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Analysis of an extended scalar sector with S_3 symmetry

Dipankar Das[1,*] and Ujjal Kumar Dey[2,†]

[1]*Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India*

[2]*Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211019, India*

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We investigate the scalar potential of a general S_3-symmetric three-Higgs-doublet model. The outcome of our analysis does not depend on the fermionic sector of the model. We identify a decoupling limit for the scalar spectrum of this scenario. In view of the recent LHC Higgs data, we show our numerical results only in the decoupling limit. Unitarity and stability of the scalar potential demand that many new scalars must be lurking below 1 TeV. We provide numerical predictions for $h \to \gamma\gamma$ and $h \to Z\gamma$ signal strengths, which can be used to falsify the theory.

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I. INTRODUCTION

The newly observed boson at the Large Hadron Collider (LHC) [1,2] fits very well to the description of the Higgs scalar in the Standard Model (SM). The SM relies on the minimal choice that a single Higgs doublet provides masses to all particles. But unexplained phenomena like neutrino masses and existence dark matter motivate us to contemplate other avenues beyond the SM (BSM). Majority of these BSM scenarios extend the SM Higgs sector predicting a richer scalar spectrum. One of them—the S_3 flavor model—stems from an effort to answer the aesthetic question as to why there are precisely three fermion generations [3]. Keeping the fermions in appropriate S_3 multiplets, it is possible to reproduce all the measured parameters of the CKM and PMNS matrices as well as make testable predictions for the unknown parameters of the PMNS matrix [4–24]. But one needs at least three Higgs doublets to achieve this goal [6]. However, the S_3 invariant scalar potential contains some new parameters which are difficult to constrain phenomenologically. Although some lower bounds on the additional scalar masses can be placed from the Higgs mediated flavor changing neutral current (FCNC) processes [25], these bounds rely heavily on the Yukawa structure of the model. In this paper we will present some new bounds on the physical scalar masses which do not depend on the parameters of the Yukawa sector. To achieve this, we will employ the prescription of tree unitarity, which is known to be able to set upper limits on different scalar masses [26]. Although various aspects of the S_3 scalar potential have been discussed in the literature [27,28], to the best of our knowledge, this is the first attempt to derive the exact unitarity constraints on the quartic couplings in the S_3 invariant three-Higgs-doublet model (S3HDM) scalar potential. We also identify a *decoupling limit* in the context of S3HDM, where a CP-even Higgs with SM-like properties can be obtained. Since the recent LHC Higgs data seem to increasingly leaned towards the SM expectations, our numerical analysis will be restricted to this limit.

The paper is organized as follows: in Sec. II we discuss the scalar potential and derive necessary conditions for the potential to be bounded from below. In Sec. III we minimize the potential and calculate the physical scalar masses. In this section we also figure out a decoupling limit in which one neutral CP-even physical scalar behaves exactly like the SM Higgs. In Sec. IV we derive the exact constraints arising from the considerations of tree level unitarity and use them to constrain the nonstandard scalar masses. In Sec. V we quantitatively investigate the effect of the charged scalar induced loops on $h \to \gamma\gamma$ and $h \to Z\gamma$ signal strengths. Finally, we summarize our findings in Sec. VI.

II. THE SCALAR POTENITAL

S_3 is the permutation group involving three objects, $\{\phi_a, \phi_b, \phi_c\}$. The three dimensional representation of S_3 is not an irreducible one simply because we can easily construct a linear combination of the elements, $\phi_a + \phi_b + \phi_c$, which remains unaltered under the permutation of the indices. We choose to decompose the three dimensional representation into a singlet and doublet as follows :

$$\mathbf{1}\colon\ \phi_3 = \frac{1}{\sqrt{3}}(\phi_a + \phi_b + \phi_c), \tag{1a}$$

$$\mathbf{2}\colon\ \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_a - \phi_b) \\ \frac{1}{\sqrt{6}}(\phi_a + \phi_b - 2\phi_c) \end{pmatrix}. \tag{1b}$$

The elements of S_3 for this particular doublet representation are given by

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}, \quad \text{for} \left(\theta = 0, \pm\frac{2\pi}{3}\right). \tag{2}$$

*d.das@saha.ac.in
†ujjaldey@hri.res.in

The most general renormalizable potential invariant under S_3 can be written in terms of ϕ_3, ϕ_1 and ϕ_2 as follows [27–31]:

$$V(\phi) = V_2(\phi) + V_4(\phi), \tag{3a}$$

$$\text{where, } V_2(\phi) = \mu_1^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + \mu_3^2\phi_3^\dagger\phi_3, \tag{3b}$$

$$\begin{aligned} V_4(\phi) = {} & \lambda_1(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2)^2 + \lambda_2(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1)^2 \\ & + \lambda_3\{(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)^2 + (\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2\} \\ & + \lambda_4\{(\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) + (\phi_3^\dagger\phi_2)(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2) + \text{H.c.}\} \\ & + \lambda_5(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + \lambda_6\{(\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_3) + (\phi_3^\dagger\phi_2)(\phi_2^\dagger\phi_3)\} \\ & + \lambda_7\{(\phi_3^\dagger\phi_1)(\phi_3^\dagger\phi_1) + (\phi_3^\dagger\phi_2)(\phi_3^\dagger\phi_2) + \text{H.c.}\} + \lambda_8(\phi_3^\dagger\phi_3)^2. \end{aligned} \tag{3c}$$

