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Triplet-extended scalar sector and the naturalness problem

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We consider the extension of the standard model by a complex scalar triplet field, which occurs naturally in several models of leptogenesis and the seesaw mechanism for neutrino mass generation, in the context of ameliorating the fine-tuning problem of the fundamental scalars through the Veltman condition, i.e. by demanding the sum of the quadratically divergent corrections to vanish (or be at a reasonable level) by virtue of some possible symmetry of the underlying theory. We show that it is possible to cancel all the scalar one-loop quadratic divergences, and hence obtain a viable solution for the fine-tuning problem, while satisfying the electroweak precision observables, including the ρ parameter, and successfully generating the neutrino masses. The stability of the scalar potential puts important constraints on the model.

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I. INTRODUCTION

Notwithstanding its experimental successes, the standard model (SM) is widely believed to be an effective theory valid up to a certain scale, above which some new physics (NP) takes over. There are several motivations for such an ultraviolet completing NP, e.g. the fine-tuning problem of the physical scalar mass, the existence of massive neutrinos and the cold dark matter, etc.

The scalar mass receives a quadratically divergent quantum correction, because there is no symmetry to protect the scalar mass, like the gauge symmetry for gauge bosons or chiral symmetry for fermions. If the SM is valid up to an energy scale, Λ, the mass of the Higgs boson, instead of being 125 GeV, is expected to be of the order of Λ, unless there is an unnatural fine-tuning between the bare mass term and the quantum corrections. The standard way out is to appeal to some new symmetry or somehow bring down the Planck scale to soften the fine-tuning.

Let us assume that there is some yet-to-be-discovered symmetry, which protects the scalar mass. In fact, some well-explored mechanisms, like supersymmetry, may provide this protection, but we will be more interested to use a bottom-up approach along with the principle of Occam's razor, and try to find the minimal field content that can do the job. In the framework of cutoff regularization (which, though not the Lorentz invariant, is more intuitive as this separates the quadratic and logarithmic divergences, and the fine-tuning problem depends on the quadratically divergent terms), the sum of all quadratic divergences in the radiative corrections to the scalar self energy is set to zero, which is also known as the Veltman condition (VC) [1].[1] A slightly different VC results if one uses dimensional regularization [2]. In the SM, the VC can be written in terms of the masses of the Higgs boson, the gauge bosons, and the top quark, and, unfortunately, is far from being satisfied; the required Higgs boson mass is more than 300 GeV.

The role of the VC in looking for the possible directions of NP has been well investigated in the literature. For example, a possible extension of the SM by one or more gauge singlet scalars satisfying the VC, and its possible ramifications in collider searches as a cold dark matter candidate or as a gateway to an ultraviolet complete theory, has been discussed in detail in Refs. [3–6]. It is easy to check that to satisfy the VC for the SM Higgs boson, one needs an extension by bosonic fields that couple to the former and hence contribute to the quadratic divergence. A minimally extended scalar sector is enough if one is interested only in the fine-tuning of the SM Higgs boson; however, one would naturally expect to satisfy the VC for the new scalars too. If the new scalars do not couple to the SM fermions (like the singlet extension), one has to bring in some new fermions at the same time.

In this paper, we concentrate on the extension of the SM with a complex triplet scalar [7,8]. Why a scalar extension? As we will show, cancellation of quadratic divergences to the Higgs boson mass requires extra bosonic degrees of freedom that couple to the SM Higgs boson at the tree level. Extra gauge fields can also be invoked, but one needs more scalars anyway to give them mass in a gauge-invariant way. An alternative option, the two-Higgs boson doublet models, has been discussed elsewhere [9].

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[1] The sum need not be exactly zero, but should be some small manageable number. The higher-order contributions are subleading, and so are the contributions of the higher-dimensional operators if the Wilson coefficients are perturbative.

Triplet scalars have received a lot of attention in the literature, including a detailed study of couplings and the mass spectrum [10], radiative corrections, renormalizability issues, and precision observables [11], enhancement of the $h \to \gamma\gamma$ branching ratio [12], and collider studies [13].[2] However, their main appeal lies in neutrino mass generation through the seesaw mechanism [14] with a lepton number (L) violating interaction, and also the type-II leptogenesis scenario [15]. As the complex triplet can couple to left-hand leptons to generate Majorana masses for the neutrinos through $\Delta L = 2$ terms, there is no need to introduce any additional fermions in the model. The stability and unitarity conditions of such triplet models in the light of a 125 GeV Higgs boson have been discussed in Ref. [16].

