2}\right)^{-5/2} \times[\mathcal{J}_1F(\phi) + \mathcal{J}_2\dot{F}(\phi) + \mathcal{J}_3(Z(\phi)\dot{\phi}^2 + \ddot{F}(\phi))], \tag{27}$$

where

$$\mathcal{J}_1 = \left(\frac{K}{a^2} - \dot{H}\right)\left\{2H\left[2H^2\left(1 - \frac{1}{E}\right) + \dot{H}\left(1 - \frac{2}{E}\right) + \frac{K}{a^2}\right] - \frac{\dot{E}}{E^2}\left(H^2 + \frac{K}{a^2}\right)\right\}, \tag{28}$$

$$\mathcal{J}_2 = \frac{K}{a^2}\left(\frac{K}{a^2} + \dot{H} + 3H^2\right) + H^2\left[2H^2\left(1 - \frac{1}{E}\right) + \dot{H}\left(1 - \frac{2}{E}\right)\right] - \frac{H\dot{E}}{2E^2}\left(H^2 + \frac{K}{a^2}\right), \tag{29}$$

$$\mathcal{J}_3 = \frac{2H}{E}(\dot{H} + H^2) + \frac{\dot{E}}{2E^2}\left(H^2 + \frac{K}{a^2}\right), \tag{30}$$

and $S_{\rm tot} = S_{\rm m} + S_{\rm A}$. Equation (27) shows that the validity of the GSL, i.e., $T_{\rm A}\dot{S}_{\rm tot} \geq 0$, depends on the scalar-tensor gravity model. For instance, in the Einstein gravity, i.e., $F(\phi) = E(\phi) = 1$ and $Z(\phi) = U(\phi) = 0$, the GSL (27) yields

$$T_A\dot{S}_{tot} = \frac{H}{2G}\frac{(\dot{H}-\frac{K}{a^2})^2}{(H^2+\frac{K}{a^2})^{5/2}} \ge 0, \tag{31}$$

which shows that the GSL in Einstein's gravity is always satisfied.

Here, following [43], we try to rewrite the GSL (27) in terms of the effective equation of state (EoS) parameter w_{eff} defined as

$$w_{eff} = \frac{p_{eff}}{\rho_{eff}} = -1 - \frac{2}{3}\left(\frac{\dot{H}-\frac{K}{a^2}}{H^2+\frac{K}{a^2}}\right), \tag{32}$$

where we have used Eqs. (10) and (11). From Eqs. (19), (26), and (32), one can get

$$\frac{\dot{\tilde{r}}_A}{\tilde{r}_A} = \frac{3}{2}H(1+w_{eff}). \tag{33}$$

Substituting this into Eqs. (22) and (25) yields

$$T_A\dot{S}_m = \frac{\tilde{r}_A^3}{4G}\rho_m(1+w_m)\left[H(1+3w_{eff}) - \frac{1}{2}\frac{\dot{E}(\phi)}{E(\phi)}\right], \tag{34}$$

$$T_A\dot{S}_A = \frac{\tilde{r}_A}{8G}(1-3w_{eff})[3H(1+w_{eff})F(\phi)+\dot{F}(\phi)], \tag{35}$$

where $w_m = p_m/\rho_m$ is the EoS parameter of the ordinary matter. In the framework of Einstein gravity, i.e., $F(\phi) = E(\phi) = 1$ and $Z(\phi) = U(\phi) = 0$; for a flat FRW universe dominated by a single fluid ($H^2 = \rho_f/3$) filling the volume enclosed by the apparent (Hubble) horizon ($\tilde{r}_A = H^{-1}$), we have $w_{eff} = w_m = w_f$ and Eq. (34) reduces to Eq. (11) in [43].

In the absence of the chameleon scalar field, i.e., $E(\phi) = 1$, Eq. (34) reduces to

$$T_A\dot{S}_m = \frac{H\tilde{r}_A^3\rho_m}{4G}(1+w_m)(1+3w_{eff}). \tag{36}$$

Note that for the ordinary matter we have $w_m \ge 0$; hence, the contribution of the matter entropy in the GSL will be positive or nil for $w_{eff} \ge -1/3$ and negative otherwise.

For a chameleon scalar field minimally coupled to the Ricci scalar curvature, i.e., $F(\phi) = 1$, Eq. (35) yields

$$T_A\dot{S}_A = \frac{3H\tilde{r}_A}{8G}(1+w_{eff})(1-3w_{eff}), \tag{37}$$

which shows that for $-1 \le w_{eff} \le 1/3$ the horizon entropy has a positive or nil contribution in the GSL.

Adding Eqs. (34) and (35) gives the GSL as

$$T_A\dot{S}_{tot} = \frac{\tilde{r}_A}{8G}\left\{(1-3w_{eff})[3H(1+w_{eff})F(\phi)+\dot{F}(\phi)] + 2\tilde{r}_A^2\rho_m(1+w_m)\left[H(1+3w_{eff}) - \frac{1}{2}\frac{\dot{E}(\phi)}{E(\phi)}\right]\right\}. \tag{38}$$

In the Einstein gravity, i.e., $F(\phi) = E(\phi) = 1$ and $Z(\phi) = U(\phi) = 0$, we have $\tilde{r}_A^2\rho_m = \tilde{r}_A^2\rho_{eff} = 3$ and $w_m = w_{eff}$ and then the GSL (38) reduces to

$$T_A\dot{S}_{tot} = \frac{9H\tilde{r}_A}{8G}(1+w_{eff})^2 \ge 0, \tag{39}$$

which is always respected. This can also be obtained by replacing Eqs. (19) and (32) with (31).

Note that in general one cannot explore the validity of the GSL (38) in terms of w_{eff}, explicitly, even in some special cases like $E(\phi) = 1$ and $F(\phi) = 1$. To do so, we need to solve the set of Eqs. (7), (8), and (16), numerically, to obtain the evolutions of $a(t)$ (or H), ϕ, and ρ_m.

In what follows, we are interested in examining the validity of the GSL for some viable scalar-tensor gravity models. We further assume the Universe to be spatially flat, i.e., $K = 0$, which is compatible with the recent observations [1].

IV. MODEL I: BRANS-DICKE GRAVITY

The action of Brans-Dicke (BD) theory is given by [44]

$$I = \int d^4x\sqrt{-g}\left[\frac{1}{2k^2}\left(\phi R - \frac{\omega}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) + L_m\right], \tag{40}$$

where ω is the dimensionless BD parameter. In [45], it was shown that the BD scalar-tensor theory of gravitation (40) brings a negligible correction to the matter density component of the Friedmann equation. It was pointed out that if this correction is found to be nonzero, data can favor this model and hence this theory turns out to be the most powerful candidate in place of the standard Einstein cosmological model with the cosmological constant.

