PHYSICAL REVIEW D **89,** 101302(R) (2014)

Steps to reconcile inflationary tensor and scalar spectra

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(Received 20 March 2014; published 22 May 2014)

The recent BICEP2 B-mode polarization determination of an inflationary tensor-scalar ratio $r = 0.2^{+0.07}_{-0.05}$ is in tension with simple scale-free models of inflation due to a lack of a corresponding low multipole excess in the temperature power spectrum which places a limit of $r_{0.002} < 0.11$ (95% C.L.) on such models. Single-field inflationary models that reconcile these two observations, even those where the tilt runs substantially, introduce a scale into the scalar power spectrum. To cancel the tensor excess, and simultaneously remove the excess already present without tensors, ideally the model should introduce this scale as a relatively sharp transition in the tensor-scalar ratio around the horizon at recombination. We consider models which generate such a step in this quantity and find that they can improve the joint fit to the temperature and polarization data by up to $2\Delta \ln \mathcal{L} \approx -14$ without changing cosmological parameters. Precision E-mode polarization measurements should be able to test this explanation.

DOI: 10.1103/PhysRevD.89.101302 PACS numbers: 98.80.Cq, 98.70.Vc

I. INTRODUCTION

The recent BICEP2 measurement of a tensor-scalar ratio $r = 0.2^{+0.07}_{-0.05}$ from degree scale B-mode polarization of the cosmic-microwave background (CMB) [1] is in "moderately strong" tension with slow-roll inflation models that predict scale-free, albeit slightly tilted $(1 - n_s \ll 1)$ power-law power spectra. This tension is due to the implied excess in the temperature spectrum at low multipoles which is not observed and restricts $r_{0.002} < 0.11$ (95% C.L.) in this context [2].

These findings can be reconciled in the single-field inflationary paradigm by introducing a scale into the scalar power spectra to suppress power on these large-angular scales. For example a large running of tilt, $dn_s/d\ln k \sim -0.02$, is possible as a compromise [1]. Here the scale introduced is associated with the scalar spectrum transiently passing through a scale-invariant slope near observed scales. However, such a large running is uncomfortable in the simplest models of inflation which typically produce running of order $\mathcal{O}[(1 - n_s)^2]$. Moreover, a large running also requires further additional parameters in order that inflation does not end too quickly after the observed scales leave the horizon [3].

The temperature anisotropy excess implied by tensors is also not a smooth function of scale, but rather cut off at the horizon at recombination. To counter this excess, a transition in the scalar power spectrum that occurs more sharply, though coincidentally near these scales, would be preferred. Such a transition can occur without affecting the tensor spectrum if there is a slow-roll violating step in the tensor-scalar ratio while the Hubble rate is left nearly fixed. In this work we consider the effects of placing such a feature near scales associated with the horizon at recombination, thereby suppressing the scalar spectrum on large scales.

This slow-roll violating behavior also produces oscillations in the power spectrum [4–7] and generates enhanced non-Gaussianity [8,9] if this transition occurs in much less than an e-fold. For transitions that alleviate the tensor-scalar tension, these oscillations would violate tight constraints on the acoustic peaks and hence only transitions that occur over at least an e-fold are allowed. The resulting non-Gaussianity is then undetectable [10,11]. Throughout, we work in natural units where the reduced Planck mass $M_{\rm Pl} = (8\pi G_N)^{-1/2} = 1$ as well as $c = \hbar = 1$.

II. STEP SOLUTIONS

In slow-roll inflation, the tensor power spectrum in each gravitational wave polarization state is directly related to the Hubble scale during inflation

$$\Delta^2_{+,\times} = \frac{H^2}{2\pi^2}, \tag{1}$$

whereas the scalar or curvature power spectrum is given by

$$\Delta^2_{\mathcal{R}} = \frac{H^2}{8\pi^2 \epsilon_H c_s}, \tag{2}$$

where $\epsilon_H = -d\ln H/d\ln a$ and c_s is the sound speed, yielding a tensor-scalar ratio $r = 4\Delta^2_{+,\times}/\Delta^2_{\mathcal{R}} = 16\epsilon_H c_s$. The addition of a nearly scale-invariant tensor spectrum

