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Axial-vector $f_1(1285) - f_1(1420)$ mixing and $B_s \to J/\psi(f_1(1285), f_1(1420))$ decays

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Inspired by the very recent LHCb measurements of $B_s \to J/\psi f_1(1285)$ and the good agreement between the perturbative QCD predictions and the data for many $B \to J/\psi V$ decays, we here investigate the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays for the first time by employing the perturbative QCD (pQCD) approach, in which the 1^3P_1 states $f_1(1285)$ and $f_1(1420)$ are believed to be the mixture of flavor singlet f_1 and octet f_8 or of quark-flavor states f_{1q} and f_{1s}. We show that the pQCD predictions for the branching ratio of $B_s \to J/\psi f_1(1285)$ agree well with the data within errors for the mixing angle $\theta_{^3P_1} \approx 20°(\phi_{^3P_1} \approx 15°)$ between $f_1(f_{1q})$ and $f_8(f_{1s})$ states. Furthermore, the branching ratio of $B_s \to J/\psi f_1(1420)$ and the large transverse polarization fractions in these two considered channels are also predicted and will be tested by the LHC and the forthcoming Super-B factory experiments. Based on the decay rates of $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decay modes predicted in the pQCD approach, the extracted mixing angle between $f_1(1285)$ and $f_1(1420)$ is basically consistent with currently available experimental measurements and lattice QCD analysis within still large theoretical errors.

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Very recently, the LHCb Collaboration located at CERN reported the first observation of $B_s \to J/\psi f_1(1285)$ decay with the branching ratio [1],

$$\mathrm{Br}(B_s \to J/\psi f_1(1285))_{\mathrm{Exp}} = (7.14 \pm 0.99^{+0.83}_{-0.91} \pm 0.41) \times 10^{-5}. \tag{1}$$

Of course, the accuracy of the above data is expected to be improved rapidly with the future LHCb and Super-B experiments. By combining the first measurement of the $B_d \to J/\psi f_1(1285)$ channel with this new one, the mixing angle between the strange and nonstrange component of the wave function of $f_1(1285)$ in the $q\bar{q}$ structure model is determined to be $\pm(24.0^{+3.1+0.6}_{-2.6-0.8})°$ [1] for the first time *in B meson decays*.

In the quark model, as is well known, $f_1(1285)$ is treated as a p-wave axial-vector meson with $J^{\mathrm{PC}} = 1^{++}$, which is believed to mix with its partner $f_1(1420)$ [2,3] just like the "$\eta - \eta'$" mixing in the pseudoscalar sector. Up to now, many discussions have been presented on the mixing angle $\theta_{^3P_1}$ or $\phi_{^3P_1}$ of $f_1(1285) - f_1(1420)$ mixing, in the framework of the two popular mixing schemes: i.e., the so-called singlet-octet (SO) basis and the quark-flavor (QF) basis [4–16]. One of the most important reasons is that the mixing angle $\theta_{^3P_1}$ or $\phi_{^3P_1}$ can be utilized to constrain the magnitude of the mixing angle θ_{K_1} of the axial-vector $K_1(1270) - K_1(1400)$ system [13], which is a very special mixing between two distinct types of axial-vector mesons $K_{1A}(1^3P_1)$ and $K_{1B}(1^1P_1)$.

In the SO basis, the axial-vector $f_1(1285) - f_1(1420)$ mixing can be written in the form of [17]

