PHYSICAL REVIEW D **89,** 094008 (2014)

Azimuthal asymmetries in high-energy collisions of protons with holographic shockwaves

Jorge Noronha[1,*] and Adrian Dumitru[2,†]

[1] *Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, SP, Brazil*
[2] *Department of Natural Sciences, Baruch College, CUNY, 17 Lexington Avenue, New York, New York 10010, USA*

(Received 7 February 2014; published 8 May 2014)

Large azimuthal quadrupole and octupole asymmetries have recently been found in $p + \mathrm{Pb}$ collisions at the LHC. We argue that these might arise from a projectile dipole scattering off fluctuations in the target with a size on the order of the dipole. In a holographic scenario, parity-even angular moments v_{2n} are generated by the real part of the light-like Wilson loop due to the contribution from the background metric to the Nambu-Goto action. On the other hand, parity-odd moments v_{2n+1} must arise from the imaginary part of the light-like Wilson loop, which is naturally induced by a fluctuating Neveu-Schwarz 2-form.

DOI: 10.1103/PhysRevD.89.094008 PACS numbers: 12.38.Mh, 25.75.Nq, 11.25.Tq

I. INTRODUCTION

Recent measurements of the azimuthal momentum distributions of particles produced in high-multiplicity $p + \mathrm{Pb}$ collisions at the LHC have revealed large asymmetries $v_n = \langle \cos n\phi \rangle$ [1]. Remarkably, the octupole asymmetry is of the same order of magnitude as the quadrupole. In this paper, we argue that these asymmetries in the final state could reflect a snapshot of the fluctuations in the Pb target taken in the instant of the collision. Rotational symmetry is spontaneously broken (locally) by a fluctuation in the target of arbitrary shape. Such random fluctuations generically lead to large local azimuthal anisotropies in coordinate space [2,3].

In this paper we propose an alternative to (almost) dissipationless hydrodynamic expansion in pA collisions [4,5] for the conversion of coordinate-space fluctuations into asymmetries in momentum space. Here, nonzero v_n's emerge due to the orientation of a projectile "dipole" relative to the global orientation of the event determined by a fluctuation in the target. We show that parity-even moments v_{2n} are generated by the real part of the dipole forward scattering amplitude while parity-odd moments v_{2n+1} arise from its imaginary part. While this argument for the generation of even and odd v_n's is quite general, the actual calculation of these moments directly from QCD is quite challenging. In fact, calculations of v_2 in proton-nucleus collisions have been carried out within the "color glass condensate" approach (see Ref. [6] for a recent review), though at present it is not clear if a large v_3 emerges as well.

In this aspect, the holographic correspondence [7] may provide an interesting alternative where calculations can be done in a nonperturbative manner in strongly coupled non-Abelian gauge theories using a (higher-dimensional) effective theory in the semiclassical approximation which includes gravity (among other fields). In the holographic approach, the real and imaginary parts of the dipole forward scattering amplitude in a proton-nucleus collision can be extracted by studying a classical string described by the Nambu-Goto action (which corresponds to the probe dipole) coupled to the underlying nontrivial background fields (such as the metric, the dilaton, the Neveu-Schwarz 2-form, etc.) that give support to a single holographic shockwave solution, used here to model the traveling nucleus. We outline such a calculation below and show how parity-even angular moments v_{2n} are generated by the real part of the light-like Wilson loop due to the contribution from the background metric to the Nambu-Goto action while parity-odd moments v_{2n+1} arise from the imaginary part of the light-like Wilson loop, which is naturally induced by a fluctuating Neveu-Schwarz 2-form.

Our treatment is distinct from that of Ref. [8] where a proton-nucleus collision is modeled as an asymmetric collision of holographic shockwaves [9], or from $\mathcal{R}$-current deep inelastic scattering [10]. In our approach the gauge-gravity correspondence is used only to evaluate the "target field averages" such as the light-like Wilson loop, analogous to the computation of the jet-quenching parameter $\hat{q}$ in Ref. [11]. We do not use it to describe the actual collision; thus, no black hole is formed and the projectile quark is not stopped (a nonstopping solution in a holographic framework has indeed been found for a thin enough projectile and target in Ref. [12]). The form of the scattering amplitude of a projectile quark with large light-cone momentum is taken from QCD.