The vacuum expectation value (VEV) of the triplet is, of course, restricted from the ρ-parameter to be at most of a few GeV [17]. However, it is more than enough to generate the neutrino masses if the corresponding Yukawa couplings are of order unity. This is why we do not consider the extension of the SM with one complex and one real triplet, keeping the custodial SU(2) intact, which may give a large VEV for the neutral triplets [7,18]. The mixing between the triplet and doublet states is proportional to the triplet VEV, which, being tiny, makes the mixing small too [19]. Thus, the 125 GeV scalar is almost a pure doublet, which is completely consistent with its production cross section and decay branching ratios.

We will show that the introduction of a complex triplet can successfully address the naturalness problem for the doublet. Further multiplets, triplets or otherwise, might also help, but here we will try to keep the life simple by considering only the minimal extension, and that too without introducing any extra fermions. One notes that as the number of scalars increases, there is a compulsion to apply the naturalness condition to all of them, unless some of them happen to be extremely heavy (in which case they get frozen and do not contribute to the radiative corrections at a low energy). For the triplet scalars, the naturalness problem is addressed through its coupling to the leptons.[3] One has also to take into account the stability conditions of the scalar potential. As will be seen, the potential of this model becomes unstable at a high energy scale; the scale depends on the initial choice of parameters but is at a few thousand TeV. One might argue that the potential could be made stable with higher-order corrections; even then, some of the couplings grow large and hit a Landau pole somewhere below 10^5 TeV, which indicates the maximum energy where some NP must supersede the effective theory.[4] We find out the parameter space for such a triplet-enhanced SM consistent with the VC as well as the stability of the scalar potential.

The paper is arranged as follows. In Sec. II, we show the complete scalar potential, the corresponding Veltman conditions, and the one-loop renormalization group (RG) equations for all the relevant couplings. In Sec. III, we study the RG evolution of the couplings and its possible consequences. Section IV is on the scalar spectrum of such a model. We summarize and conclude in Sec. V.

II. THE VELTMAN CONDITION

In the SM, with the scalar potential of the form

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \tag{1}$$

the Higgs boson self-energy receives a quadratically divergent correction,

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(6\lambda + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2\right), \tag{2}$$

where g_1 and g_2 are the $U(1)_Y$ [not grand unified theory (GUT)–normalized] and $\mathrm{SU}(2)_L$ gauge couplings, respectively, and $g_t = \sqrt{2}m_t/v$ is the top quark Yukawa coupling. We treat all other fermions as massless and use the cutoff regularization, Λ being the cutoff scale. The VC demands that the quantity inside the parentheses in Eq. (2) be made zero, or at least controllably small, by some symmetry. There are further quadratic divergences coming from two-loop diagrams, but they are suppressed from one-loop contributions by a factor of $\ln(\Lambda/\mu)/16\pi^2$, where μ is the regularization scale, and we will neglect them here.

One can say that the quadratic divergence is under control if, say, $|\delta m_h^2| \le m_h^2$, which translates into[5]

$$|m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2| \le \frac{16\pi^2}{3}\frac{v^2}{\Lambda^2}m_h^2. \tag{3}$$

This inequality is satisfied in the SM only for $v^2/\Lambda^2 \ge 0.1$, or $\Lambda \le 760$ GeV, which means that we should expect a NP at this scale. This, however, is almost ruled out by the LHC. Equation (2) also shows that one needs a bosonic contribution to satisfy the Veltman condition.

Let us now enhance the scalar sector with a complex triplet, X, with a weak hypercharge: $Y = 2$. The VEVs are

$$\langle\phi^0\rangle = \frac{v_1}{\sqrt{2}}, \qquad \langle X^0\rangle = v_2. \tag{4}$$

We can express the triplet in a bidoublet notation:

[2] Triplet scalars may also be embedded in a bigger theory, like supersymmetry, vector fermions or more scalar multiplets.

[3] So, if necessary, one can keep the triplets light, but heavy triplets can easily be accommodated.

[4] Thus, the fine-tuning of the Higgs boson mass is never more severe than 1 in 1000, which might not seem too bad, but we want to have a cancellation even less severe.

