CP violation in lepton number violating semihadronic decays of K, D, D_s, B, B_c

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We study the *CP* violation in lepton-number-violating meson decays $M^{\pm} \to \ell_1^{\pm}\ell_2^{\pm}M'^{\mp}$, where M and M' are pseudoscalar mesons, $M = K, D, D_s, B, B_c$ and $M' = \pi, K, D, D_s$, and the charged leptons are $\ell_1, \ell_2 = e, \mu$. It turns out that the *CP*-violating difference $S_{-}(M) \equiv [\Gamma(M^- \to \ell_1^- \ell_2^- M'^+) - \Gamma(M^+ \to \ell_1^+ \ell_2^+ M'^-)]$ can become appreciable when two intermediate on-shell Majorana neutrinos N_j $(j = 1, 2)$ participate in these decays. Our calculations show that the asymmetry becomes largest when the masses of N_1 and N_2 are almost degenerate, i.e., when the mass difference ΔM_N becomes comparable with the (small) decay widths Γ_N of these neutrinos: $\Delta M_N \gg \Gamma_N$. We show that in such a case, the *CP* ratio $\mathcal{A}_{CP}(M) \equiv [\Gamma(M^- \to \ell_1^- \ell_2^- M'^+) - \Gamma(M^+ \to \ell_1^+ \ell_2^+ M'^-)]/[\Gamma(M^- \to \ell_1^- \ell_2^- M'^+) + \Gamma(M^+ \to \ell_1^+ \ell_2^+ M'^-)]$ becomes a quantity ~ 1. The observation of *CP* violation in these decays would be consistent with the existence of the well-motivated νMSM model with two almost degenerate heavy neutrinos in the mass range $M_N \sim 0.1$–10^1 GeV.

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I. INTRODUCTION

At this moment, one of the main questions in neutrino physics is unresolved: whether the neutrinos are Majorana or Dirac particles. If the neutrinos are Dirac particles, the lepton number is conserved in all processes. If the neutrinos are Majorana particles, i.e., if they are indistinguishable from their antiparticles, the lepton number in the reactions involving them can be violated. The main processes whose eventual detection would decide on the nature of neutrinos are the neutrinoless double beta decays ($0\nu\beta\beta$) in nuclei [1]. Among other processes which may reflect the character of neutrinos are specific scattering processes [2–5] and rare meson decays [6–14].

Another important question is the value of the masses of neutrinos. Neutrino oscillations were predicted a long time ago [15], under the assumption that neutrinos have masses. These oscillations were later observed [16–18], leading to the conclusion that the first three neutrinos have nonzero but very light masses $\lesssim 1$ eV. They can be produced via a seesaw mechanism [19], where the light neutrinos have masses $\sim \mathcal{M}_D^2/\mathcal{M}_R (\lesssim 1\,\text{eV})$, where $\mathcal{M}_D$ is an electroweak scale or lower. The heavy Majorana neutrinos in these seesaw scenarios are very heavy, with typical masses $\mathcal{M}_R \gg 1\,\text{GeV}$, and their mixing with active neutrino flavors is very suppressed $\sim \mathcal{M}_D/\mathcal{M}_R (\ll 1)$. However, scenarios exist [3,20–23] where the heavy Majorana neutrinos can have relatively low masses ~ 1 GeV and their mixings with active neutrino flavors can be larger than in the usual seesaw scenarios.

Another important question in neutrino physics is the strength (if any) of the *CP* violation in the neutrino sector. It could be measured by neutrino oscillations [24]. However, in this work we will investigate the possibility of detection of *CP* violation in the rare lepton-number-violating (LNV) semihadronic decays of charged pseudoscalar mesons.

In general, *CP* violation is expected in both cases, whether neutrinos are Dirac or Majorana particles. Nonetheless, in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [15,25], the number of possible *CP*-violating phases is larger when the neutrinos are Majorana particles. If n is the number of neutrino generations, the number of *CP*-violating phases is $n(n-1)/2$ in the Majorana case and $(n-1)(n-2)/2$ in the Dirac case, cf. Ref. [26].

In a recent work [27], we investigated the possibility of measuring the *CP* asymmetry in the rare leptonic decays of charged pions $\pi^{\pm} \to e^{\pm}e^{\pm}\mu^{\mp}\nu$. Both lepton-number-conserving (LNC) and lepton-number-violating (LNV) processes contribute to these decays and to the *CP* violation. We concluded that the *CP* violation is appreciable when these processes are mediated by two on-shell (Majorana or Dirac) sterile neutrinos N_1 and N_2 (i.e., with masses between 106 and 140 MeV), and that the *CP* violation effect is largest when these two neutrinos are almost degenerate in their masses. It is interesting that such neutrinos fall within the regime predicted by the νMSM model [20,28]. Further, they are not ruled out by experiments [11,29].

The νMSM model [20,28] contains two almost degenerate sterile Majorana neutrinos with mass between 100 MeV and a few GeV, and in addition a light sterile Majorana neutrino of mass $\sim 10^1$ keV and the three very light neutrinos. The model is well motivated because (a) it can explain simultaneously the pattern of light neutrino masses and oscillations, (b) it can explain the baryon

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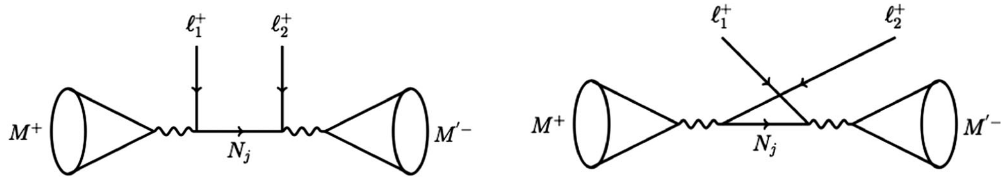

FIG. 1. The lepton-number-violating decay $M^+ \to \ell_1^+ \ell_2^+ M'^-$, e.g., with $M = K$ and $M' = \pi$. Left: The direct (D) channel. Right: The crossed (C) channel.

asymmetry of the Universe, and (c) it provides a dark matter candidate. We refer to Ref. [30] for reviews, and to Ref. [31] for the determination of the allowed range of the sterile neutrinos of the νMSM model. Remarkably, the tentative evidence of a dark matter line, recently discussed in Ref. [32], falls into the regime predicted for νMSM in Ref. [31]. It is interesting that the requirement that the lightest sterile neutrino be the dark matter candidate reduces the parameters of the model in such a way as to make the two heavier neutrinos nearly degenerate in mass. This, in turn, as demonstrated in Ref. [27], increases significantly the possible effects of CP violation.

Moreover, CERN-SPS has proposed a search of such heavy neutrinos, Ref. [33], in the leptonic and semihadronic decays of D, D_s mesons. As argued in Ref. [33] and in Refs. [6–14], such rare decays can have appreciable rates to be detected in future experiments (such as the experiment proposed at CERN-SPS).

In this work, we investigate such rare semihadronic decays of charged pseudoscalar mesons $M^\pm \to \ell_1^\pm \ell_2^\pm M'^\mp$, where $M = K, D, D_s, B, B_c$ and $M' = \pi, K, D, D_s$, and the charged leptons are $\ell_1, \ell_2 = e, \mu$. These decays are lepton-number violating, hence the neutrinos mediating them must be of Majorana type. We focus on signals of CP violation in such processes by working in scenarios with two on-shell sterile neutrinos N_1 and N_2; i.e., with masses M_{N_j} in the intervals $M_{M'} + M_{\ell_2} < M_{N_j} < M_M - M_{\ell_1}$. The signals of CP violation are represented by the CP-violating difference $S_-(M) \equiv [\Gamma(M^- \to \ell_1^- \ell_2^- M'^+) - \Gamma(M^+ \to \ell_1^+ \ell_2^+ M'^-)]$, and alternatively by the usual CP ratio $\mathcal{A}_{CP}(M) \equiv [\Gamma(M^- \to \ell_1^- \ell_2^- M'^+) - \Gamma(M^+ \to \ell_1^+ \ell_2^+ M'^-)]/[\Gamma(M^- \to \ell_1^- \ell_2^- M'^+) + \Gamma(M^+ \to \ell_1^+ \ell_2^+ M'^-)]$.

In Sec. II, we describe the formalism for calculation of the various decay widths. The details of the calculation are given in Appendix A, and the details for the total decay widths $\Gamma_N(M_N)$ of the (heavy) sterile Majorana neutrinos are given in Appendix B. In Sec. III, we present the expressions for the decay widths $S_\pm(M) \equiv [\Gamma(M^- \to \ell_1^- \ell_2^- M'^+) \pm \Gamma(M^+ \to \ell_1^- \ell_2^- M'^+)]$ and for the mentioned CP ratio $\mathcal{A}_{CP}(M)$. Additional details are given in Appendix C. In Sec. IV, we discuss the acceptance factor due to the (long) decay time of the on-shell sterile neutrinos and the resulting effective (i.e., experimental) branching ratios $\mathrm{Br}^{(\mathrm{eff})}(M)$ $[\propto S_+(M)]$ and $\mathcal{A}_{CP}(M)\mathrm{Br}^{(\mathrm{eff})}(M)$ $[\propto S_-(M)]$, and we present numerical results. In Sec. V, we summarize our results and present conclusions.

II. THE PROCESS AND FORMALISM FOR THE LNV SEMIHADRONIC DECAYS OF PSEUDOSCALARS

We consider the lepton-number-violating (LNV) processes, Fig. 1, $M^\pm \to \ell_1^\pm \ell_2^\pm M'^\mp$, where the two intermediate Majorana neutrinos (N_1, N_2) are on shell. The intermediate neutrinos have to be Majorana here, because these processes violate lepton number.

In such a case, the topology of these tree-level processes is like that of the "s channel." The processes with (two-loop) "t-channel" topology are strongly suppressed [9]. The type of processes in Fig. 1, within the models with sterile neutrinos N in the mass range of mesons, have been studied in several works, among them Refs. [6–14].

We denote the mixing coefficient for the heavy mass eigenstate N_j with the standard flavor neutrino ν_ℓ ($\ell = e, \mu, \tau$) as $B_{\ell N_j}$ ($j = 1, 2$).[1] The relevant mixing relations in our notation are

$$\nu_\ell = \sum_{k=1}^{3} B_{\ell \nu_k} \nu_k + (B_{\ell N_1} N_1 + B_{\ell N_2} N_2), \tag{1}$$

where ν_k ($k = 1, 2, 3$) are the light mass eigenstates, and the (unitary) PMNS matrix B is in this scenario a 5×5 matrix.[2]

We will use the phase conventions of Ref. [26]; i.e., all the CP-violating phases are incorporated in the PMNS matrix of mixing elements. The sum and difference of the decay widths, $S_\pm(M) \equiv [\Gamma(M^- \to \ell_1^- \ell_2^- M'^+) \pm \Gamma(M^+ \to \ell_1^- \ell_2^- M'^+)]$, of the processes of Fig. 1 will be appreciable only if the two intermediate neutrinos N_j are on shell:

$$\begin{aligned} &(M_{M'} + M_{\ell_2}) < M_{N_j} < (M_M - M_{\ell_1}), \quad \text{or/and} \\ &(M_{M'} + M_{\ell_1}) < M_{N_j} < (M_M - M_{\ell_2}). \end{aligned} \tag{2}$$

We will often use schematic notations for the decay widths of these rare processes:

[1]There exist also other notations for $B_{\ell N}$ in the literature, e.g., $U_{\ell N}$ in Ref. [12] and $V_{\ell N}$ in Ref. [11].

