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Study of the $B_s^0 \to J/\psi K^+K^-$ decay

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We calculate the branching fractions of the $B_s^0 \to J/\psi\phi(1020)$ and $B_s^0 \to J/\psi f_2'(1525)$ decays using a simple model based on the framework of the factorization approach. We also evaluate the total $B_s^0 \to J/\psi K^+K^-$ branching ratio, including the resonances and nonresonant contributions to the K^+K^- channel, by applying the Dalitz plot analysis. The largest resonant component of the final state structure in the $B_s^0 \to J/\psi K^+K^-$ decay is $\phi(1020)$, accompanied by $f_2'(1525)$ and five additional resonances. We compute the resonant contributions $B_s^0 \to J/\psi\phi(1020)(\to K^+K^-)$, $B_s^0 \to J/\psi f_2'(1525)(\to K^+K^-)$ and nonresonant decay then compare with data from Belle and LHCb. The overall branching fractions are obtained by applying our calculations to be $BR[B_s^0 \to J/\psi\phi(1020)] = (1.14 \pm 0.17) \times 10^{-3}$, $BR[B_s^0 \to J/\psi f_2'(1525)] = (0.33 \pm 0.05) \times 10^{-3}$, and $BR[B_s^0 \to J/\psi K^+K^-] = (1.03 \pm 0.09) \times 10^{-3}$.

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I. INTRODUCTION

The decay $B_s^0 \to J/\psi K^+K^-$ was investigated using 0.16 fb^{-1} of data collected with the LHCb detector using 7 TeV pp collisions. In the K^+K^- mass spectrum they observed a significant signal in the $f_2'(1525)$ region as well as a nonresonant component. After subtracting the nonresonant component, they found [1]

$$\frac{BR(B_s^0 \to J/\psi f_2'(1525))}{BR(B_s^0 \to J/\psi\phi(1020))} = (26.4 \pm 2.7 \pm 2.4)\%. \quad (1)$$

A first measurement of the entire $B_s^0 \to J/\psi K^+K^-$ decay rate (including resonant and nonresonant decays) was recently performed by LHCb with a measured branching fraction of [2]

$$BR(B_s^0 \to J/\psi K^+K^-) = (7.70 \pm 0.08 \pm 0.39 \pm 0.60 \pm 10) \times 10^{-4}. \quad (2)$$

A recent discovery in this field is the measurement of the branching fraction of the B_s^0 to $J/\psi\phi(1020)$, $J/\psi f_2'(1525)$, and $J/\psi K^+K^-$ decays, based on a 121.4 fb^{-1} data sample collected at the $\gamma(5S)$ resonance by the Belle experiment at the KEKB asymmetric-energy e^+e^- collider [3]

$$\begin{aligned} BR(B_s^0 \to J/\psi\phi(1020)) &= (1.25 \pm 0.07(\text{stat}) \pm 0.08(\text{syst}) \pm 0.22(f_s)) \times 10^{-3} \\ BR(B_s^0 \to J/\psi f_2'(1525)) &= (0.26 \pm 0.06(\text{stat}) \pm 0.02(\text{syst}) \pm 0.05(f_s)) \times 10^{-3} \\ BR(B_s^0 \to J/\psi K^+K^-) &= (1.01 \pm 0.09(\text{stat}) \pm 0.10(\text{syst}) \pm 0.18(f_s)) \times 10^{-3} \\ \frac{BR(B_s^0 \to J/\psi f_2'(1525))}{BR(B_s^0 \to J/\psi\phi(1020))} &= (21.5 \pm 4.9(\text{stat}) \pm 2.6(\text{syst}))\%. \end{aligned} \quad (3)$$

In this analysis, we study the $B_s^0 \to J/\psi K^+K^-$ decay using the Dalitz plot analysis. While a large $\phi(1020)$ contribution is well known and the $f_2'(1525)$ component has been recently observed and confirmed, as shown in Fig. 1, it is better if we examine the $B_s^0 \to J/\psi\phi(1020)$ and $B_s^0 \to J/\psi f_2'(1525)$ decays. The branching fractions of the last decays are computed by using a simple model based on the framework of the factorization approach. After the calculations are done, the resulting branching fractions are

$$\begin{aligned} BR(B_s^0 \to J/\psi\phi(1020)) &= (1.14 \pm 0.17) \times 10^{-3}, \\ BR(B_s^0 \to J/\psi f_2'(1525)) &= (0.33 \pm 0.05) \times 10^{-3}, \\ BR(B_s^0 \to J/\psi K^+K^-) &= (1.03 \pm 0.09) \times 10^{-3}, \end{aligned} \quad (4)$$

where the branching fractions $BR(\phi(1020) \to K^+K^-) = (48.9 \pm 0.5)\%$ and $BR(f_2'(1525) \to K^+K^-) = (88.7 \pm 2.2)\%$ are used [4]. Note that our $B_s \to J/\psi K^+K^-$ prediction includes also nonresonant contributions. Our calculated values are in good agreement with the Belle collaboration measurements [3] and also the results of the $B_s^0 \to J/\psi\phi(1020)$ and $B_s^0 \to J/\psi f_2'(1525)$ decays are in good agreement with the current PDG values [4] that are dominated by the CDF measurements [5].