[5] The fine-tuning condition is, of course, subjective, and one can easily allow a higher fine-tuning, but any fine-tuning defeats the motivation of the Veltman condition.

$$X = \begin{pmatrix} X^+/\sqrt{2} & X^{++} \\ X^0 & -X^+/\sqrt{2} \end{pmatrix}, \quad (5)$$

and the generic form of the $\Delta L = 2$ terms is

$$V_{\Delta L=2} = -i f_{ab} L_a^T C^{-1} \tau_2 X L_b + \text{h.c.}, \quad (6)$$

where C is the charge conjugation operator, and $L = (\nu \quad \ell)^T$ is the left-handed lepton doublet. If there is no leptonic flavor-changing neutral current, we can take the Yukawa coupling f_{ab} to be diagonal. For subsequent discussion, we will not only assume f_{ab} to be diagonal but also to be a multiple of the unit matrix: $f_{ab} = f\delta_{ab}$. While this seems to be at variance with the neutrino data, any form that correctly reproduces the neutrino masses and mixing hardly changes our conclusions.[6]

The scalar potential can be written as [8]

$$V = V_2 + V_3 + V_4, \quad (7)$$

where the individual terms are

$$\begin{aligned} V_2 &= -\mu_1^2(\Phi^\dagger\Phi) + \mu_2^2(X^\dagger X), \\ V_3 &= -a_0(\Phi\Phi X^\dagger) + \text{h.c.}, \\ V_4 &= \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2(X^\dagger X)^2 + \lambda_3(\Phi^\dagger\Phi)(X^\dagger X) \\ &\quad + \lambda_4(\Phi^\dagger\tau_i\Phi)(X^\dagger t_i X) + \lambda_5|X^T\tilde{C}X|^2, \end{aligned} \quad (8)$$

with

$$\tilde{C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (9)$$

and τ_is and t_is ($i = 1$–3) are the 2×2 and 3×3 Pauli matrices, respectively, with $t_1 = \delta_{i,i+1} + \delta_{i,i-1}$, $t_2 = -i(\delta_{i,i+1} - \delta_{i,i-1})$, and $t_3 = \text{diag}(1, 0, -1)$. Note that the triplet has a "right-sign" mass term, which ensures that the triplet VEV will arise only through the trilinear and quartic interactions, and can remain small without necessarily keeping the triplet light and hence jeopardizing the experimental constraints.[7] Without the trilinear term, there is a global O(2) symmetry in the neutral scalar sector, so that there will be a physical Goldstone boson in the spectrum if both neutral fields acquire a VEV. One needs $a_0 > 0$ to prevent the tachyonic mass of the scalars. Further ramifications of the trilinear term can be found in [8].

In terms of the real components, the fields can be written as

$$\begin{aligned} \phi^0 &= \frac{1}{\sqrt{2}}(\phi^{0R} + v_1 + i\phi^{0I}), \\ \phi^+ &= \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \\ X^0 &= \frac{1}{\sqrt{2}}(X^{0R} + \sqrt{2}v_2 + iX^{0I}), \\ X^{++} &= \frac{1}{\sqrt{2}}(X_1 + iX_2), \\ X^+ &= \frac{1}{\sqrt{2}}(X_1' + iX_2'), \end{aligned} \quad (10)$$

where the neutral components have been vacuum shifted. Only the terms in V_4 are relevant for computing quadratic divergences, so we rewrite those terms as[8]

