Lepton asymmetries for the $B_s \to \gamma l^+ l^-$ decay in a family nonuniversal Z' model

Berin Belma Şirvanlı

Department of Physics, Faculty of Arts and Science, Gazi University, 06100 Teknikokullar, Ankara, Turkey

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The exclusive $B_s \to \gamma l^+ l^-$ decay is analyzed in the framework of a family nonuniversal Z' model by calculating the differential branching ratio, double lepton polarizations, and forward-backward asymmetries. Our results are compared against those of the Standard Model. The predictions of this work are hoped to be tested in the near future at LHCb.

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I. INTRODUCTION

The rare decays in the SM proceed via the flavor-changing neutral current (FCNC), which are forbidden at the tree level. The rare decays are one of the best grounds for testing the predictions of the Standard Model (SM) at quantum level. Moreover, these decays are also very promising for establishing new physics beyond the SM indirectly. With operation of the LHCb, new windows are opened for searching of rare decays. Recently, LHCb and CMS Collaborations [1,2] have announced the observation of $B_s \to \mu^+\mu^-$ decays. This decay is helicity suppressed, and its matrix element is proportional to the lepton mass. The branching ratio for the $\mu^+\mu^-$ channel is 1.8×10^{-9} in the SM. In this sense, the observation of this decay is a great achievement in particle physics.

Another rare decay that can be measured in LHCb is the $B_s \to \ell^+\ell^-\gamma$ transition. The main feature of this decay is that the helicity suppression is overcome. For this reason, despite that the width of this decay has an extra factor of fine structure constant α, it is comparable to the decay width of the pure leptonic $B_s \to \ell^+\ell^-$ channel. Indeed, it is shown in [3] that the $B_s \to \ell^+\ell^-\gamma$ decay can have a larger branching ratio compared to that of the $B_s \to \ell^+\ell^-$ channel. As has already been noted, $B_s \to \ell^+\ell^-$ and consequently $B_s \to \ell^+\ell^-\gamma$ decays are both sensitive to the existence of new physics beyond the SM. One possible extension of the SM is the family nonuniversal Z' model, which contains family nonuniversal $U(1)$ gauge symmetries. Such type models appear in some low-energy manifestations of the string theory [4] and E_6 models [5]. Detailed information about this model can be found in [6].

In the framework of this model B_q-$\bar{B}_q$ mixing, $B \to X_s\mu^+\mu^-$, $B_s \to \mu^+\mu^-$ decays [7]; $B \to K^*\ell^+\ell^-$ [8], $B_s \to \phi\mu^+\mu^-$ [9], $B \to K_1\ell^+\ell^-$ [10], $\Lambda_b \to \Lambda\ell^+\ell^-$ [11], and $\Sigma_b \to \Sigma\mu^+\mu^-$ [12] processes have already been investigated, respectively. In the present work, we study the rare $B_s \to \ell^+\ell^-\gamma$ decay.

The work is organized as follows: In Sec. 2, we present the matrix element for the $B_s \to \ell^+\ell^-\gamma$ decay. In Sec. 3, the expressions of the differential branching ratio, double lepton polarization, as well as forward-backward asymmetries are presented. The last section is devoted to the numerical analysis and discussions.

II. THE MATRIX ELEMENT FOR THE $B_s \to \ell^+\ell^-\gamma$ DECAY

As is well known, the $B_s \to \ell^+\ell^-$ decay is described by the $b \to s\ell^+\ell^-$ transition at the quark level. In the SM, the effective Hamiltonian for the $b \to s\ell^+\ell^-$ transition can be written in the following form [13,14]:

$$\mathcal{H}_{\rm eff} = \frac{\alpha_{em}G_F}{2\sqrt{2}\pi} V_{tb}V_{ts}^* \bigg\{ C_9^{\rm eff}(\mu)[\bar{s}\gamma_\mu(1-\gamma_5)b]\bar{\ell}\gamma_\mu\ell + C_{10}(\mu)[\bar{s}\gamma_\mu(1-\gamma_5)b]\bar{\ell}\gamma_\mu\gamma_5\ell - 2C_7(\mu)\frac{i}{q^2}m_b[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b]\bar{\ell}\gamma_\mu\ell \bigg\}, \tag{1}$$

where V_{tb} and V_{ts}^* are the elements of the Cabibbo-Kobayashi-Maskawa mixing matrix, $C_7(\mu)$, $C_9^{\rm eff}(\mu)$ and $C_9^{\rm eff}(\mu)$ are the Wilson coefficients. If the mixing between Z and Z' is neglected, the contribution coming from Z' can be described by just modifying of the Wilson coefficients without introducing any new operator structure. The expression of the effective Hamiltonian describes the contribution of the Z' boson. It can be written in the following form [15,16]:

$$\mathcal{H}^{Z'} = -\frac{2G_F}{\sqrt{2}} V_{tb}V_{ts}^* \bigg[\frac{B_{sb}^L B_{\ell\ell}^L}{V_{tb}V_{ts}^*}\bar{s}\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu(1-\gamma_5)\ell + \frac{B_{sb}^L B_{\ell\ell}^R}{V_{tb}V_{ts}^*}\bar{s}\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu(1+\gamma_5)\ell \bigg], \tag{2}$$

where $B_{sb}^L = |B_{sb}^L|e^{i\varphi_S^L}$ and $B_{\ell\ell}^{L,R}$ correspond to the interaction vertex of Z' with quark and leptons, respectively.