By comparing the actions (40) and (1), one can get

$$F(\phi) = \phi, \quad Z(\phi) = \frac{\omega}{\phi}, \quad U(\phi) = 0, \quad E(\phi) = 1. \tag{41}$$

Substituting the above relations into Eqs. (8) and (16), for a flat universe ($K = 0$) one can obtain

$$\ddot{\phi} + 3H\dot{\phi} = \frac{3\phi}{\omega}(\dot{H}+2H^2) + \frac{\dot{\phi}^2}{2\phi}, \tag{42}$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \tag{43}$$

Also, the GSL (27) for a flat universe ($K = 0$) reduces to

$$T_A \dot{S}_{tot} = \frac{2\pi}{H^4}\left\{2\phi \dot{H}^2 - \dot{\phi}\dot{H}H + 2(\dot{H}+H^2)\left(\ddot{\phi} + \frac{\omega\dot{\phi}^2}{\phi}\right)\right\}, \quad (44)$$

where we take $k^2 = 8\pi G = 1$. Equation (43) for the pressureless matter (i.e., $p_m = 0$) yields

$$\rho_m = \rho_{m_0}\left(\frac{a}{a_0}\right)^{-3}. \quad (45)$$

Substituting Eqs. (41) and (45) into the second Friedmann equation (7) yields

$$-2\phi\dot{H} = \rho_{m_0}\left(\frac{a}{a_0}\right)^{-3} + \omega\frac{\dot{\phi}^2}{\phi} + \ddot{\phi} - H\dot{\phi}. \quad (46)$$

To obtain the evolutionary behavior of the GSL (44), we first need to know the time evolution of both the Hubble parameter $H(t)$ and the scalar field $\phi(t)$. To do so, one can obtain $a(t)$ and $\phi(t)$ by the numerical solving of Eqs. (42) and (46). Taking $\Omega_{m_0} = \rho_{m_0}/(3H_0^2) = 0.27$ [2] and $\omega = 1.2$ [46], and using the initial values $a(1) = 1$, $\dot{a}(1) = 0.84$, $\phi(1) = 1.5$, and $\dot{\phi}(1) = 1$ [46], the variations of the scale factor, the Hubble parameter, and the scalar field versus the redshift $z = \frac{a_0}{a} - 1$ are plotted in Figs. 1(a), 1(b), and 1(c), respectively. Also, the evolutionary behaviors of the deceleration parameter,

$$q = -1 - \frac{\dot{H}}{H^2}, \quad (47)$$

and the effective EoS parameter w_{eff}, Eq. (32), in terms of the redshift are plotted in Figs. 1(d) and 1(e), respectively.

Figures 1(a) to 1(e) show the following: (i) the scale factor, the Hubble parameter, and the scalar field, respectively, increases, decreases, and increases during the history of the Universe. (ii) The deceleration parameter shows a cosmic deceleration $q > 0$ to the acceleration $q < 0$ transition in the near past, which is compatible with the observations [47]. (iii) The effective EoS parameter w_{eff} at late times $z \to -1$ goes to -0.6, which behaves like the quintessence model [48].

With the help of numerical results obtained for the Hubble parameter and the scalar field presented in Figs. 1(b) and 1(c), the variation of the GSL (44) versus z is plotted in Fig. 1(f). The figure shows that the GSL in the BD gravity model (40) is satisfied during the late cosmological history of the Universe, i.e., $T_A\dot{S}_{tot} \geq 0$.

It is worth noting that, although in our numerical calculations following [46] we take the BD parameter $\omega = 1.2$, this clearly contradicts the solar system limit $\omega > 600$. In [49], it was shown that the BD parameter ω asymptotically acquires a small value due to having an accelerating universe at the late time. There are also other evidences in the literature where a small ω has been supported. In the extended inflationary model, La and Steinhardt [50] showed that the required value for ω is 20. The structure formation in scalar-tensor theory also contradicts the solar system bound on ω [51]. Thus, the problem seems to appear in different scales (astronomical and cosmological). Considering ω to be a variable (as we consider in Sec. VII) having both decelerating and accelerating phases at different epochs, while large ω values occur due to local inhomogeneities in the astronomical scale to satisfy the solar system bound, may give a satisfactory answer to this question [49].

V. MODEL II: BD GRAVITY WITH A SELF INTERACTING POTENTIAL

The action of BD theory with a self interacting potential and a matter field is given by [49]

$$I = \int d^4x\sqrt{-g}\left[\frac{1}{2k^2}\left(\phi R - \frac{\omega}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) + L_m\right], \quad (48)$$

with

$$V(\phi) = \lambda\phi^4 - \frac{\mu_0^2}{a(t)^n}\phi^2. \quad (49)$$

Here, λ and μ_0 are two constants and n is a positive integer. In [49], it was shown that the model described by the action (48) and potential (49) can support the late-time accelerated phase of the Universe in BD cosmology. The geometric BD scalar field ϕ can play the role of the dynamical Λ and provide the missing energy. Authors of Ref. [49] also calculated different parameters like the time variation of gravitational coupling, the age of the Universe, and the luminosity-distance redshift relation and show that all of these cosmological parameters agree quite well with the observations.

By comparing the actions (48) and (1), we find

$$F(\phi) = \phi, \quad Z(\phi) = \frac{\omega}{\phi}, \quad U(\phi) = \frac{V(\phi)}{2}, \quad E(\phi) = 1. \quad (50)$$

Inserting the above relations into Eqs. (8) and (16), for a flat universe one can obtain

$$\frac{2\omega}{\phi}(\ddot{\phi} + 3H\dot{\phi}) = 6(\dot{H} + 2H^2) + \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - 4\lambda\phi^3 + \mu_0^2\frac{\phi}{a^n}\left(2 - n\frac{\dot{a}/a}{\dot{\phi}/\phi}\right), \quad (51)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \quad (52)$$

where the evolution of ρ_m for the pressureless matter ($p_m = 0$) is the same as that obtained in (45).

