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Observation of electromagnetic Dalitz decays $J/\psi \rightarrow Pe^+e^-$

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Based on a sample of $(225.3\pm2.8)\times10^6~J/\psi$ events collected with the BESIII detector, the electromagnetic Dalitz decays of $J/\psi\to Pe^+e^-(P=\eta'/\eta/\pi^0)$ are studied. By reconstructing the pseudoscalar mesons in various decay modes, the decays $J/\psi\to\eta'e^+e^-$, $J/\psi\to\eta e^+e^-$, and $J/\psi\to\pi^0e^+e^-$ are observed for the first time. The branching fractions are determined to be $\mathcal{B}(J/\psi\to\eta'e^+e^-)=(5.81\pm0.16\pm0.31)\times10^{-5},~~\mathcal{B}(J/\psi\to\eta e^+e^-)=(1.16\pm0.07\pm0.06)\times10^{-5},$ and $\mathcal{B}(J/\psi\to\pi^0e^+e^-)=(7.56\pm1.32\pm0.50)\times10^{-7},$ where the first errors are statistical and the second ones systematic.

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I. INTRODUCTION

The study of electromagnetic (EM) decays of hadronic states plays an important role in revealing the structure of hadrons and the mechanism of the interactions between photons and hadrons [1]. Notably, the EM Dalitz decays $V \to Pe^+e^-$ of unflavored vector (V) mesons (V = ρ , ω , ϕ or J/ψ) are of interest for probing the EM structure arising at the vertex of the transition from vector to pseudoscalar (P) states. In these decays, the lepton pair can be formed by internal conversion of an intermediate virtual photon with invariant mass $M_{e^+e^-}$. Assuming pointlike particles, the variation of the decay rate with $M_{e^+e^-}$ is exactly described by QED [2]. For physical mesons, however, the rate will be modified by the dynamic transition form factor $|F_{VP}(q^2)|^2$, where q is the total 4-momentum of the lepton pair and $q^2=M_{e^+e^-}^2$ is their invariant mass squared. The general form for the q^2 -dependent differential decay width for $V \to Pe^+e^-$, normalized to the width of the corresponding radiative decay $V \rightarrow P\gamma$, is given by [1]

$$\frac{d\Gamma(V \to Pe^+e^-)}{dq^2\Gamma(V \to P\gamma)} = \frac{\alpha_{\rm em}}{3\pi} |F_{VP}(q^2)|^2 \frac{1}{q^2} \left(1 - \frac{4m_e^2}{q^2}\right)^{1/2} \\
\times \left(1 + \frac{2m_e^2}{q^2}\right) \left[\left(1 + \frac{q^2}{m_V^2 - m_P^2}\right)^2 \\
- \frac{4m_V^2 q^2}{(m_V^2 - m_P^2)^2}\right]^{3/2} \\
= |F_{VP}(q^2)|^2 \times [\text{QED}(q^2)], \tag{1}$$

where m_V is the mass of the initial vector state; m_P and m_e are the masses of the final states pseudoscalar meson and lepton, respectively; $\alpha_{\rm em}$ is the fine structure constant; and $[{\rm QED}(q^2)]$ represents the pointlike QED result. The magnitude of the form factor can be estimated based on phenomenological models of nonperturbative QCD [3–7]. For example, in the vector meson dominance (VMD) model [8], the form factor is governed mainly by the resonance interaction between photons and hadrons in the timelike region. Experimentally, the form factor is directly accessible by comparing the measured invariant-mass spectrum of the lepton pairs from Dalitz decays with the pointlike QED prediction [2]. In the simple pole approximation [9,10], the q^2 -dependent form factor is parametrized by

$$|F_{VP}(q^2)| = \frac{1}{(1 - q^2/\Lambda^2)},$$
 (2)

where the parameter Λ is the spectroscopic pole mass.

The EM Dalitz decays of the light unflavored mesons ρ , ω , and ϕ have been intensively studied by the CMD2, SND, NA60, and KLOE experiments [11–15]. For the decays of $\phi \to \eta e^+ e^-$ and $\omega \to \pi^0 e^+ e^-$, the branching fractions and slopes of the form factors Λ^{-2} are measured [12–15], and the results are in agreement with VMD predictions.