In general λ_4 and λ_7 can be complex, but we assume them to be real so that CP symmetry is not broken explicitly. For the stability of the vacuum in the asymptotic limit we impose the requirement that there should be no direction in the field space along which the potential becomes infinitely negative. The necessary and sufficient conditions for this is well known in the context of two Higgs-doublet models (2HDMs) [32]. For the potential of Eq. (3), a 2HDM equivalent situation arise if one of the doublets is made identically zero. Then it is quite straightforward to find the following necessary conditions for the global stability in the asymptotic limit:

$$\lambda_1 > 0, \tag{4a}$$

$$\lambda_8 > 0, \tag{4b}$$

$$\lambda_1 + \lambda_3 > 0, \tag{4c}$$

$$2\lambda_1 + (\lambda_3 - \lambda_2) > |\lambda_2 + \lambda_3|, \tag{4d}$$

$$\lambda_5 + 2\sqrt{\lambda_8(\lambda_1 + \lambda_3)} > 0, \tag{4e}$$

$$\lambda_5 + \lambda_6 + 2\sqrt{\lambda_8(\lambda_1 + \lambda_3)} > 2|\lambda_7|, \tag{4f}$$

$$\lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + 2\lambda_7 + \lambda_8 > 2|\lambda_4|. \tag{4g}$$

To avoid confusion, we wish to mention that an equivalent doublet representation,

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tag{5}$$

has also been used in the literature. In terms of this new doublet, the quartic part of the scalar potential is written as [33–35]

$$\begin{aligned} V_4 = {} & \frac{\beta_1}{2}(\chi_1^\dagger\chi_1 + \chi_2^\dagger\chi_2)^2 + \frac{\beta_2}{2}(\chi_1^\dagger\chi_1 - \chi_2^\dagger\chi_2)^2 + \beta_3(\chi_1^\dagger\chi_2)(\chi_2^\dagger\chi_1) \\ & + \frac{\beta_4}{2}(\phi_3^\dagger\phi_3)^2 + \beta_5(\phi_3^\dagger\phi_3)(\chi_1^\dagger\chi_1 + \chi_2^\dagger\chi_2) \\ & + \beta_6\phi_3^\dagger(\chi_1\chi_1^\dagger + \chi_2\chi_2^\dagger)\phi_3 + \beta_7\{(\phi_3^\dagger\chi_1)(\phi_3^\dagger\chi_2) + \text{H.c.}\} \\ & + \beta_8\{\phi_3^\dagger(\chi_1\chi_2^\dagger\chi_1 + \chi_2\chi_1^\dagger\chi_2) + \text{H.c.}\}. \end{aligned} \tag{6}$$

It is easy to verify that the parameters of Eq. (6) are related to the parameters of Eq. (3c) in the following way:

$$\begin{aligned} & \beta_1 = 2\lambda_1; \quad \beta_2 = -2\lambda_2; \quad \beta_3 = 4\lambda_3; \quad \beta_4 = 2\lambda_8; \\ & \beta_5 = \lambda_5; \quad \beta_6 = \lambda_6; \quad \beta_7 = 2\lambda_7; \quad \beta_8 = -\sqrt{2}\lambda_4. \end{aligned} \tag{7}$$

This mapping can be used to translate the constraints on λs into constraints on βs. In this paper we opt to work with the parametrization of Eq. (3).

III. PHYSICAL EIGENSTATES

We represent the scalar doublets in the following way:

$$\phi_k = \begin{pmatrix} w_k^+ \\ \frac{1}{\sqrt{2}}(v_k + h_k + iz_k) \end{pmatrix} \quad \text{for } k = 1, 2, 3. \tag{8}$$

We shall assume that CP symmetry is not spontaneously broken and so the vacuum expectation values (VEVs) are taken to be real. They also satisfy the usual VEV relation: $v = \sqrt{v_1^2 + v_2^2 + v_3^2} = 246$ GeV. The minimization conditions for the scalar potential of Eq. (3) reads

$$\begin{aligned} \mu_1^2 = {} & -2\lambda_1(v_1^2 + v_2^2) - 2\lambda_3(v_1^2 + v_2^2) \\ & - v_3\{6\lambda_4 v_2 + (\lambda_5 + \lambda_6 + 2\lambda_7)v_3\}, \end{aligned} \tag{9a}$$

$$\begin{aligned} \mu_1^2 = {} & -2\lambda_1(v_1^2 + v_2^2) - 2\lambda_3(v_1^2 + v_2^2) \\ & - \frac{3v_3}{v_2}\lambda_4(v_1^2 - v_2^2) - (\lambda_5 + \lambda_6 + 2\lambda_7)v_3^2, \end{aligned} \tag{9b}$$

$$\mu_3^2 = \lambda_4\frac{v_2}{v_3}(v_2^2 - v_1^2) - (\lambda_5 + \lambda_6 + 2\lambda_7)(v_1^2 + v_2^2) - 2\lambda_8 v_3^2. \tag{9c}$$

For the self-consistency of Eqs. (9a) and (9b), two possible scenarios arise[1]:

$$\lambda_4 = 0, \tag{10a}$$

$$\text{or,}\ v_1 = \sqrt{3} v_2. \tag{10b}$$

In the following subsections we shall discuss each of the above scenarios separately.