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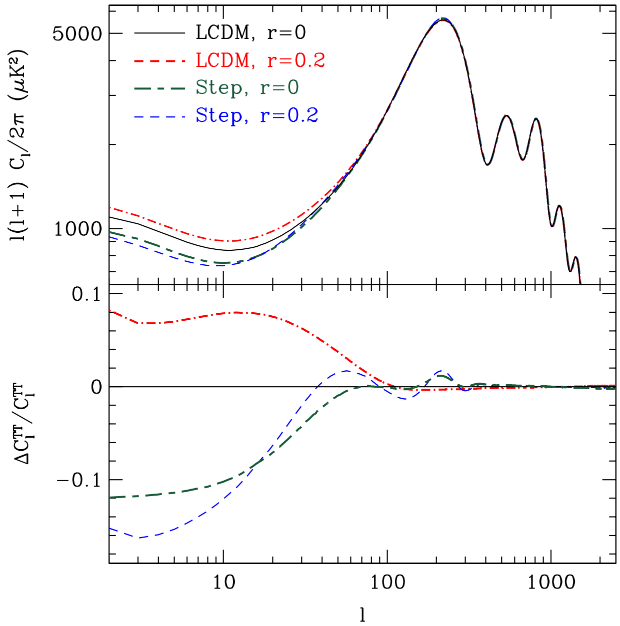

FIG. 1 (color online). Total temperature power spectra showing the unobserved excess produced by adding tensors of $r = 0.2$ to the best fit 6 parameter Λ cold dark matter (ΛCDM) model and its removal by adding a step in the tensor-scalar parameter $\epsilon_H c_s$. Planck data in fact favor removing more power than the tensor excess, preferring a step even if $r = 0$. Step model parameters are given in Table I.

to the CMB temperature anisotropy produces excess power below $\ell \approx 100$ which at $r = 0.2$ is difficult to accommodate in slow-roll inflation where the scalar spectrum is, to a good approximation, a scale-free power law (see Fig. 1).

The scalar power spectrum can be changed largely without affecting the tensors if the quantity $\epsilon_H c_s$ changes while ϵ_H remains small. As shown in Fig. 1, the excess power resembles a step in this quantity on scales near the horizon at recombination. Hence to alleviate the tension between the tensor inference from the BICEP2 experiment, $r = 0.2^{+0.07}_{-0.05}$, and the upper limits from the combined CMB temperature power spectrum $r_{0.002} < 0.11$ (95% C.L.), we examine models where there is a step in this quantity (see Fig. 2). In this paper we quote r at the scalar pivot of $k = 0.05\ \mathrm{Mpc}^{-1}$ where it is unaffected by changes to the scalar power spectrum that we introduce whereas the upper limit is quoted at $k = 0.002\ \mathrm{Mpc}^{-1}$.

As an example, we consider a step in the warp

$$T(\phi) = \frac{\phi^4}{\lambda_B}\left\{1 + b_T\left[\tanh\left(\frac{\phi - \phi_s}{d}\right) - 1\right]\right\} \tag{3}$$

of Dirac-Born-Infeld (DBI) inflation[1] [12,13] with the Lagrangian

$$\mathcal{L} = [1 - \sqrt{1 - 2X/T(\phi)}]T(\phi) - V(\phi), \tag{4}$$

where the kinetic term $2X = -\nabla^\mu\phi\nabla_\mu\phi$, the sound speed

$$c_s(\phi, X) = \sqrt{1 - 2X/T(\phi)}. \tag{5}$$

Here $\{b_T, \phi_s, d\}$ parametrize the height, field position and field width of the step while the underlying parameters λ_B and the inflaton potential $V(\phi)$ are set to fix n_s and A_s [14]. In Ref. [7], we showed that such a model produces a step in the quantity $\epsilon_H c_s$ that controls the tensor-scalar ratio. To keep this discussion model independent, we follow Ref. [15] and quantify the amplitude of the step by the change in this quantity

$$C_1 = -\ln\frac{\epsilon_{Hb}c_{sb}}{\epsilon_{Ha}c_{sa}}, \tag{6}$$

where "b" and "a" denote the quantities before and after the step on the slow-roll attractor. For definiteness, we take $c_{sb} \approx 1$. In place of ϕ_s we quote the sound horizon

$$s = \int dN \frac{c_s}{aH} \tag{7}$$

at the step $s_s = s(\phi_s)$ and in place of the width in field space d, we take the inverse of the number of e-folds N the inflaton takes in traversing the step

$$x_d = \frac{1}{\pi d}\frac{d\phi}{d\ln s}. \tag{8}$$

See Refs. [15,16] for details of this description. We utilize the generalized slow-roll technique [17–19] to calculate the power spectra of these models since at the step the slow-roll approximation is transiently violated.