Recently, however, a measurement of $\omega \to \pi^0 \mu^+ \mu^-$ from the NA60 experiment [14] obtained a value of Λ^{-2} , which is 10 standard deviations from the expectations of VMD.

These theoretical and experimental investigations of the EM Dalitz decays of the light vector mesons motivate us to study the rare charmonium decays $J/\psi \to Pe^+e^-$, which should provide useful information on the interaction of the charmonium states with the electromagnetic field. At present, there is no experimental information on these decays. In Ref. [16], by assuming a simple pole approximation, the decay rates are estimated to be 10^{-5} and 10^{-7} for the $J/\psi \to \eta'(\eta)e^+e^-$ and $\pi^0e^+e^-$, respectively. In this paper, we present measurements of the branching fractions of $J/\psi \to Pe^+e^-$. This analysis is based on $(225.3 \pm 2.8) \times 10^6 \ J/\psi$ events [17], accumulated with the Beijing Spectrometer III (BESIII) detector [18], at the Beijing Electron Positron Collider II (BEPCII).

II. BESIII EXPERIMENT AND MONTE CARLO SIMULATION

The BESIII detector and BEPCII accelerator represent major upgrades over the previous versions, BESII and BEPC; the facility is used for studies of hadron spectroscopy and τ -charm physics. The design peak luminosity of the double-ring e^+e^- collider, BEPCII, is 10^{33} cm⁻² s⁻¹ at a beam current of 0.93 A. The BESIII detector has a geometrical acceptance of 93% of 4π solid angle and consists of four main components; the inner three are enclosed in a superconducting solenoidal magnet of 1.0 T magnetic field. First, a small-celled, helium-based main drift chamber (MDC) with 43 layers provides charged particle tracking and measurements of ionization energy loss (dE/dx). The average single wire resolution is 135 μ m, and the momentum resolution for 1 GeV/c charged particles is 0.5%. Next is a time-of-flight system (TOF) for particle identification (PID) composed of a barrel part made of two layers with 88 pieces of 5 cm thick, 2.4 m long plastic scintillators in each layer and two end caps with 96 fan-shaped, 5 cm thick, plastic scintillators in each end cap. The time resolution is 80 ps in the barrel and 110 ps in the end caps, corresponding to a 2σ K/π separation for momenta up to about 1.0 GeV/c. Third is an electromagnetic calorimeter (EMC) made of 6240 CsI (Tl) crystals arranged in a cylindrical shape (barrel) plus two end caps. For 1.0 GeV photons, the energy resolution is 2.5% in the barrel and 5% in the end caps, and the position resolution is 6 mm in the barrel and 9 mm in the end caps. Finally, a muon chamber system made of 1272 m² of resistive plate chambers arranged in nine layers in the barrel and eight layers in the end caps is incorporated in the return iron of the superconducting magnet. The position resolution is about 2 cm.

Optimization of event selection and estimations of physical backgrounds are performed using Monte Carlo (MC) simulated samples. The GEANT4-based [19] simulation software BOOST includes the geometric and material

descriptions of the BESIII detector, the detector response, and digitization models and also tracks the detector running conditions and performance. The production of the J/ψ resonance is simulated by the MC event generator KKMC [20]; the known decay modes are generated by EVTGEN [21,22] with branching ratios set at the world average values [23], while unknown decays are generated by LUNDCHARM [24]. The analysis is performed in the framework of the BESIII offline software system, which takes care of the detector calibration, event reconstruction, and data persistency.

In this analysis, $J/\psi \to \eta' e^+ e^-$ is studied using $\eta' \to \gamma \pi^+ \pi^-$ and $\eta' \to \pi^+ \pi^- \eta$ with $\eta \to \gamma \gamma$; $J/\psi \to \eta e^+ e^-$ is studied using $\eta \to \gamma \gamma$ and $\eta \to \pi^+ \pi^- \pi^0$ with $\pi^0 \to \gamma \gamma$; and $J/\psi \to \pi^0 e^+ e^-$ is studied using $\pi^0 \to \gamma \gamma$. An independent data sample of approximately 2.9 fb⁻¹ taken at $\sqrt{s} = 3.773$ GeV is utilized to study a potential continuum background.