$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos\theta_{^3P_1} & \sin\theta_{^3P_1} \\ -\sin\theta_{^3P_1} & \cos\theta_{^3P_1} \end{pmatrix} \begin{pmatrix} f_1 \\ f_8 \end{pmatrix}, \tag{2}$$

with the SO states $f_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $f_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$. In the QF basis, the $f_1(1285) - f_1(1420)$ mixing can be written as the following pattern [17]:

$$\begin{aligned} f_1(1285) &= \cos\phi_{^3P_1} f_{1q} + \sin\phi_{^3P_1} f_{1s}, \\ f_1(1420) &= \sin\phi_{^3P_1} f_{1q} - \cos\phi_{^3P_1} f_{1s} \end{aligned} \tag{3}$$

with the QF states $f_{1q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $f_{1s} = s\bar{s}$. The QF mixing angle $\phi_{^3P_1}$ is related to the SO mixing angle $\theta_{^3P_1}$ by the relation $\phi_{^3P_1} = \theta_i - \theta_{^3P_1}$, where θ_i is the "ideal" mixing angle with $\theta_i = 35.3°$. Therefore, $\phi_{^3P_1}$ measures the deviation from ideal mixing.

Though the $f_1(1285)$ mixing angle has been preliminarily determined through the $B_{d/s} \to J/\psi f_1(1285)$ decays in the QF basis by the LHCb Collaboration, it is necessary to point out that the assumption of exact SU(3) flavor symmetry on the decay amplitudes of $B_{d/s} \to J/\psi f_1(1285)$ has been

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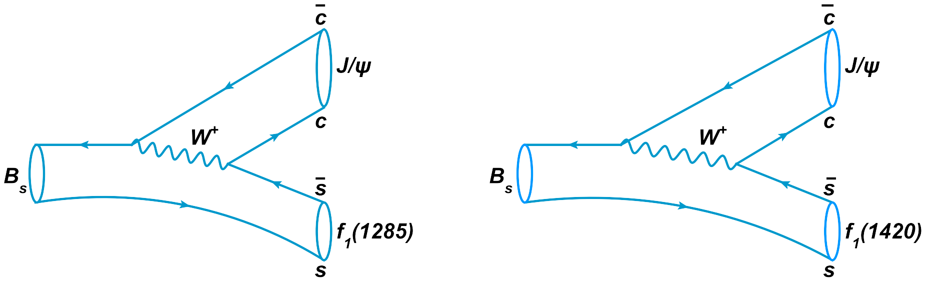

FIG. 1 (color online). Leading quark-level Feynman diagrams for the $B_s \to J/\psi f_1(1285)$ (left) and $B_s \to J/\psi f_1(1420)$ (right) decays.

adopted there [1]. In fact, at the theoretical aspect, the contributing components in the above-mentioned $B_d \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1285)$ decays at the quark level should be the QF states f_{1q} and f_{1s} respectively, whose behavior may be rather different because of the breaking of SU(3) flavor symmetry for f_{1q} and f_{1s}. Consequently, the resultant mixing angles may considerably shift away from the expected values.

It may be very interesting to study the mixing angle of $f_1(1285) - f_1(1420)$ mixing through the same components at the quark level, for example, the $f_1(1285)$ and $f_1(1420)$ mesons are produced through their strange components in the B_s meson decays, as illustrated in Fig. 1. For the $B_s \to J/\psi f_1(1285)$ [$B_s \to J/\psi f_1(1420)$] decay, the coefficient for the $s\bar{s}$ component is $\sin\phi_{{}^3P_1}$ ($-\cos\phi_{{}^3P_1}$) in the QF basis, and $\frac{\cos\theta_{{}^3P_1}}{\sqrt{3}} - \frac{\sqrt{2}\cdot\sin\theta_{{}^3P_1}}{\sqrt{3}}$ ($-\frac{\sin\theta_{{}^3P_1}}{\sqrt{3}} - \frac{\sqrt{2}\cdot\cos\theta_{{}^3P_1}}{\sqrt{3}}$) in the SO basis, respectively.