III. JOINT FIT

We jointly fit the Planck CMB temperature results, WMAP9 polarization results, and BICEP2 to models with and without steps in the tensor-scalar ratio parameter $\epsilon_H c_s$. We use the MIGRAD variable metric algorithm from the CERN Minuit2 code [20] and a modified version of CAMB [21,22] for model comparisons. The Planck likelihood includes the Planck low-ℓ spectrum (Commander, $\ell < 50$) and the high-ℓ spectrum (CAMspec, $50 < \ell < 2500$), whereas the BICEP2 likelihood[2] includes both its E and B contributions.

We begin with the baseline best fit 6 parameter slow-roll flat ΛCDM model with $r = 0$. This model sets the noninflationary cosmological parameters to $\Omega_c h^2 = 0.1200$, $\Omega_b h^2 = 0.02204$, $h = 0.672$, $\tau = 0.0895$ and the inflationary scalar amplitude at $k = 0.05\ \mathrm{Mpc}^{-1}$,

[1]Of course, we are well outside the region of validity of UV complete versions of DBI inflation. However, this is merely a phenomenological proof of principle rather than a working construction.

[2]http://bicepkeck.org/

TABLE I. Likelihood for models with tensors and steps with noninflationary parameters fixed. $\mathcal{L}_P$ is the likelihood for the Planck low-ℓ spectrum, high-ℓ spectrum and WMAP9 polarization; $\mathcal{L}_B$ is that for the BICEP2 E and B likelihood. The change in the total is quoted relative to the $r = 0.2$ no feature case.

r	C_1	s_s (Mpc)	x_d	$A_s \times 10^9$	n_s	$-2 \ln \mathcal{L}_P$	$-2 \ln \mathcal{L}_B$	$-2 \ln \Delta\mathcal{L}_{\rm tot}$
0	0	…	…	2.1972	0.961	9802.7	89.1	40.1
0	−0.15	337.1	1.58	2.2003	0.957	9798.6	89.2	36.1
0.1	0	…	…	2.1961	0.962	9806.5	47.9	2.7
0.1	−0.22	339.2	1.60	2.2000	0.958	9797.8	48.2	−5.7
0.2	0	…	…	2.1939	0.963	9812.3	39.4	0
0.2	−0.31	351.8	1.47	2.2002	0.959	9798.1	39.9	−13.7

$A_s = 2.1972 \times 10^{-9}$, and spectral tilt, $n_s = 0.961$. When considering alternate models we fix the noninflationary parameters to these values while allowing the inflationary parameters, including A_s and n_s to vary.

As shown in Table I, this $r = 0$ model is strongly penalized by the BICEP2 data. Moving to the $r = 0.2$ model with the same parameters removes this penalty at the expense of making the Planck likelihood worse by $2\Delta \ln \mathcal{L} = 9.6$ due to the excess in the $\ell \lesssim 100$ temperature power spectrum shown in Fig. 1.

Next we fit for a step with parameters C_1, s_s, x_d controlling the amplitude, location and width of the step. The best fit model at $r = 0.2$ more than removes the penalty from the temperature excess for Planck while fitting the BICEP2 BB results equally well. The net result is a preference for a step feature at the level of $2\Delta \ln \mathcal{L}_P = -14.2$ over no feature. The inclusion of BICEP2 results slightly degrades the fit to $2\Delta \ln \mathcal{L}_{\rm tot} = -13.7$ due to changes in the EE spectrum (see below). The $r = 0.2$ model with a step is very close to the global maximum with further optimization in r allowing only an improvement of $2\Delta \ln \mathcal{L}_{\rm tot} = -0.1$. With the addition of the step, there remains a small high-ℓ change in the vicinity of the first acoustic peak in Fig. 1 which is interestingly marginally favored by the data. Note that we have fixed the noninflationary parameters to their values without the step, for example τ. Thus the likelihood may in fact increase in a full fit (see Fig. 3). Conversely, we do not consider any compromise solutions where noninflationary cosmological parameters ameliorate the tension without a step. We leave these considerations to a future work.

The best fit step also predicts changes to the EE polarization. Like the TT spectrum, the excess power from the tensor contribution is partially compensated by the reduction in the scalar spectrum for $\ell \gtrsim 30$. This is a signature of the step model which requires only a moderate increase in data to test as witnessed by the change in the BICEP2 likelihood of $2\Delta \ln \mathcal{L}_B \sim 0.5$ it induces.