A. Case-I ($\lambda_4 = 0$)

Since CP symmetry is assumed to be exact in the scalar potential, the neutral physical states will be eigenstates of CP too. We find that the mass-squared matrices in the scalar(M_S^2), pseudoscalar(M_P^2) and charged(M_C^2) sectors are simultaneously block diagonalizable by the following matrix :

$$X = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with} \quad \tan\gamma = \frac{v_1}{v_2}. \tag{11}$$

For the charged mass matrix, we obtain:

$$XM_C^2X^T = \begin{pmatrix} m_{1+}^2 & 0 & 0 \\ 0 & -\frac{1}{2}v_3^2(\lambda_6+2\lambda_7) & \frac{1}{2}v_3\sqrt{v_1^2+v_2^2}(\lambda_6+2\lambda_7) \\ 0 & \frac{1}{2}v_3\sqrt{v_1^2+v_2^2}(\lambda_6+2\lambda_7) & -\frac{1}{2}(v_1^2+v_2^2)(\lambda_6+2\lambda_7) \end{pmatrix}, \tag{12}$$

where, one of the charged Higgs (H_1^+) with mass m_{1+} is defined as:

$$H_1^+ = \cos\gamma w_1^+ - \sin\gamma w_2^+, \tag{13a}$$

$$m_{1+}^2 = -\left\{2\lambda_3 \sin^2\beta + \frac{1}{2}(\lambda_6+2\lambda_7)\cos^2\beta\right\}v^2, \tag{13b}$$

$$\text{with,} \quad \tan\beta = \frac{\sqrt{v_1^2+v_2^2}}{v_3}. \tag{13c}$$

The second charged Higgs (H_2^+) along with the massless Goldstone (ω^+), which will appear as the longitudinal component of the W-boson, can be obtained by diagonalizing the remaining 2×2 block:

$$\begin{pmatrix} H_2^+ \\ \omega^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} w_2'^+ \\ w_3^+ \end{pmatrix}$$
$$\text{with,} \quad w_2'^+ = \sin\gamma w_1^+ + \cos\gamma w_2^+. \tag{14}$$

The mass of the second charged Higgs is given by:

$$m_{2+}^2 = -\frac{1}{2}(\lambda_6+2\lambda_7)v^2. \tag{15}$$

Similar considerations for the pseudoscalar part gives:

$$XM_P^2X^T = \begin{pmatrix} \frac{1}{2}m_{A1}^2 & 0 & 0 \\ 0 & -v_3^2\lambda_7 & v_3\sqrt{v_1^2+v_2^2}\lambda_7 \\ 0 & v_3\sqrt{v_1^2+v_2^2}\lambda_7 & -(v_1^2+v_2^2)\lambda_7 \end{pmatrix}, \tag{16}$$

where, the pseudoscalar state (A_1) with mass eigenvalue m_{A1} is defined as:

$$A_1 = \cos\gamma z_1 - \sin\gamma z_2, \tag{17a}$$

$$m_{A1}^2 = -2\{(\lambda_2+\lambda_3)\sin^2\beta + \lambda_7\cos^2\beta\}v^2, \tag{17b}$$

where, $\tan\beta$ has already been defined in Eq. (13c). Similar to the charged part, here also the second pseudoscalar (A_2) along with the massless Goldstone (ζ) can be obtained as follows:

$$\begin{pmatrix} A_2 \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} z_2' \\ z_3 \end{pmatrix}$$
$$\text{with,} \quad z_2' = \sin\gamma z_1 + \cos\gamma z_2, \tag{18a}$$

$$\text{and,} \quad m_{A2}^2 = -2\lambda_7 v^2. \tag{18b}$$

Finally, for the CP-even part we have

$$XM_S^2X^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A_S' & -B_S' \\ 0 & -B_S' & C_S' \end{pmatrix}, \tag{19a}$$

$$\text{where,}\ A_S' = (\lambda_1+\lambda_3)(v_1^2+v_2^2), \tag{19b}$$

$$B_S' = -\frac{1}{2}v_3\sqrt{v_1^2+v_2^2}(\lambda_5+\lambda_6+2\lambda_7), \tag{19c}$$

[1]Another possibility, $v_3 = 0$, while mathematically consistent, is unattractive. This is because, in some S_3 structure of the Yukawa sector, the S_3-singlet fermion generation will the remain massless.

$$C'_S = \lambda_8 v_3^2. \tag{19d}$$

The massless state (h^0), as also noted in [36], is given by

$$h^0 = \cos\gamma h_1 - \sin\gamma h_2. \tag{20}$$

But we wish to add here that the appearance of a massless scalar is not surprising. One can easily verify that the potential of Eq. (3) has the following $SO(2)$ symmetry for $\lambda_4 = 0$:

$$\begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \tag{21}$$

Since $SO(2)$ is a continuous symmetry isomorphic to $U(1)$, a massless physical state is expected. Other two physical scalars are obtained as follows,

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h'_2 \\ h_3 \end{pmatrix} \quad \text{with,}$$

$$h'_2 = \sin\gamma h_1 + \cos\gamma h_2, \tag{22a}$$

$$\text{and,} \quad \tan 2\alpha = \frac{2B'_S}{A'_S - C'_S}. \tag{22b}$$

We assume H and h to be the heavier and lighter CP-even mass eigenstates, respectively, with the following eigenvalues:

$$m_H^2 = (A'_S + C'_S) + \sqrt{(A'_S - C'_S)^2 + 4B'^2_S}, \tag{23a}$$

$$m_h^2 = (A'_S + C'_S) - \sqrt{(A'_S - C'_S)^2 + 4B'^2_S}. \tag{23b}$$

At this stage, it is worth noting that we can define two intermediate scalar states, H^0 and R, as

$$\begin{pmatrix} R \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h'_2 \\ h_3 \end{pmatrix}, \tag{24}$$

with the property that H^0 has the exact SM couplings with the vector boson pairs and fermions. H^0 does not take part in the flavor changing processes as well. Of course, H^0 and R are not the physical eigenstates in general but are related to them in the following way:

$$h = \cos(\beta - \alpha)R + \sin(\beta - \alpha)H^0, \tag{25a}$$

$$H = -\sin(\beta - \alpha)R + \cos(\beta - \alpha)H^0. \tag{25b}$$

In view of the fact that a 125 GeV scalar with SM like properties has already been observed at the LHC, we wish the lighter CP-even mass eigenstate (h) to coincide with H^0. Then we must require