In order to take into account the contributions coming from the Z' boson, it is sufficient to modify the Wilson coefficients $C_9^{\rm eff}(M_W)$ and $C_{10}(M_W)$ in Eqs. (1) and (2) as follows:

$$C_9^{\rm eff} \to C_9^{\rm tot} = C_9^{\rm eff} - \frac{4\pi}{\alpha_S}(28.82)\frac{B_{sb}^L}{V_{tb}V_{ts}^*}(B_{\ell\ell}^L + B_{\ell\ell}^R)$$

$$C_{10} \to C_{10}^{\rm tot} = C_{10} + \frac{4\pi}{\alpha_S}(28.82)\frac{B_{sb}^L}{V_{tb}V_{ts}^*}(B_{\ell\ell}^L - B_{\ell\ell}^R), \quad (3)$$

where α_S is the strong coupling constant. It should be noted here that C_7 receives no contribution from Z', and the evolution of $C_9^{\rm tot}$ and $C_{10}^{\rm tot}$ from weak to $\mu = m_b$ scale should be the same as in SM.

The Wilson coefficient $C_7^{\rm eff}$ in the SM is given by [17]:

$$C_7^{\rm eff}(m_b) = \eta^{\frac{16}{23}}C_7(\mu_W) + \frac{8}{3}(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}})C_8(\mu_W) + C_2(\mu_W)\sum_{i=1}^{8} h_i\eta^{a_i}, \quad (4)$$

where

$$C_2(\mu_W) = 1,$$

$$C_7(\mu_W) = -\frac{1}{2}D_0(x_t),$$

$$C_8(\mu_W) = -\frac{1}{2}E_0(x_t).$$

$D_0(x_t)$ and $E_0(x_t)$ are to be functions of (x_t) and $x_t = m_t^2/m_W^2$. m_t^2 and m_W^2 are the top quark and W boson masses, respectively. $D_0(x_t)$ and $E_0(x_t)$ are defined as

$$D_0(x_t) = -\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1-x_t)^3} + \frac{x_t^2(2-3x_t)}{2(1-x_t)^4}\ln x_t,$$

$$E_0(x_t) = -\frac{x_t(x_t^2 - 5x_t - 2)}{4(1-x_t)^3} + \frac{3x_t^2}{2(1-x_t)^4}\ln x_t. \quad (5)$$

The coefficients a_i and h_i in Eq. (4) are given as

$$a_i = \left(\frac{14}{23}, \frac{16}{23}, \frac{6}{23}, -\frac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456\right),$$

$$h_i = \left(2.2996, -1.0880, -\frac{3}{7}, -\frac{1}{14}, -0.6494, -0.0380, -0.0186, -0.0057\right), \quad (6)$$

and the parameter η is defined as

$$\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)},$$

with

$$\alpha_s(x) = \frac{0.118}{1 - \frac{23}{3}\frac{\alpha_s(m_Z)}{2\pi}\ln(\frac{m_Z}{x})}.$$

The expression for the Wilson coefficient $C_9^{\rm eff}(\hat{s})$ is given as [17]

$$C_9^{\rm eff}(\hat{s}) = C_9^{\rm NDR}\eta(\hat{s}) + h(z,\hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \frac{1}{2}h(1,\hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) - \frac{1}{2}h(0,\hat{s})(C_3 + 3C_4) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \quad (7)$$

where $\hat{s} = q^2/m_b^2$, $\hat{m}_\ell = m_\ell/m_B$ and

$$C_9^{\rm NDR} = P_0^{\rm NDR} + \frac{Y_0(x_t)}{\sin^2\theta_W} - 4Z_0(x_t) + P_E E(x_t).$$

Note that the small contribution coming from P_E is neglected in further numerical analysis. In the naive dimensional regularization scheme, we have $P_0^{\rm NDR} = 2.60 \pm 0.25$ [17], and the remaining two functions $Y(x_t)$ and $Z(x_t)$ are expressed as

$$Y_0(x_t) = \frac{x_t}{8}\left[\frac{x_t - 4}{x_t - 1} + \frac{3x_t}{(x_t-1)^2}\ln x_t\right],$$

$$Z_0(x_t) = \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t-1)^3} + \left[\frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t-1)^4} - \frac{1}{9}\right]\ln x_t. \quad (8)$$

The coefficients $\eta(\hat{s})$, $\omega(\hat{s})$, $h(y,\hat{s})$, and $h(0,\hat{s})$ in Eq. (7) are given as

$$\eta(\hat{s}) = 1 + \frac{\alpha_s(\mu_b)\omega(\hat{s})}{\pi}$$

$$\omega(\hat{s}) = -\frac{2}{9}\pi^2 - \frac{4}{3}\mathrm{Li}_2(\hat{s}) - \frac{2}{3}(\ln\hat{s})\ln(1-\hat{s}) - \frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s}) - \frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln\hat{s} + \frac{5+9\hat{s}-6\hat{s}^2}{6(1-\hat{s})(1+2\hat{s})},$$

$$h(y,\hat{s}) = -\frac{8}{9}\ln\frac{m_b}{\mu_b} - \frac{8}{9}\ln y + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2+x)|1-x|^{1/2} \times \begin{cases} \left(\ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi\right), & \text{for } x \equiv \frac{4z^2}{\hat{s}} < 1 \\ 2\arctan\frac{1}{\sqrt{x-1}}, & \text{for } x \equiv \frac{4z^2}{\hat{s}} > 1, \end{cases}$$

$$h(0,\hat{s}) = \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu_b} - \frac{4}{9}\ln\hat{s} + \frac{4}{9}i\pi, \tag{9}$$

where $y = 1$ or $z = m_c/m_b$ and $\mathrm{Li}_2(\hat{s})$ is the Spence function.

