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CP violation and CPT invariance in $B^{\pm}$ decays with final state interactions

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We show that, besides the usual short distance contribution for CP violation, final state interactions together with CPT invariance can play an important role in the recent observation of CP violation in three-body charmless $B^{\pm}$ decays. A significant part of the observed CP asymmetry distribution in the Dalitz plot is located in a region where hadronic channels are strongly coupled. We illustrate our discussion comparing the recent observation of CP violation in the $B^{\pm} \to K^{\pm}K^{+}K^{-}$ and $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ phase space, with a calculation based on $\pi\pi \to KK$ scattering.

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I. INTRODUCTION

For CP violation to occur, two interfering amplitudes with different weak phases are necessary. Until now, all observed CP violation is compatible with the Cabibbo-Kobayashi-Maskawa weak phase, however there are many modes with interfering amplitudes that produce this asymmetry. For neutral mesons, direct and indirect CP asymmetries were observed, the latter associated to $M^0 - \bar{M}^0$ oscillation, where $M^0 = K^0$ and B^0. On the other hand, for charged mesons, direct CP violation was observed only in bottom meson decays [1–6].

The most common mechanism, at the quark level, expected to give a CP asymmetry in charmless charged B decays, comes from the short distance Bander-Silverman-Soni (BSS) model [7], through the interference of the tree and penguin amplitudes. However, at the hadronic level, there are other interfering contributions with different weak phases. One of them associated with the interference between intermediate states, in three-or-more-body decays [8–12]. In general, interference occurs when two resonant intermediate states, with different weak phases, share the same kinematical region and hadronic final state. Another possibility is related to hadronic rescattering in two different states [13,14].

Wolfenstein [13], based on CPT invariance and unitarity, proposed a formalism for decay, in which the hadronic final-state interaction (FSI) and CPT constraint are considered together. From that, the sum of the partial widths for channels coupled by the strong Hamiltonian, must be equal to the corresponding sum of the partial decay widths of the associated antiparticle. It is more restrictive than the CPT condition, which equates the lifetime for a particle and its antiparticle. Then, in addition to the usual CP -violating amplitude from the BSS mechanism, one has the asymmetry induced by rescattering, namely the "compound" contribution [15].

The large number of final states with the same flavor quantum numbers, accessible for a charmless B meson decay, could wash out the "compound" contribution for a single decay channel. However, since hadronic many-body rescattering effects are far from being understood, it is evident that this phenomenological hypothesis deserves to be tested experimentally and further explored theoretically. The aim of this paper is to investigate the possible presence of the "compound" contribution in charmless three-body charged B decays presented recently by the LHCb collaboration [4,5].

II. BASIC FACTS AND OUR ASSUMPTIONS

One of the most intriguing characteristics in three-body charmless B decays, observed by Belle [2], *BABAR* [1], and now by LHCb [5], is that the two-body distributions of events are concentrated at low invariant mass taking into account the huge phase-space available, for example, in $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$. The distributions of events in $K^{\pm}\pi^{\mp}$ and $\pi^{+}\pi^{-}$ invariant masses squared are mostly concentrated below 3 GeV^2 (except charmonium intermediate states). This result confirms the old phenomenological assumption of the isobar model, in which the final state factorizes in a two body interacting system plus a bachelor. In this case, the rescattering associated to hadron-hadron interactions should be below the experimental limit of 3 GeV^2, that is basically in the elastic hadron-hadron regime [10].

The two-body elastic scattering data from different collaborations in the 1970s and 1980s can be well parametrized within S-matrix theory. The opening of new channels is encoded by the inelasticity (η), which represents the amount of two-body elastic flux lost at a given energy. For $\eta = 1$ no inelastic processes happen. In general, the S-matrix element is represented by the unitary Argand diagram, which allows us to identify resonances through phase variation and also the inelasticity. If data are around a circle, $\eta = 1$, they otherwise appear inside the circle and inelastic scattering takes place.