[2]In our work, B can be an $n \times n$ matrix with $n \geq 5$. If $n > 5$, we implicitly assume that the additional sterile neutrinos (N_3, etc.) have significantly less mixing than N_1 and N_2 with the active flavor ("light") neutrino sector; one such framework is νMSM [20,28,30], with $n = 6$.

$$\Gamma(M^{\pm}) \equiv \Gamma(M^{\pm} \to \ell_1^{\pm}\ell_2^{\pm}M'^{\mp}). \tag{3}$$

These decay widths can be written in the form

$$\Gamma(M^{\pm}) = (2-\delta_{\ell_1\ell_2})\frac{1}{2!}\frac{1}{2M_M}\frac{1}{(2\pi)^5}\int d_3|\mathcal{T}(M^{\pm})|^2, \tag{4}$$

where $1/2!$ is the symmetry factor when the two charged leptons are equal. Here, $|\mathcal{T}(M^{\pm})|^2$ is the absolute square (summed over the final helicities) of the sum of amplitudes from N_1 and N_2 neutrinos in the two channels D (direct) and C (crossed). We refer to Appendix A for details. In Eq. (4), d_3 denotes the integration over the three-particle final phase space

$$d_3 \equiv \frac{d^3\vec{p}_1}{2E_{\ell_1}(\vec{p}_1)}\frac{d^3\vec{p}_2}{2E_{\ell_2}(\vec{p}_2)}\frac{d^3\vec{p}_{M'}}{2E_{M'}(\vec{p}_{M'})} \times \delta^{(4)}(p_M - p_1 - p_2 - p_{M'}). \tag{5}$$

We denote by p_1 and p_2 the momenta of ℓ_1 and ℓ_2 from the left and the right vertices of the direct channels, respectively (in the crossed channel, ℓ_2 couples to the left vertex), cf. Fig. 1. The decay widths [Eq. (4)] can then be written as a double sum over the contributions of N_i and N_j exchanges ($i,j = 1,2$), with the mixing effects factored out:

$$\Gamma(M^{\pm}) = (2-\delta_{\ell_1\ell_2})\sum_{i=1}^{2}\sum_{j=1}^{2}k_i^{(\pm)}k_j^{(\pm)*}[\bar{\Gamma}(DD^*)_{ij} + \bar{\Gamma}(CC^*)_{ij} + \bar{\Gamma}_{\pm}(DC^*)_{ij} + \bar{\Gamma}_{\pm}(CD^*)_{ij}], \tag{6}$$

where $k_j^{(\pm)}$ are the corresponding mixing factors

$$k_j^{(-)} = B_{\ell_1 N_j}B_{\ell_2 N_j}, \qquad k_j^{(+)} = (k_j^{(-)})^*, \tag{7}$$

and $\bar{\Gamma}_{\pm}(XY^*)_{ij}$ are the normalized (i.e., without the mixing) contributions of N_i exchange in the X channel and the complex conjugate of the N_j exchange in the Y channel ($X,Y = C,D$):

$$\bar{\Gamma}_{\pm}(XY^*)_{ij} \equiv K^2\frac{1}{2!}\frac{1}{2M_M}\frac{1}{(2\pi)^5} \times \int d_3 P_i(X)P_j(Y)^* M_{N_i}M_{N_j}T_{\pm}(X)T_{\pm}(Y)^*. \tag{8}$$

Here, $T_{\pm}(X)$ ($X = D,C$) are the relevant parts of the amplitude in the X channel, which appear also in the total decay amplitudes $\mathcal{T}_{\pm}$ (see Appendix A),[3] and $P_j(X)$ ($X = D,C$) are the propagators of the intermediate neutrinos N_j in the two channels

$$P_j(D) = \frac{1}{[(p_M - p_1)^2 - M_{N_j}^2 + i\Gamma_{N_j}M_{N_j}]}, \tag{9a}$$

$$P_j(C) = \frac{1}{[(p_M - p_2)^2 - M_{N_j}^2 + i\Gamma_{N_j}M_{N_j}]}. \tag{9b}$$

The overall constant K^2 appearing in Eq. (8) is

$$K^2 = G_F^4 f_M^2 f_{M'}^2 |V_{Q_uQ_d}V_{q_uq_d}|^2, \tag{10}$$

where f_M and $f_{M'}$ are the decay constants of $M^{\pm}$ and $M'^{\mp}$, and $V_{Q_uQ_d}$ and $V_{q_uq_d}$ are the CKM elements corresponding to $M^{\pm}$ and $M'^{\mp}$ (the valence quark content of M^+ is $Q_u\bar{Q}_d$; that of M'^+ is $q_u\bar{q}_d$).

Several symmetry relations exist among the normalized decay widths $\bar{\Gamma}_{\pm}(XY^*)_{ij}$, as given in Eqs. (A6) and (A7) in Appendix A. The most important symmetry property is that the (2×2) matrices $\bar{\Gamma}(DD^*)$ and $\bar{\Gamma}(CC^*)$ are self-adjoint (and even equal if $\ell_1 = \ell_2$). The matrices $\bar{\Gamma}_{\pm}(DC^*)$ and $\bar{\Gamma}_{\pm}(CD^*)$, which represent the (normalized) D-C channel interference contributions to the decay widths $\Gamma(M^{\pm})$, will turn out to be several orders of magnitude smaller than the $\bar{\Gamma}(DD^*)$ and $\bar{\Gamma}(CC^*)$ matrices.

In our calculations, we will also need to know the total decay width $\Gamma(N_j \to \text{all}) \equiv \Gamma_{N_j}$ of the two Majorana neutrinos N_j as a function of the mass M_{N_j}, or more specifically, the corresponding mixing factor $\tilde{\mathcal{K}}_j$. The width Γ_{N_j} can be written as

$$\Gamma_{N_j} = \tilde{\mathcal{K}}_j\bar{\Gamma}(M_{N_j}), \tag{11}$$

where

$$\bar{\Gamma}(M_{N_j}) \equiv \frac{G_F^2 M_{N_j}^5}{96\pi^3}, \tag{12}$$

and the factor $\tilde{\mathcal{K}}_j$ includes the heavy-light mixing factors dependence

$$\tilde{\mathcal{K}}_j(M_{N_j}) \equiv \tilde{\mathcal{K}}_j = \mathcal{N}_{eN_j}|B_{eN_j}|^2 + \mathcal{N}_{\mu N_j}|B_{\mu N_j}|^2 + \mathcal{N}_{\tau N_j}|B_{\tau N_j}|^2), \qquad (j = 1,2). \tag{13}$$

Here, $\mathcal{N}_{\ell N}(M_N) \equiv \mathcal{N}_{\ell N}$ ($\ell = e,\mu,\tau$) are the effective mixing coefficients; they are numbers $\sim 10^0$–10^1 which depend on the mass M_N of the Majorana neutrino N ($N = N_1, N_2$). In Appendix B, we write down the relevant formulas for the calculation of these coefficients. The results of these calculations are given in Fig. 2, for the here-relevant neutrino mass interval $0.1\ \text{GeV} < M_N < 6.3\ \text{GeV}$. Some additional remarks are given in Appendix B.

[3] Since $|T_+(D)|^2 = |T_-(D)|^2$ and $|T_+(C)|^2 = |T_-(C)|^2$, we omit the subscripts $\pm$ from the DD^* and CC^* contribution terms $\bar{\Gamma}(DD^*)_{ij}$ and $\bar{\Gamma}(CC^*)_{ij}$ in Eq. (6).

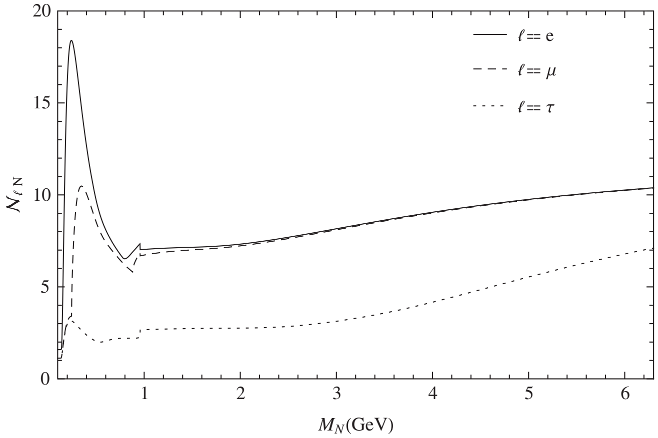

FIG. 2. The effective mixing coefficients $\mathcal{N}_{\ell N}$ ($\ell = e, \mu, \tau$) appearing in Eqs. (11)–(13) as a function of the mass M_N of the Majorana neutrino N. See the text and Appendix B for details.

On the other hand, the present upper bounds for the squares $|B_{\ell N}|^2$ of the heavy-light mixing matrix elements, in our range of interest 0.1 GeV $< M_N <$ 6.3 GeV, can be inferred from Ref. [11] (and references therein). The present upper bounds for $|B_{eN}|^2$, in the mentioned range of M_N, are largely determined by the neutrinoless double beta decay experiments [34,35] ($0\nu\beta\beta$). The upper bounds for $|B_{\mu N}|^2$ come from searches of peaks in the spectrum of μ in pion and kaon decays [36] and from decay searches [36–39]. The upper bounds for $|B_{\tau N}|^2$ come from CC interactions (if τ is produced) and from NC interactions [39,40]. In Table I, we present the upper bounds on $|B_{\ell N}|^2$ for specific chosen values of M_N in the mentioned integral. The upper bounds have in some cases strong dependence on the precise values of M_N, and for further details we refer to the corresponding figures in Ref. [11].

III. THE DECAY WIDTHS AND *CP* ASYMMETRY FOR THE LNV SEMIHADRONIC DECAYS OF PSEUDOSCALARS

Here we will use the results of Sec. II, and a combination of analytic and numerical evaluations, in order to obtain the results for the decay widths $S_\pm$ and the CP asymmetry ratios $\mathcal{A}_{CP}$ of the discussed semihadronic LNV decays of pseudoscalar mesons $M^\pm$

$$S_\pm(M) \equiv \Gamma(M^-) \pm \Gamma(M^+), \tag{14}$$

$$\mathcal{A}_{CP}(M) \equiv \frac{S_-(M)}{S_+(M)} \equiv \frac{\Gamma(M^-) - \Gamma(M^+)}{\Gamma(M^-) + \Gamma(M^+)}, \tag{15}$$

where we use the notations of Eq. (3). $S_+(M)$ represents the total (sum) of the decay widths of M^+ and M^- for these rare LNV decays, while $S_-(M)$ is the corresponding (CP-violating) difference. The ratio $\mathcal{A}_{CP}(M)$ in Eq. (15) is the usual measure of the relative CP violation effect. We adopt the convention $M_{N_2} > M_{N_1}$ and introduce the following notations related with the heavy-light neutrino mixing elements $B_{\ell_1 N_j}$ and $B_{\ell_2 N_j}$ and their phases:

$$\kappa_{\ell_1} = \frac{|B_{\ell_1 N_2}|}{|B_{\ell_1 N_1}|}, \qquad \kappa_{\ell_2} = \frac{|B_{\ell_2 N_2}|}{|B_{\ell_2 N_1}|}, \tag{16a}$$

$$B_{\ell_k N_j} \equiv |B_{\ell_k N_j}| e^{i\phi_{kj}} \qquad (k, j = 1, 2), \tag{16b}$$

$$\theta_{ij} = (\phi_{1i} + \phi_{2i} - \phi_{1j} - \phi_{2j})(i, j = 1, 2). \tag{16c}$$

For example, if $\ell_1 = \ell_2 = \mu$, then $\theta_{21} = 2(\phi_{\mu 2} - \phi_{\mu 1}) = 2(\arg(B_{\mu N_2}) - \arg(B_{\mu N_1}))$. Here we will not write explicitly the D-C channel interference contributions to the quantities in Eqs. (14) and (15), as our numerical calculations give contributions which are several orders of magnitude smaller than the contributions from the D channel and from the C channel.

The resulting sums $S_+(M) \equiv (\Gamma(M^-) + \Gamma(M^+))$ of the decay widths can then be written in terms of only the normalized decay widths $\bar{\Gamma}(XX^*)_{11}$, $\bar{\Gamma}(XX^*)_{22}$, and $\mathrm{Re}\bar{\Gamma}(XX^*)_{12}$ (where $X = D; C$), and in terms of the phase difference θ_{21}:

TABLE I. Present upper bounds for the squares $|B_{\ell N}|^2$ of the heavy-light mixing matrix elements, for various specific values of M_N.

M_N[GeV]	$\vert B_{eN}\vert^2$	$\vert B_{\mu N}\vert^2$	$\vert B_{\tau N}\vert^2$
0.1	$(1.5 \pm 0.5) \times 10^{-8}$	$(6.0 \pm 0.5) \times 10^{-6}$	$(8.0 \pm 0.5) \times 10^{-4}$
0.3	$(2.5 \pm 0.5) \times 10^{-9}$	$(3.0 \pm 0.5) \times 10^{-9}$	$(1.5 \pm 0.5) \times 10^{-1}$
0.5	$(2.0 \pm 0.5) \times 10^{-8}$	$(6.5 \pm 0.5) \times 10^{-7}$	$(2.5 \pm 0.5) \times 10^{-2}$
0.7	$(3.5 \pm 0.5) \times 10^{-8}$	$(2.5 \pm 0.5) \times 10^{-7}$	$(9.0 \pm 0.5) \times 10^{-3}$
1.0	$(4.5 \pm 0.5) \times 10^{-8}$	$(1.5 \pm 0.5) \times 10^{-7}$	$(3.0 \pm 0.5) \times 10^{-3}$
2.0	$(1.0 \pm 0.5) \times 10^{-7}$	$(2.5 \pm 0.5) \times 10^{-5}$	$(3.0 \pm 0.5) \times 10^{-4}$
3.0	$(1.5 \pm 0.5) \times 10^{-7}$	$(2.5 \pm 0.5) \times 10^{-5}$	$(4.5 \pm 0.5) \times 10^{-5}$
4.0	$(2.5 \pm 0.5) \times 10^{-7}$	$(1.5 \pm 0.5) \times 10^{-5}$	$(1.5 \pm 0.5) \times 10^{-5}$
5.0	$(3.0 \pm 0.5) \times 10^{-7}$	$(1.5 \pm 0.5) \times 10^{-5}$	$(1.5 \pm 0.5) \times 10^{-5}$
6.0	$(3.5 \pm 0.5) \times 10^{-7}$	$(1.5 \pm 0.5) \times 10^{-5}$	$(1.5 \pm 0.5) \times 10^{-5}$

$$S_+(M) \equiv (\Gamma(M^-) + \Gamma(M^+)) = 2(2-\delta_{\ell_1\ell_2})|B_{\ell_1 N_1}|^2|B_{\ell_2 N_1}|^2\Bigg\{\bar{\Gamma}(DD^*)_{11} \times\left[1+\kappa_{\ell_1}^2\kappa_{\ell_2}^2\frac{\bar{\Gamma}(DD^*)_{22}}{\bar{\Gamma}(DD^*)_{11}} + 2\kappa_{\ell_1}\kappa_{\ell_2}\cos\theta_{21}\delta_1\right] + \bar{\Gamma}(CC^*)_{11}\left[1+\kappa_{\ell_1}^2\kappa_{\ell_2}^2\frac{\bar{\Gamma}(CC^*)_{22}}{\bar{\Gamma}(CC^*)_{11}} + 2\kappa_{\ell_1}\kappa_{\ell_2}\cos\theta_{21}\delta_1\right] + (D-C\,\text{terms})\Bigg\}, \quad (17)$$

where we use the notations of Eq. (16), and the quantity δ_1 measures the effect of N_1-N_2 overlap contributions,

$$\delta_j \equiv \frac{\mathrm{Re}\bar{\Gamma}(XX^*)_{12}}{\bar{\Gamma}(XX^*)_{jj}} \qquad (X = D; C; \qquad j = 1; 2). \quad (18)$$

It is expected that $\delta_j \approx 0$ when $\Delta M_N \gg \Gamma_{N_j}$, because in such a case the overlap (interference) effects of the N_1 and N_2 exchanges are expected to be absent due to a large distance between the two "bumps" of the neutrino propagators. Numerical evaluations confirm this expectation and confirm that δ_j is practically independent of the channel $X = D, C$ (see later on in this section).