$$\begin{aligned} V_4 &= \frac{1}{4}\lambda_1[(\phi_1^2 + \phi_2^2 + \phi^{0R2} + \phi^{0I2})^2] + \frac{1}{4}\lambda_2[(X_1^2 + X_2^2 + X_1'^2 + X_2'^2 + X^{0R2} + X^{0I2})^2] \\ &\quad + \frac{1}{4}\lambda_3[(\phi_1^2 + \phi_2^2 + \phi^{0R2} + \phi^{0I2})(X_1^2 + X_2^2 + X_1'^2 + X_2'^2 + X^{0R2} + X^{0I2})] \\ &\quad + \frac{1}{4}\lambda_4[(\phi_1^2 + \phi_2^2)(X_1^2 + X_2^2) - (\phi^{0R2} + \phi^{0I2})(X_1^2 + X_2^2) - (\phi_1^2 + \phi_2^2)(X^{0R2} + X^{0I2}) \\ &\quad + (\phi^{0R2} + \phi^{0I2})(X^{0R2} + X^{0I2}) + \sqrt{2}\{(\phi_1 + i\phi_2)(X_1' + iX_2')(X_1 - iX_2)(\phi^{0R} - i\phi^{0I}) + \text{h.c.}\} \\ &\quad + \sqrt{2}(\phi_1 + i\phi_2)(X_1' - iX_2')(\phi^{0R} - i\phi^{0I})(X^{0R} - iX^{0I}) + \text{h.c.}] \\ &\quad + \lambda_5\Bigg[(X_1^2 + X_2^2)(X^{0R2} + X^{0I2}) + \frac{1}{4}(X_1'^2 + X_2'^2)^2 \\ &\quad + \frac{1}{2}(X_1' + iX_2')(X_1' + iX_2')(X_1 - iX_2)(X^{0R} + iX^{0I}) + \text{h.c.}\Bigg]. \end{aligned} \quad (11)$$

[6]For normal hierarchy, only one of the Yukawa couplings is large and the other two can be neglected; for inverted hierarchy, we have to keep two equally large couplings and neglect the third one. Off-diagonal elements are to be introduced in f_{ab} to generate the mixing angles. Anyway, a detailed discussion of the neutrino mass matrix is outside the scope of this paper.

[7]The trilinear term can be banished by invoking discrete symmetries, $\Phi \to -\Phi$ and $X \to -X$, but the latter also forbids the $\Delta L = 2$ terms.

[8]We correct a few sign mistakes in [8].

With the triplet, the VC for the SM Higgs boson is modified to

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(6\lambda_1 + 3\lambda_3 + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2\right), \quad (12)$$

with $m_h = 125$ GeV, $m_W = 80.41$ GeV, $m_Z = 91.19$ GeV, and $m_t = 174$ GeV; this fixes $\lambda_3 \approx 1.39$. This is large but still within the perturbative limit of 4π. With N identical triplets, $\lambda_3 \approx 1.39/N$.

The stability conditions of the scalar potential read

$$\lambda_1, \lambda_2 \geq 0, \qquad \lambda_2 + 2\lambda_5 \geq 0, \qquad \lambda_3 \pm \lambda_4 \geq -2\sqrt{\lambda_1\lambda_2}, \quad (13)$$

plus some other conditions that are not independent of these. Note that λ_4 and λ_5 can be negative. As we will show later, the lighter charge parity (CP)-even neutral state at 125.8 GeV is almost a pure doublet, which fixes $\lambda_1 \sim 0.13$. Thus, the stability conditions give a range for the allowed values of λ_4 and a lower limit on λ_5 for any given value of λ_2. The VC for the triplet, which couples to the leptons through the $\Delta L = 2$ interaction, reads

$$\delta m_X^2 = \frac{\Lambda^2}{16\pi^2}\left(4\lambda_2 + \lambda_3 + 2\lambda_5 + \frac{1}{2}g_1^2 + g_2^2 - 3f^2\right). \quad (14)$$

Without the Yukawa term, δm_X^2 can never be made to vanish, even with possible negative values of λ_5, due to the stability conditions. There is no contribution proportional to λ_4 in Eq. (14); the quadratically divergent contributions cancel out. Also, even in the limit $\lambda_2, \lambda_5 \to 0$, the large value of λ_3 necessitates a correspondingly large value of the Yukawa coupling f ($\sim\mathcal{O}(1)$) and hence an extremely tiny triplet VEV v_2 ($\sim\mathcal{O}(10^{-3}$ eV)), completely consistent with the ρ-parameter bound, as well as to the identification of the 125 GeV resonance as the almost-pure SM doublet. The $3f^2$ term in Eq. (14) appears because of universal leptonic Yukawa couplings. For the normal (inverted) hierarchy, we expect $3f^2 \approx f^2_{\rm normal}(2f^2_{\rm inverted})$.

