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Bispectrum of cosmological density perturbations in the most general second-order scalar-tensor theory

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We study the bispectrum of matter density perturbations induced by the large-scale structure formation in the most general second-order scalar-tensor theory that may possess the Vainshtein mechanism as a screening mechanism. On the basis of the standard perturbation theory, we derive the bispectrum being expressed by a kernel of the second-order density perturbations. We find that the leading-order kernel is characterized by one parameter, which is determined by the solutions of the linear density perturbations, the Hubble parameter, and the other function specifying nonlinear interactions. This is because our model, which may be equipped with the Vainshtein mechanism, includes only one simple function that describes mode couplings of the nonlinear interactions. This feature does not allow for varied behavior in the bispectrum of the matter density perturbations in the most general second-order scalar-tensor theory equipped with the Vainshtein mechanism. We exemplify the typical behavior of the bispectrum in a kinetic gravity braiding model.

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I. INTRODUCTION

Researchers are interested in modified gravity models as alternatives for explaining the accelerated expansion of the universe without introducing the cosmological constant [1–13]. The most general second-order scalar-tensor theory was first constructed by Horndeski [14] and was rediscovered in [15] as a generalization of the Galileon theories [16–36]. In addition to the possibility of constructing cosmological models with accelerated expansion, this theory has the following interesting features. The equation of motion is a second-order differential equation. Thus, an additional degree of freedom is not introduced, which is advantageous to avoid the appearance of ghosts. Furthermore, the Galileon theory is endowed with the Vainshtein mechanism [33], which is a screening mechanism useful for evading local gravity constraints. In the most general second-order scalar-tensor theory, the Vainshtein mechanism may work depending on the model parameters (e.g., [37–39]).

The results from the Planck satellite have shown that the primordial perturbations almost obey Gaussian statistics [40]. Even if the initial perturbations were completely Gaussian, the non-Gaussian nature of the density perturbations is induced in the large-scale structure formation through nonlinear fluid equations under the influence of the gravitational force. The bispectrum is often used to characterize the nonlinear and non-Gaussian nature of the density perturbations (e.g., [41–45]). Recently, the bispectrum and nonlinear features in the structure formation in the Galileon models have been investigated [46–52]. In the present paper, we focus on the bispectrum in the most general second-order scalar-tensor theory, which we regard as an effective theory, in order to elucidate the characteristic features of a wide class of modified gravity models. An advantage of such a general theory is that we can discuss general features of a wide class of modified gravity models, which is useful for forecasting their detectability in future large surveys.

In the present paper, we consider the bispectrum of the matter density perturbations induced in the large-scale structure formation after the matter-dominated era. We present an expression of the bispectrum in the most general second-order scalar-tensor theory based on the standard density perturbation theory, which is written in terms of a kernel of second-order perturbations. We find that the kernel is characterized by only *one* parameter, which is determined by the solutions of the linear density perturbations, the Hubble parameter, and the other function that describes the nonlinear interactions of the background universe. This paper is organized as follows. In Sec. II, we apply the standard perturbation theory to the most general second-order scalar-tensor theory that may possess the Vainshtein mechanism, and we find the solution of the second-order of density perturbations. In Sec. III, we present the expression of the bispectrum of the density perturbations, and we investigate the influence of the modification of gravity. The results are applied to a simple kinetic gravity braiding model in Sec. IV. Section V presents a summary and conclusions.

II. FORMULATION

We consider the most general second-order scalar-tensor theory on the expanding universe background. The action is given by

$$S = \int d^4x\sqrt{-g}(\mathcal{L}_{\rm GG} + \mathcal{L}_{\rm m}), \quad (1)$$

where we define

$$\mathcal{L}_{\rm GG} = K(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3], \quad (2)$$

with four arbitrary functions of ϕ and $X \coloneqq -(\partial\phi)^2/2$, K, G_3, G_4, and G_5. Furthermore, G_{iX} stands for $\partial G_i/\partial X$, R is the Ricci scalar, $G_{\mu\nu}$ is the Einstein tensor, and $\mathcal{L}_{\rm m}$ is the matter Lagrangian, which is assumed to be minimally coupled to gravity. This theory is found in [15] as a generalization of the Galileon theory, but is shown to be equivalent to Horndeski's theory in [16]. We consider a spatially flat expanding universe and the metric perturbations in the Newtonian gauge, whose line element is written as

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 - 2\Psi)d\mathbf{x}^2. \quad (3)$$

We define the scalar field with perturbations by

$$\phi \to \phi(t) + \delta\phi(t, \mathbf{x}), \quad (4)$$

with which we introduce $Q \coloneqq H\delta\phi/\dot{\phi}$.

We consider the case where the Vainshtein mechanism may work as a screening mechanism. The basic equations for the cosmological density perturbations are derived in Ref. [37]. Here we briefly review the method and the results. The basic equations of the gravitational and scalar fields are derived on the basis of the quasistatic approximation of the subhorizon scales. The models for which the Vainshtein mechanism works can be found as follows. The equations are derived by keeping the leading terms schematically written as $(\partial\partial Y)^n$, with $n \geq 1$, where ∂ denotes a spatial derivative and Y denotes any of Φ, Ψ, or Q. Such terms make a leading contribution of the order $(L_{\rm H}^2\partial\partial Y)^n$, where $L_{\rm H}$ is a typical horizon length scale. According to Ref. [37], from the gravitational field equation, we have

$$\nabla^2(\mathcal{F}_T\Psi - \mathcal{G}_T\Phi - A_1 Q) = \frac{B_1}{2a^2H^2}\mathcal{Q}^{(2)} + \frac{B_3}{a^2H^2}(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q), \quad (5)$$

$$\mathcal{G}_T\nabla^2\Psi = \frac{a^2}{2}\rho_{\rm m}\delta - A_2\nabla^2 Q - \frac{B_2}{2a^2H^2}\mathcal{Q}^{(2)} - \frac{B_3}{a^2H^2}(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial^i\partial^j Q) - \frac{C_1}{3a^4H^4}\mathcal{Q}^{(3)}, \quad (6)$$

where $\rho_{\rm m}$ is the matter density, δ is the matter density contrast, and we define

$$\mathcal{Q}^{(2)} \coloneqq (\nabla^2 Q)^2 - (\partial_i\partial_j Q)^2, \quad (7)$$

$$\mathcal{Q}^{(3)} \coloneqq (\nabla^2 Q)^3 - 3\nabla^2 Q(\partial_i\partial_j Q)^2 + 2(\partial_i\partial_j Q)^3. \quad (8)$$

From the equation of motion of the scalar field, we have

$$A_0\nabla^2 Q - A_1\nabla^2\Psi - A_2\nabla^2\Phi + \frac{B_0}{a^2H^2}\mathcal{Q}^{(2)} - \frac{B_1}{a^2H^2}(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial^i\partial^j Q) - \frac{B_2}{a^2H^2}(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q) - \frac{B_3}{a^2H^2}(\nabla^2\Phi\nabla^2\Psi - \partial_i\partial_j\Phi\partial^i\partial^j\Psi) - \frac{C_0}{a^4H^4}\mathcal{Q}^{(3)} - \frac{C_1}{a^4H^4}\mathcal{U}^{(3)} = 0, \quad (9)$$

where we define

$$\mathcal{U}^{(3)} \coloneqq \mathcal{Q}^{(2)}\nabla^2\Phi - 2\nabla^2 Q\partial_i\partial_j Q\partial^i\partial^j\Phi + 2\partial_i\partial_j Q\partial^j\partial^k Q\partial_k\partial^i\Phi. \quad (10)$$

The coefficients ($\mathcal{F}_T$, A_1, B_1, etc.) in the field equations here and below are defined in Appendix A. A_i, B_i, and C_i are the coefficients of the linear, quadratic, and cubic terms of Ψ, Φ, and Q, respectively.