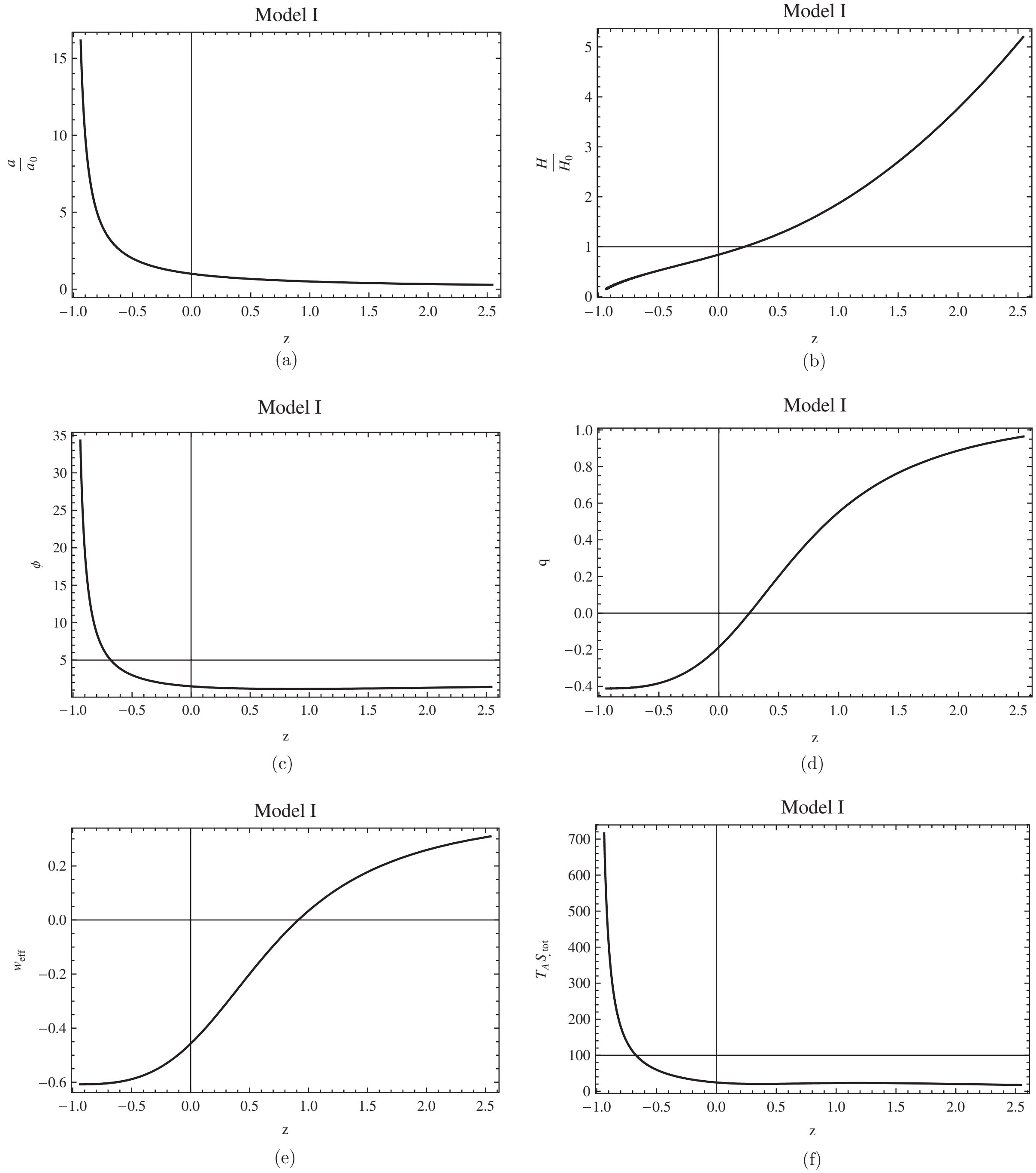

FIG. 1. Variations of the scale factor a, the Hubble parameter H, the scalar field ϕ, the deceleration parameter q, the effective EoS parameter $w_{\rm eff}$, and the GSL, $T_{\rm A}\dot{S}_{\rm tot}$, versus the redshift z for model I (40). Initial values are $a(1)=1$, $\dot{a}(1)=0.84$, $\phi(1)=1.5$, and $\dot{\phi}(1)=1$ [46]. Auxiliary parameters are $\Omega_{\rm m_0}=0.27$ [2] and $\omega=1.2$ [46]. Here, $t_0=1/H_0$.

Here, the GSL (27) for a spatially flat universe takes the form

$$T_{\rm A}\dot{S}_{\rm tot}=\frac{2\pi}{H^4}\left\{\dot{H}(2\dot{H}\phi-H\dot{\phi})+2(\dot{H}+H^2)\left(\ddot{\phi}+\frac{\omega}{\phi}\dot{\phi}^2\right)\right\}, \tag{53}$$

where $k^2=8\pi G=1$.

Inserting Eqs. (45) and (50) into the second Friedmann equation (7), one can find

$$-2\phi\dot{H}=\rho_{\rm m_0}\left(\frac{a}{a_0}\right)^{-3}+\omega\frac{\dot{\phi}^2}{\phi}+\ddot{\phi}-H\dot{\phi}, \tag{54}$$

which is the same as Eq. (46) for the BD gravity model (40) because the form of potential of the scalar field does not appear explicitly in the second Friedmann equation (7).

Taking $\Omega_{m_0} = 0.27$ [2], , $\omega = 1.2$ [46], and $n = 1$ [49], the time evolution of both the scale factor $a(t)$ and the scalar field $\phi(t)$ can be obtained by numerical solving of Eqs. (51) and (54) with the initial values $a(1) = 1$, $\dot{a}(1) = 0.84$, $\phi(1) = -1.5$, and $\dot{\phi}(1) = 1$. Also, we set $\lambda = H_0^2$ and $\mu_0 = H_0$ to recast the differential Eqs. (51) and (54) in dimensionless form, which is more suitable for numerical integration. The numerical results obtained for a, H, ϕ, q, and $w_{\rm eff}$ are plotted in Figs. 2(a), 2(b), 2(c), 2(d), and 2(e), respectively. The figures show that (i) a, H, and ϕ, respectively, increases, decreases, and increases during the history of the Universe. (ii) The deceleration parameter shows a transition from the deceleration era $q > 0$ to the acceleration regime $q < 0$. At late times ($z \to -1$), the deceleration parameter approaches a de Sitter regime (i.e., $q \to -1$), as expected. (iii) The effective EoS parameter