The EVTGEN package is used to generate $J/\psi \to \eta' e^+ e^-$, $\eta e^+ e^-$, and $\pi^0 e^+ e^-$ events, with angular distributions simulated according to the amplitude squared in Eq. (3) of Ref. [16]. A simple pole approximation is assumed for the form factor. The decay $\eta \to \pi^+ \pi^- \pi^0$ is generated according to the Dalitz plot distribution measured in Ref. [25]. For the decay $\eta' \to \gamma \pi^+ \pi^-$, the generator takes ρ - ω interference and box anomaly into account [26], while the decay $\eta' \to \pi^+ \pi^- \eta$ is generated with the phase space.

III. DATA ANALYSIS

Charged tracks in the BESIII detectors are reconstructed from ionization signals in the MDC. To select well-measured tracks, we require the polar angle to satisfy $|\cos\theta| < 0.93$ and that tracks pass within 10 cm of the interaction point in the beam direction and within 1 cm in the plane perpendicular to the beam. The number of such tracks and their net charge must exactly correspond to the particular final state understudy. For particle identification, information from dE/dx and TOF is combined to calculate the probabilities, $\operatorname{Prob}_{PID}(i)$, that these measurements are consistent with the hypothesis that a track is an electron, pion, or kaon; $i = e, \pi, K$ labels the particle type. For both electron and positron candidates, we require $\operatorname{Prob}_{PID}(e) > \operatorname{Prob}_{PID}(\pi)$ and $\operatorname{Prob}_{PID}(e) > \operatorname{Prob}_{PID}(K)$. The remaining tracks are assumed to be pions, without PID requirements.

Electromagnetic showers are reconstructed from clusters of energy depositions in the EMC crystals. The energy deposited in nearby TOF counters is included to improve the reconstruction efficiency and energy resolution. The shower energies are required to be greater than 25 MeV for the barrel region ($|\cos(\theta)| < 0.80$) and 50 MeV for the end cap region ($0.86 < |\cos(\theta)| < 0.92$). The showers in the angular range between the barrel and end cap are poorly reconstructed and excluded from the analysis. To exclude showers from charged particles, a photon candidate must be separated by at least 10° from any charged track. Cluster

TABLE I. For each decay mode, the number of observed signal events (N_S) , the number of expected total peaking background events (N_B) in the signal region, and the MC efficiency (ϵ) for signal are given. The uncertainty on N_S is statistical only, and the signal regions are defined to be within 3σ of the nominal pseudoscalar masses.

Modes	N_S	N_B	ϵ
$\overline{J/\psi \to \eta' e^+ e^- (\eta' \to \gamma \pi^+ \pi^-)}$	983.3 ± 33.0	27.4 ± 1.0	24.8%
$J/\psi \rightarrow \eta' e^+ e^- (\eta' \rightarrow \pi^+ \pi^- \eta)$	373.0 ± 19.9	8.5 ± 0.3	17.6%
$J/\psi \rightarrow \eta e^+ e^- (\eta \rightarrow \pi^+ \pi^- \pi^0)$	84.2 ± 9.6	5.3 ± 0.3	14.9%
$J/\psi \to \eta e^+ e^- (\eta \to \gamma \gamma)$	235.5 ± 16.4	8.7 ± 0.3	22.7%
$J/\psi \to \pi^0 e^+ e^- (\pi^0 \to \gamma \gamma)$	39.4 ± 6.9	1.1 ± 0.1	23.4%

timing requirements are used to suppress electronic noise and energy depositions unrelated to the event.