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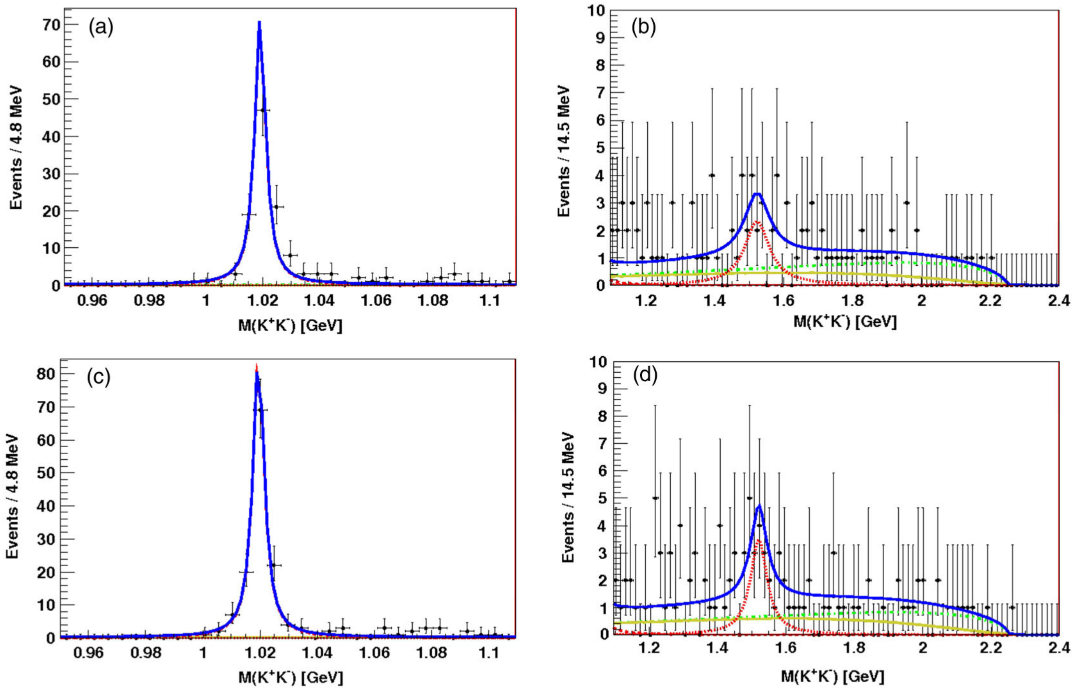

FIG. 1 (color online). Projection of the fit in $M(K^+K^-)$ for events in the signal range -0.07 GeV $< \Delta E < -0.03$ GeV. Panels (a) and (b) show the $\phi(1020)$ and $f_2'(1525)$ mass regions, respectively, for $J/\psi \to e^+e^-$ events; panels (c) and (d) are the same for $J/\psi \to \mu^+\mu^-$ events. In all plots, the upper solid line corresponds to the entire PDF model, which overlaps with the curve of the $J/\psi\phi(1020)$ component in (a) and (c) [3].

II. DECAYS AMPLITUDES

A. Amplitudes of the $B_s^0 \to J/\psi\phi(1020)$ and $B_s^0 \to J/\psi f_2'(1525)$ decays

In the factorization approaches, Feynman diagrams for the $B_s^0 \to J/\psi\phi(1020)$ and $B_s^0 \to J/\psi f_2'(1525)$ decays are shown in Fig. 2. The framework to study $B \to VV$ and $B \to TV$ decays (where V and T are the vector and tensor mesons, respectively) is the effective weak Hamiltonian [6]. For $\Delta B = 1$ transitions, it is written as