One can see that the angle $\phi_{{}^3P_1}$ of $f_1(1285) - f_1(1420)$ mixing in the QF basis could be extracted more directly and cleanly through the following ratio:

$$R_s^{QF} \equiv \frac{Br(B_s \to J/\psi f_1(1285))}{Br(B_s \to J/\psi f_1(1420))} = \frac{\Phi_{f_1(1285)} \cdot |\sin\phi_{{}^3P_1} \cdot \mathcal{A}(B_s \to J/\psi f_{1s})|^2}{\Phi_{f_1(1420)} \cdot |-\cos\phi_{{}^3P_1} \cdot \mathcal{A}(B_s \to J/\psi f_{1s})|^2} = \frac{\Phi_{f_1(1285)}}{\Phi_{f_1(1420)}} \times \tan^2\phi_{{}^3P_1}, \tag{4}$$

where $\Phi_{f_1(1285)}$ and $\Phi_{f_1(1420)}$ are the phase space factors for $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays, respectively. Once the precise measurements for the decay rates of these two channels are available, one could extract the mixing angle $\phi_{{}^3P_1}$ through Eq. (4) directly. In view of the equivalence for $f_1(1285) - f_1(1420)$ mixing in the QF basis and the SO basis, the above ratio defined in Eq. (4) can also be expressed in the SO basis as follows:

$$R_s^{SO} \equiv \frac{Br(B_s \to J/\psi f_1(1285))}{Br(B_s \to J/\psi f_1(1420))} = \frac{\Phi_{f_1(1285)}}{\Phi_{f_1(1420)}} \cdot \frac{|\frac{\cos\theta_{{}^3P_1}}{\sqrt{3}} \cdot \mathcal{A}(B_s \to J/\psi f_1) - 2 \cdot \frac{\sin\theta_{{}^3P_1}}{\sqrt{6}} \cdot \mathcal{A}(B_s \to J/\psi f_8)|^2}{|-\frac{\sin\theta_{{}^3P_1}}{\sqrt{3}} \cdot \mathcal{A}(B_s \to J/\psi f_1) - 2 \cdot \frac{\cos\theta_{{}^3P_1}}{\sqrt{6}} \cdot \mathcal{A}(B_s \to J/\psi f_8)|^2}, \tag{5}$$

which can also be used to extract out the mixing angle $\theta_{{}^3P_1}$ approximately based on the assumption [5] that $\mathcal{A}(B_s \to J/\psi f_1) \approx \mathcal{A}(B_s \to J/\psi f_8)$,[1] then extract out the mixing angle $\phi_{{}^3P_1}$ via the relation $\phi_{{}^3P_1} = \theta_i - \theta_{{}^3P_1}$.

Also, the decays of B mesons into final states containing the J/ψ charmonium state also play a special role in studies of charge parity (CP) violation physics [18]. As discussed in the literature [7–10], the behavior of the 1^3P_1 axial-vector meson is similar to that of the vector meson. It is naturally expected that the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays can serve as the alternative channels to reduce the errors in the determination of the $B_s - \bar{B}_s$ mixing phase ϕ_s effectively.

We here will investigate the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays in the perturbative QCD (pQCD) approach [19–21] with the aforementioned two mixing schemes. Because of the similar behavior between the 1^3P_1 axial-vector mesons and the vector mesons and the global agreement between the theoretical predictions in the pQCD approach and the presently existing experimental data for the $B \to J/\psi V$ decays [22], we can therefore calculate the decay amplitudes for the $B_s \to J/\psi f_1$ and $J/\psi f_8$ decays or $B_s \to J/\psi f_{1s}$ decay at next-to-leading order of the strong coupling constant α_s straightforwardly by substituting the kinematic variables and distribution amplitudes of ϕ in the $B_s \to J/\psi\phi$ mode to those of f_1 and f_8 or f_{1s} in the considered decays, apart from an overall minus sign that arises from the definitions of the wave functions for axial-vector and vector mesons.

Since the vector meson ρ and ω have the same distribution amplitudes, except for the different decay constant

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[1] Actually, as presented in Ref. [7], the two SO states f_1 and f_8 have the similar hadronic parameters, which can also be seen from the similarity of the relevant input parameters in Eq. (6) and the related phenomenological discussions.

f_ρ and f_ω, we assume that the distribution amplitude of the QF state f_{1q} is the same one as $a_1(1260)$ with decay constant $f_{f_{1q}} = 0.193^{+0.043}_{-0.038}$ GeV [23]. For the f_{1s} state, for the sake of simplicity, we adopt the same distribution amplitude as f_1 with decay constant $f_{f_{1s}} = 0.230 \pm 0.009$ GeV [23]. In fact, we have confirmed that the *CP*-averaged branching ratios just vary 3% for the change of the distribution amplitude of f_1 into that of f_8.