The Argand plot for S-wave $\pi\pi$ elastic scattering from the CERN-Munich collaboration [16] shows η close to one up to $f_0(980)$, after that $\eta < 1$ and then returns to one for masses above 1.4 GeV. The deviation from the unitary circle at 1 GeV is explained by $\pi\pi$ coupling to KK channels. Experimental results from the early 1980s show an important S-wave $\pi\pi \to KK$ scattering between 1 and 1.6 GeV [17], with a corresponding decrease of the S-wave $\pi\pi$ elastic amplitude [18]. The observed inelasticity of the $\pi\pi$ S-wave amplitude is basically associated only to the $\pi\pi \to KK$ process (see also the analysis presented in Ref. [19]). For the P-wave, the CERN-Munich experimental results show $\eta = 1$ until 1.4 GeV. Then η drops to a minimum of 0.5, due to the presence of the $\rho(1690)$, which prefers to decay into four pions. Finally, the D-wave is elastic until 1.2 GeV, after that η slowly decreases. In short, the $\pi\pi \to \pi\pi$ scattering, except for the S-wave in the invariant mass region of $1 \lesssim m_{\pi\pi} \lesssim 1.5$ GeV, the elastic scattering is the dominant contribution.

The other important study is the $K\pi \to K\pi$ scattering from the LASS experiment [20]. The S-wave has inelastic events above 1.5 GeV, and it has both isospin $1/2$ and $3/2$ states. The P-wave is elastic up to 1.41 GeV and inelastic when $K^*(1680)$ is formed, as it can decay to $K\rho$ and $K^*\pi$. Finally, the D-wave is elastic in a small region and is dominated by $K_2(1430)$, which decays to $K\pi$ about half of the time.

The conjunction between: (i) the general hypothesis of dominant $2+1$ processes in charmless three-body B decays, supported by the observed distribution of the Dalitz plot, basically, at very low hadron-hadron masses; and (ii) the observed dominance of the hadron-hadron elastic scattering, in the same region where the majority of the two-body decays are placed in the Dalitz plot, allows us to assume that the rescattering effects in three-body B decays happen essentially in $3 \to 3$ channels. Some small contributions from a D-wave can also be added to the $3 \to 5$ process, but for our general purpose it can be neglected. More sophisticated processes such as the rescattering involving the bachelor particle can be added, but they must be understood as a correction to the main contribution coming from $2+1$ processes [21].

Note that this conclusion can be used only for three-body decays, because we know well the events distribution in the Dalitz plot. The same argument does not fit for two-body charmless B decays. In that case one has to understand what is the contribution to the hadron-hadron elastic scattering in the B mass region, which is not yet available experimentally. Also for four-body decays, we do not have a clear experimental picture for two or three-body mass distributions.

Our working assumption, based on experimental evidences from $\pi\pi$ and KK scattering, is to investigate the effect of two-body rescattering contributions to the CP -violating charged B decays in the strongly coupled $\pi\pi$ and KK channels.

III. *CPT* INVARIANCE IN A DECAY

To define our notation and the framework for implementing the CPT constraint in B meson decays, we follow closely Refs. [22,23]. A hadron state $|h\rangle$ transforms under CPT as $\mathcal{CPT}|h\rangle = \chi\langle\bar{h}|$, where $\bar{h}$ is the charge conjugate state and χ a phase. The weak and strong Hamiltonians conserve CPT , therefore $(\mathcal{CPT})^{-1}H_w\mathcal{CPT} = H_w$ and $(\mathcal{CPT})^{-1}H_s\mathcal{CPT} = H_s$.The weak matrix element for the hadron decay is $\langle\lambda_{\mathrm{out}}|H_w|h\rangle$, where λ_{out} includes the distortion from the strong force due to the final state interaction. The requirement of $\mathcal{CPT}$ invariance is fulfilled for the matrix element when

$$\langle\lambda_{\mathrm{out}}|H_w|h\rangle = \chi_h\chi_\lambda\langle\bar{\lambda}_{\mathrm{in}}|H_w|\bar{h}\rangle^*. \tag{1}$$

Inserting the completeness of the strongly interacting states, eigenstates of H_s, and using hermiticity of H_w, one gets

$$\langle\lambda_{\mathrm{out}}|H_w|h\rangle = \chi_h\chi_\lambda\sum_{\bar{\lambda}'}S_{\bar{\lambda}',\bar{\lambda}}\langle\bar{\lambda}'_{\mathrm{out}}|H_w|\bar{h}\rangle^*, \tag{2}$$

where the S-matrix element is $S_{\bar{\lambda}',\bar{\lambda}} = \langle\bar{\lambda}'_{\mathrm{out}}|\bar{\lambda}_{\mathrm{in}}\rangle$.

