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Charged lepton spectrum approximation in a three body nucleon decay

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Only phase space is typically used to obtain final-state particle spectra in rare decay searches, which is a crude approximation in the case of three-body processes. We will demonstrate how both dynamics and phase space can be approximately accounted for in processes originating from grand unification models—such as nucleon decays $p \to e^+\bar{\nu}\nu$ or $p \to \mu^+\bar{\nu}\nu$—using the general effective Fermi theory formalism of electroweak muon decay, $\mu \to e^+\bar{\nu}\nu$. This approach allows for a more precise and only weakly model-dependent approximation of final particle spectra for these and similar decays, which may improve rare process searches in current and near-future experiments.

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Rare processes—such as nucleon decays that violate baryon-number conservation and that may arise in a grand unification theory (GUT) [1,2]—are essential for probing the fundamental aspects of nature and physics beyond the Standard Model (SM). Typically [3–5], experimental searches for them involve Monte Carlo (MC) simulation of the final-state particles that only utilizes phase space (4-momentum conservation) to constrain the energy spectra of the consituents. For two-body processes, such as the dominant SU(5) proton decay mode of $p \to e^+\pi^0$ [6], such an approach uniquely determines the kinematics of the decay. However, in the case of three-body decays—such as $p \to e^+\bar{\nu}\nu$ or $p \to e^+e^-e^+$ which may arise in a Pati-Salam partial unification scenario [2]—energy and momentum conservation are insufficient to uniquely constrain final-state particle spectra. The reason for this is that additional input from the interaction dynamics (matrix element), which is highly model dependent, is required. Thus, even though utilizing only phase space to represent the final decay state is a model-independent approach for rare process searches, it is a crude approximate technique if more than two resulting particles are present in the decay.

In this analysis, we will demonstrate that both dynamics and phase space may be approximately accounted for when calculating the spectrum of a charged lepton in such three-body processes as those mentioned above. Our approach utilizes the general effective Fermi theory formalism of electroweak muon decay, $\mu \to e^+\bar{\nu}\nu$. The results are predominantly model independent, assuming the absence of tensor interactions and vector interactions involving left-right mixing, which is consistent with typical GUT models [1,2].

From reviewing two- and three-body decay kinematics (see Appendix A), formulations of the respective partial decay widths outline the issue. As noted, in the parent particle rest frame, the resulting momenta in the two-body decay case are uniquely determined to be half that of original parent particle, once the 4-momentum conservation is imposed. On the other hand, in the three-body decay scenario, the energy and momenta are not uniquely distributed among the three constituents as determined by the 4-momentum conservation. Thus, the three-body partial decay width may be affected by the dependency of the matrix element. The matrix element contains information about the decay dynamics and is specific to the given model. Though using only phase space (4-momentum conservation) when determining the three-body momenta of final particles is a model-independent approximation, it may potentially be very crude. This may thus be of potential concern for experimental searches for rare processes.

Proton decay $p \to e^+\bar{\nu}\nu$ that may arise in GUTs shares a common set of final-state particles with the SM electroweak muon decay $\mu \to e^+\bar{\nu}\nu$. Noticing this fact, we will attempt to identify conditions which will allow for the well-known formalism of the latter [7] to be exploited for a reasonable approximation to the momentum spectrum of the charged lepton e^+ in the former. Since the muon decay formalism implements both dynamics and phase space, this will improve on the phase-space-only approximation typically used in simulations. Additionally, the spectrum will be known *a priori* of the searches from the formalism.

As noted, the matrix element encoding decay dynamics plays a role in determining the energy spectra of three-body decays. In the effective Fermi theory of muon decay, a specific feature of the dynamics is the vector minus axial-vector current $(\mathbf{V}-\mathbf{A})$-type interaction, which is a distinctive characteristic of the SM electroweak processes (see Appendix B). On the other hand, the formulation of muon decay can be generalized to include other types of interactions.