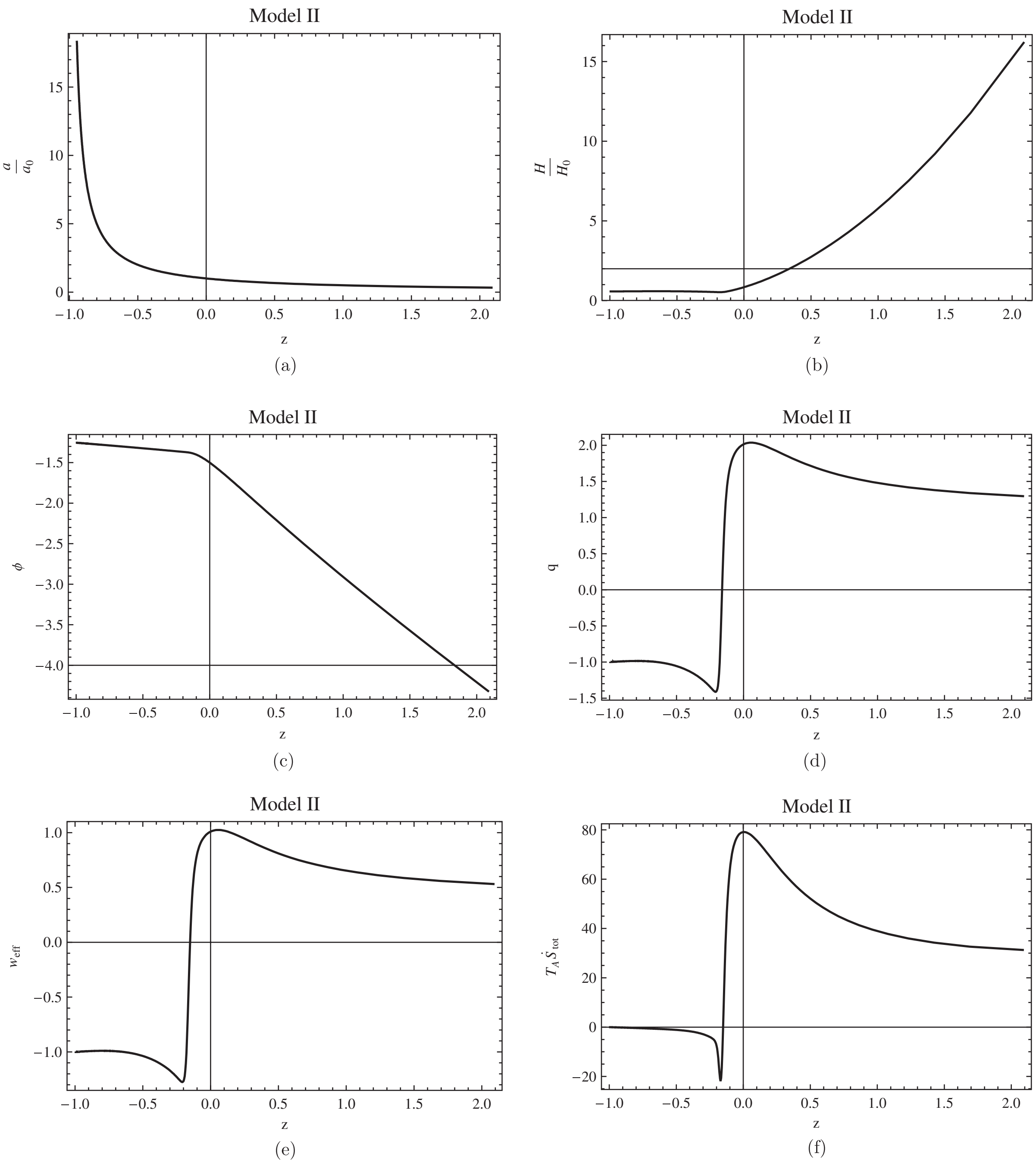

FIG. 2. Same as Fig. 1 but for model II (48). Initial values are $a(1) = 1$, $\dot{a}(1) = 0.84$, $\phi(1) = -1.5$, and $\dot{\phi}(1) = 1$. Auxiliary parameters are $\Omega_{m_0} = 0.27$ [2], $\omega = 1.2$ [46], and $n = 1$ [49]. Here, $t_0 = 1/H_0$, $\lambda = H_0^2$, and $\mu_0 = H_0$.

shows a transition from the quintessence state, $w_{\rm eff} > -1$, to the phantom regime, $w_{\rm eff} < -1$, in the future. Also, at late times we get $w_{\rm eff} \rightarrow -1$, which acts like the ΛCDM model.

The results of H and ϕ illustrated in Figs. 2(b) and 2(c) help us to obtain the variation of the GSL (53) for the BD gravity model (48) with a self interacting potential (49). The result is plotted in Fig. 2(f). The figure shows that the GSL for our model is satisfied from the past to the present epoch. But in the future the GSL is violated for $z < -0.15$.

VI. MODEL III: CHAMELEON GRAVITY

The action of chameleon gravity in the presence of matter is given by [52,53]

$$I = \int d^4x\sqrt{-g}\left[\frac{1}{2k^2}(R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi)) + f(\phi)L_{\rm m}\right], \tag{55}$$

where there is a nonminimal coupling term, $f(\phi)L_{\rm m}$, between the chameleon scalar field and the matter field. Note that the mass of the chameleon scalar field depends sensitively on the environment. In high density regions, the chameleon blends with its environment and becomes essentially invisible to searches for equivalence principle violation and the fifth force. In [16], it was pointed out that the chameleon-mediated force between the Earth and the Sun is suppressed by the thin-shell effect, which thereby ensures that solar system tests of gravity are satisfied. In [17], it was shown that the chameleons are also consistent with cosmological constraints on the existence of non-minimally coupled scalars, such as the bound on the time variation of G from nucleosynthesis.

Comparing the chameleon gravity action (55) with action (1), one can get

$$F(\phi) = 1, \quad Z(\phi) = 1, \quad U(\phi) = V(\phi), \quad E(\phi) = f(\phi). \tag{56}$$

With the help of these relations, Eqs. (8) and (16) for a flat universe read

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + \frac{1}{4}(\rho_{\rm m} - 3p_{\rm m})f_{,\phi} = 0, \tag{57}$$

$$\dot{\rho}_{\rm m} + 3H(\rho_{\rm m} + p_{\rm m}) = -\frac{3}{4}\frac{\dot{f}(\phi)}{f(\phi)}(\rho_{\rm m} + p_{\rm m}), \tag{58}$$

where we take $k^2 = 8\pi G = 1$. Taking the integration of Eq. (58) for the pressureless matter ($p_{\rm m} = 0$) gives

$$\rho_{\rm m} = \rho_{\rm m_0}\left(\frac{a}{a_0}\right)^{-3}\left(\frac{f(\phi)}{f_0}\right)^{-\frac{3}{4}}. \tag{59}$$

Substituting the relations (56) into (27) gives the GSL for a flat universe as

$$T_{\rm A}\dot{S}_{\rm tot} = \frac{2\pi}{H^4}\left\{-2\dot{H}(2H^2 + \dot{H}) + \frac{1}{2f(\phi)}(2\dot{H} + \dot{\phi}^2)\left(4(H^2 + \dot{H}) + \frac{\dot{f}}{f}H\right)\right\}. \tag{60}$$

According to [53], we consider both $f(\phi)$ and the potential $V(\phi)$ that appeared in (55) to behave exponentially as

$$f(\phi) = f_0 e^{b_1\phi}, \qquad V(\phi) = V_0 e^{b_2\phi}, \tag{61}$$

where f_0, V_0, b_1, and b_2 are arbitrary constants. There is no *a priori* physical motivation for these choices, so it is only purely phenomenological, which leads to the desired behavior of the phantom crossing model of the Universe.

Using Eqs. (59) and (61), the evolution Eq. (57) for the pressureless matter ($p_{\rm m} = 0$) gives

$$\ddot{\phi} + 3H\dot{\phi} + b_2V_0e^{b_2\phi} + \frac{1}{4}b_1f_0\rho_{\rm m_0}\left(\frac{a}{a_0}\right)^{-3}e^{b_1\phi/4} = 0. \tag{62}$$

Also, the second Friedmann equation (7) reads

$$-2\dot{H} = f_0\rho_{\rm m_0}\left(\frac{a}{a_0}\right)^{-3}e^{b_1\phi/4} + \dot{\phi}^2. \tag{63}$$

Finally, the GSL (60) takes the form

$$T_{\rm A}\dot{S}_{\rm tot} = \frac{2\pi}{H^4}\left\{-2\dot{H}(2H^2 + \dot{H}) + \frac{1}{2f_0e^{b_1\phi}}(2\dot{H} + \dot{\phi}^2)(4(H^2 + \dot{H}) + b_1\dot{\phi}H)\right\}. \tag{64}$$

From numerical solving of Eqs. (62) and (63), one can obtain $a(t)$ and $\phi(t)$. To do so, we take $\Omega_{\rm m_0} = 0.27$ [2], $f_0 = -10$, and $b_1 = b_2 = -1$ [53] and use the initial values $a(1) = 1$, $\dot{a}(1) = 1$, $\phi(1) = 1$, and $\dot{\phi}(1) = -2$. The variations of a, H, ϕ, q, and $w_{\rm eff}$ versus the redshift are plotted in Figs. 3(a) to 3(e). The figures show that (i) the scale factor increases when the time increases. The Hubble parameter decreases with increasing time and then increases to approach a constant value. The scalar field decreases with increasing time and increases at late times. (ii) The deceleration parameter shows a cosmic transition from $q > 0$ to $q < 0$ in the near past, which is compatible with the observations [47]. It also approaches a de Sitter regime at late times, as expected. (iii) The effective EoS parameter can justify the transition from the quintessence state ($w_{\rm eff} > -1$) to the phantom regime ($w_{\rm eff} < -1$) in the near past, as indicated by recent observations [54]. This is also in good agreement with that obtained in [53]. The