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{i=u,c} V_{ib}V_{iq}^*\left(c_1(\mu)O_1^i(\mu) + c_2(\mu)O_2^i(\mu) - V_{tb}V_{tq}^*\sum_{j=3}^{10} c_j(\mu)O_j(\mu)\right)\right], \tag{5}$$

where G_F is the Fermi constant, $c_i(\mu)$ are Wilson coefficients at the renormalization scale μ, $O_i(\mu)$ are local operators and V_{ij} are the respective Cabibbo-Kobayashi-Maskawa (CKM) matrix elements involved in the transitions. In order to calculate the branching ratios in this work, we use the next to leading order Wilson coefficients for $\Delta B = 1$ transitions obtained in the naive dimensional regularization scheme (NDR) at the energy scale μ. The decay amplitude of a nonleptonic two body B decay can be calculated using the effective weak Hamiltonian by

$$M(B \to M_1M_2) = < M_1M_2|H_{\rm eff}|B > = \frac{G_2}{\sqrt{2}}\sum_{i=1}^{10} C_i(\mu) < O_i(\mu) >, \tag{6}$$

where the hadronic matrix elements $< O_i(\mu) >$ are defined by $< M_1M_2|O_i(\mu)|B >$ and M_i are final state mesons. In the naive factorization hypothesis, hadronic matrix elements $< O_i(\mu) >$ are evaluated by the product of decay constants and form factors. These matrix elements are energy μ scale and renormalization scheme independent, consequently there is no term to cancel the energy μ dependency in the Wilson coefficients, and the amplitudes for nonleptonic two body B decays are scale and renormalization scheme dependent. According to Fig. 2, the decay amplitude for $B_s^0 \to J/\psi\phi(1020)$ and $B_s^0 \to J/\psi f_2'(1525)$ processes is given by

$$M(B_s^0 \to MJ/\psi) = \frac{G_2}{\sqrt{2}}(a_2V_{cb}V_{cs}^* - a_3V_{tb}V_{ts}^*) \times < J/\psi|(c\bar{c})_{V-A}|0 >< M|(s\bar{b})_{V-A}|B_s^0 > \tag{7}$$

where $M = \phi(1020)$ and $f_2'(1525)$, and

$$a_2 = c_2 + \frac{c_1}{3}, \qquad a_3 = c_3 + \frac{c_4}{3}. \tag{8}$$

The decay constant of J/ψ meson is defined as

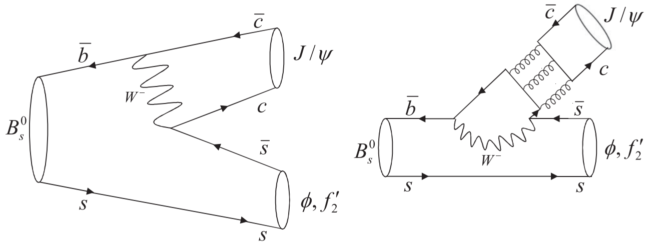

FIG. 2. Quark diagrams illustration the processes $B_s^0 \to J/\psi\phi$ and $B_s^0 \to J/\psi f_2'$ decays.

$$< J/\psi(p_{J/\psi}, \epsilon_{J/\psi})|(c\bar{c})_{V-A}|0 >= m_{J/\psi} f_{J/\psi} \epsilon_{J/\psi}, \quad (9)$$

and the polarization vectors become

$$\epsilon_{J/\psi}^{(\lambda=0)} = (|\vec{p}_{J/\psi}|, 0, 0, p_{J/\psi}^0)/m_{J/\psi},$$
$$\epsilon_{J/\psi}^{(\lambda=\pm1)} = \mp (0, 1, \pm i, 0)/\sqrt{2}. \quad (10)$$

The transitions $B_s^0 \to V$ [7] and $B_s^0 \to T$ [8] can be written in terms of form factors by the following expressions

$$< \phi(p_\phi, \epsilon_\phi)|(V_\mu - A_\mu)|B_s^0 >$$
$$= -\epsilon_{\mu\nu\alpha\beta}\epsilon_\phi^\nu p_{B_s^0}^\alpha p_\phi^\beta \frac{2V^{B_s^0\phi}(q^2)}{m_{B_s^0} + m_\phi} - i\left[\left(\epsilon_{\phi\mu} - \frac{\epsilon_\phi \cdot q}{q^2} q_\mu\right)\right.$$
$$\times (m_{B_s^0} + m_\phi) A_1^{B_s^0\phi}(q^2) - \left((p_{B_s^0} + p_\phi)_\mu\right.$$
$$\left.\left. - \frac{m_{B_s^0}^2 - m_\phi^2}{q^2} q_\mu\right)(\epsilon_\phi \cdot q)\frac{A_2^{B_s^0\phi}(q^2)}{m_{B_s^0} + m_\phi}\right], \quad (11)$$

where $q = (p_{B_s^0} - p_\phi)$, $q^2 = m_{J/\psi}^2$, and ϵ_ϕ is the polarization vector of the ϕ meson which is calculated from (10), with this difference that J/ψ becomes ϕ, and [9]