To explore the validity of the muon decay as an approximation to other processes, we begin by reviewing the most general formulation for the four-fermion decay amplitude with unspecified possible interaction couplings

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(see Appendix B). Assuming the neutrino mass to be negligible and the detector to be electron-spin insensitive, the full decay spectrum including radiative corrections is given by [8]

$$\frac{d\Gamma}{dx d\cos\theta} = \frac{D}{32}\frac{G_F^2 m_\mu^5}{192\pi^3}\cdot x^2 \left\{ \frac{1+h(x)}{1+4(m_e/m_\mu)\eta} \cdot \left[12(1-x) + \frac{4}{3}\rho(8x-6) + 24\frac{m_e}{m_\mu}\frac{(1-x)}{x}\eta \right] \pm P_\mu \cdot \xi \cdot \cos\theta \left[4(1-x) + \frac{4}{3}\delta(8x-6) + \frac{\alpha}{2\pi}\frac{g(x)}{x^2} \right] \right\}, \tag{1}$$

where G_F, m_e, m_μ, E_e, and P_μ are the Fermi constant, electron mass, muon mass, electron energy, and muon polarization, respectively. $\cos\theta$ is the angle between the electron momentum and muon spin, with $x = 2E_e/m_\mu$. The functions $g(x)$ and $h(x)$ incorporate radiative corrections [9], which in the case of muon decay have a noticeable effect on the spectrum. The parameters D, ρ, η, ξ, and δ are the Michel parameters [10,11]. At this point all the possible vector and axial-vector (V), scalar and pseudoscalar (S), and tensor (T) couplings, $g_{\epsilon\mu}^{\gamma=V,S,T}$, are allowed. The information about the couplings is encoded inside the Michel parameters, which are functions of the possible couplings. In the case of SM, only g_{LL}^V is nonzero, corresponding to a $(\mathbf{V}-\mathbf{A})$-type current, with the full set of parameters determined to be $\rho = \xi\delta = 3/4, \xi = 1$, and $\eta = 0$ [7].

To utilize the spectrum of Eq. (1) as an approximation to the three-body nucleon decay, we will substitute the mass of the proton m_p for the decaying parent particle instead of the original muon mass m_μ.

The spectrum of Eq. (1) can be separated unambiguously into isotropic and anisotropic components, with the former constituting the second line of the equation and the latter being the third. To approximate the nucleon decay spectrum—which is to be observed in the detector—only the isotropic component is of interest. Neglecting the overall normalization and assuming that the mass of the final-state charged lepton m_e is small with respect to that of the initial particle m_p, the approximate isotropic spectrum for the nucleon decay can be stated as

$$\frac{d\Gamma_{\rm nuc}}{d\bar{x}} \sim \bar{x}^2 \left\{ (1+h(\bar{x})) \cdot \left[12(1-\bar{x}) + \frac{4}{3}\rho(8\bar{x}-6) \right] \right\}, \tag{2}$$

where we have substituted the proton mass into $\bar{x} = 2E_e/m_p$. Therefore, as seen from the above, all the information about possible S, V, T couplings is encoded into a single parameter, ρ. It should be noted that the radiative correction function $h(\bar{x})$ has a similar distribution irrespective of the coupling considered [12] and Eq. (2) is thus considerably general. The term proportional to η, which governs the behavior in the low-energy region where $E_e \sim m_e \sim \frac{1}{2}$ MeV, is neglected. Given that our scenario considers the energy spectrum from 0 to $\frac{1}{2}m_p \sim 469$ MeV with a mean around $\frac{1}{3}m_p \sim 315$ MeV, the low-energy parameter, η, plays no significant role. It is thus justifiable to choose the SM value, $\eta = 0$, in our analysis.

Assuming the SM values of the Michel parameters, the only value relevant for our isotropic spectrum is $\rho = 3/4$. The value of $\rho = 3/4$ by itself is insensitive to the $((\mathbf{V}-\mathbf{A}))$ nature of the SM electroweak sector. In fact, following Ref. [13] which considered the similar decay, $\tau \to \mu\nu\bar{\nu}$ (with suppressed flavor indices), one can see that the value $\rho = 3/4$ can arise from interactions that have a structure that is different from $((\mathbf{V}-\mathbf{A}))$. The value of ρ is determined, in the presence of all possible types of the couplings, by

$$\rho = \frac{3}{4} - \frac{3}{4}[|g_{LR}^V|^2 + |g_{RL}^V|^2 + 2|g_{LR}^T|^2 + 2|g_{RL}^T|^2 + \Re(g_{LR}^S g_{LR}^{T\,*} + g_{LR}^S g_{LR}^{T\,*})]. \tag{3}$$