The remaining Wilson coefficients $C_j (j = 1, \ldots 6)$ are given as

$$C_j = \sum_{i=1}^{8} k_{ji}\eta^{a_i} \qquad (j = 1, \ldots 6),$$

and the constants k_{ji} have the values

$$k_{1i} = \left(\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0\right),$$
$$k_{2i} = \left(\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0\right),$$
$$k_{3i} = \left(-\frac{1}{14}, \frac{1}{6}, 0.0510, -0.1403, -0.0113, 0.0054\right),$$
$$k_{4i} = \left(-\frac{1}{14}, -\frac{1}{6}, 0.0984, 0.1214, 0.0156, 0.0026\right),$$
$$k_{5i} = (0, 0, -0.0397, 0.0117, -0.0025, 0.0304),$$
$$k_{6i} = (0, 0, 0.0335, 0.0239, -0.0462, -0.0112).$$

The Wilson coefficient C_9^{tot} receives also long-distance effect contribution coming from real $c\bar{c}$. In the phenomenological Breit-Wigner ansatz, the long distance part Y_{LD} is defined as

$$Y_{\mathrm{LD}} = -\frac{3\pi C^{(0)}}{\alpha^2} \sum_{V_i=\psi(1s)\ldots\psi(6s)} \kappa_i \frac{\Gamma(V_i \to \ell^+\ell^-)m_{V_i}}{q^2 - m_{V_i}^2 + i\Gamma_{V_i}m_{V_i}}, \tag{10}$$

where $C^0 = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$ and κ_i is the phenomenological factor for the lowest two resonances, which is predicted to be $\kappa_{J/\psi} = 2.3$ [18]. After these preliminary remarks, we can now proceed to study the problem under consideration.

As we already noted, the $B_s \to \ell^+\ell^-\gamma$ decay can be obtained from $B_s \to \ell^+\ell^-$, which is described by $b \to s\ell^+\ell^-$ transition, by radiating the photon from any internal and external charged particles. Having the effective Hamiltonian for the $b \to s\ell^+\ell^-$ transition, our next problem is to find the matrix element for $B_s \to \ell^+\ell^-\gamma$ decay. We have following three different contributions:

(i) The photon is emitted from the initial quark fields
(ii) The photon is emitted from final charged leptons.
(iii) The photon is radiated from charged particles in the loop.

The contributions of the diagrams when photon is emitted from internal charged particles is proportional to factor m_l^2/m_W^2. For this reason, this contribution can also be safely neglected. The contributions of the diagrams when the photon is emitted from the final state charged lepton (this part is the so-called Bremsstrahlung part and is proportional to the lepton mass, which follows from helicity arguments). The matrix element corresponding to this contribution is

$$\mathcal{M}_1 = \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb}V_{ts}^* e\imath f_B C_{10} 2m_l \left[\bar{l}\left(\frac{\not{\varepsilon}\not{P}_B}{2p_1k} - \frac{\not{P}_B\not{\varepsilon}}{2p_2k}\right)\gamma_5 l\right], \tag{11}$$

where P_B is the B meson momentum and f_B is the decay constant of the B meson.

The matrix element for the $B_s \to \ell^+\ell^-\gamma$ decay when photon is radiated from initial quarks can be written as

$$\mathcal{M}_2 = \langle\gamma|\mathcal{H}_{\mathrm{eff}}|B\rangle = \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb}V_{ts}^* \times \Big\{ C_9^{\mathrm{tot}}\bar{\ell}\gamma_\mu\ell\langle\gamma(k)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p+k)\rangle + C_{10}^{\mathrm{tot}}\bar{\ell}\gamma_\mu\gamma_5\ell\langle\gamma(k)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p+k)\rangle - i2C_7\frac{m_b}{q^2}\bar{\ell}\gamma_\mu\ell\langle\gamma(k)|\bar{s}\sigma_{\mu\nu}q_\nu(1+\gamma_5)b|B(p+k)\rangle \Big\}. \tag{12}$$

The matrix elements in the above-expression are defined in terms of the form factors as follows:

$$\langle\gamma(k)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p+k)\rangle = \frac{e}{m_B^2}\{\epsilon_{\mu\nu\lambda\sigma}\varepsilon_\mu^* q_\lambda k_\sigma g(q^2) + i[\varepsilon_\mu^*(k\cdot q) - (\varepsilon^*\cdot q)k_\mu]f(q^2)\},$$
$$\langle\gamma(k)|\bar{s}i\sigma_{\mu\nu}q_\nu(1+\gamma_5)b|B(p+k)\rangle = \frac{e}{m_B^2}\{\epsilon_{\mu\nu\lambda\sigma}\varepsilon_\nu^* q_\lambda k_\sigma g_1(q^2) + i[\varepsilon_\mu^*(k\cdot q) - (\varepsilon^*\cdot q)k_\mu]f(q^2)]\}, \tag{13}$$

where ε_μ^* and k_μ are the four vector polarization and the four vector momentum of the photon, respectively, and $g(q^2)$, $f(q^2)$, $g_1(q^2)$, and $f_1(q^2)$ are the transition form factors.