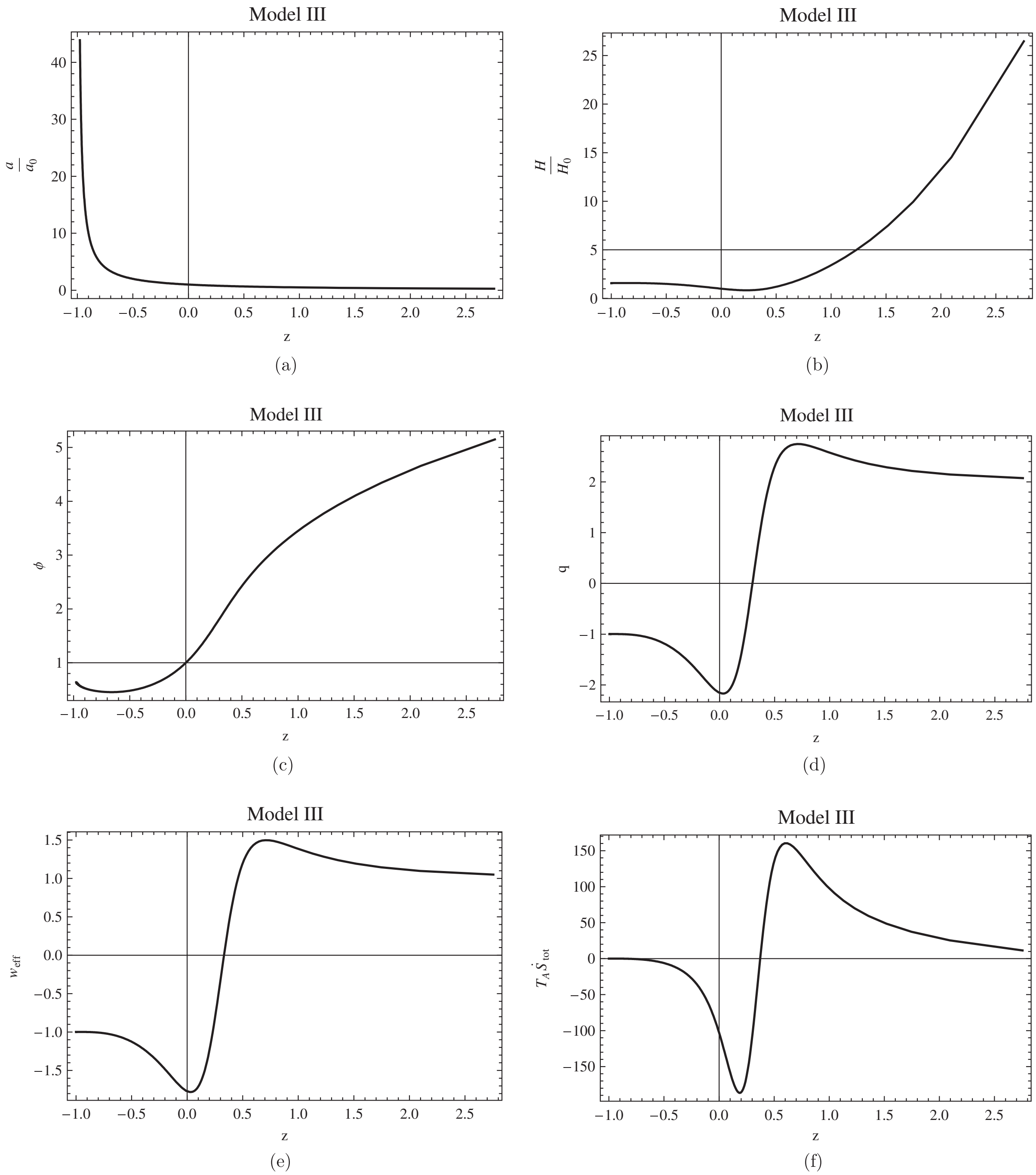

FIG. 3. Same as Fig. 1 but for model III (55). Initial values are $a(1) = 1$, $\dot{a}(1) = 1$, $\phi(1) = 1$, and $\dot{\phi}(1) = -2$. Auxiliary parameters are $\Omega_{m_0} = 0.27$ [2], $f_0 = -10$, and $b_1 = b_2 = -1$ [53]. Here, $t_0 = 1/H_0$ and $V_0 = H_0^2$.

effective EoS parameter also behaves like the ΛCDM model at late times.

With the help of numerical results obtained for the Hubble parameter and the scalar field illustrated in Figs. 3(b) and 3(c), the variation of the GSL (64) versus the redshift for the chameleon gravity model is plotted in Fig. 3(f). The figure shows that the GSL in this model is violated for the range of $-0.88 < z < 0.37$. This is in contrast with that obtained in [53]. The authors of Ref. [53] investigated the GSL in flat FRW chameleon cosmology and showed that, in an expanding universe, the GSL is always respected. This contradiction comes back to the definition of energy and pressure in the Gibbs equation (21). In [53], Farajollahi *et al.* considered the effective (total) energy $E_{\rm eff} = \rho_{\rm eff} V$ and pressure $p_{\rm eff}$ instead of $E_{\rm m} = \rho_{\rm m} V$ and $p_{\rm m}$ as we have in our case. Therefore, in

[53], the GSL is defined as $T_A\dot{S}_{tot} = T_A(\dot{S}_{eff} + \dot{S}_A)$ in which

$$T_A\dot{S}_{eff} = \frac{3H\tilde{r}_A}{4G}(1 + w_{eff})(1 + 3w_{eff}),$$

and $T_A\dot{S}_A$ is given by Eq. (37). Finally, the GSL yields Eq. (39), which is nothing but the GSL in the Einstein gravity. This confirms that the GSL investigated in [53] does not belong to the chameleon gravity.

VII. MODEL IV: CHAMELEONIC GENERALIZED BD GRAVITY

The action of the chameleonic generalized BD gravity model is given by [46]

$$I = \int d^4x\sqrt{-g}\left[\frac{1}{2k^2}\left(\phi R - \frac{\omega(\phi)}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) + f(\phi)L_m\right]. \quad (65)$$

In [55], it was pointed out that with the scalar field dependent BD parameter $\omega(\phi)$ one can have a decelerating radiation dominated era in the early time and an accelerated matter dominated era in the late time.