Equations for the matter density contrast δ and the velocity field u^i are given by

$$\frac{\partial\delta(t, \mathbf{x})}{\partial t} + \frac{1}{a}\partial_i[(1 + \delta(t, \mathbf{x}))u^i(t, \mathbf{x})] = 0, \quad (11)$$

$$\frac{\partial u^i(t, \mathbf{x})}{\partial t} + \frac{\dot{a}}{a}u^i(t, \mathbf{x}) + \frac{1}{a}u^j(t, \mathbf{x})\partial_j u^i(t, \mathbf{x}) = -\frac{1}{a}\partial^i\Phi(t, \mathbf{x}), \quad (12)$$

where the dot denotes differentiation with respect to t. Gravity exerts an effect via the gravitational potential Φ, which is determined by (5), (6), and (9). Here, we consider the scalar mode of the density perturbations, and then we introduce a scalar function by $\theta \equiv \nabla\mathbf{u}/(aH)$. Let us define the Fourier expansion of the quantities δ and θ:

$$\delta(t, \mathbf{x}) = \frac{1}{(2\pi)^3}\int d^3p\,\delta(t, \mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}}, \quad (13)$$

$$u^j(t, \mathbf{x}) = \frac{1}{(2\pi)^3}\int d^3p\,\frac{-ip^j}{p^2}aH\theta(t, \mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}}. \quad (14)$$

The Fourier expansion of Φ, Ψ, and Q is defined as in (13). Then, (5) and (6) yield

$$-p^2(\mathcal{F}_T\Psi(t,\mathbf{p})-\mathcal{G}_T\Phi(t,\mathbf{p})-A_1Q(t,\mathbf{p}))=\frac{B_1}{2a^2H^2}\Gamma[t,\mathbf{p};Q,Q]+\frac{B_3}{a^2H^2}\Gamma[t,\mathbf{p};Q,\Phi], \tag{15}$$

$$\begin{aligned}-p^2(\mathcal{G}_T\Psi(t,\mathbf{p})+A_2Q(t,\mathbf{p}))-\frac{a^2}{2}\rho_{\mathrm{m}}\delta(t,\mathbf{p})=&-\frac{B_2}{2a^2H^2}\Gamma[t,\mathbf{p};Q,Q]-\frac{B_3}{a^2H^2}\Gamma[t,\mathbf{p};Q,\Psi]\\&-\frac{C_1}{3a^4H^4}\frac{1}{(2\pi)^6}\int d\mathbf{k}_1d\mathbf{k}_2d\mathbf{k}_3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{p})\\&\times[-k_1^2k_2^2k_3^2+3k_1^2(\mathbf{k}_2\cdot\mathbf{k}_3)^2-2(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_2\cdot\mathbf{k}_3)(\mathbf{k}_3\cdot\mathbf{k}_1)]\\&\times Q(t,\mathbf{k}_1)Q(t,\mathbf{k}_2)Q(t,\mathbf{k}_3),\end{aligned} \tag{16}$$

where we define

$$\Gamma[t,\mathbf{p};Y,Z]=\frac{1}{(2\pi)^3}\int d\mathbf{k}_1d\mathbf{k}_2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})(k_1^2k_2^2-(\mathbf{k}_1\cdot\mathbf{k}_2)^2)Y(t,\mathbf{k}_1)Z(t,\mathbf{k}_2). \tag{17}$$

Here, Y and Z denote any of Q, Φ, or Ψ. Equation (9) leads to

$$\begin{aligned}&-p^2(A_0Q(t,\mathbf{p})-A_1\Psi(t,\mathbf{p})-A_2\Phi(t,\mathbf{p}))\\&=-\frac{B_0}{a^2H^2}\Gamma[t,\mathbf{p};Q,Q]+\frac{B_1}{a^2H^2}\Gamma[t,\mathbf{p};Q,\Psi]+\frac{B_2}{a^2H^2}\Gamma[t,\mathbf{p};Q,\Phi]+\frac{B_3}{a^2H^2}\Gamma[t,\mathbf{p};\Psi,\Phi]\\&\quad+\frac{C_0}{a^4H^4}\frac{1}{(2\pi)^6}\int d\mathbf{k}_1d\mathbf{k}_2d\mathbf{k}_3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{p})[-k_1^2k_2^2k_3^2+3k_1^2(\mathbf{k}_2\cdot\mathbf{k}_3)^2-2(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_2\cdot\mathbf{k}_3)(\mathbf{k}_3\cdot\mathbf{k}_1)]\\&\quad\times Q(t,\mathbf{k}_1)Q(t,\mathbf{k}_2)Q(t,\mathbf{k}_3)+\frac{C_1}{a^4H^4}\frac{1}{(2\pi)^6}\int d\mathbf{k}_1d\mathbf{k}_2d\mathbf{k}_3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{p})[-k_1^2k_2^2k_3^2+(\mathbf{k}_1\cdot\mathbf{k}_2)^2k_3^2\\&\quad+2k_1^2(\mathbf{k}_2\cdot\mathbf{k}_3)^2-2(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_2\cdot\mathbf{k}_3)(\mathbf{k}_3\cdot\mathbf{k}_1)]Q(t,\mathbf{k}_1)Q(t,\mathbf{k}_2)\Phi(t,\mathbf{k}_3).\end{aligned} \tag{18}$$

Equations (11) and (12) are rewritten as

$$\frac{1}{H}\frac{\partial\delta(t,\mathbf{p})}{\partial t}+\theta(t,\mathbf{p})=-\frac{1}{(2\pi)^3}\int d\mathbf{k}_1d\mathbf{k}_2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})\left(1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1^2}\right)\theta(t,\mathbf{k}_1)\delta(t,\mathbf{k}_2), \tag{19}$$

$$\begin{aligned}\frac{1}{H}\frac{\partial\theta(t,\mathbf{p})}{\partial t}+\left(2+\frac{\dot{H}}{H^2}\right)\theta(t,\mathbf{p})-\frac{p^2}{a^2H^2}\Phi(t,\mathbf{p})=&-\frac{1}{2}\frac{1}{(2\pi)^3}\int d\mathbf{k}_1d\mathbf{k}_2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})\left(\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)|\mathbf{k}_1+\mathbf{k}_2|^2}{k_1^2k_2^2}\right)\\&\times\theta(t,\mathbf{k}_1)\theta(t,\mathbf{k}_2).\end{aligned} \tag{20}$$

We find the solution in terms of a perturbative expansion, which can be written in the form

$$Y(t,\mathbf{p})=\sum_{n=1}Y_n(t,\mathbf{p}), \tag{21}$$

where Y denotes δ,θ,Ψ,Φ, or Q, and Y_n denotes the nth order solution of the expansion. In the present paper, we aim to solve the second-order solution. At the first order of the perturbative expansion, Φ_1,Ψ_1, and Q_1 are expressed by δ_1 as (25), (26), and (27), respectively. The modified gravity affects the matter density perturbation via Φ in the Euler equation at any order of perturbation. Using the continuity equation (33), we find that δ_1 obeys (34). At the second order of the perturbative expansion, Φ_2, Ψ_2, and Q_2 are expressed by the terms in proportion to δ_2 and $\mathcal{W}_\gamma$, (46), (47), and (48), and we find that δ_2 obeys (56). Note that the homogeneous equation of (56) is the same as the equation for δ_1. The source term of (56) is given by (64). From (56) with (64), we find that the modification due to the nonlinear interaction enters through only the function of $N_\gamma(t)$, while the other parts have the same structure as those in general relativity. These facts are important for our conclusion that the second-order kernel is characterized by only one parameter.

Now we start from the first-order equations, which can easily be solved as follows [53]. From (15), (16), and (18), we have

$$\mathcal{F}_T p^2\Psi_1(t,\mathbf{p}) - \mathcal{G}_T p^2\Phi_1(t,\mathbf{p}) - A_1 p^2 Q_1(t,\mathbf{p}) = 0, \quad (22)$$

$$\mathcal{G}_T p^2\Psi_1(t,\mathbf{p}) + A_2 p^2 Q_1(t,\mathbf{p}) = -\frac{a^2}{2}\rho_{\rm m}\delta_1(t,\mathbf{p}), \quad (23)$$

$$A_0 p^2 Q_1(t,\mathbf{p}) - A_1 p^2\Psi_1(t,\mathbf{p}) - A_2 p^2\Phi_1(t,\mathbf{p}) = 0, \quad (24)$$

which give the solutions

$$\Phi_1(t,\mathbf{p}) = -\frac{a^2H^2}{p^2}\kappa_\Phi(t)\delta_1(t,\mathbf{p}), \quad (25)$$

$$\Psi_1(t,\mathbf{p}) = -\frac{a^2H^2}{p^2}\kappa_\Psi(t)\delta_1(t,\mathbf{p}), \quad (26)$$

$$Q_1(t,\mathbf{p}) = -\frac{a^2H^2}{p^2}\kappa_Q(t)\delta_1(t,\mathbf{p}). \quad (27)$$

Here, we define

$$\kappa_\Phi(t) = \frac{\rho_{\rm m}\mathcal{R}(t)}{H^2\mathcal{Z}(t)}, \qquad \kappa_\Psi(t) = \frac{\rho_{\rm m}\mathcal{S}(t)}{H^2\mathcal{Z}(t)}, \qquad \kappa_Q(t) = \frac{\rho_{\rm m}\mathcal{T}(t)}{H^2\mathcal{Z}(t)}, \quad (28)$$

and

$$\mathcal{R}(t) = A_0\mathcal{F}_T - A_1^2, \quad (29)$$

$$\mathcal{S}(t) = A_0\mathcal{G}_T + A_1A_2, \quad (30)$$

$$\mathcal{T}(t) = A_1\mathcal{G}_T + A_2\mathcal{F}_T, \quad (31)$$

$$\mathcal{Z}(t) = 2(A_0\mathcal{G}_T^2 + 2A_1A_2\mathcal{G}_T + A_2^2\mathcal{F}_T). \quad (32)$$