This paper is organized as follows. In the next section we briefly review how the quark-nucleus elastic scattering is described within the hybrid formalism and give the general argument which shows how nonzero v_n's may be generated by a projectile dipole scattering off fluctuations in the target

*noronha@if.usp.br
†adrian.dumitru@baruch.cuny.edu

nucleus with a size on the order of the dipole. In Sec. III we perform the holographic calculation of the v_n's. Our conclusions and outlook can be found in Sec. IV.

II. QUARK-NUCLEUS ELASTIC SCATTERING IN THE HYBRID FORMALISM

In the so-called hybrid formalism the proton projectile is treated as a beam of collinear partons with a large light-cone momentum p^- which probe the field of the target. At large Feynman-x_F the contribution from quarks dominates while particles with $x_F \ll 1$ are mainly gluons. We describe here mainly the case of quark scattering but it is straightforward to obtain the contribution from gluons by considering the scattering amplitude in the adjoint representation. The main difference, however, is that the forward scattering amplitude for an adjoint dipole is real and thus can only generate parity-even moments v_{2n}.

Our approach is based on the intuitive picture that a high-energy projectile parton couples weakly to the target field. However, over some intermediate range of semihard transverse momenta the target is modeled as a holographic shockwave before it turns into a beam of noninteracting asymptotically free partons at very high transverse momentum $p_\perp$ (far exceeding its saturation scale Q_s computed below). Whether or not the proposed picture has a viable theoretical justification remains to be seen; it is not clear if the limit of eikonal, recoilless propagation of a high-energy projectile parton is well defined in the present context [13]. We shall follow Ref. [11] and assume the validity of this eikonal description in this paper.

If we assume that to leading order in $p_\perp/p^-$ projectile partons propagate on eikonal trajectories then the amplitude corresponding to elastic scattering from momentum p to q is [14]

$$< \text{out}, q|\text{in}, p > \equiv \bar{u}(q)\tau(q,p)u(p) \tag{1}$$

$$\tau(q,p) = 2\pi\delta(p^- - q^-)\gamma^- \int d^2\vec{x}[V(\vec{x}) - 1]e^{i(\vec{p}-\vec{q})\cdot\vec{x}}. \tag{2}$$

Here,

$$V(\vec{x}) = \mathcal{P}\exp\left[ig\int dx^- A^+(x^-,\vec{x})\right] \tag{3}$$

is a Wilson line along the light cone. Upon squaring the amplitude [15] the scattering cross section can be written as [16]

$$\frac{d\sigma_{qA}}{d^2\vec{b}d^2\vec{q}} = \frac{1}{(2\pi)^2}\int d^2\vec{x}e^{-i\vec{q}\cdot\vec{x}}\left(\left\langle\frac{1}{N_c}\text{tr}(W(\vec{x},\vec{b}) - V(\vec{b}-\vec{x}/2) - V^\dagger(\vec{b}+\vec{x}/2))\right\rangle + 1\right). \tag{4}$$

Here, $\vec{b}$ denotes the impact parameter of the collision and $W(\vec{x},\vec{b})$ is a light-like Wilson loop of width given by $|\vec{x}|$. In covariant gauge $W(\vec{x},\vec{b}) = V^\dagger(\vec{b}+\vec{x}/2)V(\vec{b}-\vec{x}/2)$ and this is commonly referred to as the dipole unintegrated gluon distribution [17]. The size of the dipole corresponds to the shift of the transverse coordinate of the eikonal quark line from the amplitude to the complex conjugate amplitude, respectively.

Thus, the quark-nucleus cross section is written as a Fourier transform of the dipole S-matrix,

$$\begin{aligned}\frac{d\sigma_{qA}}{d^2\vec{b}d^2\vec{q}} &= \frac{1}{(2\pi)^2}\int d^2\vec{x}e^{-i\vec{q}\cdot\vec{x}}S(\vec{x}) \\ &= \frac{1}{(2\pi)^2}\int d^2\vec{x}e^{-i\vec{q}\cdot\vec{x}}(D(\vec{x}) + iO(\vec{x})).\end{aligned} \tag{5}$$

Here, $D(\vec{x}) = \text{Re}S(\vec{x})$ and $O(\vec{x}) = \text{Im}S(\vec{x})$ denote the real and imaginary parts of the S-matrix, respectively. Since the left-hand side of this equation is manifestly real, we must have that $D(\vec{x}) = D(-\vec{x})$ is even under exchange of the quark and antiquark lines, while $O(\vec{x}) = -O(-\vec{x})$ is odd. It follows that $D(\vec{x})$ is responsible for generating nonzero $v_{2n} = \langle\cos n\phi\rangle$ which are even under $\phi \to \phi + \pi$ (resp. $\vec{x} \to -\vec{x}$). On the other hand, v_{2n+1} is odd under $\phi \to \phi + \pi$ and hence can only arise from $O(\vec{x})$.