The sum of partial decays width of the hadron decay and the correspondent sum for the charge conjugate should be identical, which follows from Eq. (2)

$$\sum_\lambda|\langle\lambda_{\mathrm{out}}|H_w|h\rangle|^2 = \sum_{\bar{\lambda}}\left|\sum\nolimits_{\bar{\lambda}'}S^*_{\bar{\lambda}',\bar{\lambda}}\langle\bar{\lambda}'_{\mathrm{out}}|H_w|\bar{h}\rangle\right|^2 = \sum_{\bar{\lambda}}|\langle\bar{\lambda}_{\mathrm{out}}|H_w|\bar{h}\rangle|^2, \tag{3}$$

and note that besides the CPT constraint we have also used the hermiticity of the weak Hamiltonian.

The CP-violating phase enters linearly at lowest order in the hadron decay amplitude. In general, the decay amplitude can be written as $\mathcal{A}^\pm = A_\lambda + B_\lambda e^{\pm i\gamma}$, where A_λ and B_λ are complex amplitudes invariant under CP , containing the strongly interacting final-state channel, i.e., $\mathcal{A}^- = \langle\lambda_{\mathrm{out}}|H_w|h\rangle$, and $\mathcal{A}^+ = \langle\bar{\lambda}_{\mathrm{out}}|H_w|\bar{h}\rangle$. The only change due to the CP transformation is the sign multiplying the weak phase γ.

IV. COUPLED-CHANNEL DECAY, *CPT* AND *CP* ASYMMETRY

Now, we discuss the example of a decay to channels coupled by rescattering, i.e., the strong S-matrix has nonvanishing off-diagonal matrix elements, $S_{\lambda',\lambda} = \delta_{\lambda',\lambda} + i t_{\lambda',\lambda}$, where $t_{\lambda',\lambda}$ is the strong scattering amplitude of $\lambda' \to \lambda$, and $\delta_{\lambda',\lambda}$ is the Kronecker delta symbol. In this case the CPT condition (3) gives

$$\sum_{\lambda} \Gamma(A_{\lambda}^{-}) = \sum_{\bar{\lambda}} \Gamma(A_{\bar{\lambda}}^{+}), \tag{4}$$

where the subindex labels the final state channels, summed up in the kinematically allowed phase-space.

The decay amplitude written in terms of the CPT constraint (2), and considering the CP violating amplitudes for the hadron and its charge conjugate, is given by

$$A_{\lambda} + e^{\mp i\gamma} B_{\lambda} = \chi_h \chi_{\lambda} \sum_{\lambda'} S_{\lambda',\lambda} (A_{\lambda'} + e^{\pm i\gamma} B_{\lambda'})^{*}. \tag{5}$$

Note that the above equation imposes a relation between A_{λ} or B_{λ} with their respective complex conjugates.

V. *CP* ASYMMETRY AND FSI AT LEADING ORDER

The full decay amplitudes A_{λ} and B_{λ} can be separated in two parts, one carrying the FSI distortion $(\delta A_{\lambda}, \delta B_{\lambda})$ and another one corresponding to a source term without FSI $(A_{0\lambda}, B_{0\lambda})$, $A_{\lambda} = A_{0\lambda} + \delta A_{\lambda}$ and $B_{\lambda} = B_{0\lambda} + \delta B_{\lambda}$. Retaining terms up to leading order (LO) in $t_{\lambda',\lambda}$ in (5), one can easily find that

$$\mathcal{A}_{\mathrm{LO}}^{+} = A_{0\lambda} + e^{i\gamma} B_{0\lambda} + i \sum_{\lambda'} t_{\lambda',\lambda} (A_{0\lambda'} + e^{i\gamma} B_{0\lambda'}), \tag{6}$$

where we have used that

$$A_{0\lambda} = \chi_h \chi_{\lambda} A_{0\lambda}^{*} \tag{7}$$

and

$$B_{0\lambda} = \chi_h \chi_{\lambda} B_{0\lambda}^{*}, \tag{8}$$

which come from (1), when the strong interaction is turned off. We point out that Eq. (6) is equivalent to the one shown in [13,14], but it was obtained with a different approach.