The first-order equation of (19) is

$$\theta_1(t,\mathbf{p}) = -\frac{1}{H}\frac{\partial\delta_1(t,\mathbf{p})}{\partial t}. \quad (33)$$

Substituting (33) and (25) into the first-order equation of (20), we have

$$\frac{\partial^2\delta_1(t,\mathbf{p})}{\partial t^2} + 2H\frac{\partial\delta_1(t,\mathbf{p})}{\partial t} + L(t)\delta_1(t,\mathbf{p}) = 0, \quad (34)$$

where we defined

$$L(t) = -\kappa_\Phi H^2 \quad (35)$$

$$= -\frac{(A_0\mathcal{F}_T - A_1^2)\rho_{\rm m}}{2(A_0\mathcal{G}_T^2 + 2A_1A_2\mathcal{G}_T + A_2^2\mathcal{F}_T)}. \quad (36)$$

This second-rank differential equation has the growing mode solution $D_+(t)$ and the decaying mode solution $D_-(t)$. Neglecting the decaying mode solution, we write the first-order solution,

$$\delta_1(t,\mathbf{p}) = D_+(t)\delta_{\rm L}(\mathbf{p}), \quad (37)$$

where $\delta_{\rm L}(\mathbf{p})$ is a constant, which is determined by the initial density fluctuations. We assume that $\delta_{\rm L}(\mathbf{p})$ obeys the Gaussian random statistics. Here we adopt the normalization $D_+(a) = a$ at $a \ll 1$. The first-order solutions for the other quantities can be expressed in terms of $\delta_1(t,\mathbf{p})$.

Then, we consider the second-order equations of the perturbative expansion. From (15), (16), and (18), the second-order equations are

$$-p^2(\mathcal{F}_T\Psi_2(t,\mathbf{p}) - \mathcal{G}_T\Phi_2(t,\mathbf{p}) - A_1Q_2(t,\mathbf{p})) = \frac{B_1}{2a^2H^2}\Gamma[t,\mathbf{p};Q_1,Q_1] + \frac{B_3}{a^2H^2}\Gamma[t,\mathbf{p};Q_1,\Phi_1], \quad (38)$$

$$-p^2(\mathcal{G}_T\Psi_2(t,\mathbf{p}) + A_2Q_2(t,\mathbf{p}) = \frac{a^2}{2}\rho_{\rm m}\delta_2(t,\mathbf{p}) - \frac{B_2}{2a^2H^2}\Gamma[t,\mathbf{p};Q_1,Q_1] - \frac{B_3}{a^2H^2}\Gamma[t,\mathbf{p};Q_1,\Psi_1], \quad (39)$$

$$-p^2(A_0Q_2(t,\mathbf{p}) - A_1\Psi_2(t,\mathbf{p}) - A_2\Phi_2(t,\mathbf{p})) = -\frac{B_0}{a^2H^2}\Gamma[t,\mathbf{p};Q_1,Q_1] + \frac{B_1}{a^2H^2}\Gamma[t,\mathbf{p};Q_1,\Psi_1] + \frac{B_2}{a^2H^2}\Gamma[t,\mathbf{p};Q_1,\Phi_1] + \frac{B_3}{a^2H^2}\Gamma[t,\mathbf{p};\Psi_1,\Phi_1]. \quad (40)$$

Using the first-order solutions (25), (26), (27), and (37), the above equations are rewritten as

$$-p^2(\mathcal{F}_T\Psi_2(t,\mathbf{p}) - \mathcal{G}_T\Phi_2(t,\mathbf{p}) - A_1Q_2(t,\mathbf{p})) = D_+^2(t)a^2H^2\left(\frac{1}{2}B_1\kappa_Q^2 + B_3\kappa_\Phi\kappa_Q\right)\mathcal{W}_\gamma(\mathbf{p}), \quad (41)$$

$$-p^2(\mathcal{G}_T\Psi_2(t,\mathbf{p}) + A_2Q_2(t,\mathbf{p})) = \frac{a^2}{2}\rho_{\rm m}\delta_2(t,\mathbf{p}) + D_+^2(t)a^2H^2\left(-\frac{1}{2}B_2\kappa_Q^2 - B_3\kappa_\Psi\kappa_Q\right)\mathcal{W}_\gamma(\mathbf{p}), \quad (42)$$

$$-p^2(A_0Q_2(t,\mathbf{p}) - A_1\Psi_2(t,\mathbf{p}) - A_2\Phi_2(t,\mathbf{p})) = D_+^2(t)a^2H^2(-B_0\kappa_Q^2 + B_1\kappa_\Psi\kappa_Q + B_2\kappa_\Phi\kappa_Q + B_3\kappa_\Phi\kappa_\Psi)\mathcal{W}_\gamma(\mathbf{p}), \quad (43)$$

where we defined

$$\mathcal{W}_\gamma(\mathbf{p}) = \frac{1}{(2\pi)^3}\int d\mathbf{k}_1 d\mathbf{k}_2 \delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})\gamma(\mathbf{k}_1,\mathbf{k}_2) \times \delta_\mathrm{L}(\mathbf{k}_1)\delta_\mathrm{L}(\mathbf{k}_2), \tag{44}$$

$$\gamma(\mathbf{k}_1,\mathbf{k}_2) = 1-\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2k_2^2}. \tag{45}$$

These equations yield

$$\Phi_2(t,\mathbf{p}) = -\frac{a^2H^2}{p^2}(\kappa_\Phi(t)\delta_2(t,\mathbf{p}) + D_+^2(t)\tau_\Phi(t)\mathcal{W}_\gamma(\mathbf{p})), \tag{46}$$

$$\Psi_2(t,\mathbf{p}) = -\frac{a^2H^2}{p^2}(\kappa_\Psi(t)\delta_2(t,\mathbf{p}) + D_+^2(t)\tau_\Psi(t)\mathcal{W}_\gamma(\mathbf{p})), \tag{47}$$

$$Q_2(t,\mathbf{p}) = -\frac{a^2H^2}{p^2}(\kappa_Q(t)\delta_2(t,\mathbf{p}) + D_+^2(t)\tau_Q(t)\mathcal{W}_\gamma(\mathbf{p})), \tag{48}$$

where we defined

$$\tau_\Phi(t) = \frac{1}{\mathcal{Z}}(2B_0\mathcal{T}\kappa_Q^2 - 3B_1\mathcal{S}\kappa_Q^2 - 3B_2\mathcal{R}\kappa_Q^2 - 6B_3\mathcal{R}\kappa_\Psi\kappa_Q), \tag{49}$$

$$\begin{aligned}\tau_\Psi(t) = \frac{1}{\mathcal{Z}}(&2B_0A_2\mathcal{G}_T\kappa_Q^2 + B_1(A_2^2\kappa_Q^2 - 2A_2\mathcal{G}_T\kappa_\Psi\kappa_Q)\\ &- B_2(\mathcal{S}\kappa_Q^2 + 2A_2\mathcal{G}_T\kappa_\Phi\kappa_Q)\\ &- 2B_3(\mathcal{S}\kappa_\Psi\kappa_Q - A_2^2\kappa_\Phi\kappa_Q + A_2\mathcal{G}_T\kappa_\Phi\kappa_\Psi)),\end{aligned} \tag{50}$$

$$\begin{aligned}\tau_Q(t) = -\frac{1}{\mathcal{Z}}(&2B_0\mathcal{G}_T^2\kappa_Q^2 + B_1(A_2\mathcal{G}_T\kappa_Q^2 - 2\mathcal{G}_T^2\kappa_\Psi\kappa_Q)\\ &+ B_2(\mathcal{T}\kappa_Q^2 - 2\mathcal{G}_T^2\kappa_\Phi\kappa_Q)\\ &+ 2B_3(\mathcal{T}\kappa_\Psi\kappa_Q + A_2\mathcal{G}_T\kappa_\Phi\kappa_Q - \mathcal{G}_T^2\kappa_\Phi\kappa_\Psi)).\end{aligned} \tag{51}$$