$$A_{1,2}^{B_s^0\phi}(q^2) = \frac{f_{1,2}(0)}{1 - \sigma_{1,2}(q^2/m_{B_s^*}^2) + \sigma_{1,2}'(q^2/m_{B_s^*}^2)^2},$$
$$V^{B_s^0\phi}(q^2) = \frac{f_V(0)}{(1 - q^2/m_{B_s^*}^2)(1 - \sigma_V(q^2/m_{B_s^*}^2) + \sigma_V'(q^2/m_{B_s^*}^2)^2)}, \quad (12)$$

and

$$< f_2'(p_{f_2'}, \epsilon_{f_2'})|(V_\mu - A_\mu)|B_s^0 >$$
$$= -h(q^2)\epsilon_{\mu\nu\alpha\beta}\epsilon_{f_2'}^{\nu\lambda} P_\lambda P^\alpha q^\beta - i[k(q^2)\epsilon_{f_2'\mu\nu} P^\nu$$
$$+ \epsilon_{f_2'\alpha\beta} P^\alpha P^\beta (b_+(q^2) P_\mu + b_-(q^2) q_\mu)], \quad (13)$$

where $P = p_{B_s^0} + p_{f_2'}$, $q = p_{B_s^0} - p_{f_2'}$, and $q^2 = m_{J/\psi}^2$, and the two sets of form factors are related via

$$V(q^2) = -m_{B_s^0}(m_{B_s^0} + m_{f_2'}) h(q^2),$$
$$A_1(q^2) = -\frac{m_{B_s^0}}{m_{B_s^0} + m_{f_2'}} k(q^2),$$
$$A_2(q^2) = m_{B_s^0}(m_{B_s^0} + m_{f_2'}) b_+(q^2),$$
$$A_0(q^2) = \frac{m_{B_s^0} + m_{f_2'}}{2m_{f_2'}} A_1(q^2) - \frac{m_{B_s^0} - m_{f_2'}}{2m_{f_2'}} A_2(q^2)$$
$$- \frac{m_{B_s^0} q^2}{2m_{f_2'}} b_-(q^2). \quad (14)$$

The spin-2 polarization tensor, which satisfies $\epsilon_{\mu\nu} p_{f_2'}^\nu = 0$, is symmetric and traceless. It can be constructed via the spin-1 polarization vector ϵ:

$$\epsilon_{\mu\nu}(\pm2) = \epsilon_\mu(\pm)\epsilon_\nu(\pm),$$
$$\epsilon_{\mu\nu}(\pm1) = \frac{1}{\sqrt{2}}(\epsilon_\mu(\pm)\epsilon_\nu(0) + \epsilon_\mu(0)\epsilon_\nu(\pm)),$$
$$\epsilon_{\mu\nu}(0) = \frac{1}{\sqrt{6}}(\epsilon_\mu(+)\epsilon_\nu(-) + \epsilon_\mu(-)\epsilon_\nu(+))$$
$$+ \sqrt{\frac{2}{3}}\epsilon_\mu(0)\epsilon_\nu(0). \quad (15)$$

In the case of the f_2' meson moving on the plus direction of the z axis, the explicit structures of ϵ in the ordinary coordinate frame are chosen as

$$\epsilon_{f_2'\mu}(0) = \frac{1}{m_{f_2'}}(|\vec{p}_{f_2'}|, 0, 0, E_{f_2'}),$$
$$\epsilon_{f_2'\mu}(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \quad (16)$$

The following modified form is more appropriate for $B_s^0 \to f_2'$ form factors:

$$F(q^2) = \frac{F(0)}{(1 - q^2/m_{B_S^0}^2)(1 - a(q^2/m_{B_S^0}^2) + b(q^2/m_{B_S^0}^2)^2)}. \quad (17)$$

B. Amplitude of the $B_s^0 \to J/\psi K^+K^-$ decay with Dalitz plot analysis

1. Nonresonant background

For three body $B_s^0 \to J/\psi K^+K^-$ decay, the Feynman diagrams are shown in Fig. 3, Under the factorization approach, the $B_s^0 \to J/\psi K^+K^-$ decay amplitude consists of a tree and a penguin processes, $< B_s^0 \to K^+K^- > \times < 0 \to J/\psi >$, where $< B_s^0 \to K^+K^- >$ denotes two-meson transition matrix element. So the matrix elements of this three body decay is given by

$$< J/\psi K^+K^-|H_{\rm eff}|B_s^0 > \propto < J/\psi|(c\bar{c})_{V-A}|0 >< K^+K^-|(s\bar{b})_{V-A}|B_s^0 > . \tag{18}$$