The CP asymmetry, $\Delta\Gamma_{\lambda} = \Gamma(h \to \lambda) - \Gamma(\bar{h} \to \bar{\lambda})$, evaluated by considering the amplitude (6) and only terms up to leading order in $t_{\lambda',\lambda}$, is given by

$$\Delta\Gamma_{\lambda} = 4(\sin\gamma)\mathrm{Im}\Big[B_{0\lambda}^{*} A_{0\lambda} + i \sum_{\lambda'} (B_{0\lambda}^{*} t_{\lambda',\lambda} A_{0\lambda'} - B_{0\lambda'}^{*} t_{\lambda',\lambda}^{*} A_{0\lambda})\Big], \tag{9}$$

where the external sum of λ' represents each channel separately. The second and third terms in the imaginary part in Eq. (9) can be associated to the "compound" CP asymmetry [15], and have the important property of canceling each other when summed with all FSI, in order to satisfies the CPT condition expressed by Eq. (4). The first term, namely $B_{0\lambda}^{*} A_{0\lambda}$, is related to the interference between two CP conserving amplitudes without FSI, as happens for the tree and penguin amplitudes in the BSS model [7]. This term must satisfy

$$\sum_{\lambda} \mathrm{Im}[B_{0\lambda} A_{0\lambda}^{*}] = 0, \tag{10}$$

as a consequence of the CPT constraint.

The cancellation in Eq. (10) reflects the stringent condition of CPT invariance given in Eq. (1), when the FSI is turned off. Therefore, the general condition given by Eq. (10) should be satisfied, with one trivial solution that the phase difference between the two CP -conserving amplitudes is zero for all decay channels. This term was neglected by Wolfenstein.

It is noteworthy to mention here that the second term in Eq. (9) also satisfies the CPT condition, which follows straightforwardly by using Eqs. (7)–(8), the symmetry of $t_{\lambda,\lambda'}$, and the fact that the strong interaction does not mix different CP eigenstates.

VI. INELASTICITY AND *CP* VIOLATION IN A TWO-CHANNEL PROBLEM

Considering the case of two body and two coupled channels, α and β, the unitarity of the S-matrix together with its symmetry ($S_{\alpha,\beta} = S_{\beta,\alpha}$), leads to $|S_{\alpha\alpha}|^2 + |t_{\beta,\alpha}|^2 = |S_{\beta\beta}|^2 + |t_{\beta,\alpha}|^2 = 1$ and $S_{\alpha\alpha} t_{\beta,\alpha}^{*} - S_{\beta\beta}^{*} t_{\beta,\alpha} = 0$. By writing the diagonal elements of the two body elastic scattering S-matrix as $S_{\alpha\alpha} = \eta_{\alpha} e^{2i\delta_{\alpha}}$ and $S_{\beta\beta} = \eta_{\beta} e^{2i\delta_{\beta}}$, where η_{α} and η_{β} are the inelasticity for the α and β channels, respectively, one gets that $\eta_{\alpha} = \eta_{\beta} = \eta$, and $|t_{\beta,\alpha}| = \sqrt{1-\eta^2}$. Furthermore, one can easily derive that $t_{\beta,\alpha} = \sqrt{1-\eta^2} e^{i(\delta_{\alpha}+\delta_{\beta})}$. Therefore, we can rewrite Eq. (9) for the α channel as a sum of two distinct terms, namely, the short distance and the compound contributions. The expression can be written as

$$\Delta\Gamma_{\alpha} = 4(\sin\gamma)(\zeta_0 + \sqrt{1-\eta^2}\zeta_1). \tag{11}$$

The term containing

$$\zeta_0 = \mathrm{Im}[B_{0\alpha}^{*} A_{0\alpha}(1 + i(t_{\alpha\alpha} - t_{\alpha\alpha}^{*}))] \tag{12}$$

corresponds to the short distance contribution to the CP asymmetry. It is widely used to calculate CP asymmetries in two-body B decays, through the interference between the tree and penguin amplitudes for single decays.