The condition for $\rho = 3/4$ is found by setting the bracket term in Eq. (3) to zero,

$$|g_{LR}^V|^2 + |g_{RL}^V|^2 + 2|g_{LR}^T|^2 + 2|g_{RL}^T|^2 = -\Re(g_{LR}^S g_{LR}^{T\,*} + g_{LR}^S g_{LR}^{T\,*}). \tag{4}$$

In the absence of tensor couplings, $g_{LR}^T = g_{RL}^T = 0$, the condition $g_{LR}^V = g_{RL}^V = 0$ follows, for arbitrary values of the remaining six couplings, g_{LL}^S, g_{LR}^S, g_{RL}^S, g_{RR}^S, g_{LL}^V, g_{RR}^V. This allows for both $(\mathbf{V}-\mathbf{A})$-type (i.e., $g_{LL}^V \neq 0$) interactions as well as $(\mathbf{V}+\mathbf{A})$-type (i.e., $g_{RR}^V \neq 0$) interactions, along with arbitrary scalar couplings. From the above formalism, the SM muon decay corresponds to $g_{LL}^V = 1$ with all other couplings being zero.

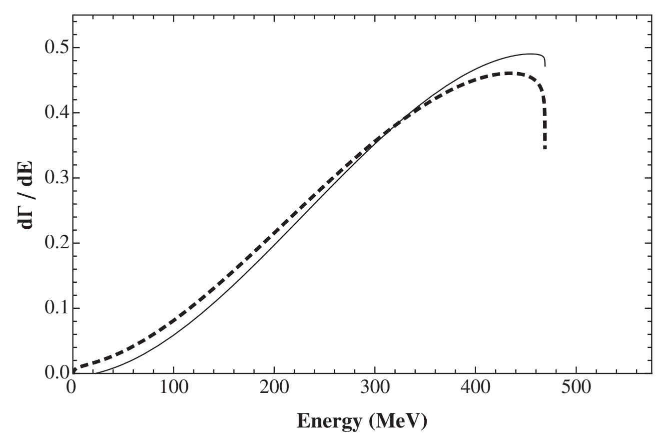

FIG. 1. Decay spectra of charge leptons e^+ (dotted line) and μ^+ (continuous line) in respective $p \to e^+\nu\bar{\nu}$ and $p \to \mu^+\nu\bar{\nu}$ decays.

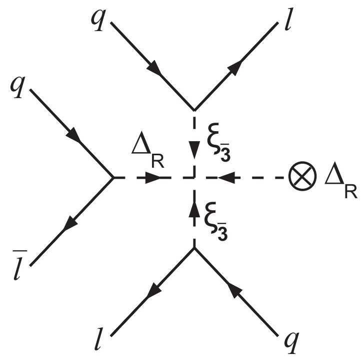

FIG. 2. Trilepton nucleon decay $p \to 2l + \bar{l}$ originating from a Pati-Salam GUT model.

Assuming the absence of the tensor interactions and vector couplings that involve left-right mixing, we can then take the value $\rho = 3/4$. Taking into account the radiative corrections as well as the charged lepton and initial particle masses of $m_e = 0.511$ MeV and $m_p = 938.2$ MeV, the isotropic spectrum up to overall normalization as a function of energy is shown in Fig. 1, for the approximate e^+ spectrum in $p \to e^+ \nu\bar{\nu}$ decay and the approximate μ^+ spectrum in $p \to \mu^+ \nu\bar{\nu}$ decay. The μ^+ spectrum is also reasonably approximated since the condition that the final-state charged lepton mass m_μ is significantly smaller than the original parent particle mass m_p still holds, given that the mass of the muon is $m_\mu = 105.7$ MeV.

The allowed general coupling combination using the validity of assuming the SM value $\rho = 3/4$ as stated above is consistent with the usual nucleon decay and similar processes predicted by popular models of grand unification, such as SU(5) [1] and Pati-Salam theories [2]. As an example, the three-body decay, $p \to e^+(\mu^+)\nu\bar{\nu}$, can arise through a typical mediation by the scalar fields in the extended Higgs sector in GUT models based on the Pati-Salam partial unification [14], as shown in Fig. 2. This process is mediated by the Higgs fields, transforming as $\xi = (2, 2, 15)$ and $\Delta_R = (1, 3, 10)$ under the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)^c$ left-right symmetric Pati-Salam gauge group. Here, $\xi_{\bar{3}}$ is the $\mathrm{SU}(3)^c$ triplet component of the ξ multiplet.