Comparing Eq. (65) with action (1) gives

$$F(\phi) = \phi, \quad Z(\phi) = \frac{\omega(\phi)}{\phi}, \quad U(\phi) = 0, \quad E(\phi) = f(\phi). \quad (66)$$

Using these, Eqs. (8) and (16) for a flat universe read

$$\ddot{\phi} + 3H\dot{\phi} = \frac{1}{2\omega(\phi) + 3}\left[(\rho_m - 3p_m)\left(f(\phi) - \frac{1}{2}\phi f_{,\phi}\right) - \omega_{,\phi}\dot{\phi}^2\right], \quad (67)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -\frac{3}{4}\frac{\dot{f}(\phi)}{f(\phi)}(\rho_m + p_m), \quad (68)$$

where $k^2 = 8\pi G = 1$. The solution of Eq. (68) for the pressureless matter ($p_m = 0$) yields the same result obtained in (59).

With the help of relations (66), the GSL (27) for a flat universe yields

$$T_A\dot{S}_{tot} = \frac{2\pi}{H^4}\Bigg\{(2H^2 + \dot{H})(\dot{\phi}H - 2\phi\dot{H}) + \frac{1}{2f(\phi)}\left(4(H^2 + \dot{H}) + H\frac{\dot{f}}{f}\right) \times \left(2\phi\dot{H} - \dot{\phi}H + \ddot{\phi} + \omega(\phi)\frac{\dot{\phi}^2}{\phi}\right)\Bigg\}. \quad (69)$$

According to [46], we take

$$f(\phi) = f_0e^{b\phi}, \qquad \omega(\phi) = \omega_0\phi^n. \quad (70)$$

In [46], it was shown that the model (70) can predict the late-time acceleration and phantom divide line crossing as well as fit the observational data for velocity drift and distance modulus better that the Chevallier-Polarski-Linder [56] and ΛCDM models, respectively, subject to constraints on the model parameters.

Substituting Eqs. (59) and (70) into (67) for the pressureless matter ($p_m = 0$) gives

$$\ddot{\phi} + 3H\dot{\phi} = \frac{1}{3 + 2\omega_0\phi^n} \times \left[f_0\rho_{m_0}\left(\frac{a}{a_0}\right)^{-3}\left(1 - \frac{b}{2}\phi\right)e^{b\phi/4} - n\omega_0\phi^{n-1}\dot{\phi}^2\right]. \quad (71)$$

Also, the second Friedmann equation (7) gives

$$-2\phi\dot{H} = f_0\rho_{m_0}\left(\frac{a}{a_0}\right)^{-3}e^{b\phi/4} + \omega_0\phi^{n-1}\dot{\phi}^2 + \ddot{\phi} - H\dot{\phi}. \quad (72)$$

Moreover, the GSL (69) reduces to

$$T_A\dot{S}_{tot} = \frac{2\pi}{H^4}\Bigg\{(2H^2 + \dot{H})(\dot{\phi}H - 2\phi\dot{H}) + \frac{1}{2f_0e^{b\phi}}(4(H^2 + \dot{H}) + b\dot{\phi}H) \times (2\phi\dot{H} - \dot{\phi}H + \ddot{\phi} + \omega_0\phi^{n-1}\dot{\phi}^2)\Bigg\}. \quad (73)$$

Taking $\Omega_{m_0} = 0.27$ [2], $\omega_0 = 1.2$, $n = -2$, $f_0 = -7$, and $b = -0.4$ [46], the time evolution of both the scale factor $a(t)$ and the scalar field $\phi(t)$ can be obtained by numerical solving of Eqs. (71) and (72) with the initial values $a(1) = 1$, $\dot{a}(1) = 1$, $\phi(1) = -6.5$, and $\dot{\phi}(1) = 0.1$. Figures 4(a) to 4(e) show the following: (i) the scale factor and the Hubble parameter, respectively, increases and decreases with increasing time. (ii) The scalar field with increasing time decreases to a minimum and then increases to approach a constant value. (iii) The deceleration parameter shows the cosmic transition $q > 0 \to q < 0$ in the near past, as indicated by recent observations [47]. It also approaches a de Sitter regime at late times, as expected. (iv) The effective EoS parameter at late times behaves like the ΛCDM model ($w_{eff} \to -1$).

The results of H and ϕ illustrated in Figs. 4(b) and 4(c) help us to obtain the variation of the GSL (73) for the chameleonic generalized BD gravity model (65). The result is plotted in Fig. 4(f). The figure shows that the GSL for this