The second-order equations of (19) and (20) are

$$\begin{aligned}&\frac{1}{H}\frac{\partial\delta_2(t,\mathbf{p})}{\partial t} + \theta_2(t,\mathbf{p})\\ &= -\frac{1}{(2\pi)^3}\int d\mathbf{k}_1 d\mathbf{k}_2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})\alpha(\mathbf{k}_1,\mathbf{k}_2)\\ &\quad\times\theta_1(t,\mathbf{k}_1)\delta_1(t,\mathbf{k}_2),\end{aligned} \tag{52}$$

$$\begin{aligned}&\frac{1}{H}\frac{\partial\theta_2(t,\mathbf{p})}{\partial t} + \left(2+\frac{\dot{H}}{H^2}\right)\theta_2(t,\mathbf{p}) - \frac{p^2}{a^2H^2}\Phi_2(t,\mathbf{p})\\ &= -\frac{1}{(2\pi)^3}\int d\mathbf{k}_1 d\mathbf{k}_2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})\beta(\mathbf{k}_1,\mathbf{k}_2)\\ &\quad\times\theta_1(t,\mathbf{k}_1)\theta_1(t,\mathbf{k}_2),\end{aligned} \tag{53}$$

where we define

$$\alpha(\mathbf{k}_1,\mathbf{k}_2) = 1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1^2}, \tag{54}$$

$$\beta(\mathbf{k}_1,\mathbf{k}_2) = \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)|\mathbf{k}_1+\mathbf{k}_2|^2}{2k_1^2k_2^2} \quad . \tag{55}$$

Combining (52) and (53), and using the first-order solution and (46), we have

$$\frac{\partial^2\delta_2(t,\mathbf{p})}{\partial t^2} + 2H\frac{\partial\delta_2(t,\mathbf{p})}{\partial t} + L(t)\delta_2(t,\mathbf{p}) = S_\delta(t,\mathbf{p}), \tag{56}$$

where we define

$$\begin{aligned}S_\delta(t,\mathbf{p}) = &(\dot{D}_+^2(t) - L(t)D_+^2(t))\mathcal{W}_\alpha(\mathbf{p}) + \dot{D}_+^2(t)\mathcal{W}_\beta(\mathbf{p})\\ &+ N_\gamma(t)D_+^2(t)\mathcal{W}_\gamma(\mathbf{p}),\end{aligned} \tag{57}$$

$$\mathcal{W}_\alpha(\mathbf{p}) = \frac{1}{(2\pi)^3}\int d\mathbf{k}_1 d\mathbf{k}_2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p}) \times\alpha(\mathbf{k}_1,\mathbf{k}_2)\delta_\mathrm{L}(\mathbf{k}_1)\delta_\mathrm{L}(\mathbf{k}_2), \tag{58}$$

$$\mathcal{W}_\beta(\mathbf{p}) = \frac{1}{(2\pi)^3}\int d\mathbf{k}_1 d\mathbf{k}_2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p}) \times\beta(\mathbf{k}_1,\mathbf{k}_2)\delta_\mathrm{L}(\mathbf{k}_1)\delta_\mathrm{L}(\mathbf{k}_2), \tag{59}$$

and

$$\begin{aligned}N_\gamma(t) &= \tau_\Phi H^2\\ &= \frac{H^4}{\rho_\mathrm{m}}(2B_0\kappa_Q^3 - 3B_1\kappa_\Psi\kappa_Q^2 - 3B_2\kappa_\Phi\kappa_Q^2 - 6B_3\kappa_\Phi\kappa_\Psi\kappa_Q).\end{aligned} \tag{60}$$

In deriving (57), we use (34). Because of the symmetry with respect to the interchange of $\mathbf{k}_1$ and $\mathbf{k}_2$, we define $\alpha^{(s)}(\mathbf{k}_1,\mathbf{k}_2)$ as follows:

$$\alpha^{(s)}(\mathbf{k}_1,\mathbf{k}_2) = 1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2(k_1^2+k_2^2)}{2k_1^2k_2^2}. \tag{61}$$

Using the symmetry, we redefine $\mathcal{W}_\alpha(\mathbf{p})$ as

$$\mathcal{W}_\alpha(\mathbf{p}) = \frac{1}{(2\pi)^3}\int d\mathbf{k}_1 d\mathbf{k}_2\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})\alpha^{(s)}(\mathbf{k}_1,\mathbf{k}_2) \times\delta_\mathrm{L}(\mathbf{k}_1)\delta_\mathrm{L}(\mathbf{k}_2). \tag{62}$$

By the relation

$$\begin{aligned}&\beta(\mathbf{k}_1,\mathbf{k}_2) = \alpha^{(s)}(\mathbf{k}_1,\mathbf{k}_2) - \gamma(\mathbf{k}_1,\mathbf{k}_2) \quad \text{or}\\ &\mathcal{W}_\beta(\mathbf{p}) = \mathcal{W}_\alpha(\mathbf{p}) - \mathcal{W}_\gamma(\mathbf{p}),\end{aligned} \tag{63}$$

Eq. (57) reduces to

$$S_\delta(t,\mathbf{p}) = D_+^2(t)\{(2f^2H^2 - L(t))\mathcal{W}_\alpha(\mathbf{p}) + (N_\gamma(t) - f^2H^2)\mathcal{W}_\gamma(\mathbf{p})\}, \tag{64}$$

where we define the growth rate as $f = d\ln D_+(t)/d\ln a$.

Note that the homogeneous equation of (56) is the same as that of the first-order equation. Therefore, we have the second-order solution:

$$\delta_2(t,\mathbf{p}) = c_+(\mathbf{p})D_+(t) + c_-(\mathbf{p})D_-(t) + \int_0^t dt' \frac{D_+(t')D_-(t) - D_+(t)D_-(t')}{W(t')} S_\delta(t',\mathbf{p}), \tag{65}$$

where $c_+(\mathbf{p})$ and $c_-(\mathbf{p})$ are constants, and $W(t)$ is the Wronskian $W(t) = D_+(t)\dot{D}_-(t) - \dot{D}_+(t)D_-(t)$. From equations for $D_+(t)$ and $D_-(t)$, Eq. (34), the Wronskian obeys $\dot{W}(t) + 2HW(t) = 0$, which yields

$$W(t) = \frac{C}{a^2}, \tag{66}$$

where C is a constant. In the present paper, we assume the initial density perturbations obey the Gaussian statistics, and we set $c_\pm(\mathbf{p}) = 0$. Then, the second-order solution is written in the form

$$\delta_2(t,\mathbf{p}) = D_+^2(t)\left(\kappa(t)\mathcal{W}_\alpha(\mathbf{p}) - \frac{2}{7}\lambda(t)\mathcal{W}_\gamma(\mathbf{p})\right), \tag{67}$$

with

$$\kappa(t) = \frac{1}{D_+^2(t)}\int_0^t \frac{D_-(t)D_+(t') - D_+(t)D_-(t')}{W(t')} \times D_+^2(t')(2f^2H^2 - L(t'))dt', \tag{68}$$

$$\lambda(t) = \frac{7}{2D_+^2(t)}\int_0^t \frac{D_-(t)D_+(t') - D_+(t)D_-(t')}{W(t')} \times D_+^2(t')(f^2H^2 - N_\gamma(t'))dt'. \tag{69}$$

These expressions are a generalization of the results in Ref. [51]. In Sec. IV, we numerically evaluate the function $\lambda(t)$ without neglecting the decaying model in a specific model.

In the case of the matter-dominated universe within general relativity, $a(t) \propto t^{2/3}$, $D_+(t) = a$, $D_-(t) = a^{-3/2}$, $L(t) = -3/(2H^2)$, and $N_\gamma(t) = 0$, then the second-order solution reduces to

$$\delta_2(t,\mathbf{p}) = D_+^2(t)\left(\mathcal{W}_\alpha(\mathbf{p}) - \frac{2}{7}\mathcal{W}_\gamma(\mathbf{p})\right). \tag{70}$$

That is, one finds $\kappa(t) = \lambda(t) = 1$ in the Einstein–de Sitter universe. Even in the general second-order scalar-tensor theory, we may consider models where the matter-dominated era is realized in the early stage of the universe. In this stage, the effect of the scalar field perturbations would be negligible, and we may naturally expect that the matter density perturbations grow in the same way as those in general relativity. In this case, $\kappa(t) = 1$ and $\lambda(t) = 1$ at $a \ll 1$.

Interestingly, we can show that (68) generally reduces to $\kappa(t) = 1$ for all times. Using the expression for the Wronskian (66), Eq. (68) is rewritten as

$$\kappa(t) = \frac{1}{CD_+^2(t)}\int_0^t a^2(t')(D_-(t)D_+(t') - D_+(t)D_-(t')) \times \{2\dot{D}_+^2(t') + D_+(t')(\ddot{D}_+(t') + 2H\dot{D}_+(t'))\}dt', \tag{71}$$

where we used the fact that $D_+(t')$ satisfies (34) to eliminate the term $L(t')$. Partially integrating the term $\ddot{D}_+(t')$ in (71), we have

$$\kappa(t) = \frac{1}{CD_+(t)}\int_0^t a^2(t')\{\dot{D}_-(t')D_+(t') - D_-(t')\dot{D}_+(t')\}\dot{D}_+(t')dt'. \tag{72}$$

Using the Wronskian, we finally obtain

$$\kappa(t) = 1, \tag{73}$$

for all times. Therefore, the kernel (81) depends on only the parameter $\lambda(t)$, which is determined by the solution of the linear density perturbation, $H(t)$ and the function $N_\gamma(t)$. This conclusion is a generalization of the results in Ref. [46]. The authors of Ref. [46] investigated the standard density perturbation theory in the Dvali, Gabadadze and Porrati model, and a similar result is obtained at the second order of perturbation (Appendix B.1 in their paper). The result is explained by $\alpha^{(s)}(\mathbf{k}_1,\mathbf{k}_2)$ and $\gamma(\mathbf{k}_1,\mathbf{k}_2)$ being independent of each other and the modification of gravity coming through only the terms in proportion to $\gamma(\mathbf{k}_1,\mathbf{k}_2)$ at the second order of perturbation. Therefore, the term in proportion to $\alpha^{(s)}(\mathbf{k}_1,\mathbf{k}_2)$ is not modified.