The (CP-violating) difference $S_-(M) \equiv (\Gamma(M^-) - \Gamma(M^+))$ of the LNV rare decays is

$$S_-(M) \equiv (\Gamma(M^-) - \Gamma(M^+)) = 4(2-\delta_{\ell_1\ell_2})|B_{\ell_1 N_1}||B_{\ell_2 N_1}||B_{\ell_1 N_2}||B_{\ell_2 N_2}| \times\{\sin\theta_{21}[\mathrm{Im}\bar{\Gamma}(DD^*)_{12} + \mathrm{Im}\bar{\Gamma}(CC^*)_{12}] + (D-C\,\text{terms})\}. \quad (19)$$

We can see that CP violation in these decays is proportional to the CP-odd phase difference θ_{21} defined in Eq. (16c). The other factor in this CP violation is the imaginary part of $\bar{\Gamma}(DD^*)_{12} + \bar{\Gamma}(CC^*)_{12}$; this factor will be investigated later on in this section.

The decay widths Γ_{N_j} are very small in comparison with the masses M_{N_j} due to the mixing suppression, cf. Eqs. (11)–(13) (in general, $\Gamma_{N_j} \ll 1$ eV). Therefore, the absolute value of the square of the intermediate neutrino propagator can be approximated to a high degree of accuracy by the delta function

$$|P_j(D)|^2 = \left|\frac{1}{(p_M - p_1)^2 - M_{N_j}^2 + i\Gamma_{N_j}M_{N_j}}\right|^2 \approx \frac{\pi}{M_{N_j}\Gamma_{N_j}}\delta((p_M-p_1)^2 - M_{N_j}^2); \quad (j = 1, 2; \Gamma_{N_j} \ll M_{N_j}), \quad (20)$$

with an analogous equation for $|P_j(C)|^2$. Therefore, in the integration d_3, the part of integration dp_N^2 ($p_N = p_M - p_1$ in the D channel; $p_N = p_M - p_2$ in the C channel) becomes a trivial integration over a delta function, and the expressions for the diagonal elements $\bar{\Gamma}(DD^*)_{jj}$ and $\bar{\Gamma}(CC^*)_{jj}$ can be calculated analytically, cf. Appendix C,

$$\bar{\Gamma}(DD^*)_{jj} = \frac{K^2 M_M^5}{128\pi^2}\frac{M_{N_j}}{\Gamma_{N_j}}\lambda^{1/2}(1, x_j, x_{\ell_1})\lambda^{1/2}\left(1, \frac{x'}{x_j}, \frac{x_{\ell_2}}{x_j}\right) \times Q(x_j; x_{\ell_1}, x_{\ell_2}, x') \qquad (j = 1 \quad \text{or} \quad j = 2), \quad (21)$$

and $\bar{\Gamma}(CC^*)_{jj}$ is obtained from Eq. (21) by the simple exchange $x_{\ell_1} \leftrightarrow x_{\ell_2}$:

$$\bar{\Gamma}(CC^*)_{jj} = \bar{\Gamma}(DD^*)_{jj}(x_{\ell_1} \leftrightarrow x_{\ell_2}). \quad (22)$$

In Eq. (21) we use the notations

$$\lambda(y_1, y_2, y_3) = y_1^2 + y_2^2 + y_3^2 - 2y_1y_2 - 2y_2y_3 - 2y_3y_1, \quad (23a)$$

$$x_j = \frac{M_{N_j}^2}{M_M^2}, \qquad x_{\ell_s} = \frac{M_{\ell_s}^2}{M_M^2}, \qquad x' = \frac{M_{M'}^2}{M_M^2}, \quad (j = 1, 2; \ell_s = \ell_1, \ell_2), \quad (23b)$$

and the function $Q(x_j; x_{\ell_1}, x_{\ell_2}, x')$ is given in Appendix C. In the special case $\ell_1 = \ell_2$, the expression for $\bar{\Gamma}(DD^*)_{jj}$ is somewhat simpler and can be deduced, e.g., from Ref. [13]. Equations (21) and (22) are used in the evaluation of the sum $S_+(M)$, Eq. (17), of the rare decay widths of $M^\pm$. In Eq. (17), the contributions of the N_1-N_2 overlap effects are parametrized in the function δ_1 defined in Eq. (18), and will be evaluated later on numerically.

In order to evaluate the CP-violating difference $S_-(M)$ [Eq. (19)] of the rare decay widths $M^\pm$, the evaluation of the quantity $\mathrm{Im}\bar{\Gamma}(XX^*)_{12}$ ($X = D; C$) is of central importance. In the integrand of $\mathrm{Im}\bar{\Gamma}(XX^*)_{12}$ we have, according to Eq. (8), as a factor the following combination of the propagators of N_1 and N_2:

$$\mathrm{Im}P_1(D)P_2(D)^* = \frac{(p_N^2 - M_{N_1}^2)\Gamma_{N_2}M_{N_2} - \Gamma_{N_1}M_{N_1}(p_N^2 - M_{N_2}^2)}{[(p_N^2 - M_{N_1}^2)^2 + \Gamma_{N_1}^2 M_{N_1}^2][(p_N^2 - M_{N_2}^2)^2 + \Gamma_{N_2}^2 M_{N_2}^2]} \quad (24a)$$

$$\approx \mathcal{P}\left(\frac{1}{p_N^2 - M_{N_1}^2}\right)\pi\delta(p_N^2 - M_{N_2}^2) - \pi\delta(p_N^2 - M_{N_1}^2)\mathcal{P}\left(\frac{1}{p_N^2 - M_{N_2}^2}\right) \quad (24b)$$

$$= \frac{\pi}{M_{N_2}^2 - M_{N_1}^2}[\delta(p_N^2 - M_{N_2}^2) + \delta(p_N^2 - M_{N_1}^2)], \quad (24c)$$

TABLE II. Values of $\delta(y)$, $\eta(y)$, and $\eta(y)/y$ correction parameters as a function of $y \equiv \Delta M_N/\Gamma_N$.

$y \equiv \frac{\Delta M_N}{\Gamma_N}$	$\log_{10} y$	$\delta(y)$	$\eta(y)$	$\frac{\eta(y)}{y}$
1.00	0.000	0.500 ± 0.004	0.500 ± 0.001	0.500 ± 0.001
1.25	0.097	0.390 ± 0.003	0.610 ± 0.003	0.488 ± 0.002
1.67	0.222	0.264 ± 0.003	0.736 ± 0.002	0.441 ± 0.001
2.50	0.398	0.138 ± 0.001	0.862 ± 0.001	0.345 ± 0.001
5.00	0.699	0.038 ± 0.001	0.962 ± 0.002	0.192 ± 0.001
10.0	1.000	0.0098 ± 0.0010	0.990 ± 0.002	$0.0990 \pm 2 \times 10^{-4}$

where we have $p_N = (p_M - p_1)$ in the direct (D) channel. In Eqs. (24b) and (24c), we assumed $\Gamma_{N_j} \ll |\Delta M_N| \equiv M_{N_2} - M_{N_1}$. The expression (24) has formally the same structure with Dirac delta functions as Eq. (20), but the factors in front of these Dirac delta functions are different now. Hence, we can perform the integration over the final particle phase space in the same way, but now under the more stringent assumption $\Gamma_{N_j} \ll |\Delta M_N|$ (and not just $\Gamma_{N_j} \ll M_{N_j}$, which is always fulfilled),[4] leading to the result

$$\mathrm{Im}\bar{\Gamma}(DD^*)_{12} = \eta \frac{K^2 M_M^5}{128\pi^2} \frac{M_{N_1} M_{N_2}}{(M_{N_2} + M_{N_1})\Delta M_N} \times \sum_{j=1}^{2} \lambda^{1/2}(1, x_j, x_{\ell_1}), \lambda^{1/2}\left(1, \frac{x'}{x_j}, \frac{x_{\ell_2}}{x_j}\right) Q(x_j; x_{\ell_1}, x_{\ell_2}, x'), \tag{25a}$$

$$\mathrm{Im}\bar{\Gamma}(CC^*)_{12} = \mathrm{Im}\bar{\Gamma}(DD^*)_{12}(x_{\ell_1} \leftrightarrow x_{\ell_2}), \tag{25b}$$

where we denoted $\Delta M_N \equiv M_{N_2} - M_{N_1} > 0$. In Eq. (25), we introduced an overall factor η which accounts for the effects $\Delta M_N \ggg \Gamma_N$; i.e., for the situation when the approximation in Eq. (24b) of $\mathrm{Im} P_1(D) P_2(D)^*$ in terms of Dirac delta functions is not justified. Later on in this section, we will evaluate numerically the factor η. When $\Delta M_N \gg \Gamma_{N_j}$, i.e., when the identity in Eq. (24b) can be applied, the factor η is equal to unity, $\eta = 1$.

The normalized decay matrix elements $\bar{\Gamma}(XY^*)_{ij}$, Eq. (8), were evaluated also numerically, by versions of Monte Carlo integration, independently by the two authors, using finite (small) widths Γ_{N_j} in the propagators. We confirmed numerically the analytic expression [Eq. (21)] for $\bar{\Gamma}^{(X)}(DD^*)_{jj}$ ($\propto 1/\Gamma_{N_j}$), as well as the analytic expression [Eq. (25)] with $\eta = 1$ for $\mathrm{Im}\bar{\Gamma}(DD^*)_{12}$ ($\propto 1/\Delta M_N$) when $\Delta M_N \gg \Gamma_{N_j}$.

Further, our numerical evaluations lead us to the conclusion that the direct-crossed channel (DC^* and CD^*) interference contributions to the sum and the difference of the rare decay widths $S_\pm(M)$ of $M^\pm$ are by several orders of magnitude smaller than the corresponding direct (DD^*) and crossed (CC^*) channel contributions to these quantities, in all cases.[5]

In addition, our numerical evaluations give us values of the parameters δ_j of Eq. (18), and of the η correction parameters of Eq. (25). In the cases when $\Delta M_N \ggg \Gamma_{N_j}$, these values differ appreciably from their limiting values $\delta_j = 0$ and $\eta = 1$ of the $\Delta M_N \gg \Gamma_{N_j}$ limit. It turns out that the parameters δ_j are practically independent of the channel contribution considered (DD^* or CC^*), and of the type of pseudoscalar mesons ($M^\pm$, $M'^\mp$), and of the light leptons ($\ell_1, \ell_2 = e, \mu$) involved in the considered decays, and the same is true for the parameter η. Further, numerical calculations show that, in the considered case $\Delta M_N \ggg \Gamma_{N_j}$ (i.e., when N_1 and N_2 are almost degenerate), the parameters η and $\delta \equiv (1/2)(\delta_1 + \delta_2)$ are functions of only one parameter $y \equiv \Delta M_N/\Gamma_N$, where $\Delta M_N \equiv M_{N_2} - M_{N_1}$ (> 0) and $\Gamma_N = (1/2)(\Gamma_{N_1} + \Gamma_{N_2})$:

[4]We note that this mechanism is central to the CP violation effects in the considered LNV semihadronic decays of charged pseudoscalar mesons. This mechanism was presented in Ref. [27] and applied there to the CP violation of the rare leptonic decays of charged pions.