$$\cos(\beta - \alpha) \approx 0. \tag{26}$$

In analogy with the 2HDM case [32], this limit can be taken as the *decoupling limit* in the context of a 3HDM with an S_3 symmetry. We must emphasize though, the term "decoupling limit" does not necessarily imply the heaviness of the additional scalars. Considering Eqs. (20) and (24), it is also interesting to note that the state h^0, being orthogonal to H^0, does not have any trilinear h^0VV ($V = W$, Z) coupling. But, in general, it will have flavor changing coupling in the Yukawa sector. This type of neutral massless state with flavor changing fermionic coupling will be ruled out from the well measured values of neutral meson mass differences. This means that the choice $\lambda_4 = 0$ is phenomenologically unacceptable and we shall not pursue this scenario any further.

B. Case-II ($v_1 = \sqrt{3}v_2$)

This situation has recently been analyzed in [37]. We, however, use a convenient parametrization that can provide intuitive insight into the scenario and additionally, we also discuss the possibility of a decoupling limit in the same way as done in the previous subsection.

The definitions for the angles, γ and β, and the digonalizing matrix, X, remain the same as before. Only difference is that, due to the VEV alignment ($v_1 = \sqrt{3}v_2$), $\tan\gamma$ $(= \sqrt{3})$ and hence X is determined completely. Now only two of the VEVs, v_2 and v_3 (say), can be considered independent and $\tan\beta$ is given in terms of them as follows:

$$\tan\beta = \frac{2v_2}{v_3}. \tag{27}$$

The charged and pseudoscalar mass eigenstates have the same form as before; only the mass eigenvalues get modified due to the presence of λ_4,

$$m_{1+}^2 = -\left\{2\lambda_3\sin^2\beta + \frac{5}{2}\lambda_4\sin\beta\cos\beta + \frac{1}{2}(\lambda_6 + 2\lambda_7)\cos^2\beta\right\}v^2, \tag{28a}$$

$$m_{2+}^2 = -\frac{1}{2}\{\lambda_4\tan\beta + (\lambda_6 + 2\lambda_7)\}v^2, \tag{28b}$$

$$m_{A1}^2 = -\left\{2(\lambda_2 + \lambda_3)\sin^2\beta + \frac{5}{2}\lambda_4\sin\beta\cos\beta + 2\lambda_7\cos^2\beta\right\}v^2, \tag{28c}$$

$$m_{A2}^2 = -\left(\frac{1}{2}\lambda_4\tan\beta + 2\lambda_7\right)v^2. \tag{28d}$$

In the presence of λ_4, analysis of the scalar part will be slightly different:

$$XM_S^2X^T = \begin{pmatrix} \frac{1}{2}m_{h0}^2 & 0 & 0 \\ 0 & A_S & -B_S \\ 0 & -B_S & C_S \end{pmatrix}, \quad (29a)$$

where, $A_S = (\lambda_1 + \lambda_3)v^2\sin^2\beta + \frac{3}{4}\lambda_4 v^2 \sin\beta\cos\beta$, (29b)

$$B_S = -\frac{1}{2}\left\{\frac{3}{2}\lambda_4\sin^2\beta + (\lambda_5 + \lambda_6 + 2\lambda_7)\sin\beta\cos\beta\right\}v^2, \quad (29c)$$

$$C_S = -\frac{\lambda_4}{4}v^2\sin^2\beta\tan\beta + \lambda_8 v^2\cos^2\beta. \quad (29d)$$

The state, h^0, will no longer be massless; in fact,

$$m_{h0}^2 = -\frac{9}{2}\lambda_4 v^2 \sin\beta\cos\beta. \quad (30)$$

The angle α, which was used to rotate from (h_2', h_3) basis to the physical (H, h) basis, should be redefined as

$$\tan 2\alpha = \frac{2B_S}{A_S - C_S}, \quad (31)$$

and corresponding mass eigenvalues should have the following expressions:

$$m_H^2 = (A_S + C_S) + \sqrt{(A_S - C_S)^2 + 4B_S^2}, \quad (32a)$$

$$m_h^2 = (A_S + C_S) - \sqrt{(A_S - C_S)^2 + 4B_S^2}. \quad (32b)$$

The conclusion of the previous subsection still holds: in the decoupling limit, $\cos(\beta - \alpha) = 0$, h possesses SM-like gauge and Yukawa couplings. It should be emphasized that the Yukawa couplings of h in this limit, resembles that of the SM, do not depend on the transformation properties of the fermions under S_3. Also, the self-couplings of h coincides with the corresponding SM expressions in the decoupling limit :

$$\mathcal{L}_h^{\text{self}} = -\frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4. \quad (33)$$

Similar to the case described in the previous subsection, h^0 will not have any h^0VV ($V = W, Z$) couplings, but in the present scenario, we may identify a symmetry which forbids such couplings. Note that when the specified relation between v_1 and v_2 is taken, there exists a two-dimensional representation of $\mathbb{Z}_2$,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}, \quad (34)$$

TABLE I. $\mathbb{Z}_2$ parity assignments to the physical mass eigenstates.

Physical states	Transformation under $\mathbb{Z}_2$
h^0, $H_1^\pm$, A_1	Odd
H^0, R, $H_2^\pm$, A_2	Even

which was initially a subgroup of the original S_3 symmetry, remains intact even after the spontaneous symmetry breaking; i.e., the vacuum is invariant under this $\mathbb{Z}_2$ symmetry. This allows us to assign a $\mathbb{Z}_2$ parity for different physical states, and this should be conserved in the theory. The state h^0 is odd under this $\mathbb{Z}_2$ and this is what forbids it to couple with the VV pair. In fact, using the assignments of Table I, together with CP symmetry, many of the scalar self-couplings can be inferred to be zero.