The following input parameters, such as the QCD scale (GeV), masses (GeV), decay constants (GeV) and B_s meson lifetime (ps) as given in Refs. [7,11,17,23], will be used in the numerical calculations:

$$\begin{aligned}
&\Lambda^{(f=4)}_{\overline{\rm MS}} = 0.287, \quad && m_W = 80.41, \quad && m_b = 4.8, \quad && m_{B_s} = 5.37;\\
&m_{J/\psi} = 3.097, && m_{f_1} = 1.28, && m_{f_8} = 1.29, && m_c = 1.50;\\
&f_{J/\psi} = 0.405, && f_{B_s} = 0.23, && f_{f_1} = 0.245, && f_{f_8} = 0.239;\\
&\tau_{B_s} = 1.497, && f_{f_{1s}} = 0.230, && \theta_{^3P_1} = 20^\circ, && \phi_{^3P_1} = 15.3^\circ.
\end{aligned} \tag{6}$$

For the mixing angle of the $f_1(1285) - f_1(1420)$ system, we here adopt the recently updated value $\theta_{^3P_1} \approx 20^\circ$ and $\phi_{^3P_1} \approx 15.3^\circ$ extracted from the $f_1(1285) \to \rho\gamma, \phi\gamma$ decays [11], to calculate the physical quantities for the two considered B_s decays. For the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, we adopt the Wolfenstein parametrization up to corrections of $\mathcal{O}(\lambda^5)$ and the updated parameters $A = 0.811$, $\lambda = 0.22535$, $\bar\rho = 0.131^{+0.026}_{-0.013}$ and $\bar\eta = 0.345^{+0.013}_{-0.014}$ as given in PDG 2012 [17].

The pQCD predictions for the *CP*-averaged branching ratios of the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays within errors in the standard model with the two mixing schemes are the following:

(i) In the QF basis

$$\begin{aligned}
{\rm Br}(B_s \to J/\psi f_1(1285)) &= 7.70^{+2.30}_{-1.74}(\omega_B)^{+1.05}_{-0.99}(f_M)^{+3.33}_{-2.50}(a_i)^{+1.22}_{-1.25}(m_c)^{+4.38}_{-3.45}(\phi_{^3P_1})^{+0.22}_{-0.30}(a_t)\\
&= \left[7.70^{+6.18}_{-4.88}\right] \times 10^{-5},
\end{aligned} \tag{7}$$

$$\begin{aligned}
{\rm Br}(B_s \to J/\psi f_1(1420)) &= 0.97^{+0.30}_{-0.21}(\omega_B)^{+0.14}_{-0.12}(f_M)^{+0.42}_{-0.31}(a_i)^{+0.17}_{-0.15}(m_c)^{+0.04}_{-0.04}(\phi_{^3P_1})^{+0.04}_{-0.04}(a_t)\\
&= \left[0.97^{+0.56}_{-0.42}\right] \times 10^{-3}.
\end{aligned} \tag{8}$$

(ii) In the SO basis

$$\begin{aligned}
{\rm Br}(B_s \to J/\psi f_1(1285)) &= 8.71^{+2.59}_{-1.99}(\omega_B)^{+2.46}_{-2.23}(f_M)^{+9.26}_{-5.40}(a_i)^{+1.25}_{-1.34}(m_c)^{+4.96}_{-3.91}(\theta_{^3P_1})^{+0.23}_{-0.34}(a_t)\\
&= \left[8.71^{+11.17}_{-7.44}\right] \times 10^{-5},
\end{aligned} \tag{9}$$