[5]For example, when $M^\pm = K^\pm$ and $M'^\mp = \pi^\mp$, and we choose in numerical calculation $\Gamma_N \sim 10^{-3}$ GeV $\sim \Delta M_N$, the $\bar{\Gamma}(DD^*)_{ij}$ and $\bar{\Gamma}(CC^*)_{ij}$ contributions are about 2 orders of magnitude larger than the D-C interference contributions $\bar{\Gamma}_\pm(DC^*)_{ij}$. When Γ_N and ΔM_N are decreased further ($\Gamma_N \sim \Delta M_N$), the $\bar{\Gamma}(DD^*)_{ij}$ and $\bar{\Gamma}(CC^*)_{ij}$ contributions increase (they are $\propto 1/\Gamma_N$, or $\propto 1/\Delta M_N$), while the D-C interference contributions $\bar{\Gamma}_\pm(DC^*)_{ij}$ remain approximately unchanged and become thus relatively insignificant.

$$\eta = \eta(y), \qquad y \equiv \frac{\Delta M_N}{\Gamma_N}, \qquad \Gamma_N \equiv \frac{1}{2}(\Gamma_{N_1} + \Gamma_{N_2}), \tag{26a}$$

$$\delta = \delta(y), \qquad \delta \equiv \frac{1}{2}(\delta_1 + \delta_2),$$
$$\frac{\delta_1}{\delta_2} = \frac{\bar{\Gamma}(DD^*)_{22}}{\bar{\Gamma}(DD^*)_{11}} = \frac{\Gamma_{N_1}}{\Gamma_{N_2}} = \frac{\tilde{\mathcal{K}}_1}{\tilde{\mathcal{K}}_2}. \tag{26b}$$

The numerical integration gives us these values, which are tabulated in Table II as a function of y. The uncertainties indicate the numerical uncertainties and the small variations from the various considered LNV semihadronic decays $M^\pm \to \ell_1^\pm \ell_2^\pm M'^\mp$, where M and M' are pseudoscalar mesons, $M = K, D, D_s, B, B_c$ and $M' = \pi, K, D, D_s$, and the charged leptons are $\ell_1, \ell_2 = e, \mu$. It is interesting that the values in Table II are almost equal to the values of the parameters $\delta(y)$ and $\eta(y)$ for the rare leptonic decays of the charged pions $\pi^\pm \to e^\pm N \to e^\pm e^\pm \mu^\mp \nu$ (Ref. [27]). The uncertainties in the present table are in general smaller, though, because of the high statistics applied in Monte Carlo calculations, which practically eliminates the numerical uncertainty part.

The rare LNV semihadronic decay widths of $M^\pm$, cf. $S_+(M)$ of Eq. (17), at first sight appear to be quartic in the heavy-light mixing elements $|B_{\ell N}|$, and thus very suppressed. However, they are proportional to the expressions $\bar{\Gamma}(DD^*)_{jj}$ in Eq. (21), which are proportional to $1/\Gamma_{N_j}$ due to the on-shell-ness of the intermediate N_j's [cf. also Eq. (20)]. This $1/\Gamma_{N_j}$ is proportional to $1/\tilde{\mathcal{K}}_j \sim 1/|B_{\ell N_j}|^2$ according to Eqs. (11)–(13). Hence, this on-shell-ness of N_j's makes these rare process decay widths significantly less suppressed:

$$\bar{\Gamma}(DD^*)_{jj} \propto 1/\Gamma_{N_j} \propto 1/\tilde{\mathcal{K}}_j \propto 1/|B_{\ell N_j}|^2 \Rightarrow$$
$$S_+(M) \propto |B_{\ell N_j}|^2. \tag{27}$$

However, the expressions in Eq. (25), which appear in the CP-violating decay width difference $S_-(M)$ [Eq. (19)], are suppressed by mixings as $\sim |B_{\ell N}|^4$. This means that in general, $S_-(M)$ is much smaller than the decay width $S_+(M) \propto |B_{\ell N_j}|^2$. Nonetheless, Eq. (25) shows that $S_-(M)$ is proportional to $1/\Delta M_N$, and it is this aspect that represents the opportunity to detect appreciable CP violation in such decays when ΔM_N is sufficiently small. While in general we expect $\Delta M_N \gg \Gamma_{N_j}$, there exists a well-motivated model [20,28,30] with two sterile, almost degenerate neutrinos (where the relation $\Delta M_N \not\gg \Gamma_{N_j}$ is possible) in the mass range 0.1 GeV $\lesssim M_{N_j} \lesssim 10^1$ GeV. Our calculations thus suggest that in such a model the CP violation effects may be appreciable—namely, for $\Delta M_N \sim \Gamma_N$, we obtain $S_-(M) \sim S_+(M)$ and thus $\mathcal{A}_{CP}(M) \sim 1$.

For these reasons, from now on we consider the case of near degeneracy: $\Delta M_N \not\gg \Gamma_N$ (i.e., $\Delta M_N \sim \Gamma_N$). In this case, several formulas written by now in this section become even more simplified—in particular, the expressions (21), (18), and (25). Namely, they can be written in terms of the common canonical decay width $\bar{S}$ ratio

$$\bar{S}(x; x_{\ell_1}, x_{\ell_2}, x') \equiv \frac{3\pi}{4} \frac{K^2 M_M}{G_F^2} \frac{1}{x^2} \lambda^{1/2}(1, x, x_{\ell_1}) \lambda^{1/2}\left(1, \frac{x'}{x}, \frac{x_{\ell_2}}{x}\right) Q(x; x_{\ell_1}, x_{\ell_2}, x'), \tag{28}$$

where we use the notations of Eq. (23) and

$$x \equiv \frac{M_N^2}{M_M^2} \equiv x_2 \approx x_1, \tag{29}$$

where $M_N \equiv M_{N_2} \approx M_{N_1}$. The function Q is the same as in Eqs. (21) and (25), and is given explicitly in Appendix C. In practice, we will need two variants of this function $\bar{S}$, namely the one for the DD^* contributions ($\bar{S}^{(D)}$) and the one for the CC^* contributions ($\bar{S}^{(C)}$):

$$\bar{S}^{(D)}(x) \equiv \bar{S}(x; x_{\ell_1}, x_{\ell_2}, x'), \tag{30a}$$

$$\bar{S}^{(C)}(x) \equiv \bar{S}(x; x_{\ell_2}, x_{\ell_1}, x'). \tag{30b}$$

When $\ell_1 = \ell_2$ (e.g., when both final leptons are electrons, or both are muons), the two functions $\bar{S}^{(D)}$ and $\bar{S}^{(C)}$ coincide. It is straightforward to check that the expressions of Eqs. (21), (18), and (25) can then be rewritten in the considered case of nearly degenerate N_1 and N_2, in terms of these common functions $\bar{S}^{(X)}$ $(X = D, C)$ and of the heavy-light mixing expressions $\tilde{\mathcal{K}}_j$ $(\sim |B_{\ell N_j}|^2)$ of Eq. (13),

$$\bar{\Gamma}(DD^*)_{jj} = \frac{1}{\tilde{\mathcal{K}}_j} \bar{S}^{(D)}(x),$$
$$\bar{\Gamma}(CC^*)_{jj} = \frac{1}{\tilde{\mathcal{K}}_j} \bar{S}^{(C)}(x), \tag{31a}$$

$$\mathrm{Re}\bar{\Gamma}(DD^*)_{12} = \delta(y) \frac{2}{(\tilde{\mathcal{K}}_1 + \tilde{\mathcal{K}}_2)} \bar{S}^{(D)}(x),$$
$$\mathrm{Re}\bar{\Gamma}(CC^*)_{12} = \delta(y) \frac{2}{(\tilde{\mathcal{K}}_1 + \tilde{\mathcal{K}}_2)} \bar{S}^{(C)}(x), \tag{31b}$$

$$\mathrm{Im}\bar{\Gamma}(DD^*)_{12} = \frac{\eta(y)}{y}\frac{2}{(\tilde{\mathcal{K}}_1+\tilde{\mathcal{K}}_2)}\bar{S}^{(D)}(x), \qquad \mathrm{Im}\bar{\Gamma}(CC^*)_{12} = \frac{\eta(y)}{y}\frac{2}{(\tilde{\mathcal{K}}_1+\tilde{\mathcal{K}}_2)}\bar{S}^{(C)}(x), \tag{31c}$$

where the definition $y \equiv \Delta M_N/\Gamma_N$ is kept.

After some straightforward algebra, we can rewrite the sum and difference $S_\pm(M)$ of decay widths [Eq. (14)] as expressions proportional to these canonical decay widths $\bar{S}^{(X)}$ $(X = D, C)$. The proportionality factors involve the heavy-light mixing factors $|B_{\ell N_j}|$ and $\tilde{\mathcal{K}}_j$ [cf. Eq. (13)] and the overlap functions $\delta(y)$ and $\eta(y)/y$ tabulated in Table II. The resulting expressions are

$$\begin{aligned} S_+(M) &\equiv \Gamma(M^- \to \ell_1^-\ell_2^- M'^+) + \Gamma(M^+ \to \ell_1^+\ell_2^+ M'^-) \\ &= 2(2-\delta_{\ell_1\ell_2})\left[\sum_{j=1}^{2}\frac{|B_{\ell_1 N_j}|^2|B_{\ell_2 N_j}|^2}{\tilde{\mathcal{K}}_j} + 4\delta(y)\frac{|B_{\ell_1 N_1}||B_{\ell_2 N_1}||B_{\ell_1 N_2}||B_{\ell_2 N_2}|}{(\tilde{\mathcal{K}}_1+\tilde{\mathcal{K}}_2)}\cos\theta_{21}\right](\bar{S}^{(D)}(x)+\bar{S}^{(C)}(x)), \end{aligned} \tag{32a}$$

$$\begin{aligned} S_-(M) &\equiv \Gamma(M^- \to \ell_1^-\ell_2^- M'^+) - \Gamma(M^+ \to \ell_1^+\ell_2^+ M'^-) \\ &= 8(2-\delta_{\ell_1\ell_2})\frac{|B_{\ell_1 N_1}||B_{\ell_2 N_1}||B_{\ell_1 N_2}||B_{\ell_2 N_2}|}{(\tilde{\mathcal{K}}_1+\tilde{\mathcal{K}}_2)}\sin\theta_{21}\frac{\eta(y)}{y}(\bar{S}^{(D)}(x)+\bar{S}^{(C)}(x)). \end{aligned} \tag{32b}$$

The resulting CP violation ratio $\mathcal{A}_{CP}(M)$ [Eq. (15)] can then be written in a form involving only the heavy-light mixing factors $|B_{\ell N_j}|$ and $\tilde{\mathcal{K}}_j$ [cf. Eq. (13)] and the overlap functions $\delta(y)$ and $\eta(y)/y$ tabulated in Table II:

$$\begin{aligned} \mathcal{A}_{CP}(M) &\equiv \frac{S_-(M)}{S_+(M)} \equiv \frac{\Gamma(M^- \to \ell_1^-\ell_2^- M'^+) - \Gamma(M^+ \to \ell_1^+\ell_2^+ M'^-)}{\Gamma(M^- \to \ell_1^-\ell_2^- M'^+) + \Gamma(M^+ \to \ell_1^+\ell_2^+ M'^-)} \\ &= \frac{\sin\theta_{21}}{\left[\frac{1}{4}\sum_{j=1}^{2}\frac{|B_{\ell_1 N_j}|^2|B_{\ell_2 N_j}|^2}{|B_{\ell_1 N_1}||B_{\ell_2 N_1}||B_{\ell_1 N_2}||B_{\ell_2 N_2}|}\frac{(\tilde{\mathcal{K}}_1+\tilde{\mathcal{K}}_2)}{\tilde{\mathcal{K}}_j} + \delta(y)\cos\theta_{21}\right]}\frac{\eta(y)}{y} \end{aligned} \tag{33a}$$

$$= \frac{\sin\theta_{21}}{\{\frac{1}{4}[\kappa_{\ell_1}\kappa_{\ell_2}(1+\frac{\tilde{\mathcal{K}}_1}{\tilde{\mathcal{K}}_2}) + \frac{1}{\kappa_{\ell_1}\kappa_{\ell_2}}(1+\frac{\tilde{\mathcal{K}}_2}{\tilde{\mathcal{K}}_1})] + \delta(y)\cos\theta_{21}\}}\frac{\eta(y)}{y}. \tag{33b}$$

In Eq. (33b), we use the notations of Eq. (16a).

When $\ell_1 = \ell_2$ $(\equiv \ell)$, the formulas in Eqs. (32) and (33) simplify, because then $\bar{S}^{(D)} = \bar{S}^{(C)} = \bar{S}$, and $B_{\ell_1 N_j} = B_{\ell_2 N_j} = B_{\ell N_j}$, $\kappa_{\ell_1} = \kappa_{\ell_2} = \kappa_\ell$:

$$S_+(M) = = 4\left[\sum_{j=1}^{2}\frac{|B_{\ell N_j}|^4}{\tilde{\mathcal{K}}_j} + 4\delta(y)\frac{|B_{\ell N_1}|^2|B_{\ell N_2}|^2}{(\tilde{\mathcal{K}}_1+\tilde{\mathcal{K}}_2)}\cos\theta_{21}\right]\bar{S}(x), \tag{34a}$$

$$S_-(M) = 16\frac{|B_{\ell N_1}|^2|B_{\ell N_2}|^2}{(\tilde{\mathcal{K}}_1+\tilde{\mathcal{K}}_2)}\sin\theta_{21}\frac{\eta(y)}{y}\bar{S}(x), \tag{34b}$$

$$\mathcal{A}_{CP}(M) = \frac{\sin\theta_{21}}{\{\frac{1}{4}[\kappa_\ell^2(1+\frac{\tilde{\mathcal{K}}_1}{\tilde{\mathcal{K}}_2}) + \frac{1}{\kappa_\ell^2}(1+\frac{\tilde{\mathcal{K}}_2}{\tilde{\mathcal{K}}_1})] + \delta(y)\cos\theta_{21}\}}\frac{\eta(y)}{y}. \tag{34c}$$

From these expressions and Table II, we can deduce the following:

(1) When y becomes large ($y > 10$, i.e., $\Delta M_N > 10\Gamma_N$), the CP asymmetries [Eqs. (32b) and (33)] become suppressed by the small $\eta(y)/y$ factor.