Finally, in this section, we present the expression of the velocity divergence at the second order of perturbation. We obtain the expression by inserting $\delta_1(t,\mathbf{p})$, $\theta_1(t,\mathbf{p})$, and $\delta_2(t,\mathbf{p})$ into (52),

$$\theta_2(t,\mathbf{p}) = -D_+^2(t)f\left(\mathcal{W}_\alpha(\mathbf{p}) - \frac{4}{7}\lambda_\theta(t)\mathcal{W}_\gamma(\mathbf{p})\right), \tag{74}$$

where we defined

$$\lambda_\theta(t) = \lambda(t) + \frac{\dot{\lambda}(t)}{2fH}. \tag{75}$$

In the Einstein–de Sitter universe, we have $\lambda_\theta(t) = 1$.

III. BISPECTRUM

In this section, we consider the bispectrum of the density perturbations in the most general second-order scalar-tensor theory on the cosmological background. The power spectrum and the bispectrum are defined by

$$\langle \delta(t, \mathbf{k}_1)\delta(t, \mathbf{k}_2)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)P(t, k_1), \tag{76}$$

$$\langle \delta(t, \mathbf{k}_1)\delta(t, \mathbf{k}_2)\delta(t, \mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B(t, k_1, k_2, k_3), \tag{77}$$

respectively. The three-point function at the lowest order of the standard perturbation theory is evaluated as

$$\langle \delta(t, \mathbf{k}_1)\delta(t, \mathbf{k}_2)\delta(t, \mathbf{k}_3)\rangle = D_+^4(t)(\langle \delta_{\rm L}(\mathbf{k}_1)\delta_{\rm L}(\mathbf{k}_2)\delta_{\rm 2K}(t, \mathbf{k}_3)\rangle + 2 \text{ cyclic terms}), \tag{78}$$

where we define

$$\delta_{\rm 2K}(t, \mathbf{k}) = \mathcal{W}_\alpha(\mathbf{k}) - \frac{2}{7}\lambda(t)\mathcal{W}_\gamma(\mathbf{k}). \tag{79}$$

The first term in parentheses in the right-hand side of (78) is

$$\langle \delta_{\rm L}(\mathbf{k}_1)\delta_{\rm L}(\mathbf{k}_2)\delta_{\rm 2K}(t, \mathbf{k}_3)\rangle = \int \frac{d^3q_1}{(2\pi)^3} F_2(t, \mathbf{q}_1, \mathbf{k}_3 - \mathbf{q}_1) \times \langle \delta_{\rm L}(\mathbf{k}_1)\delta_{\rm L}(\mathbf{k}_2)\delta_{\rm L}(\mathbf{q}_1)\delta_{\rm L}(\mathbf{k}_3 - \mathbf{q}_1)\rangle, \tag{80}$$

where we define the kernel

$$F_2(t, \mathbf{k}_1, \mathbf{k}_2) \equiv \alpha^{(s)}(\mathbf{k}_1, \mathbf{k}_2) - \frac{2}{7}\lambda(t)\gamma(\mathbf{k}_1, \mathbf{k}_2). \tag{81}$$

Using the definition of the linear matter power spectrum,

$$\langle \delta_{\rm L}(\mathbf{k}_1)\delta_{\rm L}(\mathbf{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)P_{11}(k_1), \tag{82}$$

and Wick's theorem, we have

$$\langle \delta_{\rm L}(\mathbf{k}_1)\delta_{\rm L}(\mathbf{k}_2)\delta_{\rm 2K}(t, \mathbf{k}_3)\rangle = 2(2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)F_2(t, \mathbf{k}_1, \mathbf{k}_2)P_{11}(k_1)P_{11}(k_2), \tag{83}$$

where we use $F_2(t, -\mathbf{k}_1, -\mathbf{k}_2) = F_2(t, \mathbf{k}_1, \mathbf{k}_2)$. Finally, we have the expression for the bispectrum at the lowest order of the perturbation theory,

$$B(t, k_1, k_2, k_3) = D_+^4(t)B_4(t, k_1, k_2, k_3) \tag{84}$$

with

$$B_4(t, k_1, k_2, k_3) = 2F_2(t, \mathbf{k}_1, \mathbf{k}_2)P_{11}(k_1)P_{11}(k_2) + 2 \text{ cyclic terms}. \tag{85}$$

The reduced bispectrum is given by

$$Q_{123}(t, k_1, k_2, \theta_{12}) = \frac{B_4(t, k_1, k_2, k_3)}{P_{11}(k_1)P_{11}(k_2) + P_{11}(k_2)P_{11}(k_3) + P_{11}(k_3)P_{11}(k_1)}, \tag{86}$$

at the lowest order of perturbations. Note that the (reduced) bispectrum is described by the kernel (81), which depends on only the parameter $\lambda(t)$, which is given by (69).

Because $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$ is satisfied, the reduced bispectrum is a function of only three parameters, which we take as $k_1 = |\mathbf{k}_1|$, $k_2 = |\mathbf{k}_2|$, and the angle θ_{12} between $\mathbf{k}_1$ and $\mathbf{k}_2$. Explicit expressions for $\alpha^{(s)}(\mathbf{k}_i, \mathbf{k}_j)$, and $\gamma(\mathbf{k}_i, \mathbf{k}_j)$, where (i, j) denotes any of (1,2), (2,3), or (3,1), are summarized in Appendix B. Each panel of Fig. 1 shows a typical behavior of Q_{123} as a function of θ_{12} with fixed k_1 and k_2, whose values are described in the caption. In each panel, we adopt a different value of $\lambda(t) = 1$ (blue solid curve), $\lambda(t) = 1.2$ (red dotted curve), and $\lambda(t) = 0.8$ (yellow dashed curve), assuming a spatially flat universe with the CDM model and the cosmological constant Λ, whose density parameters are $\Omega_0 = 0.3$ and $\Omega_\Lambda = 0.7$, for the linear matter power spectrum $P_{11}(k)$. Note that the reduced bispectrum depends on time t through only $\lambda(t)$. One can see the following features. First, the overall amplitude of Q_{123} depends on the value of k_1 and k_2. However, when the values of k_1 and k_2 are fixed, the reduced bispectrum is enhanced for $\lambda < 1$ but reduced for $\lambda > 1$. This feature is explained by kernel (81) and the fact $\gamma(\mathbf{k}_i, \mathbf{k}_j) \geq 0$.

With the limit $\theta_{12} = 0$, we have $\gamma(\mathbf{k}_1, \mathbf{k}_2) = \gamma(\mathbf{k}_2, \mathbf{k}_3) = \gamma(\mathbf{k}_3, \mathbf{k}_1) = 0$ (see also Appendix B). Then, Q_{123} is independent of λ at $\theta_{12} = 0$. With the limit $\theta_{12} = \pi$, Q_{123} behave differently depending on the conditions $k_1 = k_2$ and $k_1 \neq k_2$. If $k_1 \neq k_2$, then we have $\gamma(\mathbf{k}_1, \mathbf{k}_2) = \gamma(\mathbf{k}_2, \mathbf{k}_3) = \gamma(\mathbf{k}_3, \mathbf{k}_1) = 0$, which is the same as with the limit $\theta_{12} = 0$. In the case $k_1 = k_2$, however, we have $\gamma(\mathbf{k}_1, \mathbf{k}_2) = 0$, $\gamma(\mathbf{k}_2, \mathbf{k}_3) = \gamma(\mathbf{k}_3, \mathbf{k}_1) = 1$, and $k_3 = 0$; that is, $P_{11}(k_3) = 0$. Then the bispectrum approaches zero with this limit, though the rate of convergence depends on $\lambda(t)$, as is discussed in the next section.