The matrix element for the $B_s \to \ell^+\ell^-\gamma$ decay is the sum of $\mathcal{M}_1$ and $\mathcal{M}_2$; i.e., $\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$. Using Eqs. (11)–(13) for the differential decay width, we get

$$\frac{d\Gamma}{d\hat{s}} = \left|\frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb}V_{ts}^*\right|^2 \frac{\alpha}{(2\pi)^3} m_B^5 \pi \left\{ \frac{1}{12}\int_\delta^{1-4r} x^3 dx \sqrt{1-\frac{4r}{1-x}} m_B^2 [(|A|^2+|B|^2)(1-x+2r) \right.$$
$$+ (|C|^2+|D|^2)(1-x-4r)] - 2C_{10} f_B r \int_\delta^{1-4r} x^2 dx \mathrm{Re}(A) \ln \frac{1+\sqrt{1-\frac{4r}{1-x}}}{1-\sqrt{1-\frac{4r}{1-x}}}$$
$$\left. - 4|f_B C_{10}|^2 r \frac{1}{m_B^2} \int_\delta^{1-4r} dx \left[\left(2+\frac{4r}{x}-\frac{2}{x}-x\right) \ln \frac{1+\sqrt{1-\frac{4r}{1-x}}}{1-\sqrt{1-\frac{4r}{1-x}}} + \frac{2}{x}(1-x)\sqrt{1-\frac{4r}{1-x}} \right] \right\}, \tag{14}$$

where the f_B is the leptonic decay constant of the B meson, $x = \frac{2E_\gamma}{m_B}$ is a dimensionless parameter with E_γ being the photon energy, and $r = \frac{m_l^2}{m_B^2}$. The lower limit of integration over x comes from imposing a cut δ on the photon energy (for details, see [3]).

In further numerical analysis, we use the results of [3] for the form factors, which are given as

$$g(q^2) = \frac{1\ \mathrm{GeV}}{(1-\frac{q^2}{5.6^2})^2}, \qquad f(q^2) = \frac{0.8\ \mathrm{GeV}}{(1-\frac{q^2}{6.5^2})^2}, \qquad g_1(q^2) = \frac{3.74\ \mathrm{GeV}^2}{(1-\frac{q^2}{40.5})^2}, \qquad f_1(q^2) = \frac{0.68\ \mathrm{GeV}^2}{(1-\frac{q^2}{30})^2}. \tag{15}$$

Since Z' boson contributions are introduced by just modifying the Wilson coefficients C_9^{eff} and C_{10}, the expressions of the doubly and singly polarized leptons are the same as given in [19–21], in which the explicit expressions are as follows:

$$P_{LL}(\hat{s}) = \frac{1}{\Delta(\hat{s})} \left\{ \frac{1}{2} f_B^2 m_B^4 \{(1-\hat{s})^2(\mathcal{I}_1+\mathcal{I}_4) - [2\hat{s}+(1+\hat{s}^2)v^2]\mathcal{I}_3 + [2\hat{s}-(1+\hat{s}^2)v^2]\mathcal{I}_6\}|F|^2 - \frac{1}{2\hat{m}_\ell} f_B m_B \hat{s}[8(1+\hat{s})v^2 \right.$$
$$+ m_B^2(1-\hat{s})(2-2\hat{s}-2v^2+2\hat{s}v^2+v^4+\hat{s}v^4)\mathcal{I}_8 - m_B^2(1-\hat{s}^2)v^2\mathcal{I}_9]\mathrm{Re}[(A_1^*+B_1^*)F]$$
$$\left. -\frac{1}{3\hat{m}_\ell^2} m_B^2 \hat{s}^2 (1-\hat{s})^2(1-v^2)^2 \mathrm{Re}[A_1^*B_1 + A_2^*B_2] - \frac{2}{3} m_B^2 \hat{s}(1-\hat{s})^2(1+3v^2)(|A_1|^2+|A_2|^2+|B_1|^2+|B_2|^2) \right\}, \tag{16}$$

$$P_{LN}(\hat{s}) = \frac{1}{\Delta(\hat{s})} \{ f_B m_B^3 \sqrt{\hat{s}}(1-\hat{s}^2)v^2 \mathrm{Im}[A_1^*F - B_1^*F]\mathcal{I}_7 - 4\pi f_B m_B \sqrt{\hat{s}}(1-\hat{s})(1-\sqrt{1-v^2})\mathrm{Im}[(A_2^*+B_2^*)F]\}, \tag{17}$$

$$P_{NL}(\hat{s}) = \frac{1}{\Delta(\hat{s})} \{ f_B m_B^3 \sqrt{\hat{s}}(1-\hat{s}^2)v^2 \mathrm{Im}[-A_1^*F + B_1^*F]\mathcal{I}_7 + 4\pi f_B m_B \sqrt{\hat{s}}(1-\hat{s})(1-\sqrt{1-v^2})\ \mathrm{Im}[-(A_2^*+B_2^*)F]\}, \tag{18}$$