$$\begin{aligned}
{\rm Br}(B_s \to J/\psi f_1(1420)) &= 1.06^{+0.32}_{-0.23}(\omega_B)^{+0.16}_{-0.14}(f_M)^{+0.31}_{-0.25}(a_i)^{+0.19}_{-0.18}(m_c)^{+0.04}_{-0.04}(\theta_{^3P_1})^{+0.04}_{-0.04}(a_t)\\
&= \left[1.06^{+0.51}_{-0.41}\right] \times 10^{-3},
\end{aligned} \tag{10}$$

where the total errors are obtained by adding the errors from different sources in quadrature. The individual theoretical errors are induced by the variation of the shape parameter $\omega_B = 0.50 \pm 0.05$ GeV [24] for the B_s meson wave function, of the J/ψ meson decay constant $f_{J/\psi} = 0.405 \pm 0.014$ GeV [25,26] and the $f_1(f_8)$ state decay constant $f_{f_1} = 0.245 \pm 0.013 (f_{f_8} = 0.239 \pm 0.013)$ GeV [7] or the f_{1s} state decay constant $f_{f_{1s}} = 0.230 \pm 0.009$ GeV [23], of the Gegenbauer moments $a_2^\parallel = -0.04 \pm 0.03$ and $a_1^\perp = -1.06 \pm 0.36$ $(a_2^\parallel = -0.07 \pm 0.04$ and $a_1^\perp = -1.11 \pm 0.31)$ for the $f_1(f_8)$ distribution amplitudes [7], of the charm quark mass $m_c = 1.50 \pm 0.15$ GeV, and of the mixing angle $\phi_{^3P_1} = (15.3 \pm 4)^\circ$ or $\theta_{^3P_1} = (20 \pm 4)^\circ$ in the QF or SO basis [11], respectively. Moreover, as displayed in Eqs. (7)–(10), the higher order contributions are also simply investigated by varying the hard scale $t_{\rm max}$ from $0.8t$ to $1.2t$ (not changing $1/b_i, i = 1, 2, 3$) in the hard kernel, which has been counted as one of the sources of theoretical uncertainties. It is found that the higher order corrections to these considered $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays are indeed small as the naive expectation. It is worthwhile to stress that the variation of the CKM parameters has almost no effects on the *CP*-averaged branching ratios and polarization fractions of these considered $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays in the pQCD approach and thus will be neglected in the numerical results as shown in Eqs. (7)–(10) and Table I.

It is easy to see that the pQCD predictions for the branching ratios $\mathrm{Br}(B_s \to J/\psi f_1(1285))$, in both the SO and QF mixing schemes, agree well with currently available data $(7.14^{+1.36}_{-1.41}) \times 10^{-5}$ [1] within the theoretical errors. Meanwhile, we observe that the pQCD predictions for the branching ratios of $B_s \to J/\psi f_1(1420)$ decay mode are at the order of 10^{-3}, very similar to the decay rate of $B_s \to J/\psi\phi$, and can be accessed and tested easily at the running LHCb and forthcoming Super-B experiments in the near future. The slightly larger central value of $\mathrm{Br}(B_s \to J/\psi f_1(1285))$ and $\mathrm{Br}(B_s \to J/\psi f_1(1420))$ in the SO basis than the one in the QF basis is due to the larger decay constants of f_1 and f_8 than that of f_{1s}, which can be clearly seen in Eq. (6).

When the very recently measured value of the mixing angle $\phi_{^3P_1} = 24°$ [1] is used in the numerical calculations, we find the pQCD predictions for the branching ratios: $\mathrm{Br}(B_s \to J/\psi f_1(1285)) = 18.29 \times 10^{-5}$ (20.71×10^{-5}) and $\mathrm{Br}(B_s \to J/\psi f_1(1420)) = 0.87 \times 10^{-3}$ (0.95×10^{-3}) in the QF (SO) basis. One can see that the central values of the above pQCD predictions for the decay rates $\mathrm{Br}(B_s \to J/\psi f_1(1285))$ in both mixing schemes exceed the measured value as listed in Eq. (1).