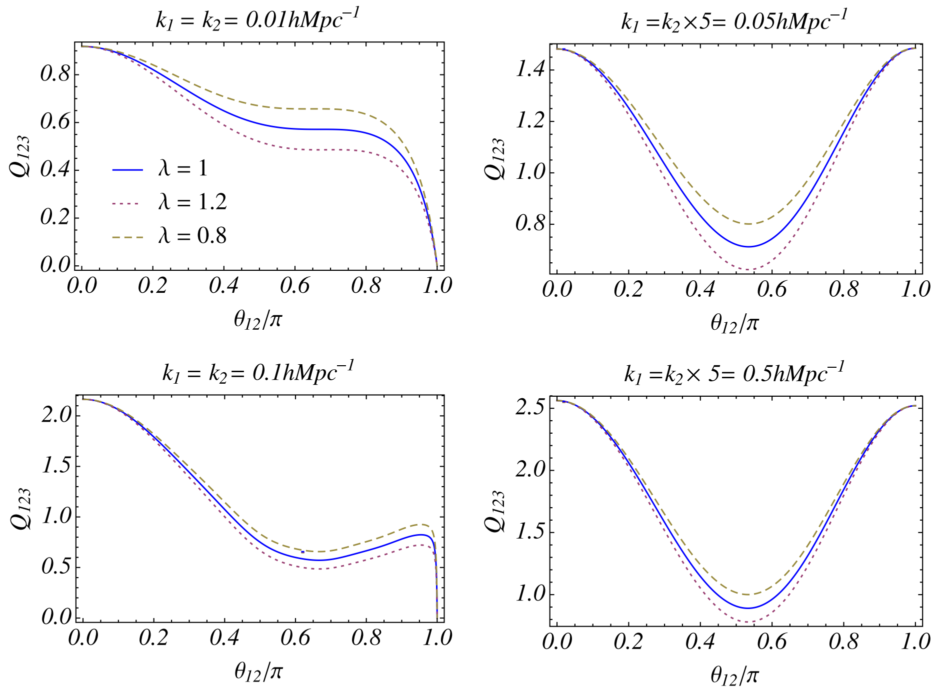

FIG. 1 (color online). Q_{123} as a function of θ_{12} with $k_1 = k_2 = 0.01h\ \mathrm{Mpc}^{-1}$ (upper left panel), $k_1 = k_2 = 0.1h\ \mathrm{Mpc}^{-1}$ (lower left panel), $k_1 = 5 \times k_2 = 0.05h\ \mathrm{Mpc}^{-1}$ (upper right panel), and $k_1 = 5 \times k_2 = 0.5h\ \mathrm{Mpc}^{-1}$ (lower right panel). For the linear matter power spectrum $P_{11}(k)$, we adopt the spatially flat universe with the cold dark matter (CDM) model and the cosmological constant Λ, whose density parameters are $\Omega_0 = 0.3$ and $\Omega_\Lambda = 0.7$, respectively. Note that the reduced bispectrum depends on time t through only $\lambda(t)$, for which we adopt different values of $\lambda(t) = 1$ (blue solid curve), $\lambda(t) = 1.2$ (red dotted curve), and $\lambda(t) = 0.8$ (yellow dashed curve), irrespective of the ΛCDM model.

All the influence of the nonlinear interaction of the modified gravity arises through only the parameter $\lambda(t)$, which appears as the term in proportion to $\gamma(\mathbf{k}_1, \mathbf{k}_2)$ in the kernel (81). The bispectrum of the matter density perturbations behaves in a restricted way only, which is a feature of the general second-order scalar-tensor theory equipped with the Vainshtein mechanism.

IV. KINETIC GRAVITY BRAIDING MODEL

In this section, we consider a simple example to demonstrate how the modification of gravity influences the behavior of the bispectrum at a quantitative level. We consider the kinetic gravity braiding model investigated in Refs. [31,52], whose action is written as

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + K - G_3\Box\phi + \mathcal{L}_{\rm m}\right], \quad (87)$$

with the Planck mass $M_{\rm pl}$, which is related with the gravitational constant G_N by $8\pi G_N = 1/M_{\rm pl}^2$. Comparing this action (87) with that of the most general second-order scalar-tensor theory, the action of the kinetic gravity braiding model is produced by setting

$$G_4 = \frac{M_{\rm pl}^2}{2}, \qquad G_5 = 0. \quad (88)$$

In Ref. [52], K and G_3 are chosen as

$$K = -X, \qquad G_3 = M_{\rm pl}\left(\frac{r_c^2}{M_{\rm pl}^2}X\right)^n, \quad (89)$$

where n and r_c are parameters. In this model, we have

$$L(t) = -\frac{A_0\mathcal{F}_T\rho_{\rm m}}{2(A_0\mathcal{G}_T + A_2^2\mathcal{F}_T)}, \quad (90)$$

$$N_\gamma(t) = \frac{B_0A_2^3\mathcal{F}_T^3\rho_{\rm m}^2}{4(A_0\mathcal{G}_T^2 + A_2^2\mathcal{F}_T)^3H^2}. \quad (91)$$

Useful expressions of the kinetic gravity braiding model are summarized in Appendix A.

When we consider the attractor solution, which satisfies

$$3\dot{\phi}HG_{3X} = 1, \quad (92)$$

the Friedmann equation is written in the form

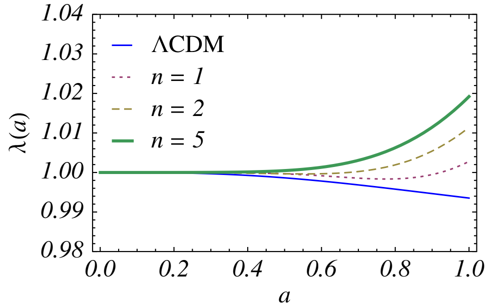

FIG. 2 (color online). $\lambda(t)$ as a function of a in the ΛCDM model (blue solid curve) and the kinetic gravity braiding model with $n = 1$ (red dotted curve), $n = 2$ (yellow dashed curve), and 5 (green thick curve).

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_0}{a^3} + (1 - \Omega_0)\left(\frac{H}{H_0}\right)^{-2/(2n-1)}, \tag{93}$$

where H_0 is the Hubble constant and Ω_0 is the density parameter at the present time, and the model parameters must satisfy

$$H_0 r_c = \left(\frac{2^{n-1}}{3n}\right)^{1/2n}\left[\frac{1}{6(1 - \Omega_0)}\right]^{(2n-1)/4n}. \tag{94}$$

On the attractor solution, $L(t)$ and $N_\gamma(t)$ reduce to

$$L(t) = -\frac{3}{2}\frac{2n + (3n - 1)\Omega_m(t)}{5n - \Omega_m(t)}H^2, \tag{95}$$

$$N_\gamma(t) = -\frac{9}{4}\frac{(1 - \Omega_m(t))(2n - \Omega_m(t))^3}{\Omega_m(t)(5n - \Omega_m(t))^3}H^2, \tag{96}$$

where $\Omega_m(a)$ is defined by $\Omega_m(a) = \Omega_0 H_0^2/H(a)^2 a^3$. Note that the quasistatic approximation on the scales of the large-scale structure holds for $n \lesssim 10$ (see [52]).

Figure 2 shows the evolution of $\lambda(t)$ as a function of a for the kinetic gravity braiding model with $n = 1, 2, 5$ and the ΛCDM model. For $a \ll 1$, we have $\lambda(t) = 1$, which is the prediction of the Einstein–de Sitter universe. However, the accelerated expansion arises due to domination of the Galileon field as a approaches 1, and so the value of $\lambda(t)$ starts to deviate from 1.

The deviation of $\lambda(t)$ from 1 is small. The value of $\lambda(t)$ at the present epoch is 0.994 under the ΛCDM model with the density parameter $\Omega_0 = 0.3$. The value of $\lambda(t)$ at the present epoch is 1.003, 1.011, and 1.019 under the kinetic gravity braiding (KGB) model with $n = 1, 2, 5$, respectively. Our results demonstrate the validity of the approximation setting $\lambda(t) = 1$, which is usually adopted in the standard density perturbations theory.

Figure 3 shows the relative deviation of the bispectrum at the present epoch under the KGB model from that under the ΛCDM model, $Q_{123}(t, k_1, k_2, \theta_{12})/Q_{123\Lambda}(t, k_1, k_2, \theta_{12}) - 1$, as a function of θ_{12}, where $Q_{123\Lambda}(t, k_1, k_2, \theta_{12})$ is the reduced bispectrum of the ΛCDM model. The relative deviation from the ΛCDM model is less than 2%. For the case $k_1 \neq k_2$, the deviation between the models does not appear at $\theta_{12} = 0, \pi$, which is simply understood by the fact that $\gamma(\mathbf{k}_i, \mathbf{k}_j) = 0$ there.