The two-meson transition matrix element $< K^+K^-|(s\bar{b})_{V-A}|B_s^0 >$ has the general expression as follow [10]

$$< K^-(p_1)K^+(p_2)|(s\bar{b})_{V-A}|B_s^0 > = ir(p_{B_s^0} - p_1 - p_2)_\mu + i\omega_+(p_2 + p_1)_\mu + i\omega_-(p_2 - p_1)_\mu. \tag{19}$$

The r, ω_+, and ω_- form factors are computed from pointlike and pole diagrams, we also need the strong coupling of $B^*B_s^{0(*)}K$ and $B_s^0B_s^0KK$ vertices. These form factors are given by [10]

$$\begin{aligned} r &= \frac{f_{B_s^0}}{2f_K^2} - \frac{f_{B_s^0}}{f_K^2}\frac{p_{B_s^0}.(p_2 - p_1)}{(p_{B_s^0} - p_1 - p_2)^2 - m_{B_s^0}^2} + \frac{2gf_{B^*}}{f_K^2}\sqrt{\frac{m_{B_s^0}}{m_{B^*}}}\frac{(p_{B_s^0} - p_1).p_1}{(p_{B_s^0} - p_1)^2 - m_{B^*}^2} \\ &\quad - \frac{4g^2f_{B_s^0}}{f_K^2}\frac{m_{B_s^0}m_{B^*}}{(p_{B_s^0} - p_1 - p_2)^2 - m_{B_s^0}^2}\frac{p_1.p_2 - p_1.(p_{B_s^0} - p_1)p_2.(p_{B_s^0} - p_1)/m_{B^*}^2}{(p_{B_s^0} - p_1)^2 - m_{B^*}^2}, \\ \omega_+ &= -\frac{g}{f_K^2}\frac{f_{B^*}m_{B^*}\sqrt{m_{B^*}m_{B_s^0}}}{(p_{B_s^0} - p_1)^2 - m_{B^*}^2}\left[1 - \frac{(p_{B_s^0} - p_1).p_1}{m_{B^*}^2}\right] + \frac{f_{B_s^0}}{2f_K^2}, \\ \omega_- &= \frac{g}{f_K^2}\frac{f_{B^*}m_{B^*}\sqrt{m_{B_s^0}m_{B^*}}}{(p_{B_s^0} - p_1)^2 - m_{B^*}^2}\left[1 + \frac{(p_{B_s^0} - p_1).p_1}{m_{B^*}^2}\right]. \end{aligned} \tag{20}$$

Then the matrix elements read

$$< K^-(p_1)K^+(p_2)J/\psi(\epsilon_3, p_3)|H_{\rm eff}|B_s^0 > \propto if_{J/\psi}m_{J/\psi}(\epsilon_3 \cdot p_3 r + (\epsilon_3 \cdot p_2 + \epsilon_3 \cdot p_1)\omega_+ + (\epsilon_3 \cdot p_2 - \epsilon_3 \cdot p_1)\omega_-)), \tag{21}$$

where under the Lorentz condition $\epsilon_3 \cdot p_3 = 0$. Consider the decay of B meson into three particles of masses m_1, m_2, and m_3. Denote their 4-momenta by p_B, p_1, p_2, and p_3, respectively. Energy-momentum conservation is expressed by

$$p_B = p_1 + p_2 + p_3. \tag{22}$$

Define the following invariants

$$\begin{aligned} s_{12} &= (p_1 + p_2)^2 = (p_B - p_3)^2, \\ s_{13} &= (p_1 + p_3)^2 = (p_B - p_2)^2, \\ s_{23} &= (p_2 + p_3)^2 = (p_B - p_1)^2. \end{aligned} \tag{23}$$

The three invariants s_{12}, s_{13}, and s_{23} are not independent, it follows from their definitions together with 4-momentum conservation that

$$s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2. \tag{24}$$

We take $s_{12} = s$ and $s_{23} = t$, so we have $s_{13} = m_B^2 + m_1^2 + m_2^2 + m_3^2 - s - t$. In the center of mass of $K^-(p_1)$ and $K^+(p_2)$, according to Fig. 4, we find

$$\begin{aligned} |\vec{p}_1| &= |\vec{p}_2| = \frac{1}{2}\sqrt{s - 4m_1^2}, \\ p_1^0 &= p_2^0 = \frac{1}{2}\sqrt{s}, \\ |\vec{p}_3| &= p_3^3 = \frac{1}{2\sqrt{s}}\sqrt{(m_{B_s^0}^2 - m_3^2 - s)^2 - 4sm_3^2}, \\ p_3^0 &= \frac{1}{2\sqrt{s}}(m_{B_s^0}^2 - m_3^2 - s), \\ |\vec{\epsilon}_3| &= \frac{1}{2m_3\sqrt{s}}(m_{B_s^0}^2 - m_3^2 - s), \\ \epsilon_3^0 &= \frac{1}{2m_3\sqrt{s}}\sqrt{(m_{B_s^0}^2 - m_3^2 - s)^2 - 4sm_3^2}, \end{aligned} \tag{25}$$