$$P_{LT}(\hat{s}) = \frac{1}{\Delta(\hat{s})} \left\{ -\frac{1}{\sqrt{\hat{s}}} f_B^2 m_B^4 \hat{m}_\ell (1-\hat{s}) v[(1+\hat{s})|F|^2](\mathcal{I}_2+\mathcal{I}_4) + \frac{4}{v}\pi f_B m_B \sqrt{\hat{s}}(1-\hat{s})(1-\sqrt{1-v^2}) \right.$$
$$\left. \times \mathrm{Re}[(A_2^*-B_2^*)F] + 2m_B\hat{m}_\ell \mathrm{Re}[A_1^*A_2 - B_1^*B_2] - \frac{4}{v}\pi f_B m_B \sqrt{\hat{s}}(1+\hat{s})(1-\sqrt{1-v^2})\mathrm{Re}[(A_1^*+B_1^*)F] \right\}, \tag{19}$$

$$P_{TL}(\hat{s}) = \frac{1}{\Delta(\hat{s})} \left\{ -\frac{1}{\sqrt{\hat{s}}} f_B^2 m_B^4 \hat{m}_\ell (1-\hat{s}) v[(1+\hat{s})|F|^2](\mathcal{I}_2+\mathcal{I}_4) - \frac{4}{v}\pi f_B m_B \sqrt{\hat{s}}(1-\hat{s})(1-\sqrt{1-v^2}) \right.$$
$$\left. \times \mathrm{Re}[(A_2^*-B_2^*)F] - 2m_B\hat{m}_\ell \mathrm{Re}[A_1^*A_2 - B_1^*B_2] - \frac{4}{v}\pi f_B m_B \sqrt{\hat{s}}(1+\hat{s})(1-\sqrt{1-v^2})\mathrm{Re}[(A_1^*+B_1^*)F] \right\}, \tag{20}$$

$$P_{NT}(\hat{s}) = \frac{1}{\Delta(\hat{s})}\bigg\{2f_B m_B^3 \hat{m}_\ell (1-\hat{s})^2 v \mathrm{Im}[-A_1^* F + B_1^* F](\mathcal{I}_8 - \mathcal{I}_9) - 2f_B m_B^3 \hat{m}_\ell (1-\hat{s}^2) v \mathrm{Im}[(A_2^* + B_2^*)F](\mathcal{I}_8 - \mathcal{I}_9)$$
$$- \frac{8}{3} m_B (1-\hat{s})^2 v \mathrm{Im}[-m_B \hat{s}(A_1^* B_1 + A_2^* B_2)]\bigg\}, \tag{21}$$

$$P_{TN}(\hat{s}) = \frac{1}{\Delta(\hat{s})}\bigg\{2f_B m_B^3 \hat{m}_\ell (1-\hat{s})^2 v \mathrm{Im}[A_1^* F - B_1^* F](\mathcal{I}_8 - \mathcal{I}_9) - 2f_B m_B^3 \hat{m}_\ell (1-\hat{s}^2) v \mathrm{Im}[(A_2^* + B_2^*)F](\mathcal{I}_8 - \mathcal{I}_9)$$
$$+ \frac{8}{3} m_B (1-\hat{s})^2 v \mathrm{Im}[-m_B \hat{s}(A_1^* B_1 + A_2^* B_2)]\bigg\}, \tag{22}$$

$$P_{NN}(\hat{s}) = \frac{1}{\Delta(\hat{s})}\bigg\{f_B^2 m_B^4 \hat{s}[(1+v^2)\mathcal{I}_3 - (1-v^2)\mathcal{I}_6]|F|^2 + \frac{4}{3} m_B^2 \hat{s}(1-\hat{s})^2 v^2 (2\mathrm{Re}[A_1^* B_1 + A_2^* B_2])\bigg\}, \tag{23}$$

$$P_{TT}(\hat{s}) = \frac{1}{\Delta(\hat{s})}\bigg\{\frac{1}{2} f_B^2 m_B^4 \{-(1-\hat{s})^2(1-v^2)\mathcal{I}_1 + [1-v^2-4\hat{s}+\hat{s}^2(1-v^2)]\mathcal{I}_3 - (1-v^2)(1-\hat{s})^2\mathcal{I}_4 + (1-v^2)(1-\hat{s}^2)\mathcal{I}_6\}|F|^2$$
$$-4f_B m_B^3 \hat{m}_\ell (1-\hat{s})^2 \mathrm{Re}[(A_1^* + B_1^*)F](\mathcal{I}_8 - \mathcal{I}_9) + m_B \hat{m}_\ell (|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_1|^2)]$$
$$+ \frac{8}{3} m_B^2 (1-\hat{s})^2 (\hat{s}\mathrm{Re}[A_1^* B_1 + A_2^* B_2])\bigg\}, \tag{24}$$

where

$$\Delta(\hat{s}) = 16 m_B \hat{m}_\ell (1-\hat{s})^2 (\mathrm{Re}[m_B \hat{m}_\ell (A_1^* B_1 + A_2^* B_1)]) + \frac{2}{3}(1-\hat{s})^2 [m_B^2 \hat{s}(3+v^2)(|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2)]$$
$$- \frac{1}{2} f_B^2 m_B^4 |F|^2 \{(1-\hat{s})^2 v^2 (\mathcal{I}_1 + \mathcal{I}_4) - (1+\hat{s}^2 + 2\hat{s}v^2)\mathcal{I}_3 - [1-\hat{s}(4-\hat{s}-2v^2)]\mathcal{I}_6\}$$
$$+ 2f_B m_B \hat{m}_\ell \mathrm{Re}[(A_1^* + B_1^*)F][8(1+\hat{s}) + m_B^2(1-\hat{s}^2)v^2 \mathcal{I}_8 + m_B^2(1-\hat{s})(1-3\hat{s})\mathcal{I}_9].$$