Events with the decay modes shown in Table I are selected. Every particle in the final state must be explicitly found. For each mode, a vertex fit is performed on the charged tracks; a loose χ^2 cut ensures that they are consistent with originating from the interaction point. In η'/η channels with $\eta' \to \pi^+\pi^-\eta$ and $\eta \to \pi^+\pi^-\pi^0$, photon pairs are used to reconstruct η or π^0 candidates if the invariant mass satisfies $m_{\gamma\gamma} \in (480, 600) \text{ MeV}/c^2$ or $(100, 160) \text{ MeV}/c^2$, respectively. To improve resolution and reduce backgrounds, a four-constraint (4C) energy-momentum conserving kinematic fit is performed. For states with extra photon candidates, the combination with the least χ^2_{4C} is selected, and in all cases χ^2_{4C} is required to be less than 100.

In the analysis, one of the most important backgrounds comes from events of the radiative decay $J/\psi \rightarrow P\gamma$ followed by a γ conversion in the material in front of the MDC, including the beam pipe and the inner wall of the MDC. To suppress these backgrounds, a photon-conversion finder [27] was developed to reconstruct the photonconversion point in the material. The distance from this reconstructed conversion point to the origin in the x-y plane, defined as $\delta_{xy} = \sqrt{R_x^2 + R_y^2}$, is used to distinguish photon conversion background from signal; R_x and R_y are the distances projected in the x and y directions, respectively. A scatter plot of R_v vs R_x is shown in Fig. 1(a) for the MC simulated decay $J/\psi \rightarrow \eta' \gamma (\eta' \rightarrow \gamma \pi^+ \pi^-)$, in which one of the photons undergoes conversion to an e^+e^- pair. As indicated in Fig. 1(a), the inner circle matches the position of the beam pipe, while the outer circle corresponds to the position of the inner wall of the MDC. Figure 1(b) shows the δ_{xy} distributions for the MC simulated $J/\psi \rightarrow \eta' e^+ e^-$ and $\eta' \gamma$ events, as well as the selected events in the data for comparison. In the δ_{xy} distributions, the two peaks above 2.0 cm correspond to the photon conversion of the γ from $J/\psi \to \eta' \gamma$ events in the material of the beam pipe and inner wall of the MDC, while the events near $\delta_{xy} = 0$ cm are from the EM Dalitz decay. The selected events from data are in good agreement with

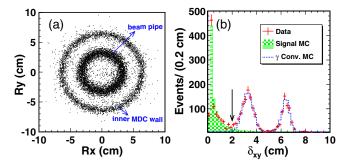


FIG. 1 (color online). Veto of γ -conversion events. (a) a scatter plot of R_y vs R_x for the MC-simulated $J/\psi \to \eta' \gamma$ ($\eta' \to \gamma \pi^+ \pi^-$) events. (b) δ_{xy} distributions. The (green) shaded histogram shows the MC-simulated $J/\psi \to e^+ e^- \eta'$ ($\eta' \to \gamma \pi^+ \pi^-$) signal events. The (red) dots with error bars are data. The (blue) dotted histogram shows the background from the γ -conversion events. In (b), the solid arrow indicates the requirement on δ_{xy} .

the MC simulations as shown in Fig. 1(b). Thus, we require $\delta_{xy} < 2$ cm to suppress the photon-conversion backgrounds for all signal modes. This requirement retains about 80% of the signal events and removes about 98% of the photon-conversion events from the decay $J/\psi \rightarrow \eta' \gamma$. The ability of this requirement to veto the photon-conversion events is the same for the other decay modes. The normalized number of the peaking background events from $J/\psi \rightarrow P\gamma$ and the corresponding selection efficiencies are listed in Table II.

In addition to $J/\psi \to P\gamma$, further peaking backgrounds arise from $J/\psi \to \phi P$, ωP , and ρP ($P=\eta', \eta, \text{ or } \pi^0$), where ϕ , ω , and ρ decay into e^+e^- . Studies based on MC simulations predict 2.2 ± 0.4 , 0.8 ± 0.1 , 2.8 ± 0.3 , and 0.4 ± 0.1 background events for $J/\psi \to \eta' e^+ e^- \times (\eta' \to \gamma \pi^+ \pi^-)$, $J/\psi \to \eta' e^+ e^- (\eta' \to \pi^+ \pi^- \eta)$, $J/\psi \to \eta e^+ e^- (\eta \to \pi^+ \pi^- \pi^0)$, and $J/\psi \to \pi^0 e^+ e^- (\pi^0 \to \gamma \gamma)$ modes, respectively.