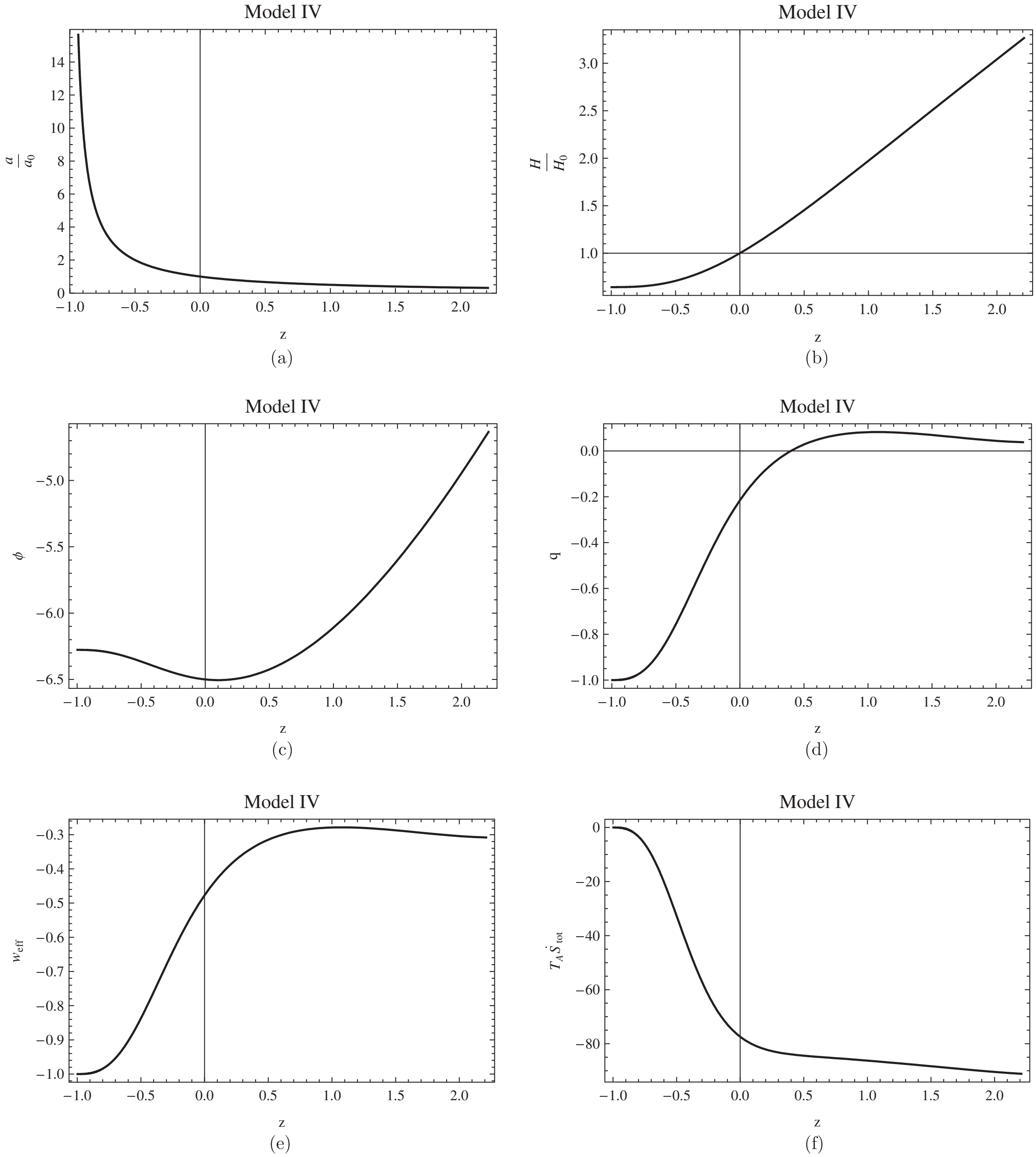

FIG. 4. Same as Fig. 1 but for model IV (65). Initial values are $a(1) = 1$, $\dot{a}(1) = 1$, $\phi(1) = -6.5$, and $\dot{\phi}(1) = 0.1$. Auxiliary parameters are $\Omega_{m_0} = 0.27$ [2], $\omega_0 = 1.2$, $n = -2$, $f_0 = -7$, and $b = -0.4$ [46]. Here, $t_0 = 1/H_0$.

model is violated during the late cosmological history of the Universe.

VIII. MODEL V: CHAMELEONIC BD GRAVITY WITH A SELF INTERACTING POTENTIAL

Within the framework of chameleonic BD gravity with a self interacting potential, the action is given by [57]

$$I = \int d^4x\sqrt{-g} \times \left[\frac{1}{2k^2}\left(\phi R - \frac{\omega}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) + f(\phi)L_{\rm m}\right]. \quad (74)$$

In [57], the cosmological applications of interacting holographic DE in BD theory with the chameleon scalar field that is nonminimally coupled to the matter field described by action (74) was investigated. It was found that in this model the phantom crossing can be constructed if the model parameters are chosen suitably.

Here, in comparison with action (1), we have

$$F(\phi)=\phi,\quad Z(\phi)=\frac{\omega}{\phi},\quad U(\phi)=\frac{V(\phi)}{2},\quad E(\phi)=f(\phi). \tag{75}$$

Using the above relations, the evolution Eq. (8) and the continuity Eq. (16) for a flat universe take the forms

$$\ddot{\phi}+3H\dot{\phi}=\frac{1}{2\omega+3}\left[(\rho_{\rm m}-3p_{\rm m})\left(f(\phi)-\frac{1}{2}\phi f_{,\phi}\right)+2V(\phi)-\phi V_{,\phi}\right], \tag{76}$$

$$\dot{\rho}_{\rm m}+3H(\rho_{\rm m}+p_{\rm m})=-\frac{3}{4}\frac{\dot{f}(\phi)}{f(\phi)}(\rho_{\rm m}+p_{\rm m}), \tag{77}$$

where $k^2=8\pi G=1$. The solution of Eq. (77) for the pressureless matter ($p_{\rm m}=0$) is same as that obtained in (59).

Inserting the relations (75) into (27) gives the GSL for the chameleonic BD gravity with a self interacting potential as

$$T_{\rm A}\dot{S}_{\rm tot}=\frac{2\pi}{H^4}\left\{(2H^2+\dot{H})(\dot{\phi}H-2\phi\dot{H})+\frac{1}{2f(\phi)}\left(4(H^2+\dot{H})+H\frac{\dot{f}}{f}\right)\times\left(2\phi\dot{H}-\dot{\phi}H+\ddot{\phi}+\omega\frac{\dot{\phi}^2}{\phi}\right)\right\}. \tag{78}$$

According to [16], we consider the inverse power-law potential

$$V(\phi)=\frac{M^{n+4}}{\phi^n}, \tag{79}$$

where M has units of mass and n is a positive constant. This kind of potential has the desired features for quintessence models of the Universe [58]. In [16], it was found that the energy scale M is generally constrained to be of the order of $(1\ \mathrm{mm})^{-1}$. Also the resulting bounds on the range of chameleon-mediated interactions in the atmosphere, in the solar system, and on cosmological scales today show that $n\le 2$.

We further take [46]

$$f(\phi)=f_0e^{b\phi}, \tag{80}$$

where f_0 and b are constant parameters.

Inserting Eqs. (59), (79), and (80) into (76), for the pressureless matter ($p_{\rm m}=0$), one can obtain

$$\ddot{\phi}+3H\dot{\phi}=\frac{1}{3+2\omega}\left[f_0\rho_{{\rm m}_0}\left(\frac{a}{a_0}\right)^{-3}\left(1-\frac{b}{2}\phi\right)e^{b\phi/4}+\frac{(n+2)M^{n+4}}{\phi^n}\right]. \tag{81}$$

Also, the second Friedmann equation (7) reduces to

$$-2\phi\dot{H}=f_0\rho_{{\rm m}_0}\left(\frac{a}{a_0}\right)^{-3}e^{b\phi/4}+\omega\frac{\dot{\phi}^2}{\phi}+\ddot{\phi}-H\dot{\phi}. \tag{82}$$

Furthermore, the GSL (78) yields

$$T_{\rm A}\dot{S}_{\rm tot}=\frac{2\pi}{H^4}\left\{(2H^2+\dot{H})(\dot{\phi}H-2\phi\dot{H})+\frac{1}{2f_0e^{b\phi}}(4(H^2+\dot{H})+b\dot{\phi}H)\times\left(2\phi\dot{H}-\dot{\phi}H+\ddot{\phi}+\omega\frac{\dot{\phi}^2}{\phi}\right)\right\}. \tag{83}$$

From Eqs. (81) and (82), the scale factor $a(t)$ and the scalar field $\phi(t)$ can be obtained, numerically. To do so, we take $\Omega_{{\rm m}_0}=0.27$ [2], $\omega=1.2$, $f_0=-7$, $b=-0.4$ [46], and $n=2$ [16] and use the initial values $a(1)=1$, $\dot{a}(1)=1$, $\phi(1)=1$, and $\dot{\phi}(1)=-1.4$. The results are plotted in Fig. 5. The figures show that (i) the scale factor is an increasing function of time, as expected for an expanding universe. (ii) The Hubble parameter and the scalar field decrease with increasing time, approach to a minimum in the future, and then increase when the time increases. (iii) The deceleration parameter at late times goes to -1 which acts like the de Sitter model. It also shows a cosmic transition from $q>0$ to $q<0$ in the future. (iv) The effective EoS parameter at late times behaves like the ΛCDM model. It also shows the phantom divide line crossing in the future.

Using the numerical results obtained for H and ϕ, the evolutionary behavior of the GSL (83) for the chameleonic BD gravity with a self interacting potential is plotted in Fig. 5(f). The figure shows that the GSL for this model is satisfied from the past to the present epoch. But in the future the GSL is violated for $z<-0.53$.