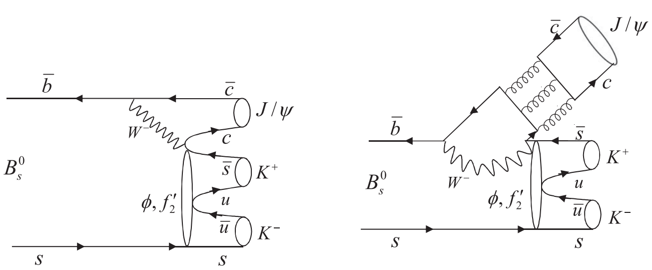

FIG. 3. Quark diagrams illustration the processes $B_s^0 \to J/\psi K^+K^-$ decay.

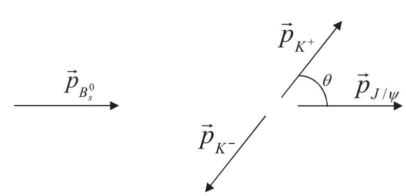

FIG. 4. Definition of helicity angle θ, for the decay of $B_s^0 \to J/\psi K^+K^-$.

and the cosine of the helicity angle θ between the direction of $\vec{p}_2$ and that of $\vec{p}_3$ reads

$$cos\theta = \frac{1}{4|\vec{p}_2||\vec{p}_3|}(m_{B_s^0}^2 + m_3^2 + 2m_2^2 - s - 2t). \quad (26)$$

With these definitions, we obtain multiplication of the 4-momentum as

$$\begin{aligned} \epsilon_3 \cdot (p_2 + p_1) &= 2p_1^0\epsilon_3^0, \\ \epsilon_3 \cdot (p_2 - p_1) &= 2|\vec{\epsilon}_3||\vec{p}_1|cos\theta. \end{aligned} \quad (27)$$

Now we can derive the nonresonant amplitude as

$$\begin{aligned} &M_{NR}(B_s^0 \to J/\psi K^+K^-) \\ &= i\frac{G_F}{2\sqrt{2}}(a_2 V_{cb}^* V_{cs} - a_3 V_{tb} V_{ts}^*) \\ &\times \left(\omega_+ \sqrt{(m_{B_s^0}^2 - m_{J/\psi}^2 - s)^2 - 4sm_{J/\psi}^2} \right. \\ &\left. + \omega_-(m_{B_s^0}^2 - m_{J/\psi}^2 - s)\sqrt{1 - 4m_K^2/s}cos\theta\right) f_{J/\psi}. \end{aligned} \quad (28)$$

2. Resonant contributions

According to Fig. 3, the decay channels of $B_s^0 \to J/\psi K^+K^-$ can also receive contributions through intermediate resonances $\phi(1020)$ and $f_2'(1525)$. Resonant effects are described in terms of the usual Breit-Wigner formalism

$$\begin{aligned} &< K^-(p_1)K^+(p_2)|(\bar{s}b)_{V-A}|B_s^0 >^R \\ &= \sum_R \frac{g^{R\to K^+K^-}}{m_R^2 - (p_2+p_1)^2 - im_R\Gamma^{R\to K^+K^-}} \sum_{pol} \epsilon_R \cdot (p_2 - p_1) \\ &\times < R|(\bar{s}b)_{V-A}|B_s^0 >, \end{aligned} \quad (29)$$

where $R = \phi(1020)$ and $f_2'(1525)$, and the transitions $B_s^0 \to \phi$ and $B_s^0 \to f_2'$ should be written as Eqs. (10) and (13), and

$$g^{R\to K^+K^-} = \sqrt{\frac{12\pi m_R^2 \Gamma^{R\to K^+K^-}}{2p_c^3}}, \quad (30)$$

where p_c is the c.m. momentum. In determining the coupling of $R \to K^+K^-$, we have used the partial width $\Gamma^{\phi(1020)\to K^+K^-} = (2.08 \pm 0.04)$ MeV and $\Gamma^{f_2'(1525)\to K^+K^-} = (64.75^{+7.06}_{-5.93})$ MeV Mev measured by PDG [4]. Then the decay amplitude through resonance intermediate reads