Moreover, according to the theoretical predictions in the pQCD approach, one can see that the decay rate for $B_s \to J/\psi f_1(1285)$ is more sensitive to the variation of the mixing angle $\theta_{^3P_1}(\phi_{^3P_1})$ than that for $B_s \to J/\psi f_1(1420)$, since the $f_1(1285)$ meson is dominated by the $u\bar{u} + d\bar{d}$ component while the $f_1(1420)$ meson is determined by the $s\bar{s}$ component.

With the help of Eq. (4), by combining the decay rate of $B_s \to J/\psi f_1(1285)$ as given in Eq. (1) and $\tan^2\phi = 0.1970 \pm 0.053^{+0.014}_{-0.012}$ [1], one can find that $\mathrm{Br}(B_s \to J/\psi f_1(1420)) = (3.42^{+1.15}_{-1.16}) \times 10^{-4}$, which is only about 35% of our pQCD predictions in both mixing schemes as given in Eqs. (8) and (9). Once the future measurements confirm this estimation, it may imply the existence of a large exotic gluonic component in the $f_1(1420)$ meson, something similar to the case of η' [27] in the $\eta - \eta'$ mixing system, which would need further studies in the future, although there are now no any signals observed at the experiments.

TABLE I. The theoretical predictions for the CP-averaged polarization fractions of $B_s \to J/\psi f_1(1285)$ and $J/\psi f_1(1420)$ decays in the pQCD approach with different mixing schemes.

Decay modes	QF basis (%)	SO basis (%)	Data
$B_s \to J/\psi f_1(1285)$	$34.3^{+14.7}_{-9.9}(L)$	$36.3^{+37.2}_{-17.7}(L)$	⋯
	$40.7^{+6.2}_{-8.7}(\parallel)$	$39.8^{+10.4}_{-22.5}(\parallel)$	
	$24.9^{+3.8}_{-5.8}(\perp)$	$23.9^{+7.3}_{-15.5}(\perp)$	
$B_s \to J/\psi f_1(1420)$	$34.7^{+14.3}_{-10.0}(L)$	$33.9^{+9.8}_{-8.5}(L)$	⋯
	$42.5^{+6.7}_{-9.0}(\parallel)$	$42.7^{+5.9}_{-6.3}(\parallel)$	
	$22.8^{+3.5}_{-5.3}(\perp)$	$23.4^{+2.7}_{-3.5}(\perp)$	

Based on the above theoretical predictions for the CP-averaged branching ratios of $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays in the pQCD approach, the ratios of the decay rates between these two modes can be obtained directly as follows:

$$\mathrm{R}_s^{\mathrm{QF;th}} \equiv \frac{\mathrm{Br}(B_s \to J/\psi f_1(1285))}{\mathrm{Br}(B_s \to J/\psi f_1(1420))} = 0.079^{+0.078}_{-0.061} \tag{11}$$

and

$$\mathrm{R}_s^{\mathrm{SO;th}} \equiv \frac{\mathrm{Br}(B_s \to J/\psi f_1(1285))}{\mathrm{Br}(B_s \to J/\psi f_1(1420))} = 0.082^{+0.113}_{-0.077}, \tag{12}$$

where we have kept the masses of $f_1(1285)$ and $f_1(1420)$ mesons in the phase space factors for the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decay rates. The good consistency between these two ratios $\mathrm{R}_s^{\mathrm{QF;th}}$ and $\mathrm{R}_s^{\mathrm{SO;th}}$ verifies the equivalence of the QF basis and SO basis for the $f_1(1285) - f_1(1420)$ mixing in the pQCD calculations. Therefore, one can extract out the mixing angle $\phi_{^3P_1}$ from the ratio of the branching ratios for $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ modes in the SO basis theoretically. The mixing angles for the $f_1(1285) - f_1(1420)$ system extracted through Eq. (4) are $\phi_{^3P_1} = (15.3^{+13.8}_{-12.1})°$ in the QF basis and $\phi_{^3P_1} = (15.5^{+17.3}_{-14.2})°$ in the SO basis, respectively. Here, we should point out that the errors induced by the variation of the input mixing angle are not considered in the extraction of the QF mixing angle $\phi_{^3P_1}$. The tiny deviation between the central values of these two QF mixing angles arises from the very small differences between the decay amplitudes $\mathcal{A}(B_s \to J/\psi f_1)$ and $\mathcal{A}(B_s \to J/\psi f_8)$ in the SO basis.