Equation (4) can be turned into a physical $pA \to h + X$ single inclusive cross section for production of a hadron of type h via a convolution with a proton-parton distribution and a corresponding $q \to h$ fragmentation function [18–20]. Here we will only need Eq. (4). Below, we employ the holographic correspondence [7] to compute the light-like Wilson loop $W(\vec{x},\vec{b})$ in the field of a shockwave in strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory with a large number of colors, N_c. The essential point is to consider scattering of a dipole whose angular orientation couples to fluctuations in the target.

III. LIGHT-LIKE WILSON LOOP IN A HOLOGRAPHIC SHOCKWAVE BACKGROUND

The nucleus is traveling along the x^+ axis with a light-cone momentum p^+. We are interested in holographic shockwave solutions of the form

$$\langle\hat{T}_{--}(x^-,\vec{x})\rangle = \frac{N_c^2}{2\pi^2}p^+\delta(x^-)\mu^2 f(\vec{x}), \tag{6}$$

where $f(\vec{x})$ describes the energy density distribution of the target in the transverse plane, and $\mu^2 \sim A^{1/3}$ [21,22] is the transverse density scale of the shockwave. This nonuniform shockwave at the boundary can be obtained from a source $J(\vec{x})\delta(z-1/\mu)$ in the bulk convoluted with the Green's function found in Ref. [23],

$$f(\vec{x}) = \int d^2\vec{x}' \frac{J(\vec{x}')}{(1+\mu^2|\vec{x}-\vec{x}'|^2)^3}. \tag{7}$$

The metric is a solution of the five-dimensional Einstein's equations with a negative cosmological constant $\sim 1/L^2$ with a source [23,24]. It has the form

$$ds^2 = \frac{L^2}{z^2}(p^+\delta(x^-)\mu^2\mathcal{F}(z,\vec{x})z^4 dx^{-2} - 2dx^+dx^- + d\vec{x}^2 + dz^2), \tag{8}$$

where $\lim_{z\to 0}\mathcal{F}(z,\vec{x}) = f(\vec{x})$. We note that Ref. [25] found a family of regular shockwave solutions which, in the homogeneous limit, have vanishing $\langle \hat{T}_{--}(x^-,\vec{x})\rangle$. In fact, those bulk solutions do not approach $\sim z^4$ near the boundary and are, thus, qualitatively distinct from the type of solutions found in Refs. [9,23]. In this paper, we consider that the nucleus is, on average, essentially uniform in the transverse plane over scales probed by the dipole and, thus, we shall restrict to solutions of the form (8).

One can now compute the light-like Wilson loop in this background [26]. The rectangular loop $\mathcal{C}$ is defined on the x^- axis with transverse size $\vec{d}$ and it is the boundary for a minimal surface in the bulk [27,28]. When the radius of AdS_5 is much larger than the string length ℓ_s, i.e., $L^2/\alpha' \gg 1$, where $\alpha' = \ell_s^2$, this is obtained by minimizing the Nambu-Goto (NG) action

$$S_{\rm NG} = \frac{1}{2\pi\alpha'}\int_\Omega d^2\sigma\sqrt{-\det h_{ab}}, \tag{9}$$

where Ω denotes the world sheet, $h_{ab} = g_{\rm MN}\partial_a X^M \partial_b X^N$ is the world-sheet metric, and $X^M(\sigma)$ is the embedding function that describes the string world sheet in the bulk. In principle, the effective action for the string should also include the coupling to other background fields such as the dilaton and the Neveu-Schwarz (NS) 2-form [29] from the NS-NS sector but these are taken to be either vanishing or pure gauge (more on that below).

Our calculation for the light-like Wilson loop closely follows the one performed in Ref. [11] to obtain the jet-quenching parameter $\hat{q}$ coefficient in the strongly coupled $\mathcal{N}=4$ SYM plasma. However, for our shockwave the gauge theory is not at finite temperature and, thus, the background geometry does not have a horizon. The light-like Wilson loop is given in terms of the on-shell action as $\langle {\rm tr}W(\mathcal{C})\rangle/N_c = e^{iS_{\text{on-shell NG}}}$.