$$\begin{aligned} &M_R(B_s^0 \to K^-(p_1)K^+(p_2)J/\psi(\epsilon_3, p_3)) \\ &= \frac{G_F}{\sqrt{2}} m_{J/\psi} f_{J/\psi}(a_2 V_{cb}^* V_{cs} - a_3 V_{tb} V_{ts}^*) \\ &\times \Bigg[(\epsilon_\phi \cdot (p_2 - p_1))\Bigg(-\epsilon_{\mu\nu\alpha\beta}\epsilon_3^\mu \epsilon_\phi^\nu p_{B_s^0}^\alpha p_\phi^\beta \frac{2V^{B_s^0\phi}(q^2)}{m_{B_s^0} + m_\phi} - i(\epsilon_\phi \cdot \epsilon_3)(m_{B_s^0} + m_\phi)A_1^{B_s^0\phi}(q^2) \\ &+ i(\epsilon_\phi \cdot p_3)(\epsilon_3 \cdot (p_{B_s^0} + p_\phi))\frac{A_2^{B_s^0\phi}(q^2)}{m_{B_s^0} + m_\phi}\Bigg)\frac{g^{\phi\to K^+K^-}}{m_\phi^2 - s - im_\phi\Gamma^{\phi\to K^+K^-}} \\ &+ (\epsilon_{f_2'} \cdot (p_2 - p_1))(-h(q^2)\epsilon_{\mu\nu\alpha\beta}\epsilon_3^\mu \epsilon_{f_2'}^{\nu\lambda} P_\lambda P^\alpha q^\beta - ik(q^2)\epsilon_{f_2'\mu\nu}\epsilon_3^\mu P^\nu \\ &- i\epsilon_{f_2'\alpha\beta}P^\alpha P^\beta(b_+(q^2)(\epsilon_3 \cdot P) + b_-(q^2)(\epsilon_3 \cdot q)))\frac{g^{f_2'\to K^+K^-}}{m_{f_2'}^2 - s - im_{f_2'}\Gamma^{f_2'\to K^+K^-}}\Bigg]. \end{aligned} \quad (31)$$

Finally by using the full amplitude, the decay rate of $B_s^0 \to J/\psi K^+K^-$ is then given by [11]

$$\Gamma(B_s^0 \to J/\psi K^+K^-) = \frac{1}{(2\pi)^3 32m_{B_s^0}^3}\int_{s_{\min}}^{s_{\max}}\int_{t_{\min}}^{t_{\max}} |M_{NR}(B_s^0 \to J/\psi K^+K^-) + M_R(B_s^0 \to J/\psi K^+K^-)|^2 dsdt, \quad (32)$$

where

$$\begin{aligned} t_{\min,\max}(s) &= m_1^2 + m_3^2 - \frac{1}{2s}((m_{B_s^0}^2 - s - m_3^2)(s - m_2^2 + m_1^2) \mp \sqrt{\lambda(s, m_{B_s^0}^2, m_3^2)}\sqrt{\lambda(s, m_1^2, m_2^2)}), \\ s_{\min} &= (m_1 + m_2)^2, \\ s_{\max} &= (m_{B_s^0} - m_3)^2, \end{aligned} \quad (33)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

III. NUMERICAL RESULTS

The Wilson coefficients c_i have been calculated in different schemes. In this paper we will use consistently the naive dimensional regularization (NDR) scheme. The values of c_i at the scales $\mu = m_b/2$, $\mu = m_b$, and $\mu = 2m_b$ at the next to leading order (NLO) are shown in Table I. The parameter g in the form factors, determined from the $D^* \to D\pi$ decay, is [10]

$$g = 0.3 \pm 0.1. \tag{34}$$

TABLE I. Wilson coefficients c_i in the NDR scheme at three different choices of the renormalization scale μ [12].

NLO	c_1	c_2	c_3	c_4
$\mu = m_b/2$	1.137	−0.295	0.021	−0.051
$\mu = m_b$	1.081	−0.190	0.014	−0.036
$\mu = 2m_b$	1.045	−0.113	0.009	−0.025

For the elements of the CKM matrix, we use the values of the Wolfenstein parameters and obtain

$$\begin{aligned}
&|V_{ud}| = 0.97425 \pm 0.00022, \quad |V_{us}| = 0.2252 \pm 0.0009, \quad |V_{ub}| = 0.00415 \pm 0.00049,\\
&|V_{cd}| = 0.230 \pm 0.011, \quad |V_{cs}| = 1.006 \pm 0.023, \quad |V_{cb}| = 0.0409 \pm 0.0011,\\
&|V_{td}| = 0.0084 \pm 0.0006, \quad |V_{ts}| = 0.0429 \pm 0.0026, \quad |V_{tb}| = 0.89 \pm 0.07.
\end{aligned} \tag{35}$$