(2) When y is smaller ($y < 10$, i.e., $\Gamma_N < \Delta M_N < 10\Gamma_N$), then the factor $\eta(y)/y$ is comparable with unity, the expressions $S_\pm(M)$ become $\sim|B_{\ell N_j}|^2\bar{S}^{(D)}(x)$ (where $x \equiv M_N^2/M_M^2$; $\ell = e, \mu$; note that $\tilde{\mathcal{K}}_j \sim |B_{\ell N_j}|^2$), and the CP violation ratio $\mathcal{A}_{CP}(M)$ becomes ~ 1.

We present in Fig. 3 the numerical results of Table II for the suppression factor $\eta(y)/y$ and for the overlap factor $\delta(y)$ as a function of $y \equiv \Delta M_N/\Gamma_N$.

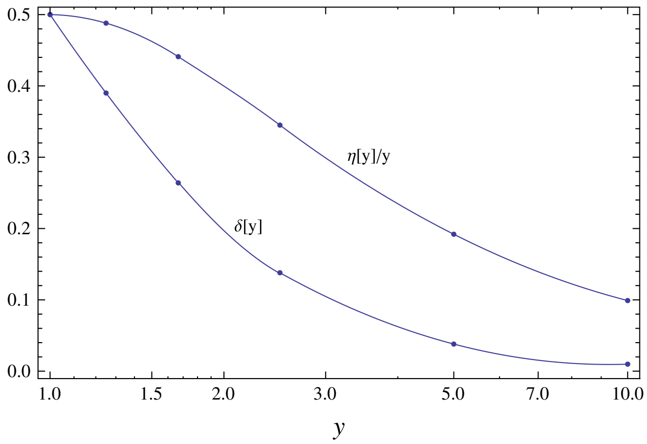

FIG. 3 (color online). The suppression factors $\eta(y)/y$ and $\delta(y)$, due to the overlap of the propagator "resonances" of N_1 and N_2, as a function of $y \equiv \Delta M_N/\Gamma_N$, for $1 < y < 10$.

In Ref. [13], the decay widths for these processes, in the case of one (on-shell) neutrino N, $\Gamma(M^+) \equiv \Gamma(M^+ \to \ell^+\ell^+M'^-)$, were considered. Since in our case $S_+(M) \approx 2\Gamma(M^+)$,[6] the conclusions in Ref. [13] on the size and measurability of $\Gamma(M^+)$ can be carried over as the conclusions on the size and measurability of $S_+(M)$ here. If, in addition, $\Delta M_N \gg \Gamma_N$ (say, $y \equiv \Delta M_N/\Gamma_N < 5$), these conclusions are valid also for the measurability of the CP-violating decay width difference $S_-(M)$, provided that the phase difference $|\theta_{21}| \sim 1$.[7]

IV. THE ACCEPTANCE FACTOR IN THE MEASUREMENT OF THE CONSIDERED DECAYS

In experiments which try to detect and investigate the LNV decay modes of the mesons $M^\pm$, the (expected) number $N_M \sim 10^N$ of produced mesons $M^\pm$ (per year, for example) is known. The value of the corresponding branching ratios of the LNV decay modes, $\mathrm{Br}(M^\pm \to \ell_1^\pm\ell_2^\pm M'^\mp) \equiv \Gamma(M^\pm \to \ell_1^\pm\ell_2^\pm M'^\mp)/\Gamma(M^\pm \to \text{all})$, then becomes important. In principle, if $\mathrm{Br}(M^\pm \to \ell_1^\pm\ell_2^\pm M'^\mp) > 10^{-N}$, then such decay modes could be detected. Further, if an experiment produces approximately equal numbers of M^+ and M^- mesons, then the branching ratios of experimental significance for the LNV decays $M^\pm \to \ell_1^\pm\ell_2^\pm M'^\mp$ are

$$\mathrm{Br}(M) \equiv \frac{S_+(M)}{[\Gamma(M^- \to \text{all}) + \Gamma(M^+ \to \text{all})]} \approx \frac{S_+(M)}{2\Gamma(M^- \to \text{all})}, \tag{35a}$$

$$\mathcal{A}_{CP}(M)\mathrm{Br}(M) = \frac{S_-(M)}{[\Gamma(M^- \to \text{all}) + \Gamma(M^+ \to \text{all})]} \approx \frac{S_-(M)}{2\Gamma(M^- \to \text{all})}, \tag{35b}$$

where we use the notation of Eqs. (14), (15), and (3). We also use the fact that in the considered cases of pseudoscalar mesons $M^\pm$, the total decay widths $\Gamma(M^- \to \text{all})$ and $\Gamma(M^+ \to \text{all})$ are practically equal. $\mathrm{Br}(M)$ represents the average of the branching ratios of M^+ and M^- for these LNV decays, while $\mathcal{A}_{CP}(M)\mathrm{Br}(M)$ is the corresponding branching ratio for the (CP-violating) difference. The corresponding canonical branching fraction $\overline{\mathrm{Br}}(M)$ is obtained by dividing the canonical decay width [Eq. (28)] by $2\Gamma(M^- \to \text{all})$,

$$\overline{\mathrm{Br}}(x; x_{\ell_1}, x_{\ell_2}, x') \equiv \frac{\bar{S}(x; x_{\ell_1}, x_{\ell_2}, x')}{2\Gamma(M^- \to \text{all})} = \frac{3\pi}{8}\frac{K^2 M_M}{G_F^2\Gamma(M^- \to \text{all})}\frac{1}{x^2}\lambda^{1/2}(1, x, x_{\ell_1})\lambda^{1/2}\left(1, \frac{x'}{x}, \frac{x_{\ell_2}}{x}\right)Q(x; x_{\ell_1}, x_{\ell_2}, x'), \tag{36}$$

where the notations in Eqs. (23) and (29) are used. We have two variants of this function: the one for the DD^* contributions [$\bar{\mathrm{Br}}^{(D)}$] and the one for the CC^* contributions [$\overline{\mathrm{Br}}^{(C)}$], which are obtained by dividing by $2\Gamma(M^- \to \text{all})$ the expressions $\bar{S}^{(D)}$ and $\bar{S}^{(C)}$, respectively, of Eq. (30). When $\ell_1 = \ell_2$, the two functions $\overline{\mathrm{Br}}^{(D)}$ and $\overline{\mathrm{Br}}^{(C)}$ coincide ($\equiv\overline{\mathrm{Br}}$).

Nonetheless, in experiments we must also take into account the acceptance (suppression) factor in the detection of these decays, which appears due to the small length of the detector in comparison to the relatively large lifetime of the (on-shell) sterile neutrinos N_j. Stated otherwise, most of the on-shell neutrinos, produced in the decay $M^\pm \to \ell_1^\pm N_j$, are expected to survive a long enough time to travel through the detector and decay (into $\ell_2^\pm M'^\mp$) outside the detector.[8] This effect suppresses the number of detected decays and should be taken into account, cf. Refs. [4,14,27,33,41]. The acceptance (suppression) factor is the probability of the on-shell neutrino N to decay inside a detector of length L:

[6]When neglecting the N_1-N_2 overlap effects $\propto \delta(y)$ in $S_+(M)$.
[7]We recall that if $y < 5$, we have $\mathcal{A}_{CP}(M) \sim 1$, and thus $S_-(M) \sim S_+(M)$.

[8]Only when $M = B$ or B_c can a large part of the produced neutrinos N_j decay within the detector (see the arguments later on).

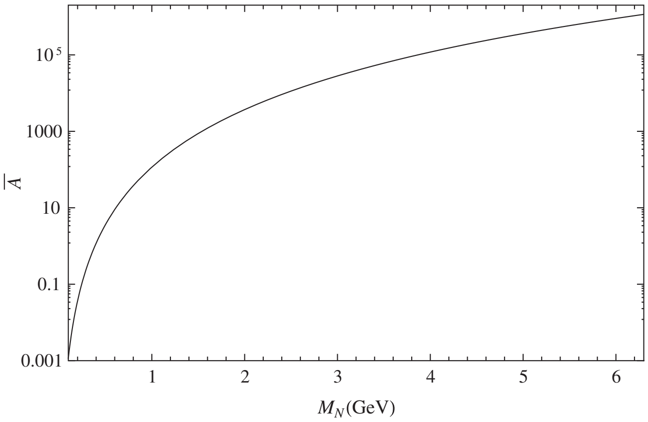

FIG. 4. The canonical acceptance $\bar{A}(M_N) \equiv (L\bar{\Gamma}(M_N)/\gamma_N)$ as a function of the neutrino mass M_N. In the curve, we take for the length of the detector the value $L = 1$ m and for the time dilation factor the value $\gamma_N = 2$.

$$P_{N_j} \approx \frac{L}{\gamma_{N_j}\tau_{N_j}\beta_{N_j}} \sim \frac{L}{\gamma_{N_j}\tau_{N_j}} = \frac{L\Gamma_{N_j}}{\gamma_{N_j}} = \frac{L\bar{\Gamma}(M_{N_j})}{\gamma_{N_j}}\tilde{\mathcal{K}}_j \equiv \bar{A}(M_{N_j})\tilde{\mathcal{K}}_j, \tag{37}$$

where γ_{N_j} is the time dilation (Lorentz) factor $\gamma_{N_j} = (1-\beta_{N_j}^2)^{-1/2}$ (~1–10) in the lab system. We take into account that the speed of the neutrino is $\beta_{N_j} \sim 1$. The quantity $\bar{\Gamma}(M_{N_j})$ ($\propto M_{N_j}^5$) and the factor $\tilde{\mathcal{K}}_j$ ($\propto |B_{\ell N_j}|^2$) were defined in Eqs. (12) and (13), respectively. The quantity $\bar{A}(M_{N_j}) \equiv (L\bar{\Gamma}(M_{N_j})/\gamma_{N_j})$ can be called "canonical acceptance" and depends heavily on the neutrino mass: $\bar{A} \propto M_{N_j}^5$. In Fig. 4, we present the values of this canonical acceptance as a function of the neutrino mass M_N, for the choice $L = 1$ m ($= 5.064 \times 10^{15}$ GeV^{-1}) and $\gamma_N = 2$. The values of $\bar{A}$ for other cases of the values of L and γ_N are obtained directly from the presented curve by taking into account that $\bar{A} \propto L/\gamma_N$. The realistic acceptance factor is then obtained by Eq. (37), where $\tilde{\mathcal{K}}_j \sim |B_{\ell N_j}|^2$ ($j = 1, 2$) are the heavy-light mixing factors defined in Eq. (13) with coefficients $\mathcal{N}_{\ell N}$ there of ~10 according to Fig. 2. Combining the results of Fig. 2 with Eq. (13), we can write rough approximations for $\tilde{\mathcal{K}}_j$:

$$\tilde{\mathcal{K}}_j \approx 15|B_{eN_j}|^2 + 8|B_{\mu N_j}|^2 + 2|B_{\tau N_j}|^2 (K \text{ decays}), \tag{38a}$$

$$\tilde{\mathcal{K}}_j \approx 7(|B_{eN_j}|^2 + |B_{\mu N_j}|^2) + 2|B_{\tau N_j}|^2 (D, D_s \text{ decays}), \tag{38b}$$

$$\tilde{\mathcal{K}}_j \approx 8(|B_{eN_j}|^2 + |B_{\mu N_j}|^2) + 3|B_{\tau N_j}|^2 (B, B_c \text{ decays}). \tag{38c}$$

The rough upper bounds for $|B_{\ell N}|^2$ for $\ell = e, \mu, \tau$ are given in Table III for the typical ranges of our interest: M_N around 0.25, 1, and 3 GeV—relevant for the decays of K, (D, D_s), and (B, B_c), respectively (see also Table I for several specific values of M_N). The corresponding values of the canonical acceptance factor $\bar{A}(M_N)$ are also included. Combining Eq. (37) with Eq. (38) and Table III, we obtain for the acceptance factor P_{N_j} the following estimates and upper bounds relevant for the K decays ($M_N \approx 0.25$ GeV), D and D_s decays ($M_N \approx 1$ GeV), and B and B_c decays ($M_N \approx 3$ GeV):

$$P_{N_j}(M_N \approx 0.25 \text{ GeV}) \approx 1.7|B_{eN_j}|^2 + 0.9|B_{\mu N_j}|^2 (+0.2|B_{\tau N_j}|^2) \lesssim 10^{-8} + 10^{-7}(+10^{-5}), \tag{39a}$$

$$P_{N_j}(M_N \approx 1 \text{ GeV}) \approx 0.8 \times 10^3|B_{eN_j}|^2 + 0.8 \times 10^3|B_{\mu N_j}|^2 (+2 \times 10^2|B_{\tau N_j}|^2) \lesssim 10^{-4} + 10^{-4}(+10^0), \tag{39b}$$

$$P_{N_j}(M_N \approx 3 \text{ GeV}) \approx 3 \times 10^5|B_{eN_j}|^2 + 3 \times 10^5|B_{\mu N_j}|^2 (+1 \times 10^5|B_{\tau N_j}|^2) \lesssim 10^0 + 10^0(+10^0). \tag{39c}$$

The upper bounds for P_{N_j} in Eq. (39) are written as a sum of the contributions of upper bounds from $|B_{eN_j}|^2$, $|B_{\mu N_j}|^2$, and $|B_{\tau N_j}|^2$ separately. Further, the contributions of $|B_{\tau N_j}|^2$ are included in Eq. (39) optionally, in the parentheses, because the upper bounds of the mixings $|B_{\tau N_j}|^2$ are still very high and are expected to be reduced significantly in the foreseeable future. The upper bounds which give results higher than 1 are replaced by 1 (10^0), because the acceptance (decay probability) P_{N_j} can never be higher than 1 by definition.