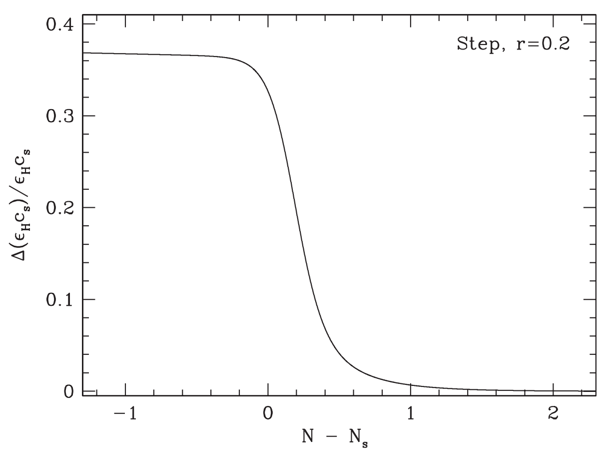

FIG. 2. Step in tensor-scalar ratio parameter $\epsilon_H c_s$ relative to no step, from the best fit $r = 0.2$ solution centered at the e-fold N_s at which the inflaton crosses the step. Planck data favor a step that is traversed in about an e-fold.

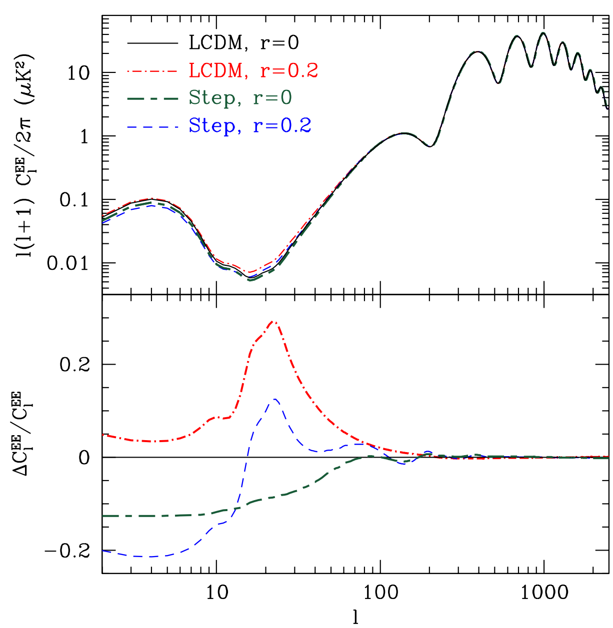

FIG. 3 (color online). EE power spectrum for the models in Fig. 1 showing the change from the best fit $r = 0$ ΛCDM power spectrum. Excess E modes from the tensors at $r = 0.2$ are partially compensated by the step at $\ell \gtrsim 30$ while changes at lower ℓ can be altered by changing the reionization history. Preference for removing power at substantially smaller r would predict a deficit of power as the $r = 0$ model shows.

Differences at $\ell \lesssim 30$, shown here at fixed τ, are largely degenerate with changes in the ionization history [23]

Due to potential contributions from foregrounds in the BICEP2 data which may imply a shift to $r = 0.16^{+0.06}_{-0.05}$ [1], we also test models at $r = 0.1$ which would formally be in tension with the BICEP2 likelihood without foreground subtraction. Even in this case, the Planck portion of the likelihood improves with the inclusion of a step though the preference is weakened to $2\Delta \ln \mathcal{L}_P = -8.6$ versus no step. At $r = 0$, the Planck data still prefers a step to remove power at a reduced improvement of $2\Delta \ln \mathcal{L}_P = -4.1$, a fact that was already evident in the Planck Collaboration analysis of anticorrelated isocurvature perturbations [2]. Such an explanation should also help resolve the tensor-scalar tension albeit outside of the context of single-field inflation. Interestingly, the addition of tensors at both $r = 0.1$ and 0.2 in fact further helps step models fit the Planck data due to the changes shown in Fig. 1 independent of the BICEP2 result.

IV. DISCUSSION

A transient violation of slow roll which generates a step in the scalar power spectrum at scales near to the horizon size at recombination can alleviate problems of predicted excess power in the temperature spectrum, present already in the best fit ΛCDM spectrum, and greatly exacerbated by tensor contributions implied by the BICEP2 measurement. Such a step may be generated by a sharp change in the speed of the rolling of the inflaton ϵ_H or by a sharp change in the speed of sound c_s over a period of around an e-folding which combine to form the tensor-scalar ratio. Preference for a step from the temperature power spectrum is at a level of $2\Delta \ln \mathcal{L}_P = -14.2$ if $r = 0.2$ and is still -8.6 at $r = 0.1$, the lowest plausible value that would fit the BICEP2 data.