The term corresponding to the compound contribution in Eq. (11) contains

$$\zeta_1 = |K_\alpha| \cos(\delta_\alpha + \delta_\beta + \Phi_\alpha), \qquad (13)$$

where $K_\alpha = B^*_{0\alpha} A_{0\beta} - B_{0\beta} A^*_{0\alpha}$ and $\Phi_\alpha = -i \ln(K_\alpha/|K_\alpha|)$. This nondiagonal term gives a close relation between the region for CP-violation and inelastic $\alpha \to \beta$ scattering, presented above. We recall that the opposite sign of $\Delta\Gamma_\beta$ in respect to $\Delta\Gamma_\alpha$ comes from Eqs. (7)–(8), and that the strong interaction does not mix states with different phases χ_λ, which leads to

$$K_\beta = -K_\alpha \quad \text{and} \quad \Phi_\beta = \Phi_\alpha + \pi. \qquad (14)$$

Note that from Eq. (10) applied to the two-channel case, the short distance term satisfies $\Delta\Gamma_\alpha = -\Delta\Gamma_\beta$, which is also verified for the compound contribution as a consequence of Eq. (14), discussed above.

Indeed, looking at the LHCb results [4,5], a direct and complementary relation between different charmless three-body decay channels coupled by the strong interaction emerges for $B^\pm \to K^\pm \pi^+ \pi^-$ and $B^\pm \to K^\pm K^+ K^-$, and for the decays $B^\pm \to \pi^\pm \pi^+ \pi^-$ and $B^\pm \to \pi^\pm K^+ K^-$. Even though the tree and penguin composition in the total decay amplitudes for each pair of coupled channels are expected to be different, the CP asymmetry distribution in the Dalitz plot for these channels shows the prevalence of CP violation in the mass region where the $\pi\pi \to KK$ scattering is important. As a matter of fact, the $\pi^+\pi^-$ and K^+K^- channels are coupled to $\pi^0\pi^0$ and $K\bar{K}$. Besides that, the two channels with two or more kaons in the final state have CP asymmetries with opposite signs with respect to the ones with two or more pions. These facts motivate us to look more closely to the compound contribution to the partial decay widths in the three-body B decays.

VII. ESTIMATE OF THE COMPOUND CONTRIBUTION TO $\Delta\Gamma_{KK(\pi\pi)}$ IN $B^\pm \to K^\pm K^+ K^-$ ($K^\pm \pi^+ \pi^-$) DECAYS

To perform a simple test of the compound contribution [second term of Eq. (11)] to CP asymmetry using only a single angular momentum channel, namely, the S-wave, the best place is to look to the asymmetry in decays involving KK and $\pi\pi$ channels. Beyond the ϕ mass region, there are no other significant resonance contributions with a strong KK coupling before the $f_2(1525)$ resonance. Therefore as an illustration, we estimate the compound contribution to the asymmetry $\Delta\Gamma_{KK(\pi\pi)}$ in $B^\pm \to K^\pm K^+ K^-$ ($K^\pm \pi^+ \pi^-$) decays, presented by the LHCb collaboration [5].

As a remark, the three-body rescattering effect at the two-loop level is small compared to the first two-body collision contribution, as suggested by the three-body model calculation for the $D^\pm \to K^\pm \pi^+ \pi^-$ decays [21]. We assume that this approximation for charmless three-body B decays must be valid at least for some regions of the phase space.

In order to get a quantitative insight on the enhancement of the CP asymmetry from the coupling between the $\pi\pi$ and KK channels in the compound contribution, we start by defining the channels $\alpha \equiv K^+K^-$ and $\beta \equiv \pi^+\pi^-$ and consider the main isospin channel $I = 0$ and $J^P = 0^+$. From the second term of Eq. (11) with ζ_1 from Eq. (13), we can write the compound contribution to the CP asymmetry as

$$\Delta\Gamma^{\mathrm{comp}}_{KK} \approx \mathcal{C}\sqrt{1-\eta^2}\cos(\delta_{KK} + \delta_{\pi\pi} + \Phi_{KK}) F(M^2_{KK}), \qquad (15)$$