In connection with the number of independent parameters in the Higgs potential, we note that there were ten to start with ($\mu_{1,3}$ and $\lambda_{1,2,\ldots,8}$). μ_1 and μ_3 can be traded for v_2 and v_3 or, equivalently for v and $\tan\beta$. The remaining eight λs can be traded for seven physical Higgs masses and α. The connections are given below:

$$\lambda_1 = \frac{1}{2v^2\sin^2\beta}\left\{(m_h^2\cos^2\alpha + m_H^2\sin^2\alpha) + \left(m_{1+}^2 - m_{2+}^2\cos^2\beta - \frac{1}{9}m_{h0}^2\right)\right\}, \quad (35a)$$

$$\lambda_2 = \frac{1}{2v^2\sin^2\beta}\{(m_{1+}^2 - m_{A1}^2) - (m_{2+}^2 - m_{A2}^2)\cos^2\beta\}, \quad (35b)$$

$$\lambda_3 = \frac{1}{2v^2\sin^2\beta}\left(\frac{4}{9}m_{h0}^2 + m_{2+}^2\cos^2\beta - m_{1+}^2\right), \quad (35c)$$

$$\lambda_4 = -\frac{2}{9}\frac{m_{h0}^2}{v^2}\frac{1}{\sin\beta\cos\beta}, \quad (35d)$$

$$\lambda_5 = \frac{1}{v^2}\left\{\frac{\sin\alpha\cos\alpha}{\sin\beta\cos\beta}(m_H^2 - m_h^2) + 2m_{2+}^2 + \frac{1}{9}\frac{m_{h0}^2}{\cos^2\beta}\right\}, \quad (35e)$$

$$\lambda_6 = \frac{1}{v^2}\left(\frac{1}{9}\frac{m_{h0}^2}{\cos^2\beta} + m_{A2}^2 - 2m_{2+}^2\right), \quad (35f)$$

$$\lambda_7 = \frac{1}{2v^2}\left(\frac{1}{9}\frac{m_{h0}^2}{\cos^2\beta} - m_{A2}^2\right), \quad (35g)$$

$$\lambda_8 = \frac{1}{2v^2\cos^2\beta}\left\{(m_h^2\sin^2\alpha + m_H^2\cos^2\alpha) - \frac{1}{9}m_{h0}^2\tan^2\beta\right\}. \quad (35h)$$

In passing, we wish to state that for the analysis purpose we will always be working in the decoupling limit with $v_1 = \sqrt{3}v_2$.

IV. CONSTRAINTS FROM UNITARITY

In this context, the pioneering work has been done by Lee, Quigg and Thacker (LQT) [26]. They have analyzed several two-body scatterings involving longitudinal gauge bosons and physical Higgs in the SM. All such scattering amplitudes are proportional to Higgs quartic coupling in the high energy limit. The $\ell = 0$ partial wave amplitude (a_0) is then extracted from these amplitudes and cast in the form of an S-matrix having different two-body states as rows and columns. The largest eigenvalue of this matrix is bounded by the unitarity constraint, $|a_0| < 1$. This restricts the quartic Higgs self-coupling and therefore the Higgs mass to a maximum value.

The procedure has been extended to the case of a 2HDM scalar potential [38–41]. We take it one step further and apply it in the context of 3HDMs. Here also same types of two-body scattering channels are considered. Thanks to the equivalence theorem [42,43], we can use unphysical Higgses instead of actual longitudinal components of the gauge bosons when considering the high energy limit. So, we can use the Goldstone-Higgs potential of Eq. (3) for this analysis. Still it will be a much involved calculation. But we notice that the diagrams containing trilinear vertices will be suppressed by a factor of E^2 coming from the intermediate propagator. Thus they do not contribute at high energies, only the quartic couplings contribute. Clearly the physical Higgs masses that could come from the propagators, do not enter this analysis. Since we are interested only in the eigenvalues of the S-matrix, this allows us to work with the original fields of Eq. (3c) instead of the physical mass eigenstates. After an inspection of all the neutral and charged two-body channels, we find the following eigenvalues to be bounded from unitarity:

$$|a_i^\pm|, \quad |b_i| \leq 16\pi, \quad \text{for } i = 1, 2, \ldots, 6. \tag{36}$$

The expressions for the individual eigenvalues in terms of λs are given below:

$$a_1^\pm = \left(\lambda_1 - \lambda_2 + \frac{\lambda_5 + \lambda_6}{2}\right) \pm \sqrt{\left(\lambda_1 - \lambda_2 + \frac{\lambda_5 + \lambda_6}{2}\right)^2 - 4\left\{(\lambda_1 - \lambda_2)\left(\frac{\lambda_5 + \lambda_6}{2}\right) - \lambda_4^2\right\}}, \tag{37a}$$

$$a_2^\pm = (\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8) \pm \sqrt{(\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4\{\lambda_8(\lambda_1 + \lambda_2 + 2\lambda_3) - 2\lambda_7^2\}}, \tag{37b}$$

$$a_3^\pm = (\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8) \pm \sqrt{(\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4\left\{\lambda_8(\lambda_1 - \lambda_2 + 2\lambda_3) - \frac{\lambda_6^2}{2}\right\}}, \tag{37c}$$

$$a_4^\pm = \left(\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7\right) \pm \sqrt{\left(\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7\right)^2 - 4\left\{(\lambda_1 + \lambda_2)\left(\frac{\lambda_5}{2} + \lambda_7\right) - \lambda_4^2\right\}}, \tag{37d}$$