TABLE III. Present rough upper bounds for $|B_{\ell N}|^2$ ($\ell = e, \mu, \tau$) for M_N in the ranges around the values 0.25, 1, 3 GeV, and the canonical acceptance factor $\bar{A}(M_N)$ (for $L = 1$ m and $\gamma_N = 2$).

M_N [GeV]	$\lvert B_{eN}\rvert^2$	$\lvert B_{\mu N}\rvert^2$	$\lvert B_{\tau N}\rvert^2$	$\bar{A}$
≈0.25	10^{-8}	10^{-7}	10^{-4}	0.11
≈1.0	10^{-7}	10^{-7}	10^{-2}	115.
≈3.0	10^{-6}	10^{-4}	10^{-4}	3×10^4

From now on in this section, we will assume the following:

$$|B_{\ell N_1}|^2 \sim |B_{\ell N_2}|^2 \equiv |B_{\ell N}|^2 \tag{40a}$$

$$\Rightarrow \tilde{\mathcal{K}}_1 \sim \tilde{\mathcal{K}}_2 \equiv \tilde{\mathcal{K}}. \tag{40b}$$

In addition, we consider that it is the flavor ℓ which has the dominant (largest) mixing $|B_{\ell N}|^2$. Then we have

$$\tilde{\mathcal{K}} \approx \mathcal{N}_{\ell N}|B_{\ell N}|^2 \sim 10|B_{\ell N}|^2. \tag{41}$$

The dominant branching ratios $\mathrm{Br}(M)$ and $\mathcal{A}_{CP}(M)\mathrm{Br}(M)$ will then be, according to the obtained expressions (32) and (34) [together with the definitions (35) and (36)], those which have in the final state two equal charged leptons ℓ with dominant mixing: $M^\pm \to \ell^\pm \ell^\pm M'^\mp$.

The theoretical branching ratios $\mathrm{Br}(M)$ and $\mathcal{A}_{CP}(M)\mathrm{Br}(M)$ [Eq. (35)] can be obtained by dividing Eqs. (34a) and (34b) by $2\Gamma(M^- \to \mathrm{all})$. Using in addition Eqs. (40) and (41) and the definition (36), this gives

$$\mathrm{Br}(M) \sim 8\frac{|B_{\ell N}|^4}{\tilde{\mathcal{K}}}\overline{\mathrm{Br}}(x) \sim \overline{\mathrm{Br}}(x)|B_{\ell N}|^2, \tag{42a}$$

$$\begin{aligned}\mathcal{A}_{CP}(M)\mathrm{Br}(M) &\sim 8\frac{|B_{\ell N}|^4}{\tilde{\mathcal{K}}}\sin\theta_{21}\frac{\eta(y)}{y}\overline{\mathrm{Br}}(x)\\ &\sim \overline{\mathrm{Br}}(x)|B_{\ell N}|^2\sin\theta_{21},\end{aligned} \tag{42b}$$

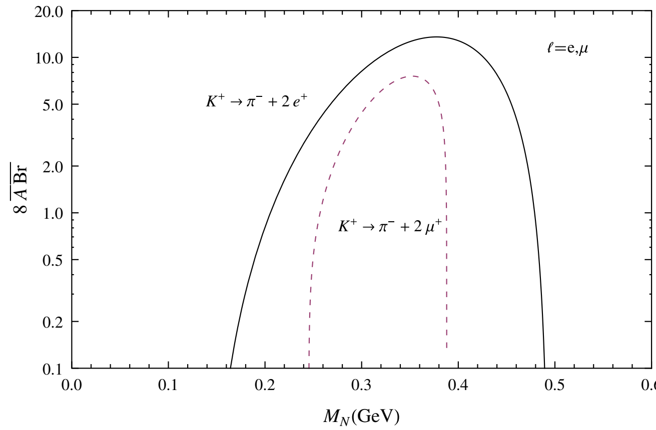

FIG. 5 (color online). The effective canonical branching ratio [Eq. (44)] for the $K^\pm \to \ell^\pm \ell^\pm \pi'^\mp$ decays ($\ell = e, \mu$) as a function of the Majorana neutrino mass M_N.

where in the last relation we took into account that $\eta(y)/y \sim 1$ (since $\Delta M_N \gg \Gamma_N$ in our considered cases).

The effective (i.e., experimental) branching ratios $\mathrm{Br}^{(\mathrm{eff})}(M) = P_N \mathrm{Br}(M)$ and $\mathcal{A}_{CP}(M)\mathrm{Br}^{(\mathrm{eff})}(M)$ can be estimated, in the considered case of Eqs. (40) and (41), in the following way [using Eqs. (37) and (42)]:

$$\mathrm{Br}^{(\mathrm{eff})}(M) \equiv P_N \mathrm{Br}(M) \sim \bar{A}(M_N)\tilde{\mathcal{K}}\mathrm{Br}(M) \sim \bar{A}(M_N)\tilde{\mathcal{K}}\left(\frac{8|B_{\ell N}|^4}{\tilde{\mathcal{K}}}\overline{\mathrm{Br}}(x)\right) = [8\bar{A}(M_N)\bar{\mathrm{Br}}(x)]|B_{\ell N}|^4, \tag{43a}$$

$$\begin{aligned}\mathcal{A}_{CP}(M)\mathrm{Br}^{(\mathrm{eff})}(M) &\equiv P_N \mathcal{A}_{CP}(M)\mathrm{Br}(M) \sim \bar{A}(M_N)\tilde{\mathcal{K}}\mathrm{Br}_-(M) \sim \bar{A}(M_N)\tilde{\mathcal{K}}\left(\frac{8|B_{\ell N}|^4}{\tilde{\mathcal{K}}}\sin\theta_{21}\frac{\eta(y)}{y}\overline{\mathrm{Br}}(x)\right)\\ &= 8\bar{A}(M_N)|B_{\ell N}|^4\sin\theta_{21}\frac{\eta(y)}{y}\overline{\mathrm{Br}}(x) \sim [8\bar{A}(M_N)\overline{\mathrm{Br}}(x)]|B_{\ell N}|^4\sin\theta_{21},\end{aligned} \tag{43b}$$

where in the last line of Eq. (43b) we take into account that $\eta(y)/y \sim 1$ (true when $\Delta M_N \gg \Gamma_N$). Furthermore, since $\ell_1 = \ell_2 = \ell$ in the considered case, the canonical branching fractions are equal: $\overline{\mathrm{Br}}^{(C)}(x) = \overline{\mathrm{Br}}^{(D)}(x) \equiv \overline{\mathrm{Br}}(x)$, and we recall that $x \equiv (M_N/M_M)^2$. We see that in Eq. (43) the most important factor at $|B_{\ell N}|^4$ is the "effective" canonical branching ratio

$$\overline{\mathrm{Br}}_{\mathrm{eff}}(M_N) \equiv 8\bar{A}(M_N)\overline{\mathrm{Br}}(x). \tag{44}$$

Only in the case of $B^\pm$ and $B_c^\pm$ LNV decays could we have $P_N \sim 1$ [Eq. (39c)], and in such a case Eq. (43) does not apply, but rather Eq. (42). In Figs. 5–8 we present the effective canonical branching ratios [Eq. (44)] as a function of the neutrino mass M_N for various considered LNV decays of the type $M^\pm \to \ell^\pm \ell^\pm M'^\mp$, where $M = K$ in Fig. 5, $M = D, D_s$ in Figs. 6(a) and 6(b); and $M = B, B_c$ in Figs. 7(a) and 8(a), respectively. In general, $\ell = e, \mu$. We take $L = 1$ m and $\gamma_N = 2$. In addition, for the case when $P_N \sim 1$, and consequently the estimates in Eq. (42) apply, we present in Figs. 7(b) and 8(b) the theoretical branching ratios $\overline{\mathrm{Br}}(x)$ as a function of M_N for $B^\pm$ and $B_c^\pm$ decays, respectively.[9] For the CKM matrix elements and the meson decay constants, appearing in the K^2 factor defined in Eq. (10), and for masses and lifetimes of the mesons, we use the values of Ref. [29], and for the decay constants f_B and f_{B_c}, we use the values of Ref. [42]: $f_B = 0.196$ GeV, $f_{B_c} = 0.322$ GeV.

[9] Our formulas permit evaluation of $\overline{\mathrm{Br}}_{\mathrm{eff}}$ and $\overline{\mathrm{Br}}(x)$ for the decays $M^\pm \to \ell_1^\pm \ell_2^\pm M'^\mp$ when $\ell_1 \neq \ell_2$, and also when the final leptons are τ leptons (and $M^\pm = B^\pm$ or $B_c^\pm$), with the values similar to those in Figs. 7 and 8, except that the range of M_N is now significantly shorter: $M_{M'} + M_\tau < M_N < M_M - M_\tau$.

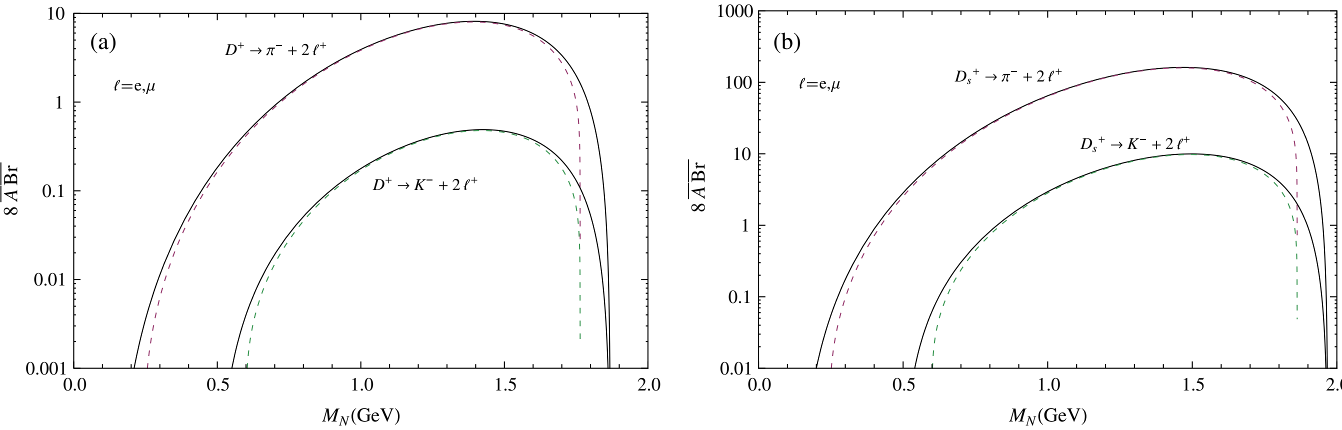

FIG. 6 (color online). The effective canonical branching ratio [Eq. (44)] as a function of the Majorana neutrino mass M_N for the LNV decays of (a) $D^{\pm}$ mesons and (b) $D_s^{\pm}$ mesons. The solid lines are for $\ell = e$, and the dashed lines for $\ell = \mu$.

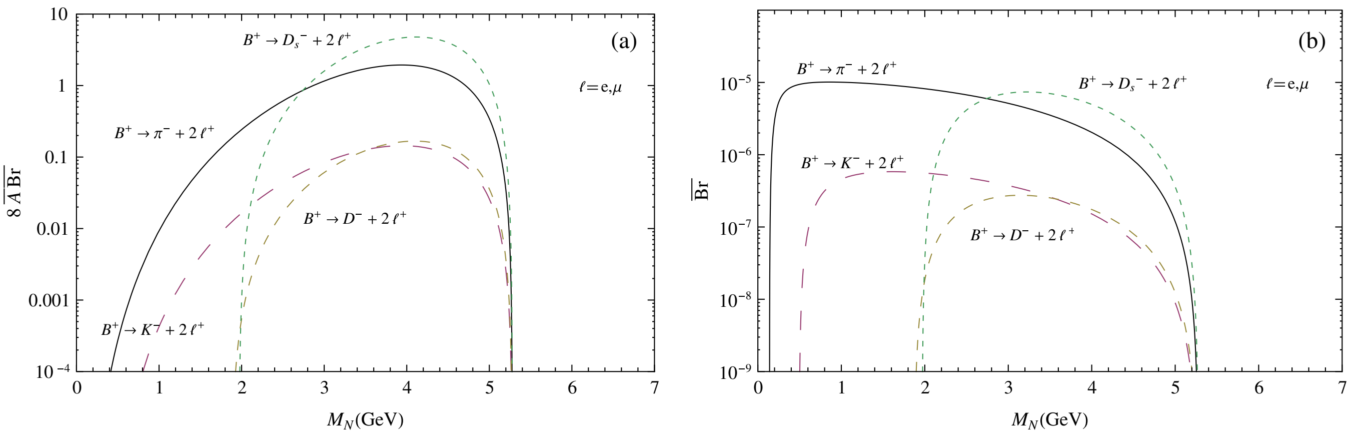

FIG. 7 (color online). (a) The effective canonical branching ratio [Eq. (44)] as a function of the Majorana neutrino mass M_N for the LNV decays of $B^{\pm}$ mesons, $B^{\pm} \to \ell^{\pm}\ell^{\pm}M'^{\mp}$, where $\ell = e, \mu$ (no discernible difference between the two cases). (b) The corresponding curves for the theoretical canonical branching ratio $\overline{\mathrm{Br}}$.