In the expressions given above, $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$ is the lepton velocity, and

$$A_1 = A_1(\hat{s}) = \frac{-2C_7^{\mathrm{eff}}(\hat{s})}{q^2}(m_b + m_s)g_1(q^2) + (C_9^{\mathrm{eff}}(\hat{s}) - C_{10}(\hat{s}))g(q^2),$$
$$A_2 = A_2(\hat{s}) = \frac{-2C_7^{\mathrm{eff}}(\hat{s})}{q^2}(m_b - m_s)f_1(q^2) + (C_9^{\mathrm{eff}}(\hat{s}) - C_{10}(\hat{s}))f(q^2),$$
$$B_1 = B_1(\hat{s}) = \frac{-2C_7^{\mathrm{eff}}(\hat{s})}{q^2}(m_b + m_s)g_1(q^2) + (C_9^{\mathrm{eff}}(\hat{s}) + C_{10}(\hat{s}))g(q^2),$$
$$B_2 = B_2(\hat{s}) = \frac{-2C_7^{\mathrm{eff}}(\hat{s})}{q^2}(m_b - m_s)f_1(q^2) + (C_9^{\mathrm{eff}}(\hat{s}) + C_{10}(\hat{s}))f(q^2),$$
$$F = F(\hat{s}) = 4m_\ell C_{10}(\hat{s}),$$

where $\mathcal{I}_i$ is determined as

$$\mathcal{I}_i = \int_{-1}^{+1} \mathcal{F}_i(z)dz,$$

where

$$\mathcal{F}_1 = \frac{z^2}{(p_1 \cdot k)(p_2 \cdot k)}, \qquad \mathcal{F}_2 = \frac{z}{(p_1 \cdot k)(p_2 \cdot k)}, \qquad \mathcal{F}_3 = \frac{1}{(p_1 \cdot k)(p_2 \cdot k)}, \qquad \mathcal{F}_4 = \frac{z^2}{(p_1 \cdot k)^2},$$
$$\mathcal{F}_5 = \frac{z}{(p_1 \cdot k)^2}, \qquad \mathcal{F}_6 = \frac{1}{(p_1 \cdot k)^2}, \qquad \mathcal{F}_7 = \frac{z}{(p_2 \cdot k)^2}, \qquad \mathcal{F}_8 = \frac{z^2}{p_1 \cdot k}, \qquad \mathcal{F}_9 = \frac{1}{p_1 \cdot k}.$$

Finally, we present the expressions of the polarized forward-backward asymmetry, which are very sensitive to the new physics effects. The explicit expressions for $\mathcal{A}_{FB}$ are given as

$$\mathcal{A}_{FB}^{LL} = \frac{1}{\Delta}\left\{-4m_B^2\hat{s}(1-\hat{s})^2 v \mathrm{Re}[A_1^* A_2 - B_1^* B_2] + \frac{4}{\hat{m}_\ell v} f_B m_B \hat{s}(1-\hat{s})(1-v^2)\ln \mathrm{Re}[(A_2^* - B_2^*)F]\right\},$$
$$\mathcal{A}_{FB}^{LN} = \frac{1}{\Delta}\left\{-\frac{2}{3\hat{m}_\ell} m_B^2 \sqrt{\hat{s}^3}(1-\hat{s})^2 v(1-v^2)(\mathrm{Im}[A_1^* B_1 + A_2^* B_2]) - f_B m_B^2 \sqrt{\hat{s}}(1-\hat{s})(+m_B \mathrm{Im}[(A_1^* - A_2^* - B_1^* - B_2^*)F\right.$$
$$\left. - \hat{s}(A_1^* + A_2^* - B_1^* + B_2^*)F])I_7\right\},$$
$$\mathcal{A}_{FB}^{NL} = \frac{1}{\Delta}\left\{+\frac{2}{3\hat{m}_\ell} m_B^2 \sqrt{\hat{s}^3}(1-\hat{s})^2 v(1-v^2)(-\mathrm{Im}[A_1^* B_1 + A_2^* B_2]) - m_B \mathrm{Im}[(A_1^* + A_2^* - B_1^* + B_2^*)F\right.$$
$$\left. - \hat{s}(A_1^* - A_2^* - B_1^* - B_2^*)F]I_7\right\},$$
$$\mathcal{A}_{FB}^{LT} = \frac{1}{\Delta}\left\{\frac{4}{3\sqrt{\hat{s}}}\hat{m}_\ell(1-\hat{s})^2[m_B^2\hat{s}(|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2)] + \frac{8}{3}m_B^2 \hat{m}_\ell \sqrt{\hat{s}}(1-\hat{s})^2(\mathrm{Re}[A_1^* B_1 + A_2^* B_2])\right.$$
$$+ \frac{1}{\sqrt{\hat{s}}} f_B^2 m_B^4 \hat{m}_\ell (1-\hat{s})[(1-\hat{s})(|F|^2)(\mathcal{J}_1 + \mathcal{J}_2)] - f_B m_B^3 \sqrt{\hat{s}}(1-\hat{s}^2)v^2 \mathrm{Re}[(A_2^* - B_2^*)F^*]\mathcal{J}_4$$
$$\left. + f_B m_B^3 \sqrt{\hat{s}}(1-\hat{s})^2(2-v^2)\mathrm{Re}[(A_1^* + B_1^*)F]\mathcal{J}_4\right\}. \tag{25}$$