$$a_5^\pm = (5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8) \pm \sqrt{(5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8)^2 - 4\left\{3\lambda_8(5\lambda_1 - \lambda_2 + 2\lambda_3) - \frac{1}{2}(2\lambda_5 + \lambda_6)^2\right\}}, \tag{37e}$$

$$a_6^\pm = \left(\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7\right) \pm \sqrt{\left(\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7\right)^2 - 4\left\{(\lambda_1 + \lambda_2 + 4\lambda_3)\left(\frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7\right) - 9\lambda_4^2\right\}}, \tag{37f}$$

$$b_1 = \lambda_5 + 2\lambda_6 - 6\lambda_7, \tag{37g}$$

$$b_2 = \lambda_5 - 2\lambda_7, \tag{37h}$$

$$b_3 = 2(\lambda_1 - 5\lambda_2 - 2\lambda_3), \tag{37i}$$

$$b_4 = 2(\lambda_1 - \lambda_2 - 2\lambda_3), \tag{37j}$$

$$b_5 = 2(\lambda_1 + \lambda_2 - 2\lambda_3), \tag{37k}$$

$$b_6 = \lambda_5 - \lambda_6. \tag{37l}$$

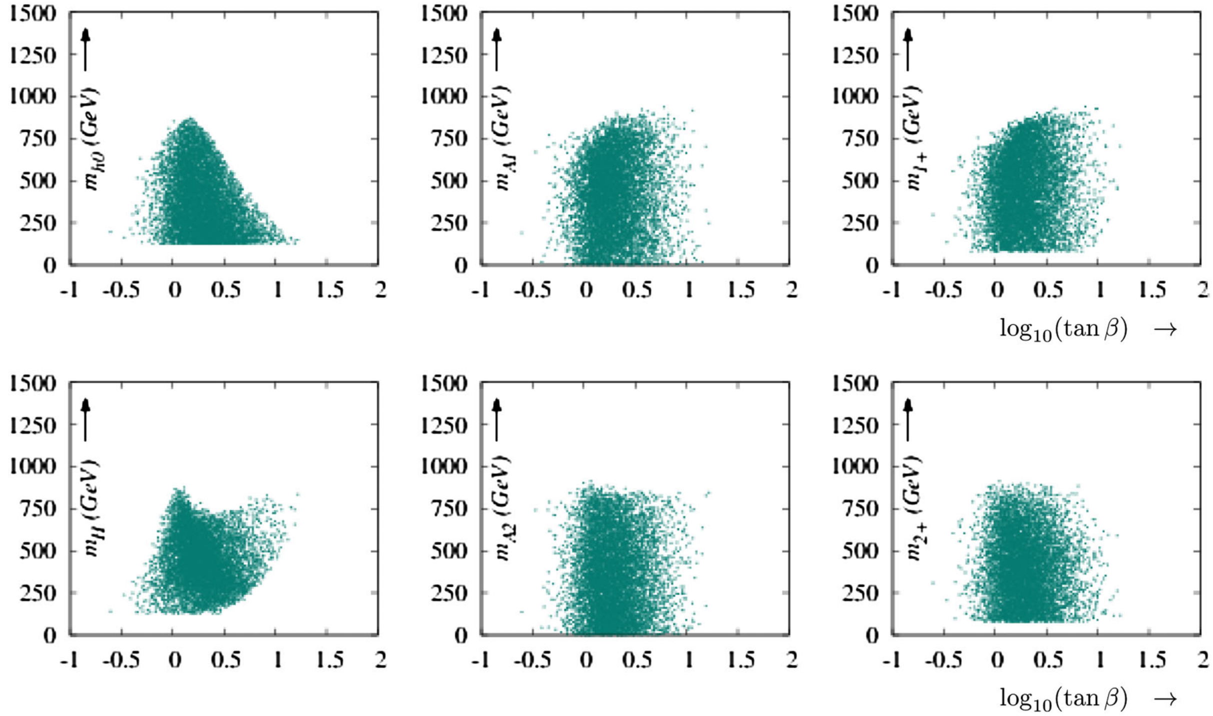

FIG. 1 (color online). (Case-II) Regions allowed from unitarity and stability. We have fixed m_h at 125 GeV and taken m_{1+}, $m_{2+} > 80$ GeV and $m_H, m_{h0} > m_h$.

In passing, we remark that the perturbativity criteria, $|\lambda_i| < 4\pi$, coming from the requirement that the leading order contribution to the physical amplitude must have higher magnitude than the subleading order, may have some ambiguity in this context. This is due to the fact the individual λs do not appear in the quartic couplings involving the physical scalars. Hence the combination of λs, that constitute the physical couplings, should be used for this purpose and it does not necessarily imply that the individual λs should be bounded. We have presented here the exact constraints on λs which should be satisfied for unitarity not to be violated.

Equations (4) and (37) can be used to put limits on the physical Higgs masses. For this purpose, we work in the decoupling limit taking the lightest scalar (h) to be the SM-like Higgs that has been found at the LHC and we set its mass at 125 GeV. We also assume the charged scalars (m_{1+} and m_{2+}) to be heavier than 80 GeV to respect the direct search bound from LEP2 [44]. To collect sufficient number of plot points we have generated fifty million random sets of $\{\tan\beta, m_{h0}, m_H, m_{A1}, m_{A2}, m_{1+}, m_{2+}\}$ by varying $\tan\beta$ from 0.1 to 100 and filter them through the combined constraints from unitarity and stability. The sets that survive the filtering are plotted in Figure 1. The bounds that follow from these figures are listed below:

(i) $\tan\beta \in [0.3, 17]$,

(ii) $m_{h0} < 870$, $m_H < 880$, $m_{A1} < 940$, $m_{A2} < 910$, $m_{1+} < 940$, $m_{2+} < 910$ GeV.