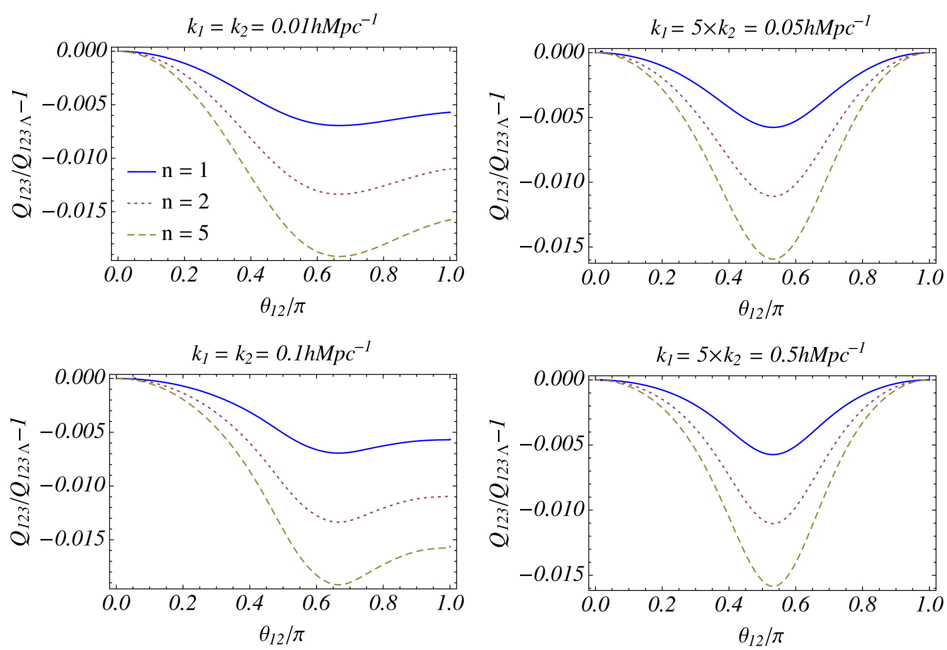

FIG. 3 (color online). Relative deviation of the reduced bispectrum at the present epoch under the kinetic gravity braiding model with $n = 1$ (blue solid curve), $n = 2$ (red dotted curve), $n = 5$ (yellow dashed curve) from that under the ΛCDM model $Q_{123\Lambda}$, as a function of θ_{12}, where k_1 and k_2 are fixed, whose values are noted on each panel. Here the density parameter is fixed as $\Omega_0 = 0.3$.

In the case $k_1 = k_2$ with the limit $\theta_{12} = \pi$, we have $\alpha^{(s)}(\mathbf{k}_1, \mathbf{k}_2) \sim (\pi - \theta_{12})^2$, $\alpha^{(s)}(\mathbf{k}_2, \mathbf{k}_3) = \alpha^{(s)}(\mathbf{k}_3, \mathbf{k}_1) = 3/4$, $\gamma(\mathbf{k}_1, \mathbf{k}_2) \sim (\pi - \theta_{12})^2$, $\gamma(\mathbf{k}_2, \mathbf{k}_3) = \gamma(\mathbf{k}_3, \mathbf{k}_1) = 1$, and $P(k_3) \propto k_3^{n_s} \propto (\pi - \theta_{12})^{n_s}$, where n_s is the spectral index. (See Appendix B for details.)

Then, the bispectrum has the asymptotic form

$$B_4(t, k_1, k_1, \theta_{12}) \sim 4\left(\frac{3}{4} - \frac{2}{7}\lambda(t)\right) P_{11}(k_3) P_{11}(k_1) \quad (97)$$

around the limit $\theta_{12} = \pi$. This leads to the ratio of the reduced bispectrum in this limit,

$$\frac{Q_{123}(t, k_1, k_1, \theta_{12})}{Q_{123\Lambda}(t, k_1, k_1, \theta_{12})} = \frac{21 - 8\lambda(t)}{21 - 8\lambda_\Lambda(t)}, \quad (98)$$

where $\lambda_\Lambda(t)$ is the parameter $\lambda(t)$ of the ΛCDM model, which explains the behavior shown in the left panels of Fig. 3.

The behavior of the reduced bispectrum is almost the same when the ratio k_1/k_2 is the same. This is because the functions $\alpha^{(s)}(\mathbf{k}_i, \mathbf{k}_j)$ and $\gamma(\mathbf{k}_i, \mathbf{k}_j)$ depend only on the ratio k_1/k_2 and θ_{12} (see also Appendix B). Recently, the bispectrum in the covariant cubic galileon cosmology is investigated in Ref. [51]. Our kinetic gravity braiding model with $n = 1$ is a cubic Galileon model; however, there is the difference between our model and the covariant cubic Galileon cosmology in Ref. [51]. The cosmic accelerated expansion in the covariant cubic Galileon model is derived by a potential of the scalar field. This causes the differences in the evolution of the background universe and the linear density perturbations.

V. SUMMARY AND CONCLUSIONS

In the present paper, we investigated the bispectrum of the matter density perturbations induced by gravitational instability in the most general second-order scalar-tensor theory that may possess the Vainshtein mechanism. We discussed a general feature of this wide class of modified gravity models in the most general second-order scalar-tensor theory. We analytically obtained the expression of the bispectrum of the second-order perturbations on the basis of the standard density perturbation theory. The bispectrum is expressed by the kernel (81), depending on only the parameter $\lambda(t)$, which is determined by the growing and decaying solutions of the linear density perturbations $D_\pm(t)$, the Hubble parameter $H(t)$, and the other function $N_\gamma(t)$ for the nonlinear interactions. These simple results come from the fact that the basic equations for the gravitational and scalar fields have the same form as the nonlinear mode couplings, which are derived as the leading terms under the quasistatic approximation within the subhorizon scales. Thus, all the effects of the modified gravity in the bispectrum come via the parameter $\lambda(t)$ in the kernel (81), which has a simple structure. This makes the behavior of the bispectrum less complex. As an application of our results, we exemplified the behavior of the bispectrum in the kinetic gravity braiding model proposed in Ref. [52]. We investigated the evolution of $\lambda(t)$ in this model and demonstrated the deviation of the reduced bispectrum from that of the ΛCDM model is less than 2%. Higher order solutions of the density perturbations can be obtained in a similar way, which is left as a future problem.

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APPENDIX A: DEFINITION OF THE COEFFICIENTS

We first summarize the definitions of the coefficients in the field equations presented in Sec. II.

$$A_0 = \frac{\dot{\Theta}}{H^2} + \frac{\Theta}{H} + \mathcal{F}_T - 2\mathcal{G}_T - 2\frac{\dot{\mathcal{G}}_T}{H} - \frac{\mathcal{E} + \mathcal{P}}{2H^2}, \quad (A1)$$

$$A_1 = \frac{\dot{\mathcal{G}}_T}{H} + \mathcal{G}_T - \mathcal{F}_T, \quad (A2)$$

$$A_2 = \mathcal{G}_T - \frac{\Theta}{H}, \quad (A3)$$

$$B_0 = \frac{X}{H}\{\dot\phi G_{3X} + 3(\dot X + 2HX)G_{4XX} + 2X\dot X G_{4XXX} - 3\dot\phi G_{4\phi X} + 2\dot\phi X G_{4\phi XX} + (\dot H + H^2)\dot\phi G_{5X} + \dot\phi[2H\dot X + (\dot H + H^2)X]G_{5XX} + H\dot\phi X\dot X G_{5XXX} - 2(\dot X + 2HX)G_{5\phi X} - \dot\phi X G_{5\phi\phi X} - X(\dot X - 2HX)G_{5\phi XX}\}, \quad (A4)$$

$$B_1 = 2X[G_{4X} + \ddot\phi(G_{5X} + XG_{5XX}) - G_{5\phi} + XG_{5\phi X}], \quad (A5)$$

$$B_2 = -2X(G_{4X} + 2XG_{4XX} + H\dot\phi G_{5X} + H\dot\phi X G_{5XX} - G_{5\phi} - XG_{5\phi X}), \quad (A6)$$

$$B_3 = H\dot\phi X G_{5X}, \quad (A7)$$

$$C_0 = 2X^2 G_{4XX} + \frac{2X^2}{3}(2\ddot\phi G_{5XX} + \ddot\phi X G_{5XXX} - 2G_{5\phi X} + XG_{5\phi XX}), \quad (A8)$$

$$C_1 = H\dot\phi X(G_{5X} + XG_{5XX}), \quad (A9)$$

where we also defined

$$\mathcal{F}_T = 2[G_4 - X(\ddot\phi G_{5X} + G_{5\phi})], \quad (A10)$$

$$\mathcal{G}_T = 2[G_4 - 2XG_{4X} - X(H\dot\phi G_{5X} - G_{5\phi})], \quad (A11)$$

$$\Theta = -\dot\phi X G_{3X} + 2HG_4 - 8HXG_{4X} - 8HX^2 G_{4XX} + \dot\phi G_{4\phi} + 2X\dot\phi G_{4\phi X} - H^2\dot\phi(5XG_{5X} + 2X^2 G_{5XX}) + 2HX(3G_{5\phi} + 2XG_{5\phi X}), \quad (A12)$$