We would, of course, like the VCs for both the doublet and the triplet to be stable over the range of validity of the theory. We do not expect the VC combinations to remain exactly zero, because higher-order effects were not taken into account, but we would like a more or less stable behavior.[9] The one-loop RG equations for the couplings are as follows:

$$\begin{aligned}
16\pi^2\beta_{\lambda_1} &= 12\lambda_1^2 + \frac{3}{2}\lambda_3^2 + \lambda_4^2 + 6g_t^2\lambda_1 - \frac{3}{2}\lambda_1(g_1^2 + 3g_2^2) - 3g_t^4 + \frac{3}{16}(g_1^4 + 2g_1^2g_2^2 + 3g_2^4),\\
16\pi^2\beta_{\lambda_2} &= 14\lambda_2^2 + \lambda_3^2 + \lambda_4^2 + 8\lambda_5^2 + 8\lambda_2\lambda_5 + 2f^2\lambda_2 - 6\lambda_2(g_1^2 + 2g_2^2) + \frac{3}{2}(2g_1^4 + 3g_2^4 + 4g_1^2g_2^2) - f^4,\\
16\pi^2\beta_{\lambda_3} &= 6\lambda_1\lambda_3 + 8\lambda_2\lambda_3 + 4\lambda_3\lambda_5 + 2\lambda_3^2 + 2\lambda_3(f^2 + 3g_t^2) + \frac{3}{2}g_1^4 + 3g_2^4 - \frac{15}{2}\lambda_3g_1^2 - \frac{33}{2}\lambda_3g_2^2,\\
16\pi^2\beta_{\lambda_4} &= 2\lambda_1\lambda_4 + 2\lambda_2\lambda_4 - 4\lambda_4\lambda_5 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_4(f^2 + 3g_t^2) - \frac{15}{2}g_1^2\lambda_4 - \frac{33}{2}g_2^2\lambda_4 + 3g_1^2g_2^2,\\
16\pi^2\beta_{\lambda_5} &= 12\lambda_2\lambda_5 + 2\lambda_5^2 - \lambda_4^2 + 2f^2\lambda_5 - 6\lambda_5g_1^2 - 12\lambda_5g_2^2 + \frac{3}{2}g_2^4 - 6g_1^2g_2^2 + \frac{1}{2}f^4,\\
16\pi^2\beta_f &= 6f^3 - \frac{1}{4}f(3g_1^2 + 9g_2^2),
\end{aligned} \quad (15)$$

where $\beta_h \equiv dh/dt$, and $t \equiv \ln(q^2/\mu^2)$, μ being the regularization scale. Note that our definition of t differs by a factor of 2 from that used by some authors.

III. ANALYSIS

To ensure that the VC for the doublet scalar is respected, one needs to fix only the value of $\lambda_3 \approx 1.39$. The rest of the couplings are free parameters of the theory, except that Eq. (14) provides a relationship between λ_2, λ_5, and f. The only constraint on λ_4 comes from the stability condition. We, of course, assume all couplings to be perturbative ($\leq 4\pi$) over the entire range of validity of the theory.

A scan over the free couplings is needed because their initial values, consistent with the stability conditions, fix the range of the validity $\mathcal{R}$ of the theory. This is particularly true for λ_2. Over the entire parameter space, λ_2 initially increases and then reverses and becomes negative, indicating some other new physics.[10] A typical evolution is shown in Fig. 1. The reason for such a turning behavior of λ_2 is easy to follow from the RG equations. The value of f at the electroweak scale is fixed by the triplet VC

[9]In a generic Yukawa theory, if the Higgs boson mass correction at one loop remains zero at all scales, the leading two-loop quadratic corrections also vanish [2].

[10]One must remember that we are using only one-loop RG equations. However, the drop of λ_2 is so sharp, thanks to the rapidly increasing value of f, that we do not expect a qualitative change in the outcome even if we include higher-order terms.

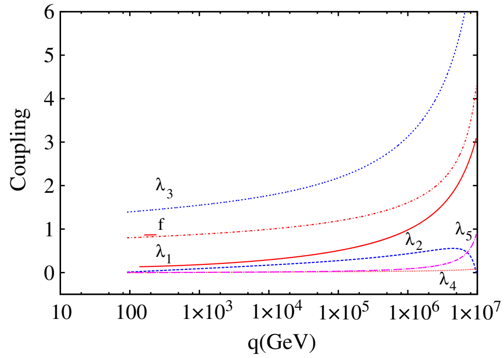

FIG. 1 (color online). The running of the couplings, using one-loop RG equations. The values at $q^2 = m_Z^2$ are $\lambda_2 = 0.01$, $\lambda_4 = \lambda_5 = 0$, and the rest are fixed by physical masses and/or Veltman conditions.