It is interesting to note that if the observed scalar at the LHC has its root in the S3HDM, then there must be several other nonstandard scalars with masses below 1 TeV.

V. IMPACT ON LOOP-INDUCED HIGGS DECAYS

As already has been pointed out, in the decoupling limit the lightest scalar (h) couples with fermions and gauge bosons exactly in the SM way. Consequently, the production cross section as well as tree level decay branching ratios will not alter from their respective SM values. However, the loop induced decay modes like, $h \to \gamma\gamma$ and $h \to Z\gamma$, will pick up additional contributions due to the presence of nonstandard charged scalar loops. Note that the change in total Higgs decay width will be negligibly small as the branching fractions of such decays are tiny.

To display the contribution of the charged scalar loops to the decay amplitudes in a convenient form, we define dimensionless parameters, $\kappa_i (i = 1, 2)$, in the following way:

$$g_{hH_i^+H_i^-} = \kappa_i \frac{g m_{i+}^2}{M_W}. \tag{38}$$

The standard expression for the diphoton decay width is given by [45]

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2 g^2}{2^{10}\pi^3} \frac{m_h^3}{M_W^2} \left| \mathcal{F}_W + \frac{4}{3}\mathcal{F}_t + \sum_{i=1}^{2} \kappa_i \mathcal{F}_{i+} \right|^2, \tag{39}$$

where, using the notation $\tau_x \equiv (2m_x/m_h)^2$, the expressions for $\mathcal{F}_W$, $\mathcal{F}_t$ and $\mathcal{F}_{i+} (i = 1, 2)$ are given by

$$\mathcal{F}_W = 2 + 3\tau_W + 3\tau_W(2 - \tau_W) f(\tau_W), \tag{40a}$$

$$\mathcal{F}_t = -2\tau_t[1 + (1 - \tau_t) f(\tau_t)], \tag{40b}$$

$$\mathcal{F}_{i+} = -\tau_{i+}[1 - \tau_{i+} f(\tau_{i+})]. \tag{40c}$$

For the values of masses that we are dealing with, makes $\tau_x > 1$ for $x = W, t, H_i^\pm$ and then

$$f(\tau) = [\sin^{-1}(\sqrt{1/\tau})]^2. \tag{41}$$

The decay width for $h \to Z\gamma$ is given by

$$\Gamma(h \to Z\gamma) = \frac{\alpha^2 g^2}{2^9 \pi^3} \frac{m_h^3}{M_W^2} \left| \mathcal{A}_W + \mathcal{A}_t + \sum_{i=1}^{2} \kappa_i \mathcal{A}_{i+} \right|^2 \left(1 - \frac{M_Z^2}{m_h^2}\right)^3, \tag{42}$$

where, using $\eta_x = (2m_x/M_Z)^2$, the expressions for $\mathcal{A}_W$, $\mathcal{A}_t$ and $\mathcal{A}_{i+}$ are given by [45],

$$\mathcal{A}_W = \cot\theta_w \left[4(\tan^2\theta_w - 3) I_2(\tau_W, \eta_W) + \left\{ \left(5 + \frac{2}{\tau_W}\right) - \left(1 + \frac{2}{\tau_W}\right)\tan^2\theta_w \right\} I_1(\tau_W, \eta_W) \right], \tag{43a}$$

$$\mathcal{A}_t = \frac{4(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w)}{\sin\theta_w \cos\theta_w} [I_2(\tau_t, \eta_t) - I_1(\tau_t, \eta_t)], \tag{43b}$$

$$\mathcal{A}_{i+} = \frac{(2\sin^2\theta_w - 1)}{\sin\theta_w \cos\theta_w} I_1(\tau_{i+}, \eta_{i+}). \tag{43c}$$

The functions I_1 and I_2 are defined as

$$I_1(\tau, \eta) = \frac{\tau\eta}{2(\tau - \eta)} + \frac{\tau^2\eta^2}{2(\tau - \eta)^2}[f(\tau) - f(\eta)] + \frac{\tau^2\eta}{(\tau - \eta)^2}[g(\tau) - g(\eta)], \tag{44a}$$

$$I_2(\tau, \eta) = -\frac{\tau\eta}{2(\tau - \eta)}[f(\tau) - f(\eta)], \tag{44b}$$

where the function f has the same definition as in Eq. (41). Since $\tau_x, \eta_x > 1$ for $x = W, t, H_i^\pm$, the function g takes the following form:

$$g(x) = \sqrt{x - 1}\sin^{-1}(\sqrt{1/x}). \tag{45}$$

In the decoupling limit, the parameters, $\kappa_i (i = 1, 2)$, which appear in Eqs. (38), (39) and (42), are given by

$$\kappa_i = -\left(1 + \frac{m_h^2}{2m_{i+}^2}\right). \tag{46}$$

In our case, the signal strengths $\mu_{\gamma\gamma}$ and $\mu_{Z\gamma}$, defined through the equations,

$$\mu_{\gamma\gamma} = \frac{\sigma(pp \to h)}{\sigma^{\rm SM}(pp \to h)} \cdot \frac{{\rm BR}(h \to \gamma\gamma)}{{\rm BR}^{\rm SM}(h \to \gamma\gamma)}, \tag{47}$$

$$\mu_{Z\gamma} = \frac{\sigma(pp \to h)}{\sigma^{\rm SM}(pp \to h)} \cdot \frac{{\rm BR}(h \to Z\gamma)}{{\rm BR}^{\rm SM}(h \to Z\gamma)}, \tag{48}$$

assume the following forms:

$$\mu_{\gamma\gamma} = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{\rm SM}(h \to \gamma\gamma)} = \frac{|\mathcal{F}_W + \frac{4}{3}\mathcal{F}_t + \sum_{i=1}^{2} \kappa_i \mathcal{F}_{i+}|^2}{|\mathcal{F}_W + \frac{4}{3}\mathcal{F}_t|^2}, \tag{49}$$