The string world-sheet coordinates are $\tau = x^-$ and σ and the world-sheet embedding function is $X^M = (x^-, \vec{b}+\sigma\vec{d}, 0, du(\sigma))$ with $\sigma \in (-1/2, 1/2)$. The endpoints of the string are located at the boundary at $\vec{b}-\vec{d}/2$ and $\vec{b}+\vec{d}/2$. With the metric (8) the Nambu-Goto action becomes

$$iS_{\rm NG} = -\frac{\sqrt{\lambda}}{2\pi}A^{1/6}\mu d\int_{-1/2}^{1/2} d\sigma\sqrt{\mathcal{F}(du(\sigma),\vec{b}+\sigma\vec{d})}\sqrt{1+u'(\sigma)^2}. \tag{10}$$

The explicit factor of $A^{1/6}$ arises from the integration over the longitudinal coordinate x^- [21,22,30].

Due to the properties of the function $\mathcal{F}$, the integrand of the action is finite at the boundary, $u \to 0$, as opposed to the case of a time-like rectangular Wilson loop in vacuum [27] where it diverges as $\sim 1/u^2$. Therefore, any configuration where $u'(\sigma) \neq 0$ will necessarily increase the world-sheet area. Hence, the minimal surface must be the one in which the string remains at the boundary for all σ, i.e., the string does not fall into the bulk. In fact, $u(\sigma)=0$ is clearly a solution of the equations of motion that satisfy the boundary conditions, which is consistent with the fact that a light-like string configuration costs zero energy in AdS_5 [31].

Therefore, the on-shell action is obtained by setting $u'(\sigma)=0$ and $u(\sigma)=0$ which leads to

$$iS_{\text{on-shell NG}} = -\frac{\sqrt{\lambda}}{2\pi}A^{1/6}\mu d\int_0^{1/2} d\sigma\left(\sqrt{f(\vec{b}+\sigma\vec{d})} + \sqrt{f(\vec{b}-\sigma\vec{d})}\right). \tag{11}$$

Note the $\vec{d} \to -\vec{d}$ symmetry and the fact that $iS_{\rm NG}$ is real. In what follows we assume that the nucleus is much larger than the dipole and that its density over large distance scales is homogeneous. Thus, we can set $\vec{b}=0$. Furthermore, in the absence of fluctuations we have $f=1$ and so the forward dipole scattering amplitude becomes

$$W(d) = e^{-\frac{\sqrt{\lambda}}{2\pi}A^{1/6}\mu d}. \tag{12}$$

The "saturation scale" where $W \sim e^{-1}$ therefore is

$$Q_s = \frac{\sqrt{\lambda}}{2\pi} A^{1/6}\mu. \tag{13}$$

Q_s obtained by averaging the Wilson loop in a shockwave background increases very rapidly with the thickness of the nucleus, $Q_s \sim A^{1/3}$ [21].

The transverse momentum distribution of scattered quarks is

$$\frac{d\sigma_{qA}}{d^2\vec{b}d^2\vec{q}_\perp} = \frac{Q_s}{(Q_s^2 + p_\perp^2)^{3/2}}. \tag{14}$$

We should stress that this result is not supposed to apply at very large $p_\perp$ where from perturbative QCD $d\sigma_{qA}/d^2p_\perp \sim \alpha_s^2/p_\perp^4$ [32]. However, in the nonlinear regime at intermediate p_T one does indeed expect a "flatter" transverse momentum distribution similar to Eq. (14).

A. Fluctuations in the holographic shockwave and even moments v_{2n}

We introduce fluctuations of the density of the shockwave in terms of their Fourier spectrum,

$$f(\vec{b} + \sigma\vec{d}) = 1 + \delta f(\vec{b} + \sigma\vec{d}) \tag{15}$$

$$\delta f(\vec{x}) = \int \frac{d^2k}{(2\pi)^2} \delta f(\vec{k}) e^{i\vec{k}\cdot\vec{x}}. \tag{16}$$