Such an explanation makes several concrete predictions. Since slow roll is transiently violated in this scenario, there will be an enhancement in the associated three-point correlation function. However, we do not expect this signal to be observable as it impacts only a small number of modes [10,11]. E-mode fluctuations on similar scales would be predicted to have a smaller enhancement then with tensors alone. This prediction should soon be testable; in the BICEP2 data it brings down the total likelihood improvement to $2\Delta \ln \mathcal{L}_{\rm tot} = -13.7$ with a step at $r = 0.2$.

While we have used a DBI-type Lagrangian to illustrate the impact of a change in the tensor-scalar ratio parameter $\epsilon_H c_s$ due to a step in the sound speed, we do not expect that our results require this form, though precise details of the fit may change. Transient shifts in the speed of sound have been found to occur in inflationary models where additional heavy degrees of freedom have been integrated out [24]. We leave investigation of specific constructions to future work.

ACKNOWLEDGMENTS

We thank Maurício Calvão, Cora Dvorkin, Dan Grin, Chris Sheehy and Ioav Waga for useful conversations. This work was supported in part by the Kavli Institute for Cosmological Physics at the University of Chicago through Grant No. NSF PHY-1125897 and an endowment from the Kavli Foundation and its founder Fred Kavli. W. H. was additionally supported by U.S. Department of Energy Contract No. DE-FG02-13ER41958 and V. M. by the Brazilian Research Agency CAPES Foundation and by U.S. Fulbright Organization.

Note added.—While this work was in preparation, the work [25] appeared which has some overlap with the work presented here.

[1] P. Ade *et al.* (BICEP2 Collaboration), arXiv:1403.3985.
[2] P. Ade *et al.* (Planck Collaboration), arXiv:1303.5082.
[3] R. Easther and H. Peiris, J. Cosmol. Astropart. Phys. 09 (2006) 010.
[4] J. A. Adams, B. Cresswell, and R. Easther, Phys. Rev. D **64**, 123514 (2001).
[5] H. Peiris *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **148**, 213 (2003).
[6] M. Park and L. Sorbo, Phys. Rev. D **85**, 083520 (2012).
[7] V. Miranda, W. Hu, and P. Adshead, Phys. Rev. D **86**, 063529 (2012).
[8] X. Chen, R. Easther, and E. A. Lim, J. Cosmol. Astropart. Phys. 06 (2007) 023.
[9] X. Chen, R. Easther, and E. A. Lim, J. Cosmol. Astropart. Phys. 04 (2008) 010.
[10] P. Adshead, W. Hu, C. Dvorkin, and H. V. Peiris, Phys. Rev. D **84**, 043519 (2011).
[11] P. Adshead and W. Hu, Phys. Rev. D **85**, 103531 (2012).
[12] E. Silverstein and D. Tong, Phys. Rev. D **70**, 103505 (2004).
[13] M. Alishahiha, E. Silverstein, and D. Tong, Phys. Rev. D **70**, 123505 (2004).
[14] P. Adshead, W. Hu, and V. Miranda, Phys. Rev. D **88**, 023507 (2013).
[15] V. Miranda and W. Hu, Phys. Rev. D **89**, 083529 (2014).
[16] P. Adshead, C. Dvorkin, W. Hu, and E. A. Lim, Phys. Rev. D **85**, 023531 (2012).
[17] E. D. Stewart, Phys. Rev. D **65**, 103508 (2002).
[18] C. Dvorkin and W. Hu, Phys. Rev. D **81**, 023518 (2010).
[19] W. Hu, Phys. Rev. D **84**, 027303 (2011).

[20] F. James and M. Roos, Comput. Phys. Commun. **10**, 343 (1975).

[21] A. Lewis, A. Challinor, and A. Lasenby, Astrophys. J. **538**, 473 (2000).

[22] C. Howlett, A. Lewis, A. Hall, and A. Challinor, J. Cosmol. Astropart. Phys. 04 (2012) 027.

[23] M. J. Mortonson, C. Dvorkin, H. V. Peiris, and W. Hu, Phys. Rev. D **79**, 103519 (2009).

[24] A. Achúcarro, V. Atal, S. Céspedes, J.-O. Gong, G. A. Palma, and S. P. Patil, Phys. Rev. D **86**, 121301 (2012).

[25] C. R. Contaldi, M. Peloso, and L. Sorbo, arXiv:1403.4596.