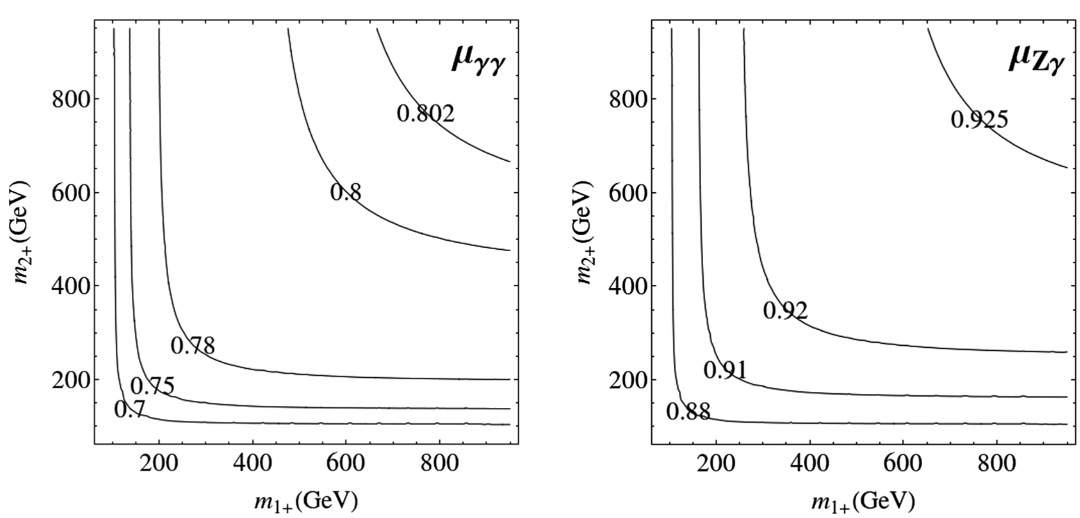

FIG. 2. Signal strengths for diphoton and Z-photon decay modes within the allowed range for charged Higgs masses.

$$\mu_{Z\gamma} = \frac{\Gamma(h \to Z\gamma)}{\Gamma^{\mathrm{SM}}(h \to Z\gamma)} = \frac{|\mathcal{A}_W + \mathcal{A}_t + \sum_{i=1}^{2} \kappa_i \mathcal{A}_{i+}|^2}{|\mathcal{A}_W + \mathcal{A}_t|^2}. \quad (50)$$

As the charged Higgs becomes heavy, the quantity $\mathcal{F}_{i+}$, for example, saturates to $\frac{1}{3}$. So the decoupling of charged Higgs from loop-induced Higgs decay depends on how κ_i behaves with increasing m_{i+}. It follows from Eq. (46) that $\kappa_i \to -1$ if $m_{i+} \gg m_h$. Consequently, the charged Higgs never decouples from the diphoton or Z-photon decay amplitudes. In fact, it reduces the decay widths from their corresponding SM expectations. These features have been displayed in Fig. 2, where we have made a contour plot by varying the charged Higgs masses within the allowed ranges coming from unitarity and vacuum stability. We find that $\mu_{\gamma\gamma}$ and $\mu_{Z\gamma}$ should lie within [0.42, 0.80] and [0.73, 0.93] for $m_{1+} \in [80, 950]$ and $m_{2+} \in [80, 950]$. We must admit, though, this nondecoupling of charged scalar is not a unique feature of a S3HDM as it is also known to be present in the context of 2HDMs [46–49]. Currently the ATLAS data favor an enhancement, whereas the data from CMS favor a suppression in the diphoton decay channel [50]. Thus a precise measurement of the diphoton and Z-photon signal strengths can pin down the difference between the SM Higgs and a SM-like Higgs arising from an extended scalar sector.

VI. CONCLUSIONS

In this paper we have analyzed in detail the scalar sector of an S3HDM. Our findings are listed below:

(i) The minimization of the scalar potential leads to a specific relation between the VEVs of the first two doublets, $v_1 = \sqrt{3} v_2$ in particular.

(ii) In this limit we find a $\mathbb{Z}_2$ subgroup of S_3 that remains unbroken even after the spontaneous symmetry breaking. The different scalar mass eigenstates can then be assigned with appropriate $\mathbb{Z}_2$ parity which can help us understand why certain couplings do not appear in the theory.

(iii) Additionally, we have identified a decoupling limit for this model where the lightest CP-even scalar has the exact same coupling as the SM Higgs with the other SM particles.

(iv) We have also derived the exact tree-unitarity constraints and exploited them, in the decoupling limit, to put new bounds on the physical nonstandard Higgs masses, which we consider to be an important development in the multi-Higgs context.

(v) From unitarity and stability $\tan\beta$ is likely to be in the range [0.3,17] and all the nonstandard Higgs masses lie below 1 TeV.

(vi) Regarding the decay of the SM-like S_3 Higgs, we have observed that the charged Higgs never decouples from the diphoton or Z-photon decay modes. The additional contributions from the charged Higgs loops to the decay amplitudes actually reduces the signal strengths of these modes. Although this depletion may not be a unique property of this scenario, but any statistically significant enhancement in $h \to \gamma\gamma$ and $h \to Z\gamma$ modes will certainly disfavor the possibility of an SM-like Higgs arising from an S3HDM.

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