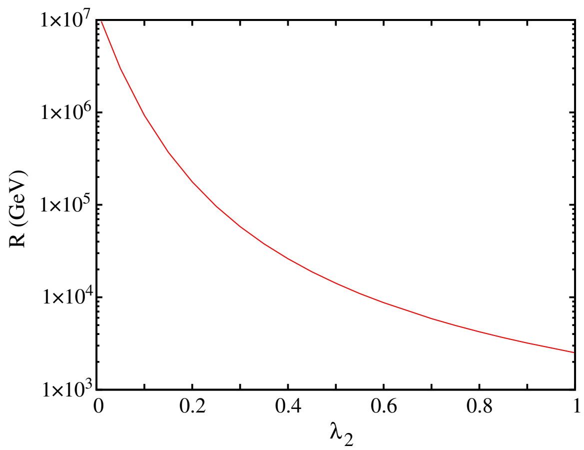

FIG. 2 (color online). The range of validity of the theory as a function of initial values of λ_2 keeping $\lambda_4 = \lambda_5 = 0$.

$$f = \sqrt{(8\lambda_2 + 2\lambda_3 + 4\lambda_5 + g_1^2 + 2g_2^2)/6}, \qquad (16)$$

and this keeps the β_{λ_2} positive. However, with an increasing q^2, the Yukawa coupling f increases so rapidly that the $-f^4$ term causes λ_2 to turn back, and ultimately the theory becomes unstable. The range $\mathcal{R}$ as a function of λ_2, keeping $\lambda_4 = \lambda_5 = 0$, is shown in Fig. 2.

Figure 2 might seem counterintuitive; with increasing λ_2, β_{λ_2} starts out from a more positive value, but $\mathcal{R}$ appears to shrink. This is because larger values of λ_2 need correspondingly larger values of f to satisfy the triplet VC, and thus the turning of λ_2 occurs at a lower energy scale. Thus, we do not envisage λ_2 to be very large.

$\mathcal{R}$ also depends on the initial values of λ_4 and λ_5, as shown in Fig. 3. With an increasing $|\lambda_4|$, the range increases. This is easy to understand; β_{λ_2} picks up another positive contribution, λ_4^2, which keeps λ_2 positive for higher values of q^2. For λ_5, the deciding factor is the initial value of f; the lower the starting value of f, the higher the range of validity.

All the other quartic couplings except λ_2 hit the Landau pole almost simultaneously because of the coupled nature of the RG equations. This, however, occurs beyond $\mathcal{R}$ but typically between (2–4) $\mathcal{R}$. Thus, the fine-tuning problem is never as severe as that of the SM.

As a last thing, we show, in Fig. 4, how the radiative corrections behave as we go up the energy scale. What is plotted is $\delta m_{h,X}^2$ times $16\pi^2/\Lambda^2$, or, in other words, the combinations of the couplings in Eqs. (12) and (14) as a function of the energy scale q. We find that the doublet VC is more or less stable while the triplet VC shows a sharp drop because of the steep increase in f.