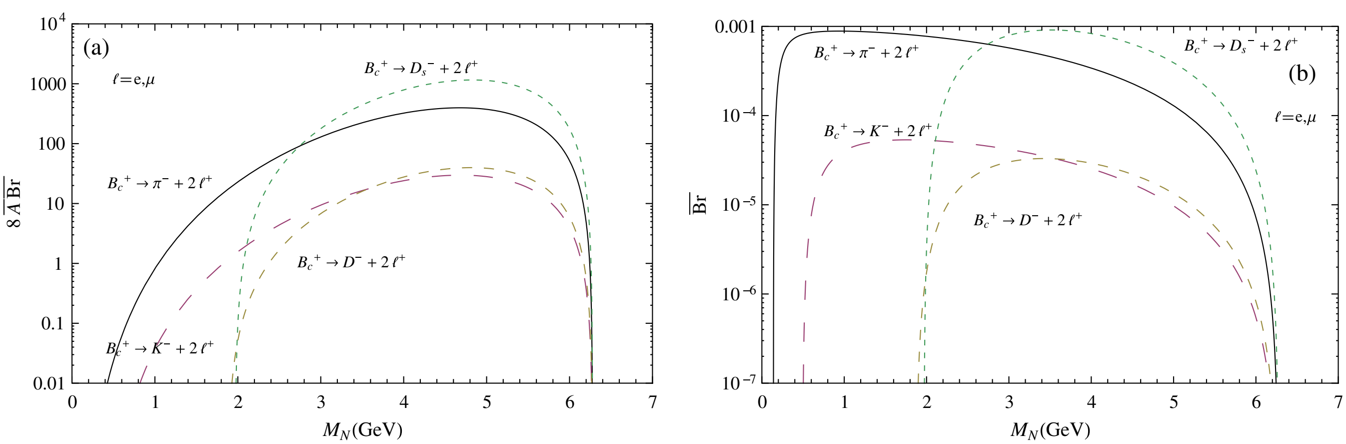

FIG. 8 (color online). The same as in Fig. 7, but for the LNV decays of the charmed mesons $B_c^{\pm}$.

TABLE IV. Values of the factor $8\bar{A}(M_N)\overline{\mathrm{Br}}(x)$ (with $L = 1$ m and $\gamma_N = 2$) for some of the considered LNV decays: $M^{\pm} \to \ell^{\pm}\ell^{\pm}\pi'^{\mp}$. We choose M_N such that the maximal value is obtained (this value of M_N is given in parentheses, in GeV). For the K decay, the two different values are given for $\ell = e$ and $\ell = \mu$. For all other decays, $\ell = \mu$ is chosen (the values for $\ell = e$ are similar).

$M^{\pm}$:	$K^{\pm}$ ($\ell = e$)	$K^{\pm}$ ($\ell = \mu$)	$D^{\pm}$	$D_s^{\pm}$	$B^{\pm}$	$B_c^{\pm}$
$8\bar{A}\,\overline{\mathrm{Br}}$:	13.5 (0.38)	7.5 (0.35)	8. (1.39)	159. (1.47)	1.93 (3.9)	395. (4.7)

In Table IV we display some values of the factor $\overline{\mathrm{Br}}_{\mathrm{eff}}$ for the representative values of M_N in the decays $M^{\pm} \to \ell^{\pm}\ell^{\pm}M'^{\mp}$.

Let us now take, as an example, the decays $D_s^{\pm} \to \mu^{\pm}\mu^{\pm}\pi^{\mp}$,[10] and let us assume that $|B_{\mu N}|^2$ is the dominant mixing (i.e., $\ell = \mu$). Then Eq. (43) and Table IV imply that the effective (experimentally measurable) sum $P_N\mathrm{Br}(D_s)$ and difference $P_N\mathcal{A}_{CP}(D_s)\mathrm{Br}(D_s)$ of the branching ratios for these decays are

$$\mathrm{Br}^{(\mathrm{eff})}(D_s) \equiv P_N\mathrm{Br}(D_s) \sim 10^2|B_{\mu N}|^4, \tag{45a}$$

$$\begin{aligned}\mathcal{A}_{CP}(D_s)&\mathrm{Br}^{(\mathrm{eff})}(D_s)\\ &\equiv P_N\mathcal{A}_{CP}(D_s)\mathrm{Br}(D_s)\\ &\sim 10^2|B_{\ell N}|^4 \sin\theta_{21}\frac{\eta(y)}{y} \sim 10^2|B_{\ell N}|^4 \sin\theta_{21}.\end{aligned} \tag{45b}$$

Taking into account that in such decays the present rough upper bound on the mixing is $|B_{\mu N}|^2 \lesssim 10^{-7}$ (cf. Table III), Eq. (45) implies that $P_N\mathrm{Br}(D_s) \lesssim 10^{-12}$. The proposed experiment at CERN-SPS [33] would produce numbers of D and D_s mesons several orders higher than 10^{12} and would thus be able to explore whether there is a production of the sterile Majorana neutrinos N_j. Furthermore, if there are two almost degenerate neutrinos (as is the case in the νMSM model [20,28]), then in such a case it is possible that $y(\equiv\Delta M_N/\Gamma_N)\lll 1$, and thus $\eta(y)/y \sim 1$. Then the estimate in Eq. (45b) would imply that the CP-violating difference of effective branching ratios $P_N\mathcal{A}_{CP}(D_s)\mathrm{Br}(D_s)$ is of the same order as the sum $P_N\mathrm{Br}(D_s)$ (provided that the phase difference $|\theta_{21}|\not\lll 1$). This means that if experiments discover the aforementioned νMSM-type Majorana neutrinos, they will possibly also discover CP violation in the Majorana neutrino sector.

V. CONCLUSIONS

We investigated the possibility of detection of CP violation in lepton-number-violating (LNV) semihadronic decays $M^{\pm} \to \ell_1^{\pm}\ell_2^{\pm}M'^{\mp}$, where M and M' are pseudoscalar mesons, $M = K, D, D_s, B, B_c$ and $M' = \pi, K, D, D_s$, and the charged leptons are $\ell_1, \ell_2 = e, \mu$. The decay widths of such decays, mediated by on-shell sterile Majorana neutrinos N with masses $M_N \sim 1$ GeV, have been studied by various authors, cf. Refs. [6–13], with a view of possible detection in future experiments such as the proposed CERN-SPS experiment [33]. In the present work, we investigated the possibility of detecting the CP-violating decay width difference $S_-(M) \equiv [\Gamma(M^- \to \ell_1^-\ell_2^-M'^+) - \Gamma(M^+ \to \ell_1^+\ell_2^+M'^-)]$ in such processes, in the scenarios of two on-shell sterile Majorana neutrinos N_1, N_2. We used the same approach as in our previous work [27], where CP violation was investigated in purely leptonic rare decays $\pi^{\pm} \to e^{\pm}e^{\pm}\mu^{\mp}\nu$: the crucial aspect is the expression for the imaginary part of the product of the propagators of two Majorana neutrinos in Eq. (24). A central point, as in Ref. [27], is that when the difference of masses $\Delta M_N \equiv M_{N_2} - M_{N_1}$ (> 0) of the two sterile neutrinos becomes small enough, comparable to the (small) total decay widths of these neutrinos, $\Delta M_N \not\ggg \Gamma_N$, the mentioned imaginary part becomes large and leads to a large CP-violating decay width difference $S_-(M)$. We show that in such a case, and provided that a specific CP-violating difference θ_{21} of the phases of heavy-light neutrino mixings is not very small ($|\theta_{21}|\not\lll 1$), the decay width difference $S_-(M)$ becomes comparable with the sum of the decay widths of the LNV decays $S_+(M) \equiv [\Gamma(M^- \to \ell_1^-\ell_2^-M'^+) + \Gamma(M^+ \to \ell_1^+\ell_2^+M'^-)]$, and the corresponding CP ratio $\mathcal{A}_{CP}(M) \equiv S_-(M)/S_+(M)$ thus becomes $\mathcal{A}_{CP}(M) \sim 1$. It is interesting that the requirement of the near degeneracy of the two sterile neutrinos (with $M_{N_j} \sim 1$ GeV), at which we arrive by requiring appreciable CP violation, fits well into the well-motivated νMSM model [20,28,30], where the near degeneracy of the two sterile neutrinos with mass $M_{N_j} \sim 1$ GeV is obtained by requiring that the third (the lightest) sterile neutrino be the dark matter candidate. The results of our calculation can thus be interpreted in the framework of the νMSM model, namely that if the model is experimentally confirmed, then it is possible that significant neutrino sector CP violation effects will be detected as well.

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[10]This is one of the preferred decay modes proposed at CERN-SPS [33].

APPENDIX A: EXPLICIT FORMULAS FOR THE $M^{\pm} \to \ell_1^{\pm}\ell_2^{\pm}M'^{\mp}$ DECAY WIDTH

The matrix element $\mathcal{T}(M^{\pm})$ for the decay of Fig. 1 can be written in the form

$$\mathcal{T}(M^{\pm}) = K_{\pm}\sum_{j=1}^{2} k_j^{(\pm)} M_{N_j}[P_j(D)T_{\pm}(D) + P_j(C)T_{\pm}(C)], \tag{A1}$$

where $j = 1, 2$ refer to the contributions of the exchanges of the two intermediate neutrinos N_j, and $X = D, C$ refer to the contributions of the direct and crossed channels, respectively, cf. Fig. 1. In Eq. (A1), $k_j^{(\pm)}$ are the heavy-light mixing factors defined in Eq. (7); $P_j(X)$ ($j = 1, 2; X = D, C$) are the propagator functions of the N_j neutrino for the D and C channels [Eq. (9)], and $K_{\pm}$ are the constants coming from the vertices

$$K_{-} = -G_F^2 V_{Q_uQ_d} V_{q_uq_d} f_M f_{M'}, \qquad K_{+} = (K_{-})^*, \tag{A2}$$

where f_M and $f_{M'}$ are the decay constants of $M^{\pm}$ and $M'^{\mp}$, and $V_{Q_uQ_d}$ and $V_{q_uq_d}$ are the CKM elements for $M^{\pm}$ and $M'^{\mp}$: M^{+} has the valence quark content $Q_u\bar{Q}_d$; M'^{+} has $q_u\bar{q}_d$. The functions $T_{\pm}(D)$ and $T_{\pm}(C)$ appearing in the amplitude [Eq. (A1)] can be written as

$$T_{\pm}(D) = \bar{u}_{\ell_2}(p_2)\not{p}_{M'}\not{p}_M(1\mp\gamma_5)v_{\ell_1}(p_1), \tag{A3a}$$

$$T_{\pm}(C) = \bar{u}_{\ell_2}(p_2)\not{p}_M\not{p}_{M'}(1\mp\gamma_5)v_{\ell_1}(p_1), \tag{A3b}$$

where the spinors are written in the helicity basis. Squaring and summing over the final helicities leads to the square $|\mathcal{T}(M^{\pm})|^2$ of the total decay amplitude [Eq. (A1)] as given in Eq. (6) in conjunction with Eqs. (7)–(10), where the quadratic expressions $T_{\pm}(X)T_{\pm}(Y)^*$ ($X, Y = D, C$) appearing in the normalized decay widths $\bar{\Gamma}_{\pm}(XY^*)_{ij}$ in Eq. (8) are

$$\begin{aligned} T_{\pm}(D)T_{\pm}(D)^* = 8[&M_M^2M_{M'}^2(p_1\cdot p_2) - 2M_M^2(p_1\cdot p_{M'})(p_2\cdot p_{M'}) - 2M_{M'}^2(p_1\cdot p_M)(p_2\cdot p_M) \\ &+ 4(p_1\cdot p_M)(p_2\cdot p_{M'})(p_M\cdot p_{M'})] \equiv T(D)T(D)^*, \end{aligned} \tag{A4a}$$

$$\begin{aligned} T_{\pm}(C)T_{\pm}(C)^* = 8[&M_M^2M_{M'}^2(p_1\cdot p_2) - 2M_M^2(p_1\cdot p_{M'})(p_2\cdot p_{M'}) - 2M_{M'}^2(p_1\cdot p_M)(p_2\cdot p_M) \\ &+ 4(p_2\cdot p_M)(p_1\cdot p_{M'})(p_M\cdot p_{M'})] \equiv T(C)T(C)^*, \end{aligned} \tag{A4b}$$

$$\begin{aligned} T_{\pm}(D)T_{\pm}(C)^* = 16\Big\{&M_M^2(p_1\cdot p_{M'})(p_2\cdot p_{M'}) + M_{M'}^2(p_1\cdot p_M)(p_2\cdot p_M) - \frac{1}{2}M_M^2M_{M'}^2(p_1\cdot p_2) \\ &+ (p_M\cdot p_{M'})[-(p_1\cdot p_M)(p_2\cdot p_{M'}) - (p_2\cdot p_M)(p_1\cdot p_{M'}) + (p_M\cdot p_{M'})(p_1\cdot p_2)] \\ &\mp i(p_M\cdot p_{M'})\epsilon(p_M, p_1, p_2, p_{M'})\Big\}, \end{aligned} \tag{A4c}$$

$$T_{\pm}(C)T_{\pm}(D)^* = (T_{\pm}(D)T_{\pm}(C)^*)^* = T_{\mp}(D)T_{\mp}(C)^* = (T_{\mp}(C)T_{\mp}(D)^*)^*, \tag{A4d}$$

where in these expressions the summation over the (final) helicities of the leptons ℓ_1 and ℓ_2 is implied, and we denote

$$\epsilon(q_1, q_2, q_3, q_4) \equiv \epsilon^{\eta_1\eta_2\eta_3\eta_4}(q_1)_{\eta_1}(q_2)_{\eta_2}(q_3)_{\eta_3}(q_4)_{\eta_4}, \tag{A5}$$

and $\epsilon^{\eta_1\eta_2\eta_3\eta_4}$ is the totally antisymmetric Levi-Civita tensor with the sign convention $\epsilon^{0123} = +1$.