Peaking background may also come from $J/\psi \to \pi^+\pi^- P$ with two pions misidentified as an e^+e^- pair. The predicted background levels are 0.2, 0.1, 0.4, and 0.3 events (with negligible errors) for $J/\psi \to \eta' e^+ e^- \times (\eta' \to \gamma \pi^+ \pi^-)$, $J/\psi \to \eta' e^+ e^- (\eta' \to \pi^+ \pi^- \eta)$, $J/\psi \to \eta e^+ e^- (\eta \to \pi^+ \pi^- \pi^0)$, and $J/\psi \to \eta e^+ e^- (\eta \to \gamma \gamma)$, respectively. For $J/\psi \to \pi^0 e^+ e^- (\pi^0 \to \gamma \gamma)$, the potential peaking background from $J/\psi \to \pi^+ \pi^- \pi^0$ (which has a large branching fraction of $(2.07 \pm 0.12)\%$ [23]) is rejected by

TABLE II. The normalized number of peaking background events $(N_{\gamma\text{-conv}})$ from $J/\psi \to P\gamma$ with the photon subsequently converted into an electron-positron pair and the corresponding MC efficiency $(\epsilon_{\gamma\text{-conv}})$ for each background mode.

Mode	$N_{\gamma ext{-conv}}$	$\epsilon_{\gamma ext{-conv}}$
$J/\psi o \eta' \gamma (\eta' o \gamma \pi^+ \pi^-)$	25.0 ± 0.9	7.4×10^{-5}
$J/\psi o \eta' \gamma (\eta' o \pi^+ \pi^- \eta)$	7.6 ± 0.3	3.9×10^{-5}
$J/\psi o \eta \gamma (\eta o \pi^+\pi^-\pi^0)$	2.1 ± 0.1	3.7×10^{-5}
$J/\psi \to \eta \gamma (\eta \to \gamma \gamma)$	8.4 ± 0.3	8.6×10^{-5}
$J/\psi \to \pi^0 \gamma (\pi^0 \to \gamma \gamma)$	0.7 ± 0.1	8.8×10^{-5}

requiring $M_{e^+e^-} \leq 0.4~{\rm GeV}/c^2$. About 80% of signal events are retained, and the remaining background is negligible. Background from $J/\psi \to \phi P$ ($\phi \to K^+K^-$) with two kaons misidentified as an e^+e^- pair is also negligible based on the MC simulation. The total expected peaking backgrounds from all sources are summarized in Table I.

For the $J/\psi \rightarrow \eta' e^+ e^- (\eta' \rightarrow \gamma \pi^+ \pi^-)$ and $J/\psi \rightarrow$ $\eta e^+ e^- (\eta \to \pi^+ \pi^- \pi^0)$ modes, there are nonpeaking backgrounds mainly coming from two sources. One is from $J/\psi \to \gamma \pi^+ \pi^- \pi^+ \pi^-$ and $J/\psi \to \pi^0 \pi^+ \pi^- \pi^+ \pi^-$. With two pions misidentified as an electron-positron pair, this produces a smooth background under the η' or η mass. The other contribution is from $J/\psi \to \pi^+\pi^-\eta$, $\eta \to \gamma e^+e^-$ and $J/\psi \to \pi^+\pi^-\pi^0,\, \pi^0 \to \gamma e^+e^-$ with the same final states as the signal mode $J/\psi \to \eta' e^+ e^- (\eta' \to \gamma \pi^+ \pi^-)$. The combined decay rate of $J/\psi \to \pi^+\pi^-\eta$, $\eta \to \gamma e^+e^-$ is at the rate of 10^{-6} ; the net contribution is negligible according to the MC simulations. To reject background from $J/\psi \rightarrow$ $\pi^+\pi^-\pi^0(\pi^0\to\gamma e^+e^-)$, we veto candidates with an invariant γe^+e^- mass in the interval [0.10, 0.16] GeV/ c^2 ; the remaining background contributes a smooth shape under the η' mass.