δf describes "classical" fluctuations in the target which contribute $\sim N_c^2$ to the energy-momentum tensor, c.f. Eq. (6). For simplicity here we assume that

$$\delta f(\vec{k}) = \frac{1}{2}(2\pi)^2 \mathcal{A}(1/|\vec{k}_0|)[\delta(\vec{k} - \vec{k}_0) + \delta(\vec{k} + \vec{k}_0) + i(\delta(\vec{k} - \vec{k}_0) - \delta(\vec{k} + \vec{k}_0))], \tag{17}$$

i.e., that the fluctuation is dominated by a single wave number and direction though one could also average over some suitable distribution. $\mathcal{A}(1/|\vec{k}_0|)$ is the amplitude of the fluctuation at the scale k_0; we shall denote the typical length scale $1/|\vec{k}_0|$ of fluctuations as ℓ, and the azimuthal orientation of the dipole as ϕ so that $\vec{d}\cdot\vec{k}_0 = d/\ell \cos\phi$. Note that in order to obtain v_n one only averages over this relative angle ϕ while the global orientation is fixed; alternatively, the moments could be defined from two-particle cumulants [33],

$$v_n^2 = \langle e^{in(\phi_1 - \phi_2)} \rangle, \tag{18}$$

where ϕ_1 and ϕ_2 are the azimuthal angles of any two particles from the same event.

Expanding the square root in Eq. (11) for small amplitude fluctuations we find

$$iS_{\text{on-shell}} = -Q_s d\left[1 + \mathcal{A}(\ell)\frac{\sin(\frac{d}{2\ell}\cos\phi)}{\frac{d}{\ell}\cos\phi}\right]. \tag{19}$$

The fluctuations generate asymmetries for the multipole moments of the $p_\perp$ distribution,

$$\frac{d\sigma_{qA}}{d^2\vec{b}p_\perp dp_\perp d\phi_p} = \frac{1}{(2\pi)^2}\int x_\perp dx_\perp d\phi e^{-ip_\perp x_\perp \cos(\phi-\phi_p)} e^{iS_{\text{on-shell NG}}}, \tag{20}$$

where we denoted the transverse size of the dipole as $\vec{x}_\perp$. Parametrically, nonzero moments of this distribution will be of order $\sim Q_s\ell\mathcal{A}(\ell)$. However, the action (19) is even under $\phi \to \phi + \pi$ and, thus, it can only generate even moments of the angular distribution.

B. Fluctuations of the NS 2-form and odd moments v_{2n+1}

Odd moments of the angular distribution can be generated in this approach from the imaginary part of the dipole-nucleus S-matrix $\langle V^\dagger(\vec{x})V(\vec{y})\rangle$ which includes C-even "pomeron" and C-odd "odderon" exchanges. Projecting on odd-even in-out states, the latter corresponds to the imaginary part $\Im m S(\vec{x},\vec{y}) = \langle O(\vec{x},\vec{y})\rangle$ where the odderon operator is given by [34]

$$O(\vec{x},\vec{y}) = \frac{1}{2iN_c}\text{tr}(V^\dagger(\vec{x})V(\vec{y}) - V^\dagger(\vec{y})V(\vec{x})). \tag{21}$$

The odderon has been identified with the fluctuations of the anti-symmetric NS-NS 2-form B_{MN} in the bulk [35,36]. In the dual holographic description used here, the contribution to the effective action of the string that is odd under the $\vec{d} \to -\vec{d}$ operation should arise from the coupling of the NS 2-form field to the string in the Nambu-Goto action,

$$S_{\text{NS-NS}} = \frac{1}{4\pi\alpha'}\int_\Omega d^2\sigma B_{\text{MN}}\epsilon_{ab}\partial_a X^M \partial_b X^N, \tag{22}$$

where ϵ_{ab} is the Levi-Civita symbol on the world sheet [37]. For a single shockwave we consider a pure gauge NS-field B_{MN} which does not alter the equations of motion of supergravity [29] so that the solutions for the background metric and for the string remain valid. [After a collision of two shockwaves this is no longer the case, just as the metric is no longer of the form (8).]