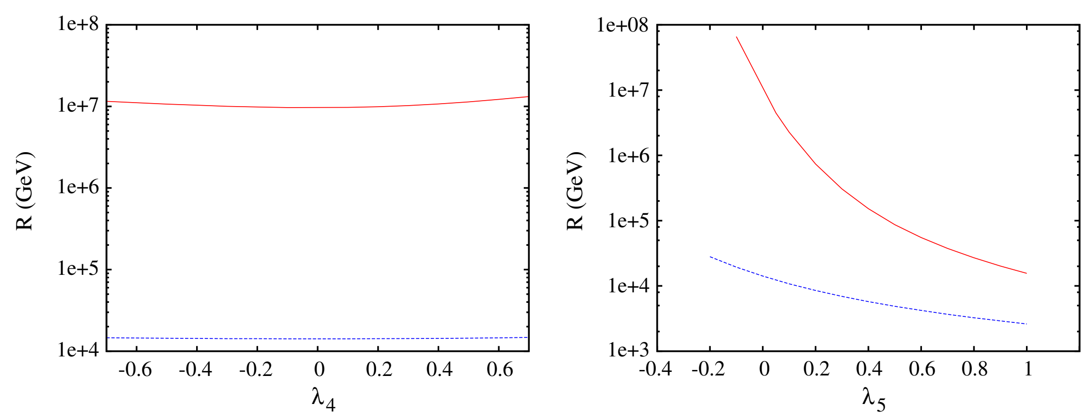

FIG. 3 (color online). Left panel: Range of validity as a function of initial values of λ_4, with $\lambda_5 = 0$, and $\lambda_2 = 0.01(0.5)$ for the upper red (lower blue) line. Right panel: The same as a function of initial values of λ_5, with $\lambda_4 = 0$, and $\lambda_2 = 0.01(0.5)$ for the upper blue (lower red) line.

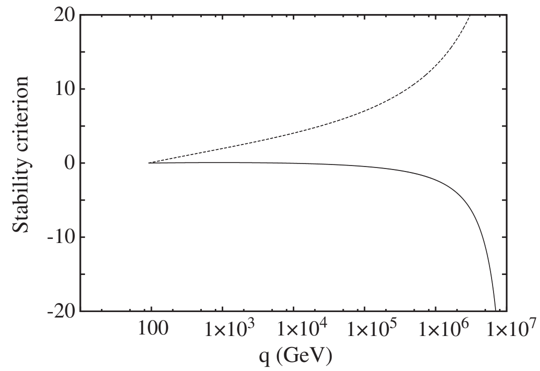

FIG. 4. Stability of the VC: the lower red line is for the triplet VC and the upper blue line is for the doublet VC. On the y-axis we plot $\delta m^2_{h(X)}(16\pi^2/\Lambda^2)$. Drawn for $\lambda_2 = 0.01$, $\lambda_4 = \lambda_5 = 0$.

IV. SCALAR SPECTRUM

Let us first note that there exists a strong hierarchy between v_1 and v_2; $v_2/v_1 \sim \mathcal{O}(10^{-14})$. This has nothing to do with fine-tuning; it is but a reflection of the hierarchy between the neutrino mass and the electroweak scale.

The doubly charged scalar H^{++} is a pure triplet and its mass can be directly read off from Eq. (8):

$$m^2_{H^{++}} = \mu_2^2 + \frac{1}{2}(\lambda_3 - \lambda_4)v_1^2 + (2\lambda_2 + 4\lambda_5)v_2^2 = 4\lambda_5 v_2^2 - \lambda_4 v_1^2 + \frac{v_1^2}{2}\frac{a_0}{v_2}. \tag{17}$$

There are two singly charged fields. After diagonalizing the mass matrix, one of them turns out to be the Goldstone boson (which, in the limit $v_2 \ll v_1$, is almost a pure doublet), and the other has a mass:

$$m^2_{H^+} = \frac{1}{2}(v_1^2 + 4v_2^2)\left(\frac{a_0}{v_2} - \lambda_4\right). \tag{18}$$

To get Eqs. (17) and (18), we have used the minimization conditions [8]:

$$-\mu_1^2 + v_1^2\lambda_1 + v_2^2(\lambda_3 + \lambda_4) = 2a_0 v_2,$$
$$\mu_2^2 + 2v_2^2\lambda_2 + \frac{1}{2}v_1^2(\lambda_3 + \lambda_4) = \frac{a_0 v_1^2}{2v_2}. \tag{19}$$

If $a_0/v_2 \gg 1$, H^+ and H^{++} are almost mass degenerate, and their masses can be large; the quartic couplings hardly have any effect on their masses.

The CP-odd neutral scalar, A, is again almost entirely the triplet component X^{0I}, whose mass is given by

$$m_A^2 = \frac{1}{2}\frac{a_0}{v_2}(v_1^2 + 8v_2^2). \tag{20}$$

Thus, not only A is almost degenerate with H^+ and H^{++} in the limit $a_0/v_2 \gg 1$; a_0 has to be nonzero in order to prevent the Goldstone boson [8] and hence v_2 must be nonzero, albeit small, for the theory to be consistent.