with $\mathcal{C} = 4|K|(\sin\gamma)$ considered energy independent. We still approximate the kaon-kaon S-wave phase shift as $\delta_{KK} \approx \delta_{\pi\pi}$ in the region where the channels are strongly coupled. The Dalitz phase-space factor is $F(M^2_{KK}) = (M^2_{K^+K^-})_{\max} - (M^2_{K^+K^-})_{\min}$, for the $B^\pm \to K^\pm K^+ K^-$ channel (see e.g., [24]). The masses $(M^2_{K^+K^-})_{\max}$ and $(M^2_{K^+K^-})_{\max}$ depend on the KK subsystem mass, M^2_{KK}. Also the symmetrization of the decay amplitude in the two equally charged kaons is disregarded as the low mass regions for each possible neutral KK subsystem are widely separated in phase space.

Following Ref. [25], we have used the parametrization for the pion-pion inelasticity and phase-shift, for the $I = 0$ and $J^p = 0^+$ dominant channel, in order to evaluate Eq. (15). The used parametrizations are given in Ref. [25] by Eqs. (2.15a), (2.15b), (2.15b'), (2.16), and the quoted errors. We also use the CPT condition given by Eq. (4), restricted to two channels, to obtain the asymmetry in the $\pi\pi$ decay channel, which in this case is given by $\Delta\Gamma^{\mathrm{comp}}_{\pi\pi} = -\Delta\Gamma^{\mathrm{comp}}_{KK}$.

In order to compare the asymmetries $\Delta\Gamma^{\mathrm{comp}}_{KK}$ and $\Delta\Gamma^{\mathrm{comp}}_{\pi\pi}$ to experimental data, we extracted the difference $B^- - B^+$, respectively for the $B^\pm \to K^\pm K^+ K^-$ and $B^\pm \to K^\pm \pi^+ \pi^-$ decays, from the recent LHCb results presented in Ref. [5]. The results are shown in Fig. 1 for an arbitrary normalization fitted to $\Delta\Gamma^{\mathrm{comp}}_{KK}$. Our calculations are presented from the subsystem mass (M^2_{sub}) above the KK mass threshold. Indeed, $M^2_{\mathrm{sub}} = M^2_{K^+K}$ ($M^2_{\pi^+\pi}$) for $B^\pm \to K^\pm K^+ K^-$ ($K^\pm \pi^+ \pi^-$).

The width of the band represents the errors in the parametrizations of the isoscalar S-wave $\pi\pi$ phase shift, and inelasticity parameter, both taken from Ref. [25]. The phase Φ_{KK} was chosen to be zero, which emphasizes the role of the strong phases in CP violation process. Note that this assumption is accompanied by $\Phi_{\pi\pi} = \pi$ according to the relation given in Eq. (14), therefore, it is ensured that $\Delta\Gamma^{\mathrm{comp}}_{KK} = -\Delta\Gamma^{\mathrm{comp}}_{\pi\pi}$.

We can see a qualitative agreement between the model parametrized with the $\pi\pi$ elastic phase-shift with data, mainly in the sense that the CP violation distribution observed in both $B^\pm \to K^\pm K^+ K^-$ and $B^\pm \to K^\pm \pi^+ \pi^-$ decays are important to the mass region where the S-wave scattering $\pi^+\pi^- \to K^+K^-$ is important, as shown in Fig. 1. A visual inspection of the Dalitz plot of the $B^\pm \to K^+K^-\pi^\pm$ and $B^\pm \to \pi^\pm\pi^+\pi^-$ decays [6], also presents an important CP violation distribution at similar masses to those

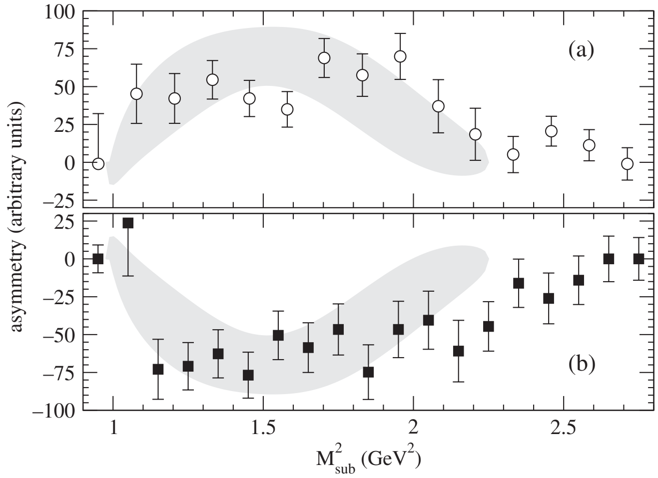

FIG. 1. Estimate (grey band) of Eq. (15) as a function of the subsystem mass compared to experimental data of (a) the asymmetry of $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ decay (circles), and of (b) the asymmetry of $B^{\pm} \to K^{\pm}K^{+}K^{-}$ decay (squares). Data extracted from Ref. [5].