Contributions such as Eq. (22) should indeed lead to parity-odd moments of the angular distribution (20). Choosing a gauge where $B_{-M} = 0$, this term in the action becomes (using the world-sheet embedding defined earlier)

$$S_{\text{NS-NS}} = \frac{1}{2\pi\alpha'}\int_{-\infty}^{\infty} dx^- \int_0^{1/2} d\sigma \vec{d} \cdot [\vec{B}_{-\vec{x}_\perp}(x^-, \vec{b} + \vec{d}\sigma, 0) + \vec{B}_{-\vec{x}_\perp}(x^-, \vec{b} - \vec{d}\sigma, 0)]. \tag{23}$$

This action is purely real. Thus, it contributes a phase to the total amplitude $\exp\{iS_{\text{NG}} + iS_{\text{NS-NS}}\}$ which is odd under $\vec{d} \to -\vec{d}$. Therefore, in this more general scenario, odd moments for the angular distribution such as v_3 should be nonzero and, again, of order ℓQ_s. Specifically, terms such as

$$i\int_{-1/2}^{1/2} d\sigma \vec{d} \cdot \vec{\nabla} f(\sigma \vec{d}) = -2i\mathcal{A}(\ell) \sin\left(\frac{d}{2\ell}\cos\phi\right) \tag{24}$$

can arise. Indeed, this is parity $(\phi \to \phi + \pi)$-odd and generates odd v_n's up to $n \sim d/\ell$.

IV. CONCLUSIONS

In summary, we have argued that azimuthal asymmetries v_n in p + A collisions may arise from scattering of a dipole on random fluctuations in the target; the fluctuations are assumed to be "classical" so that $\delta T_{--} \sim N_c^2$.

The real (imaginary) part of the dipole-nucleus S-matrix is even (odd) under exchange of the quark and antiquark lines corresponding to charge conjugation of the Wilson loop, and gives rise to parity-even (-odd) angular moments v_n. This is a simple and quite general mechanism that allows for the generation of nonzero Fourier moments of hadron yields in proton-nucleus collisions. Whether or not this is indeed the main effect behind the nonzero v_n's (in particular, of v_3) in these collisions still remains to be verified.

We have used the holographic correspondence to determine the properties of light-like Wilson loops in a shockwave background in strongly coupled $\mathcal{N} = 4$ SYM. We use this as a toy model for the actual calculation of v_n's in QCD. In the holographic description the contribution from the metric to the Nambu-Goto action produces parity-even distributions while the coupling of a fluctuating NS-NS 2-form field with the classical string can generate odd moments. More detailed numerical calculations of $v_n(p_\perp)$ could potentially provide information on the spectrum of fluctuations, such as if there is a dominant length scale and amplitude (as assumed here, for simplicity).

It would be interesting to generalize the calculation performed here to take into account other effects such as the presence of a confining scale in shockwave solutions [38]. This requires different shockwave solutions such that the string connecting the sources does sag into the bulk. We leave this to a future study.

ACKNOWLEDGEMENTS

J. N. thanks Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for support. A. D. thanks the University of São Paulo for their hospitality during a visit when this work was initiated, and gratefully acknowledges support by the DOE Office of Nuclear Physics through Grant No. DE-FG02-09ER41620 and from The City University of New York through the PSC-CUNY Research Award Program, grant 66514-0044. The authors thank Y. Hatta for discussions at the Yukawa Institute for Theoretical Physics, Kyoto University, where this work was completed during the YITP-T-13-05 workshop on "New Frontiers in QCD." We thank W. van der Schee, H. Nastase, and A. Taliotis for comments and Y. Kovchegov for a critical reading of the manuscript before publication and for helpful comments about the odderon and the eikonal limit in QCD.

[1] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **718**, 795 (2013); B. Abelev *et al.* (ALICE Collaboration), Phys. Lett. B **719**, 29 (2013); G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. Lett. **110**, 182302 (2013); Phys. Lett. B **725**, 60 (2013); S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **724**, 213 (2013).
[2] E. Avsar, C. Flensburg, Y. Hatta, J.-Y. Ollitrault, and T. Ueda, Phys. Lett. B **702**, 394 (2011).
[3] A. Bzdak, P. Bozek, and L. McLerran, arXiv:1311.7325.
[4] P. Bozek and W. Broniowski, Phys. Lett. B **718**, 1557 (2013); Phys. Rev. C **88**, 014903 (2013).
[5] E. Shuryak and I. Zahed, Phys. Rev. C **88**, 044915 (2013).
[6] R. Venugopalan, arXiv:1312.0113.
[7] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[8] J. L. Albacete, Y. V. Kovchegov, and A. Taliotis, J. High Energy Phys. 05 (2009) 060; J. Casalderrey-Solana, M. P. Heller, D. Mateos, and W. van der Schee, arXiv:1312.2956.
[9] R. A. Janik and R. B. Peschanski, Phys. Rev. D **73**, 045013 (2006).
[10] Y. Hatta, E. Iancu, and A. H. Mueller, J. High Energy Phys. 01 (2008) 026; J. High Energy Phys. 01 (2008) 063.
[11] H. Liu, K. Rajagopal, and U. A. Wiedemann, Phys. Rev. Lett. **97**, 182301 (2006); J. High Energy Phys. 03 (2007) 066.
[12] J. Casalderrey-Solana, M. P. Heller, D. Mateos, and W. van der Schee, Phys. Rev. Lett. **111**, 181601 (2013).