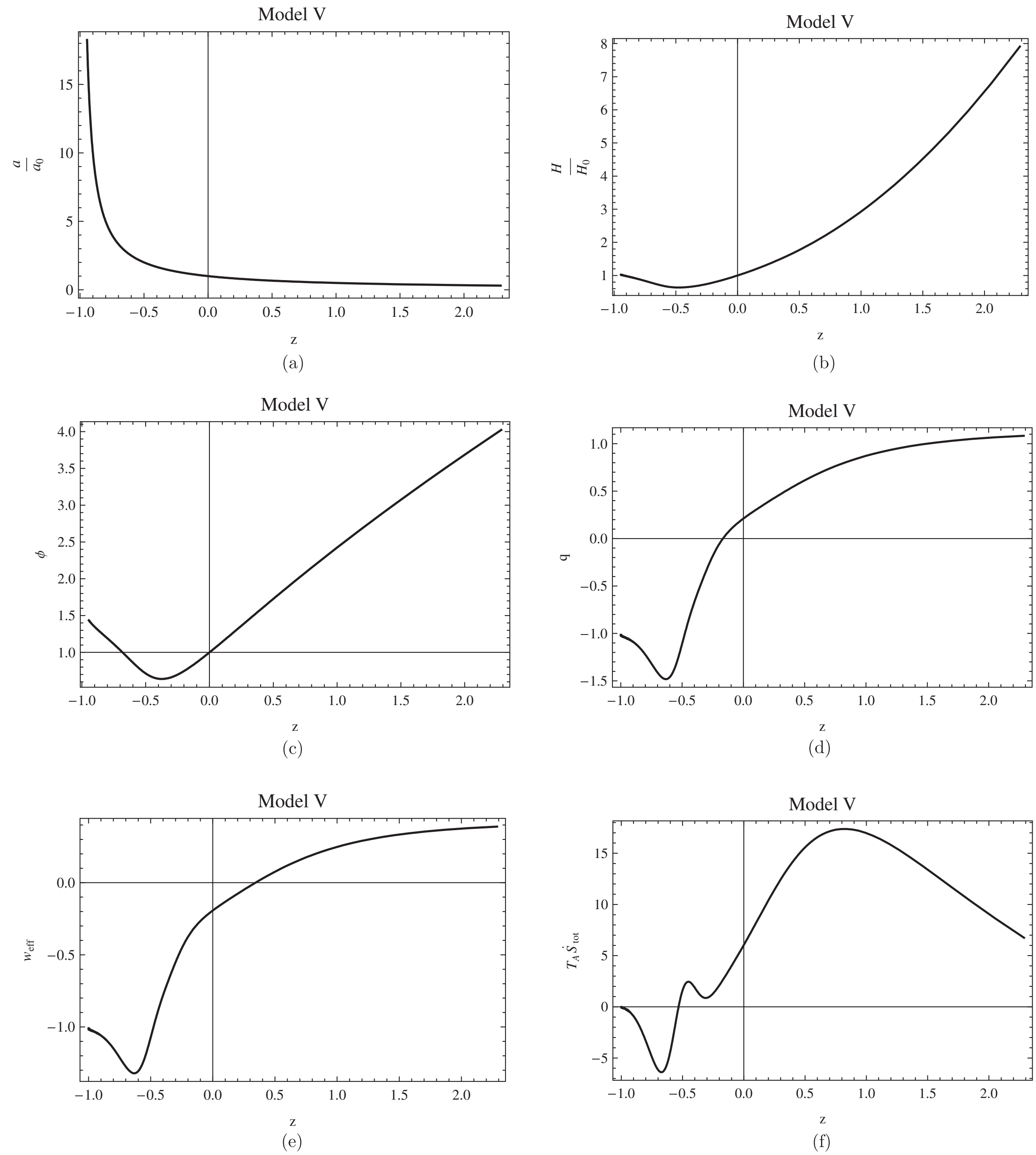

FIG. 5. Same as Fig. 1 but for model V (74). Initial values are $a(1) = 1$, $\dot{a}(1) = 1$, $\phi(1) = 1$, and $\dot{\phi}(1) = -1.4$. Auxiliary parameters are $\Omega_{m_0} = 0.27$ [2], $\omega = 1.2$, $f_0 = -7$, $b = -0.4$ [46], and $n = 2$ [16]. Here, $t_0 = 1/H_0$ and $M^{n+4} = H_0^2$.

IX. CONCLUSIONS

Here, we investigated the GSL in the framework of scalar-tensor gravity. In a general theory of scalar-tensor gravity, a scalar field can be nonminimally coupled both to the scalar curvature (as the Brans-Dicke field) and the matter Lagrangian (as the chameleon field) in the action. Hence, we extended the action of ordinary scalar-tensor gravity theory to the case in which there is a nonminimal coupling between the scalar field and the matter field. Then we derived the associated filed equations governing the gravity and the scalar field. For a FRW universe filled with the ordinary matter, we obtained the modified Friedmann equations as well as the evolution equation of the scalar field. We further assumed the boundary of the FRW universe to be enclosed by the dynamical apparent horizon that is in thermal equilibrium with the Hawking temperature. Then we obtained a general expression for the GSL of gravitational thermodynamics. For some viable scalar-tensor gravity models containing BD gravity, BD gravity with a self interacting potential, chameleon gravity, chameleonic generalized BD gravity, and chameleonic BD gravity with a self interacting potential, we first obtained the evolutionary behaviors of the matter density, the scale factor, the Hubble parameter, the scalar field, the deceleration parameter, and the effective EoS parameter. Then, we examined the validity of the GSL for the aforementioned models. Our results show the following.

(i) The aforementioned models can give rise to a late-time accelerated expansion phase for the Universe. The deceleration parameter for all the models shows a cosmic deceleration, $q > 0$, to acceleration, $q < 0$, transition. In the BD gravity model, the chameleon gravity model, and the chameleonic generalized BD gravity model, the cosmic transition from $q > 0$ to $q < 0$ occurs in the near past, which is compatible with the observations [47]. For all models but the BD gravity model, at late times ($z \to -1$), the deceleration parameter approaches a de Sitter regime (i.e., $q \to -1$), as expected.

(ii) The effective EoS parameter for the BD gravity model with a self interacting potential, the chameleon gravity model, and the chameleonic BD gravity model with a self interacting potential shows a transition from the quintessence state, $w_{\rm eff} > -1$, to the phantom regime, $w_{\rm eff} < -1$. For the chameleon gravity model, the transition from $w_{\rm eff} > -1$ to $w_{\rm eff} < -1$ occurs in the near past, as indicated by recent observations [54]. For all models but the BD gravity model, the effective EoS parameter at late times behaves like the ΛCDM model ($w_{\rm eff} \to -1$).

(iii) The GSL for the BD gravity model like the Einstein gravity is satisfied during the late cosmological history of the Universe. For the BD gravity model with a self interacting potential, the GSL is satisfied from the past to the present epoch. But in the future the GSL is violated for $z < -0.15$. For the BD gravity model with/without a self interacting potential, the contribution of the matter entropy in the GSL will be positive or nil for $w_{\rm eff} \geq -1/3$ and negative otherwise. For the chameleon gravity model, the GSL is violated for the range of $-0.88 < z < 0.37$. However, for $-1 \leq w_{\rm eff} \leq 1/3$, the horizon entropy has a positive or nil contribution in the GSL. For the chameleonic generalized BD gravity model, the GSL is violated during the late cosmological history of the Universe. Finally, for the chameleonic BD gravity model with a self interacting potential, the GSL is satisfied from the past to the present time. But in the future the GSL is violated for $z < -0.53$.

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