where *CP* violation is relevant for $B^{\pm} \to K^{\pm}K^{+}K^{-}$ and $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$. Also the *CP* asymmetry below the KK threshold in the resonance region appears appreciable, which is however outside the region where the FSI mechanism discussed here applies.

VIII. COMMENTS

Although we have focused only on the relevance of the coupling between $\pi\pi$ and KK channels in the *CP* asymmetry observables using the *CPT* constraint, one should note that three light-pseudoscalar mesons can, in principle, couple via strong interaction with channels like $D\bar{D}h$, where h can be π or K. It seems reasonable to expect that $D\bar{D}h \to hhh$ can contribute to the *CP* asymmetry in regions of large two-body invariant mass above the $D\bar{D}$ threshold, that is far from the KK threshold and above 1.6 GeV, outside the region discussed in this work. Furthermore, there is no available experimental data and even theoretical predictions for these possible long-range interactions to induce *CP* asymmetries above $D\bar{D}$ threshold in charmless three-body charged decays, as we did using the $\pi\pi \to KK$ scattering. Since that direct *CP* violation induced by the short distance interaction must be highly suppressed in double charged charm B decays, future experimental analysis could look for those asymmetries in order to observe *CP* violation induced by rescattering originated by charmless B decay channels.

The difficulty in observing this "compound" *CP* asymmetry in double charm charged B decays comes because the branching fractions of these decays are about two orders of magnitude larger than the corresponding one for charmless B decays. Therefore, in order to measure the induced *CP* asymmetries in double charm charged B decay channels, the *CP* violation must be large enough to overcome the increase in the branching fraction ratios when compared to three light-pseudoscalar channels. Despite the global suppression due the large difference in branching fractions pointed out above, double charm charged three-body B decays, can present a specific and concentrated phase-space region where the "compound" *CP* asymmetry takes place.

Although we have compared the data for the asymmetry only to the compound contribution, one must be aware of the first term in Eq. (11) containing ζ_0, that carries the short range physics. The comparison with the data suggests the importance of the rescattering, which seems to be relevant in the region of masses analyzed in Fig. 1. However, the LHCb results for charged $K\pi\pi$ and $\pi\pi\pi$ presents a clear *CP* violation below the KK threshold, and in this region it may be possible to have a more clean access to *CP* violation from short distance contributions.

IX. CONCLUSIONS

We studied *CP* violation in three-body charmless $B^{\pm}$ decays using two basic assumptions: (i) *CPT* invariance; and (ii) that part of this *CP* violation is due to the interference of two *CP*-conserving hadronic amplitudes separated by a *CP*-noninvariant phase. We have built a plausible scenario where these two assumptions lead to the observed asymmetries in both $B^{\pm} \to K^{\pm}K^{+}K^{-}$ and $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ decays as found by the LHCb collaboration [5], which are also concentrated in the low $K^{+}K^{-}$ and $\pi^{+}\pi^{-}$ mass regions. The coupling between the KK and $\pi\pi$ channels is strong in the energy range where the asymmetry in $B^{\pm} \to K^{\pm}K^{+}K^{-}(K^{\pm}\pi^{+}\pi^{-})$ decay is observed, indicating that the "compound" contribution should be taken into account to reproduce the experimental data. Modulated by a phase-space factor, the asymmetry is proportional to $\sqrt{1-\eta^2}\cos(\delta_{KK}+\delta_{\pi\pi}+\Phi)$, coming from the magnitude and phase of the $\pi\pi \to KK$ transition amplitude. In the future, the analysis of the *CP* asymmetry in charmless B decays can be extended to include corrections (expected to be small) induced by the three-body rescattering processes.

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