$\mathcal{J}_i$ represent the following integrals

$$\mathcal{J}_i = \int_0^{+1} \mathcal{G}_i(z)dz - \int_{-1}^{0} \mathcal{G}_i(z)dz,$$

where

$$\mathcal{G}_1 = \frac{z\sqrt{1-z^2}}{(p_1 \cdot k)(p_2 \cdot k)}, \qquad \mathcal{G}_2 = \frac{z\sqrt{1-z^2}}{(p_1 \cdot k)^2},$$
$$\mathcal{G}_3 = \frac{\sqrt{1-z^2}}{(p_1 \cdot k)^2}, \qquad \mathcal{G}_4 = \frac{z\sqrt{1-z^2}}{(p_1 \cdot k)}. \tag{26}$$

III. NUMERICAL ANALYSIS

For performing the numerical analysis, we use the following input parameters entering into the expressions of the branching ratio, double lepton polarization, and forward-backward asymmetries: $m_t = (173.5 \pm 0.6)$ GeV, $m_b = (4.8 \pm 0.1)$ GeV, $m_c = (1.46 \pm 0.05)$ GeV, $m_W = (80.385 \pm 0.015)$ GeV, $m_\mu = (105.658) \times 10^{-3}$ GeV, $|V_{tb}V_{ts}^*| = 0.041$, and $G_F = 1.17 \times 10^{-5}$ GeV^2.

Among the remaining input parameters of the family nonuniversal Z' model, using the latest improvement measurements on B meson decays in [22] for the Z-coupling parameters, $|B_{sb}^L|$, $B_{\ell\ell}^L$, and $B_{\ell\ell}^R$ are obtained: $|B_{sb}^L| \leq 0.96 \times 10^{-3}$ (Scenario 1), $|B_{sb}^L| \leq 0.42 \times 10^{-3}$ (Scenario 2) with $\phi_s^L = -92 \pm 30^\circ$. Here, Scenarios 1 and 2 correspond to cases that $B_{sb}^R = 0$, $B_{sb}^L = B_{sb}^R$, respectively. Using the bound on the mass of the Z' boson with what follows from the analysis of the $B \to \mu^+\mu^-$ decay (see, for example, [23] and LHC data [1]), for the $B_{\mu\mu}^L$ and $B_{\mu\mu}^R$ parameters, we get what is presented in Table I.

In Fig. 1, we present the dependence of the differential decay rate on q^2 for $B_s \to \ell^+\ell^-\gamma$ for the e, μ, and τ channels. For completeness, the result for the SM is also shown in the same figure. From this figure, we see that the

TABLE I. The values of the Z'-model parameters for two different scenarios. For mass of Z' boson, we put $m_{Z'} = 3$ TeV.

	$\lvert B_{sb}^L\rvert \times 10^{-3}$	$\varphi_S^{L(0)}$	$B_{\mu\mu}^L \times 10^{-2}$	$B_{\mu\mu}^R \times 10^{-2}$
$S1$	0.96	−72	−1.4	0.6
$S2$	0.42	−92 ± 30	−0.6	0.2

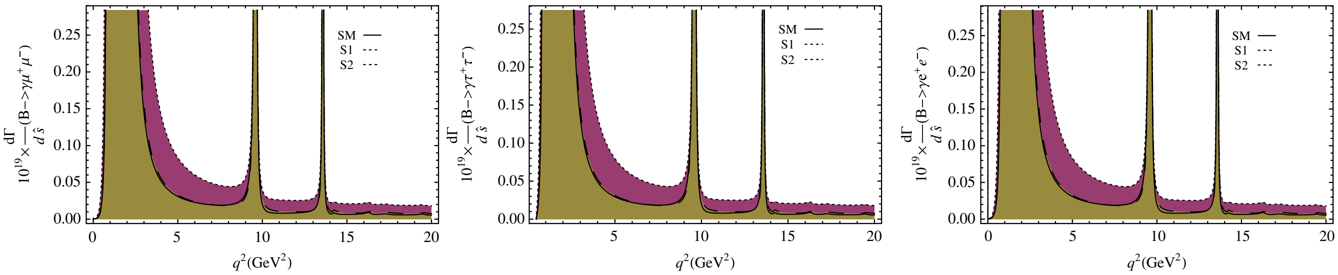

FIG. 1 (color online). The dependence of the differential decay rate of the $B \to \gamma l^+ l^-$ on q^2 for μ, τ, e leptons.