Moreover, within the still large theoretical uncertainties from the nonperturbative inputs in the pQCD approach, our extracted mixing angle $\phi_{^3P_1}$ is basically in agreement with the earlier determination $(15^{+5}_{-10})°$ by the Mark-II detector at SLAC [4], the updated lattice QCD analysis $(21 \pm 5)°$ [15], as well as the preliminary $(24.0^{+3.2}_{-2.7})°$ reported by the LHCb collaboration [1]. Strictly speaking, the nonperturbative inputs for the involved hadrons need stringent constraints from the experimental measurements, which then makes the relevant predictions theoretically reliable and comparable to the data. Of course, we know that the precision determination of the mixing angle in the $f_1(1285) - f_1(1420)$ system demands enough data samples collected from various processes.

We have also computed the CP-averaged polarization fractions for $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decay modes in the pQCD approach. The numerical results for the polarization fractions are presented in Table I, in which various errors induced by the input parameters have been added in quadrature.

From the pQCD predictions as listed in Table I, one can see the high similarity between the theoretical predictions for the three kinds of polarizations obtained for these two decay modes, and also for the two different mixing schemes. Another point is that, in the pQCD approach, the transverse polarization contributions dominate these two–decays in the QF basis and the longitudinal polarization fractions are $(24.4 - 49.0)\%$ for $B_s \to J/\psi f_1(1285)$ decay and $(24.7 - 49.0)\%$ for $B_s \to J/\psi f_1(1420)$ decay (see Table I), respectively, which seems slightly different from that for the $B_s \to J/\psi\phi$ channel [22]. Meanwhile, as can be seen from Table I, the polarization fractions calculated in the SO basis indicate that $B_s \to J/\psi f_1(1285)$ decay possibly has a little larger longitudinal contributions when the large theoretical errors induced by the less constrained hadronic parameters are taken into account. The above theoretical predictions for the *CP*-averaged polarization fractions and the related phenomenology in both mixing schemes can be tested by the near future experiments at LHCb and/or Super-B.

In summary, motivated by the very recent LHCb measurement on the $B_s \to J/\psi f_1(1285)$ decay and encouraged by the good agreement between the pQCD predictions and the available data for the $B \to J/\psi V$ decays, we studied the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays for the first time within the framework of the pQCD approach by including higher order QCD corrections. We made the first pQCD evaluation for the *CP*-averaged branching ratios for the considered $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays. The results arising from a smaller angle $\phi_{{}^3P_1} \approx 15°$ turn out to be well consistent with the current measurements within theoretical errors. By employing the ratio of the decay rates for the considered two modes, we extracted out the mixing angle $\phi_{{}^3P_1}$ of the $f_1(1285) - f_1(1420)$ system as $\phi_{{}^3P_1} = (15.3^{+13.8}_{-12.1})^\circ$ and $(15.5^{+17.3}_{-14.2})^\circ$ in the QF and SO mixing basis, which are basically consistent with currently available measurements or estimations within still large theoretical errors. Furthermore, the large transverse polarization fractions for these two decay modes are also predicted for tests by the LHCb and the forthcoming Super-B experiments. Finally, it is noted that the pQCD predictions for the considered decays still suffer from large theoretical errors induced by the uncertainties of the input parameters such as hadron decay constants and Gegenbauer moments in the distribution amplitudes of axial-vector states, which are expected to be constrained by more precision data from various channels in the future.

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