The meson masses and decay constants needed in our calculations are taken as (in units of Mev) [4]

$$\begin{aligned}
&m_{B^*} = 5325.2 \pm 0.4, \quad m_{B_s^0} = 5366.77 \pm 0.24, m_{J/\psi} = 3096.916 \pm 0.011, \quad m_{f_2'} = 1525 \pm 5,\\
&m_{\phi} = 1019.455 \pm 0.020, \quad m_{K^\pm} = 493.677 \pm 0.016,\\
&f_{B^*} = 194 \pm 6, \quad f_{B_s^0} = 206 \pm 10, \quad f_{J/\psi} = 418 \pm 9, f_K = 159.8 \pm 1.84,
\end{aligned} \tag{36}$$

and the parameters in $B_s^0 \to \phi$ form factor are taken as [9]

$$\begin{aligned}
&f_1(0) = 0.34, \quad \sigma_1 = 0.73, \quad \sigma_1' = 0.42,\\
&f_2(0) = 0.31, \quad \sigma_2 = 1.30, \quad \sigma_2' = 0.52,\\
&f_V(0) = 0.44, \quad \sigma_V = 0.62, \quad \sigma_V' = 0.20,
\end{aligned} \tag{37}$$

and for $B_s^0 \to f_2'$ form factor we use [8]

$$\begin{aligned}
V&: \quad F(0) = 0.20, \quad a = 1.75, \quad b = 0.69,\\
A_0&: \quad F(0) = 0.16, \quad a = 1.69, \quad b = 0.64,\\
A_1&: \quad F(0) = 0.12, \quad a = 0.80, \quad b = -0.11,\\
A_2&: \quad F(0) = 0.09.
\end{aligned} \tag{38}$$

Using the parameters relevant for the $B_s^0 \to J/\psi\phi(1020)$, $B_s^0 \to J/\psi f_2'(1525)$, and $B_s^0 \to J/\psi K^+K^-$ decays, we calculate the branching ratios of these decays and the numerical results are shown in Table II.

IV. CONCLUSION

In this work, we have presented a comprehensive studies of the $B_s^0 \to J/\psi\phi(1020)$, $B_s^0 \to J/\psi f_2'(1525)$, and $B_s^0 \to J/\psi K^+K^-$ decays. In fact, we have been interested to examine the $B_s^0 \to J/\psi K^+K^-$ decay by using the Dalitz plot analysis, while this decay mode has dominant $\phi(1020)$ and $f_2'(1525)$ resonances. In evaluating the $B_s^0 \to J/\psi K^+K^-$ decay, the amplitudes of the $B_s^0 \to J/\psi\phi(1020)$ and $B_s^0 \to J/\psi f_2'(1525)$ decays were required (specifically, the form factors of the vector and tensor mesons). Hence we have decided to consider these two-body decays. According to the QCD factorization approach, we have calculated the branching ratios of the $B_s^0 \to J/\psi\phi(1020)$ and $B_s^0 \to J/\psi f_2'(1525)$ decays and obtained $(1.14 \pm 0.17) \times 10^{-3}$ and $(0.33 \pm 0.05) \times 10^{-3}$, respectively. These results are in good agreement with the current PDG values [4] that are dominated by the CDF measurements [6]. Finally, we have computed the $B_s^0 \to J/\psi K^+K^-$ decay through contributions of the non-resonant and intermediate resonances $\phi(1020)$ and $f_2'(1525)$. The overall branching fraction was obtained by applying Dalitz plot analysis to be $BR(B_s^0 \to J/\psi K^+K^-) = (1.03 \pm 0.09) \times 10^{-3}$. All results are in good agreement with the Belle collaboration measurements [3].

TABLE II. Branching ratios of $B_s^0 \to J/\psi\phi(1020)$, $B_s^0 \to J/\psi f_2'(1525)$, and $B_s^0 \to J/\psi K^+K^-$ decays (in units of 10^{-3}).

Mode	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$	Exp. [3]
$B_s^0 \to J/\psi\phi(1020)$	0.26 ± 0.04	1.14 ± 0.17	2.23 ± 0.33	$1.25 \pm 0.07 \pm 0.08 \pm 0.22$
$B_s^0 \to J/\psi f_2'(1525)$	0.08 ± 0.01	0.33 ± 0.05	0.64 ± 0.10	$0.26 \pm 0.06 \pm 0.02 \pm 0.05$
$B_s^0 \to J/\psi K^+K^-$	0.24 ± 0.02	1.03 ± 0.09	2.01 ± 0.17	$1.01 \pm 0.09 \pm 0.10 \pm 0.18$

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