Thus, we have shown that starting from a general formalism for muon decay, we can obtain approximate isotropic spectra for the three-body nucleon decays $p \to e^+\nu\bar{\nu}$ and $p \to \mu^+\nu\bar{\nu}$. The validity of the approach requires the absence of the tensor-type interactions and vector-type interactions involving left-right mixing. Our approach provides a more rigorous spectrum approximation incorporating both dynamics and phase space, rather than just the typical phase-space factor as in the current nucleon decay experimental searches. Additionally, our analysis is only weakly model dependent, allowing for both types of standard nucleon decay mediation by either vector- or scalar-type currents. Further—because arbitrary combinations of such couplings are allowed as well as the fact that the current best nucleon decay experiments are insensitive to the neutrino flavor and type (such as the Super Kamiokande large water Cherenkov detector [6])—variations other than $\nu\bar{\nu}$ in the final state will lead to a similar charged lepton spectrum. To a lesser degree, the method depicted here may also serve to approximate the spectra in other decays, such as $p \to e^+e^-e^+$ and $p \to \mu^+e^-e^+$, as well as other three-body processes where the final-state particles have small masses in relation to the original parent particle.

To conclude, the method provided allows one to obtain an approximate energy spectrum for three-body nucleon decay in current and future experiments in a relatively model-independent manner using the SM electroweak formalism for muon decay. This method is more rigorous than the simple phase-space approximation typically used, leading to improved and better understood searches.

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APPENDIX A: TWO- AND THREE-BODY DECAYS

The partial decay rate in the rest frame of a particle of mass M into n constituents with a Lorentz-invariant matrix element $\mathcal{M}$ (as can be found in Ref. [7]) is

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathcal{M}|^2 d\Phi_n, \tag{A1}$$

where $d\Phi_n$ is the is the n-body phase space,

$$d\Phi_n = \delta^4\left(P - \sum_{i=1}^{n} p_i\right) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}, \tag{A2}$$

with P and p_i representing the momenta of the original and final-state particles, respectively, and E_i is their energy.

In the rest frame of the parent particle with mass M, for a two-body decay each final-state constituent will contain momentum equal to half of the original proton mass, uniquely determining the kinematics. The two-body partial decay width can be stated as

$$d\Gamma_2 = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\mathbf{p}_1|}{\mathrm{M}^2} d\Omega, \tag{A3}$$

where $\mathbf{p}_1 = \mathbf{p}_2$ are the resulting momenta of particles 1 and 2, respectively, and $d\Omega$ is the solid angle of particle 1.

In the case of three-body decay, the partial decay width is specified by

$$d\Gamma_3 = \frac{1}{(2\pi)^5}\frac{1}{16\mathrm{M}}|\mathcal{M}|^2 dE_1 dE_2 d\alpha d(\cos\beta)d\gamma, \quad \text{(A4)}$$

where dE_1, dE_2 label the energies of the resulting particles 1 and 2, respectively (with particle 3 being implicitly taken into account), and (α, β, γ) specifies the Euler angle orientation of the momenta relative to the parent particle.

APPENDIX B: MATRIX ELEMENT

The most general matrix element for a four-fermion decay with the couplings left unspecified is provided by [15]

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma = S,V,T \\ \epsilon,\mu = R,L}} g^{\gamma}_{\epsilon\mu} \langle \bar{e}_\epsilon | \Gamma^\gamma | (\nu_e)_n \rangle \langle (\bar{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle, \quad \text{(B1)}$$

where $\gamma = \mathbf{S}, \mathbf{V}, \mathbf{T}$ denote possible scalar ($\mathbf{S}$), vector ($\mathbf{V}$), and tensor ($\mathbf{T}$) interactions, and $\epsilon, \mu = R, L$ are the left- and right-handed chiralities of the electron or muon. Finally, n, m label the chiralities of neutrinos.

In the case of the Standard Model, the above simplifies to

$$\mathcal{M}_{\mathrm{muon}} = -i\frac{G_F}{\sqrt{2}}\bar{u}_3\gamma_\mu(1-\gamma^5)u_1\bar{u}_2\gamma^\mu(1-\gamma^5)v_4, \quad \text{(B2)}$$

where G_F is the Fermi constant, and $u_1, \bar{u}_2, \bar{u}_3, v_4$ stand for the usual spinor notation representing $\mu, e^+, \bar{\nu}, \nu$. The featured (**V** - **A**) current is explicitly seen.

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