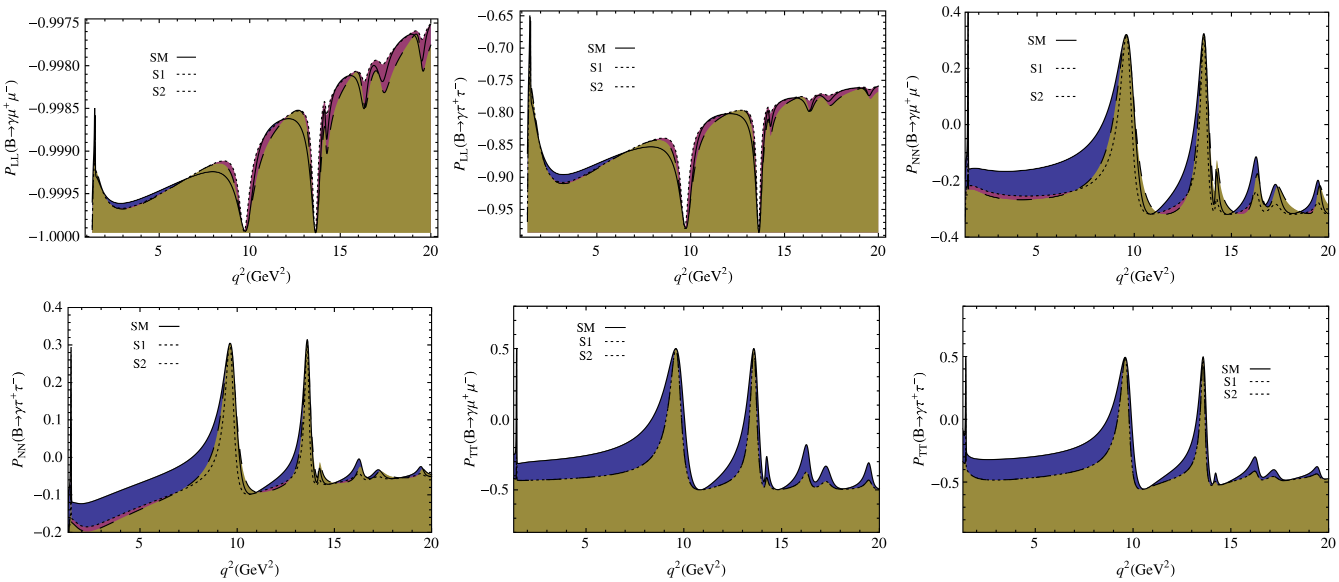

FIG. 2 (color online). The dependence of the P_{LL}, P_{NN}, and P_{TT} polarizations of the $B \to \gamma l^+ l^-$ decay on q^2 for μ and τ leptons.

differential branching ratios seem to be quite sensitive to the existence of Z', but only at the "low q^2" region. Therefore, careful analysis of differential decay rate in the low q^2 region can be useful for establishing the existence of the Z' boson.

In Fig. 2, we depict the dependence of the double lepton polarizations P_{LL}, P_{NN}, and P_{TT} on q^2 for the μ and τ channels. It follows from these figures that at low and high q^2 regions Z' gives considerable contributions to the P_{NN} and P_{TT} polarizations. In this region, a careful study of these polarizations can be useful in conforming the existence of Z'.

The dependence of the polarized forward-backward asymmetry $\mathcal{A}_{LL}$ on q^2 for the μ and τ channels is presented in Fig. 3. From these figures, we observe that at the low q^2 region, the values of $\mathcal{A}_{LL}$ are different for both channels in

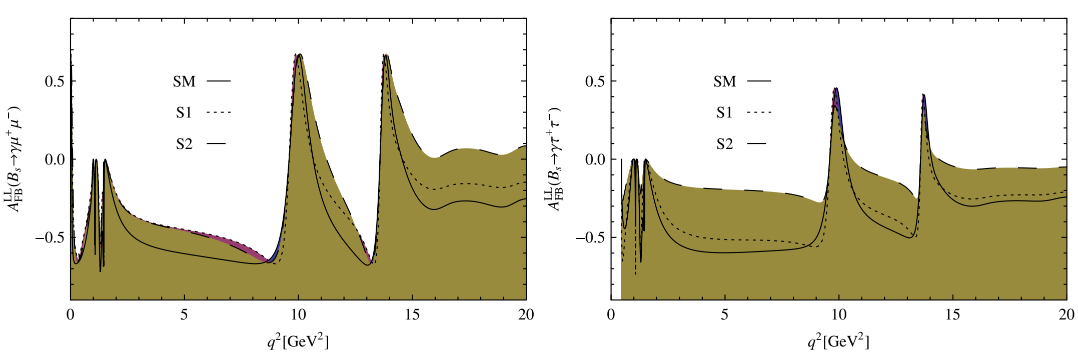

FIG. 3 (color online). The dependence of the $\mathcal{A}_{FB}^{LL}$ polarizations of the $B \to \gamma l^+ l^-$ on q^2 for μ and τ leptons.

the SM and Z' models, especially for the $S2$ scenario. This observation can serve as a useful tool in the search of the Z' boson.

IV. CONCLUSION

In the present work, we studied the $B_s \to \ell^+\ell^-\gamma$ decay in the family of nonuniversal Z' model. We investigated the possible contributions of the Z' boson to the branching ratio, double lepton polarizations, as well as polarized forward-backward asymmetries. We observe that studying these polarization effects in the low energy region $4m_\ell^2 \leq q^2 \leq 8.0$ GeV^2 can give useful information in discriminating the contributions of the Z' boson. The aforementioned measurable quantities are quite sensitive to the Z' contributions, and discrepancies between the prediction of the SM and the family of nonuniversal Z' models can be an indication for the existence of the Z' boson. We hope that the measurement of this channel can possibly be realized in the near future at LHCb since its branching ratio is of the same order as that of the $B_s \to \ell^+\ell^-$, which has already been observed at LHCb. Checking then the predictions of the family of nonuniversal Z' model might be helpful in confirming the existence of the Z' boson.

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