[13] Y. Kovchegov (private communication).

[14] J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D **3**, 1382 (1971).

[15] The amplitude and its complex conjugate must be parallel transported to $\vec{x} = 0$ unless A^+ is obtained in covariant gauge.

[16] A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett. **89**, 022301 (2002).

[17] A. H. Mueller, Nucl. Phys. **B415**, 373 (1994); **B437**107 (1995); A. H. Mueller and B. Patel, Nucl. Phys. **B425**, 471 (1994).

[18] A. Dumitru, A. Hayashigaki, and J. Jalilian-Marian, Nucl. Phys. **A765**, 464 (2006).

[19] T. Altinoluk and A. Kovner, Phys. Rev. D **83**, 105004 (2011).

[20] G. A. Chirilli, B.-W. Xiao, and F. Yuan, Phys. Rev. Lett. **108**, 122301 (2012); Phys. Rev. D **86**, 054005 (2012).

[21] J. L. Albacete, Y. V. Kovchegov, and A. Taliotis, J. High Energy Phys. 07 (2008) 074.

[22] A. Taliotis, Nucl. Phys. **A830**, 299c (2009).

[23] S. S. Gubser, S. S. Pufu, and A. Yarom, Phys. Rev. D **78**, 066014 (2008).

[24] Y. V. Kovchegov and S. Lin, J. High Energy Phys. 03 (2010) 057.

[25] G. Beuf, Phys. Lett. B **686**, 55 (2010).

[26] Given the matter content of $\mathcal{N} = 4$ SYM theory, the Wilson loop also contains the coupling to the six $SU(N_c)$ adjoint scalars X^I. We follow Ref. [39] and consider an average over all the angles θ^I on S^5 to eliminate the dependence of the string world sheet on modes that carry nonzero Kaluza-Klein charge.

[27] J. M. Maldacena, Phys. Rev. Lett. **80**, 4859 (1998).

[28] Wilson loops with light-like segments have been extensively studied within the gauge/gravity correspondence; see, e.g., L. F. Alday and J. M. Maldacena, J. High Energy Phys. 06 (2007) 064.

[29] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998).

[30] To see this one writes $\delta(x^-) \to \frac{1}{a}\Theta(x^-)\Theta(a - x^-)$ with $a \sim A^{1/3}$.

[31] K. Ito, H. Nastase, and K. Iwasaki, Prog. Theor. Phys. **120**, 99 (2008).

[32] $A \sim 1/p_\perp^4$ behavior at high $p_\perp$ is required also in order for $\langle p_\perp \rangle$ to be finite and independent of any UV cutoffs. This is a necessary condition, even at the level of purely elastic scattering, for the validity of eikonal projectile trajectories in the limit $\langle p_\perp \rangle / p^- \to 0$.

[33] N. Borghini, P. M. Dinh, and J. -Y. Ollitrault, Phys. Rev. C **63**, 054906 (2001).

[34] Y. V. Kovchegov, L. Szymanowski, and S. Wallon, Phys. Lett. B **586**, 267 (2004); Y. Hatta, E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. **A760**, 172 (2005); S. Jeon and R. Venugopalan, Phys. Rev. D **71**, 125003 (2005).

[35] R. C. Brower, M. Djuric, and C.-I. Tan, J. High Energy Phys. 07 (2009) 063.

[36] E. Avsar, Y. Hatta, and T. Matsuo, J. High Energy Phys. 03 (2010) 037.

[37] The action in Eq. (22) has to be supplemented by the boundary term $\int_{\partial\Omega} A_1$ (where A_M is a 1-form) to preserve the gauge invariance associated with B_{MN}. This term is subleading in α' and it will be omitted in this paper.

[38] E. Kiritsis and A. Taliotis, J. High Energy Phys. 04 (2012) 065.

[39] H. R. Grigoryan and Y. V. Kovchegov, Nucl. Phys. **B852**, 1 (2011).