$$\mathcal{E} = 2XK_X - K + 6X\dot\phi H G_{3X} - 2XG_{3\phi} - 6H^2 G_4 + 24H^2 X(G_{4X} + XG_{4XX}) - 12HX\dot\phi G_{4\phi X} - 6H\dot\phi G_{4\phi} + 2H^3 X\dot\phi(5G_{5X} + 2XG_{5XX}) - 6H^2 X(3G_{5\phi} + 2XG_{5\phi X}), \quad (A13)$$

$$\mathcal{P} = K - 2X(G_{3\phi} + \ddot\phi G_{3X}) + 2(3H^2 + 2\dot H)G_4 - 12H^2 XG_{4X} - 4H\dot X G_{4X} - 8\dot H X G_{4X} - 8HX\dot X G_{4XX} + 2(\ddot\phi + 2H\dot\phi)G_{4\phi} + 4XG_{4\phi\phi} + 4X(\ddot\phi - 2H\dot\phi)G_{4\phi X} - 2X(2H^3\dot\phi + 2H\dot H\dot\phi + 3H^2\ddot\phi)G_{5X} - 4H^2 X^2\ddot\phi G_{5XX} + 4HX(\dot X - HX)G_{5\phi X} + 2[2(HX)^{\cdot} + 3H^2 X]G_{5\phi} + 4HX\dot\phi G_{5\phi\phi}. \quad (A14)$$

In the kinetic gravity braiding model considered in Sec. IV, the coefficients are written as follows:

$$\mathcal{F}_T = M_{\rm pl}^2, \qquad \mathcal{G}_T = M_{\rm pl}^2, \quad (A15)$$

$$\Theta = -nM_{\rm pl}\left(\frac{r_c^2}{M_{\rm pl}^2}\right)^n \dot\phi X^n + HM_{\rm pl}^2, \quad (A16)$$

$$\dot\Theta = -n(2n+1)M_{\rm pl}\left(\frac{r_c^2}{M_{\rm pl}^2}\right)^n \ddot\phi X^n + \dot H M_{\rm pl}^2, \quad (A17)$$

$$\mathcal{E} = -X + 6nM_{\rm pl}\left(\frac{r_c^2}{M_{\rm pl}^2}\right)^n \dot\phi H X^n - 3H^2 M_{\rm pl}^2, \quad (A18)$$

$$\mathcal{P} = -X - 2nM_{\rm pl}\left(\frac{r_c^2}{M_{\rm pl}^2}\right)^n \ddot\phi X^n + (3H^2 + 2\dot H)M_{pl}^2, \quad (A19)$$

$$A_0 = \frac{X}{H^2} - 2nM_{\rm pl}\left(\frac{r_c^2}{M_{\rm pl}^2}\right)^n\left(\frac{2\dot\phi}{H} + n\frac{\ddot\phi}{H^2}\right)X^n, \quad (A20)$$

$$A_2 = B_0 = nM_{\rm pl}\left(\frac{r_c^2}{M_{\rm pl}^2}\right)^n \frac{\dot\phi}{H}X^n, \quad (A21)$$

$$A_1 = B_1 = B_2 = B_3 = C_0 = C_1 = 0. \quad (A22)$$

In the present paper, we consider the attractor solution satisfying (92), thus obtaining

$$\ddot\phi = -\frac{1}{2n-1}\frac{\dot\phi\dot H}{H}, \quad (A23)$$

$$\frac{\dot H}{H^2} = -\frac{(2n-1)3\Omega_m(a)}{2(2n - \Omega_m(a))}, \quad (A24)$$

$$A_0 = -\frac{M_{\rm pl}^2(1-\Omega_m(a))(2n+(3n-1)\Omega_m(a))}{2n-\Omega_m(a)}, \quad \text{(A25)}$$

$$A_2 = M_{\rm pl}^2(1-\Omega_m(a)), \quad \text{(A26)}$$

$$B_0 = M_{\rm pl}^2(1-\Omega_m(a)), \quad \text{(A27)}$$

where we define $\Omega_m(a) = \rho_m(a)/3M_{\rm pl}^2H^2$.

APPENDIX B: EXPLICIT EXPRESSIONS OF α AND γ

For the bispectrum, we may write the wave number vector that satisfies $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$ as follows:

$$\mathbf{k}_1 = (0, 0, k_1), \quad \text{(B1)}$$

$$\mathbf{k}_2 = (0, k_2\sin\theta_{12}, k_2\cos\theta_{12}), \quad \text{(B2)}$$

$$\mathbf{k}_3 = (0, -k_2\sin\theta_{12}, -k_1 - k_2\cos\theta_{12}), \quad \text{(B3)}$$

where θ_{12} is the angle between the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. Then, we have

$$\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2} = \cos\theta_{12}, \quad \text{(B4)}$$

$$\frac{\mathbf{k}_2\cdot\mathbf{k}_3}{k_2k_3} = \frac{-k_2 - k_1\cos\theta_{12}}{\sqrt{k_1^2 + k_2^2 + 2k_1k_2\cos\theta_{12}}}, \quad \text{(B5)}$$

$$\frac{\mathbf{k}_3\cdot\mathbf{k}_1}{k_3k_1} = \frac{-k_1 - k_2\cos\theta_{12}}{\sqrt{k_1^2 + k_2^2 + 2k_1k_2\cos\theta_{12}}}, \quad \text{(B6)}$$

where we use $k_3 = \sqrt{k_1^2 + k_2^2 + 2k_1k_2\cos\theta_{12}}$. Introducing the constant c by $k_1 = ck_2$, we have

$$k_3 = k_1\sqrt{c^2 + 2c\cos\theta_{12} + 1}, \quad \text{(B7)}$$

$$\frac{\mathbf{k}_2\cdot\mathbf{k}_3}{k_2k_3} = -\frac{c + \cos\theta_{12}}{\sqrt{c^2 + 2c\cos\theta_{12} + 1}}, \quad \text{(B8)}$$

$$\frac{\mathbf{k}_3\cdot\mathbf{k}_1}{k_3k_1} = -\frac{c\cos\theta_{12} + 1}{\sqrt{c^2 + 2c\cos\theta_{12} + 1}}. \quad \text{(B9)}$$

For convenience, we summarize the explicit expressions of $\alpha^{(s)}(\mathbf{k}_i, \mathbf{k}_j)$ and $\gamma(\mathbf{k}_i, \mathbf{k}_j)$. The above relations yield

$$\alpha^{(s)}(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{(c^2+1)\cos\theta_{12}}{2c}, \quad \text{(B10)}$$

$$\alpha^{(s)}(\mathbf{k}_2, \mathbf{k}_3) = 1 - \frac{(2c^2 + 2c\cos\theta_{12} + 1)(c + \cos\theta_{12})}{2c(c^2 + 2c\cos\theta_{12} + 1)}, \quad \text{(B11)}$$

$$\alpha^{(s)}(\mathbf{k}_3, \mathbf{k}_1) = 1 - \frac{(c^2 + 2c\cos\theta_{12} + 2)(c\cos\theta_{12} + 1)}{2(c^2 + 2c\cos\theta_{12} + 1)}, \quad \text{(B12)}$$

$$\gamma(\mathbf{k}_1, \mathbf{k}_2) = 1 - \cos^2\theta_{12}, \quad \text{(B13)}$$

$$\gamma(\mathbf{k}_2, \mathbf{k}_3) = \frac{\sin^2\theta_{12}}{c^2 + 2c\cos\theta_{12} + 1}, \quad \text{(B14)}$$

$$\gamma(\mathbf{k}_3, \mathbf{k}_1) = \frac{c^2\sin^2\theta_{12}}{c^2 + 2c\cos\theta_{12} + 1}. \quad \text{(B15)}$$

Thus, $\alpha^{(s)}$ and γ depend on only c and θ_{12}, which means that $F_2(t, \mathbf{k}_i, \mathbf{k}_j)$ depends on only c and θ_{12}, irrespective of t. It is trivial that $\alpha^{(s)}(\mathbf{k}_1, \mathbf{k}_2)$ and $\gamma(\mathbf{k}_1, \mathbf{k}_2)$ are invariant under the interchange between $\mathbf{k}_1$ and $\mathbf{k}_2$, or the replacement of c with $1/c$. Note also that $\alpha^{(s)}(\mathbf{k}_2, \mathbf{k}_3)$ and $\gamma(\mathbf{k}_2, \mathbf{k}_3)$ are transformed into $\alpha^{(s)}(\mathbf{k}_3, \mathbf{k}_1)$ and $\gamma(\mathbf{k}_3, \mathbf{k}_1)$, respectively, by the replacement of c with $1/c$.

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