The expressions in Eq. (A4), in conjunction with the definitions in Eq. (8), imply for the normalized decay widths $\bar{\Gamma}_{\pm}(XY^*)_{ij}$ of Eq. (8) various symmetry relations, among them that $\bar{\Gamma}_{\pm}(DD^*)$ and $\bar{\Gamma}_{\pm}(CC^*)$ are both self-adjoint (2×2) matrices and that elements of the D-C interference matrices $\bar{\Gamma}_{\pm}(CD^*)$ and $\bar{\Gamma}_{\pm}(DC^*)$ are related:

$$\bar{\Gamma}(DD^*)_{ij} = (\bar{\Gamma}(DD^*)_{ji})^*, \qquad \bar{\Gamma}(CC^*)_{ij} = (\bar{\Gamma}(CC^*)_{ji})^*, \tag{A6a}$$

$$\bar{\Gamma}_{\pm}(CD^*)_{ij} = (\bar{\Gamma}_{\pm}(DC^*)_{ji})^*. \tag{A6b}$$

When the two final leptons are the same ($\ell_1 = \ell_2$), we can use the fact that the integration d_3 over the final particles is symmetric under $(p_1 \leftrightarrow p_2)$ (because $M_{\ell_1} = M_{\ell_2}$), and we have additional symmetry relations

$$\bar{\Gamma}(DD^*)_{ij} = \bar{\Gamma}(CC^*)_{ij}, \tag{A7a}$$

$$\bar{\Gamma}_{\pm}(CD^*)_{ij} = \bar{\Gamma}_{\pm}(DC^*)_{ij}, \tag{A7b}$$

and the (2×2) D-C interference matrices $\bar{\Gamma}_{\pm}(CD^*)$ become self-adjoint, too.

APPENDIX B: PARTIAL DECAY WIDTHS OF NEUTRINO N

The formulas for the leptonic decay and semimesonic decay widths of a sterile Majorana neutrino N have been obtained in Ref. [11] (Appendix C there) for the masses $M_N \lesssim 1$ GeV. Nonetheless, for higher values of the masses M_N, the calculation of the semihadronic decay widths becomes increasingly complicated, because not all the resonances are known. Therefore, in Refs. [12,43], an inclusive approach was proposed for the calculation of the total contribution of the semihadronic decay width of N, by replacing the various (pseudoscalar and vector) meson channels with quark-antiquark channels. This inclusive approach, based on duality, was applied for high masses $M_N \geq M_{\eta'} \approx 0.958$ GeV. Here we summarize the formulas given in Ref. [12] for the decay width channels (see also Ref. [11]). The leptonic channels are

$$2\Gamma(N \to \ell^- \ell'^+ \nu_{\ell'}) = |B_{\ell N}|^2 \frac{G_F^2}{96\pi^3} M_N^5 I_1(y_\ell, 0, y_{\ell'})(1 - \delta_{\ell\ell'}), \tag{B1a}$$

$$\begin{aligned}\Gamma(N \to \nu_\ell \ell'^- \ell'^+) = |B_{\ell N}|^2 \frac{G_F^2}{96\pi^3} M_N^5 [&(g_L^{(\mathrm{lept})} g_R^{(\mathrm{lept})} + \delta_{\ell\ell'} g_R^{(\mathrm{lept})}) I_2(0, y_{\ell'}, y_{\ell'}) \\ &+ ((g_L^{(\mathrm{lept})})^2 + (g_R^{(\mathrm{lept})})^2 + \delta_{\ell\ell'}(1 + 2 g_L^{(\mathrm{lept})})) I_1(0, y_{\ell'}, y_{\ell'})],\end{aligned} \tag{B1b}$$

$$\sum_{\nu_\ell} \sum_{\nu'} \Gamma(N \to \nu_\ell \nu' \bar{\nu}') = \sum_\ell |B_{\ell N}|^2 \frac{G_F^2}{96\pi^3} M_N^5. \tag{B1c}$$

In Eq. (B1a), a factor of 2 was included because both decays, $N \to \ell^- \ell'^+ \nu_{\ell'}$ and $N \to \ell^+ \ell'^- \nu_{\ell'}$, contribute ($\ell \neq \ell'$).

If $M_N < M_{\eta'} \approx 0.968$ GeV, the following semimesonic decays contribute, involving presudoscalar (P) and vector (V) mesons:

$$2\Gamma(N \to \ell^- P^+) = |B_{\ell N}|^2 \frac{G_F^2}{8\pi} M_N^3 f_P^2 |V_P|^2 F_P(y_\ell, y_P), \tag{B2a}$$

$$\Gamma(N \to \nu_\ell P^0) = |B_{\ell N}|^2 \frac{G_F^2}{64\pi} M_N^3 f_P^2 (1 - y_P^2)^2, \tag{B2b}$$

$$2\Gamma(N \to \ell^- V^+) = |B_{\ell N}|^2 \frac{G_F^2}{8\pi} M_N^3 f_V^2 |V_V|^2 F_V(y_\ell, y_V), \tag{B2c}$$

$$\Gamma(N \to \nu_\ell V^0) = |B_{\ell N}|^2 \frac{G_F^2}{2\pi} M_N^3 f_V^2 \kappa_V^2 (1 - y_V^2)^2 (1 + 2 y_V^2), \tag{B2d}$$

where the factor of 2 in the charged meson channels is taken because both decays, $N \to \ell^- M'^+$ and $N \to \ell^+ M'^-$, contribute ($M' = P, V$). The factors V_P and V_V are the corresponding CKM matrix elements involving the valence quarks of the mesons, and f_P and f_V are the corresponding decay constants. The pseudoscalar mesons which may contribute are $P^\pm = \pi^\pm, K^\pm$; $P^0 = \pi^0, K^0, \bar{K}^0, \eta$. The vector mesons which may contribute are $V^\pm = \rho^\pm, K^{*\pm}$; $V^0 = \rho^0, \omega, K^{*0}, \bar{K}^{*0}$.[11] When $M_N \geq M_{\eta'}$ ($= 0.9578$ GeV), the above semimesonic decay modes are replaced [12], in the spirit of duality, with the following quark-antiquark decay modes:

$$2\Gamma(N \to \ell^- U \bar{D}) = |B_{\ell N}|^2 \frac{G_F^2}{32\pi^3} M_N^5 |V_{UD}|^2 I_1(y_\ell, y_U, y_D), \tag{B3a}$$

$$\begin{aligned}\Gamma(N \to \nu_\ell q \bar{q}) = |B_{\ell N}|^2 \frac{G_F^2}{32\pi^3} M_N^5 [&g_L^{(q)} g_R^{(q)} I_2(0, y_q, y_q) \\ &+ ((g_L^{(q)})^2 + (g_R^{(q)})^2) I_1(0, y_q, y_q)].\end{aligned} \tag{B3b}$$

In the formulas (B1)–(B3), we denoted $y_X \equiv M_X/M_N$ ($X = \ell, \nu_\ell, P, V, q$), and in Eq. (B3) we denoted $U = u, c$; $D = d, s, b$; $q = u, d, c, s, b$. The values of quark masses which we used were $M_u = M_d = 3.5$ MeV, $M_s = 105$ MeV, $M_c = 1.27$ GeV, and $M_b = 4.2$ GeV. The SM neutral current couplings in Eqs. (B1b) and (B3b) are

$$g_L^{(\mathrm{lept})} = -\frac{1}{2} + \sin^2\theta_W, \qquad g_R^{(\mathrm{lept})} = \sin^2\theta_W, \tag{B4a}$$

$$g_L^{(U)} = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W, \qquad g_R^{(U)} = -\frac{2}{3}\sin^2\theta_W, \tag{B4b}$$

$$g_L^{(D)} = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W, \qquad g_R^{(U)} = \frac{1}{3}\sin^2\theta_W. \tag{B4c}$$

The neutral current couplings κ_V of the neutral vector mesons are

[11]For the values of the decay constants f_P and f_V, see, e.g., Table 1 in Ref. [12].

$$\kappa_V = \frac{1}{3}\sin^2\theta_W (V = \rho^0, \omega), \tag{B5a}$$

$$\kappa_V = -\frac{1}{4} + \frac{1}{3}\sin^2\theta_W (V = K^{*0}, \bar{K}^{*0}). \tag{B5b}$$

The kinematical expressions I_1, I_2, F_P, and F_V are

$$I_1(x, y, z) = 12 \int_{(x+y)^2}^{(1-z)^2} \frac{ds}{s}(s - x^2 - y^2)(1 + z^2 - s)\lambda^{1/2}(s, x^2, y^2)\lambda^{1/2}(1, s, z^2), \tag{B6a}$$

$$I_2(x, y, z) = 24yz \int_{(y+z)^2}^{(1-x)^2} \frac{ds}{s}(1 + x^2 - s)\lambda^{1/2}(s, y^2, z^2)\lambda^{1/2}(1, s, x^2), \tag{B6b}$$

$$F_P(x, y) = \lambda^{1/2}(1, x^2, y^2)[(1 + x^2)(1 + x^2 - y^2) - 4x^2], \tag{B6c}$$

$$F_V(x, y) = \lambda^{1/2}(1, x^2, y^2)[(1 - x^2)^2 + (1 + x^2)y^2 - 2y^4], \tag{B6d}$$

where the λ function is written in Eq. (23a). Using these formulas, the total decay width $\Gamma(N_j \to \text{all})$ can be calculated, and coefficients $\mathcal{N}_{\ell N_j}$ of Eq. (13) at the mixing terms $|B_{\ell N_j}|^2$ can be evaluated and presented in Fig. 2. The small kink in the curves of Fig. 2 at $M_N = M_{\eta'}$ ($= 0.9578$ GeV) appears due to the replacement there (i.e., for $M_N \geq M_{\eta'}$) of the semihadronic decay channel contributions by the quark-antiquark channel contributions; we see that the duality works quite well there, with the exception of the case $\ell = \tau$ because of the large τ lepton mass.

APPENDIX C: EXPLICIT EXPRESSION FOR THE FUNCTION Q

The expression (21) can be obtained by using in the integration over the phase space of three final particles [Eqs. (4) and (5)], for the contribution of the N_j neutrino, the identity

$$d_3(M(p_M) \to \ell_1(p_1)\ell_2(p_2)M'(p_{M'})) = d_2(M(p_M) \to \ell_1(p_1)N_j(p_N))dp_N^2 d_2(N_j(p_N) \to \ell_2(p_2)M'(p_{M'})), \tag{C1a}$$

$$= d_2(M(p_M) \to \ell_2(p_2)N_j(p_N))dp_N^2 d_2(N_j(p_N) \to \ell_1(p_1)M'(p_{M'})), \tag{C1b}$$

where the first identity can be used for the DD^* contribution (where $p_N = p_M - p_1$) and the second for the CC^* contribution (where $p_N = p_M - p_2$). Using the identity in Eq. (20) in the DD^* contribution, and the analogous identity for the CC^* contribution, the integration over dp_N^2 becomes trivial, and the d_2 type of integrations are straightforward.[12] The resulting expression for $\bar{\Gamma}(DD^*)_{jj}$ is then the expression Eq. (21) with the notations in Eqs. (23) and (29), where the function Q has the form

$$Q(x; x_{\ell_1}, x_{\ell_2}, x') = \left\{\frac{1}{2}(x - x_{\ell_1})(x - x_{\ell_2})(1 - x - x_{\ell_1})\left(1 - \frac{x'}{x} + \frac{x_{\ell_2}}{x}\right) + [-x_{\ell_1}x_{\ell_2}(1 + x' + 2x - x_{\ell_1} - x_{\ell_2}) - x_{\ell_1}^2(x - x') \right.$$
$$\left. + x_{\ell_2}^2(1 - x) + x_{\ell_1}(1 + x)(x - x') - x_{\ell_2}(1 - x)(x + x')]\right\}. \tag{C2}$$

[12] This is equivalent to the factorization approach $\Gamma(M \to \ell_1 N_j)\mathrm{Br}(N_j \to \ell_2 M')$, valid when N_j is on shell.

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