The mass matrix for CP-even neutral scalars can be written, with the help of the minimization conditions of the scalar potential, as

$$\mathcal{M}^{0R} = \begin{pmatrix} v_1^2\lambda_1 & \frac{1}{\sqrt{2}}v_1 v_2\varphi \\ \frac{1}{\sqrt{2}}v_1 v_2\varphi & 2v_2^2\lambda_2 + \frac{1}{4}\frac{a_0 v_1^2}{v_2} \end{pmatrix}, \tag{21}$$

where $\varphi = \lambda_3 + \lambda_4 - a_0/v_2$. This is almost a diagonal matrix for $v_2 \ll v_1$, so that $m_h^2 = 2\lambda_1 v_1^2$. Apart from the 125 GeV scalar, all the other scalars are (almost) pure triplet and close to degenerate for $a_0/v_2 \ll 1$. The charged scalars can be pair produced at the LHC, through γ or Z exchange. Single production is suppressed by the tiny value of v_2. Once produced, they will dominantly decay into a lepton pair, irrespective of their mass. This is in contrast to the case where v_2 is sizable and digauge decay channels may be dominant. Such dilepton signals from H^{++} have been looked for by both ATLAS and CMS collaborations [20], and a bound of $m_{H^{++}} \gtrsim 400$ GeV has been established. This translates into $a_0/v_2 \gtrsim 5.3$.

Thus, the main effect of the Veltman condition for the triplets is to enforce a Yukawa coupling $\sim\mathcal{O}(1)$ and hence a tiny value of v_2. This makes the triplet decouple from the doublet, for all practical purpose, unless the dimensionless quantity a_0/v_2 falls significantly below the ATLAS and CMS limits. It also makes the triplet scalars almost mass degenerate. Consequently, the only significant production channel is through an s-channel γ or Z exchange. While $a_0/v_2 > 4\lambda_1 \approx 0.5$ ensures that the lighter CP-even neutral scalar is the doublet, even light triplets are going to be missed unless they can be pair produced.

V. SUMMARY

The SM, as it stands, is definitely not enough to address the fine-tuning problem. If we want to make a minimalistic extension of the SM to address the fine-tuning problem of the Higgs boson mass, the new degrees of freedom have to be bosonic.

The extension of the SM by scalars demands that the fine-tuning problem of all the scalars be addressed simultaneously, unless some of them are extremely heavy. While some of the scalar couplings can in principle be negative, stability of the scalar potential forces the new scalars to have some fermionic couplings. In this respect, a complex triplet is an interesting alternative as (i) it can couple to the SM leptons through $\Delta L = 2$ interactions and generate Majorana masses for the neutrinos; (ii) the smallness of the neutrino masses ensures that the triplet VEV is tiny if the new Yukawa couplings are of order unity, so that the ρ-parameter constraint is easily evaded. Moreover, the lightest CP-even scalar remains an almost pure doublet, in conformity with the LHC Higgs boson data.

Addition of the triplet gives an extra positive contribution to the Veltman condition for the doublet. The coupling

λ_3, as defined in Eq. (8), turns out to be 1.39 for exact cancellation of one-loop quadratic corrections (and $1.39/N$ if there are N numbers of identical complex triplets). Similarly, with the help of other couplings, one can satisfy the triplet VC too.

We have also checked the evolution of the couplings for the stability of the scalar potential, albeit at the one-loop level. The contribution of two-loop diagrams is suppressed by an additional factor of $\ln(\Lambda^2/m^2)/16\pi^2$, which is at most at a few per cent level to the one-loop contributions for $\Lambda \sim 10^6$ GeV. The potential becomes unstable as λ_2 becomes negative at some high scale, $\mathcal{R}$, at the ballpark of thousands of TeV. This indicates some new physics at this scale which must change the β-functions. If we neglect this feature, the other scalar quartic couplings blow up within one order of magnitude of $\mathcal{R}$, so some new physics is indicated anyway.

One might wonder about the motivation of introducing the Veltman condition to address the fine-tuning problem if the theory itself becomes invalid at, say, 10^6 GeV. We would argue that it is still a useful approach; the fine-tuning is still there in the SM, maybe not as terrible as 1 in 10^{17} but even 1 in 10^4 is bad enough and should be addressed. At this point, we do not know what the nature of the NP at $\mathcal{R}$ is, but the theory below $\mathcal{R}$ can be treated as an effective theory, with those heavy degrees of freedom integrated out. In a subsequent publication, we will discuss the role of effective higher-dimensional operators to the Veltman condition.

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