For the $J/\psi \to \eta e^+e^-(\eta \to \gamma\gamma)$ and $J/\psi \to \pi^0 e^+e^-\times (\pi^0 \to \gamma\gamma)$ modes, nonpeaking continuum backgrounds from the QED processes $e^+e^- \to e^+e^-\gamma(\gamma)$ and $e^+e^- \to 3\gamma$ (in which one γ converts into an e^+e^- pair) are studied. Since η and π^0 mesons decay isotropically, the angular

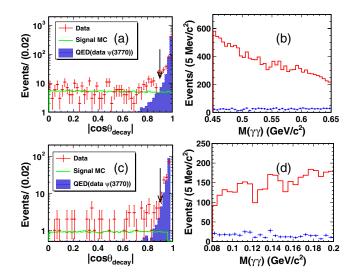


FIG. 2 (color online). The $|\cos\theta_{\rm decay}|$ distributions (a) for η and (c) for π^0 and two-photon invariant-mass distributions (b) for the $J/\psi \to \eta e^+e^-(\eta \to \gamma\gamma)$ and (d) for the $J/\psi \to \pi^0 e^+e^-(\pi^0 \to \gamma\gamma)$ modes. In (a) and(c), the (green) solid histograms are the MC-simulated signals, the (red) dots with error bars are data, and the (blue) dotted histograms are from the $\psi(3770)$ data. The arrows indicate the requirement $|\cos\theta_{\rm decay}| < 0.9$. In (b) and (d), the (red) histograms and the (blue) dots with error bars are $\psi(3770)$ data (used as a continuum sample) without and with the requirement, respectively.

distribution of photons from η or π^0 decays is flat in $\theta_{\rm decay}$, the angle of the decay photon in the η or π^0 helicity frame. However, continuum background events accumulate near $\cos\theta_{\rm decay}=\pm 1$, and thus we require $|\cos\theta_{\rm decay}|<0.9$. Figures 2(a) and 2(c) show the $|\cos\theta_{\rm decay}|$ distributions for η and π^0 decays, respectively. The (blue) dotted histogram peaking near $|\cos\theta_{\rm decay}|=1$ in Fig. 2(a) or 2(c) is from a 2.9 fb⁻¹ $\psi(3770)$ data sample taken at $\sqrt{s}=3.773$ GeV, which is dominated by QED processes. The MC events of $e^+e^- \rightarrow e^+e^-\gamma(\gamma)$ and $e^+e^- \rightarrow 3\gamma$ are generated using the Babayaga QED event generator [28], and the distributions are consistent with that from the 3.773 GeV sample. After requiring $|\cos\theta_{\rm decay}|<0.9$, as shown in Fig. 2(b) or 2(d), the background from QED processes is reduced drastically.

Mass spectra of the signal modes with all of the selection criteria applied are presented in Fig. 3. The signal efficiencies determined from MC simulations for the η' , η , and π^0 are shown in Table I.

An unbinned extended maximum likelihood (ML) fit is performed for each mode to determine the event yield. The signal probability density function (PDF) in each mode is

10³ Events/ (4 MeV/c²) Events/ $(5 \, \text{MeV/c}^2)$ 10 0.85 0.95 0.85 0.9 $M(\gamma \pi^+\pi^-)$ (GeV/c²) $M(\gamma\gamma\pi^{+}\pi^{-})$ (GeV/c²) Events/ (8 MeV/c²) Events/ (5 MeV/c²) 10 10⁻¹ 0.45 0.45 0.55 0.5 0.55 0.65 $M(\gamma\gamma\pi^{\dagger}\pi^{-})$ (GeV/c²) $M(\gamma \gamma)$ (GeV/c²) Events/ (4.8 MeV/c²) 10 0.08 0.1 0.12 0.14 0.16 0.18 $M(\gamma \gamma) (GeV/c^2)$

FIG. 3 (color online). Mass distributions of the pseudoscalar meson candidates in $J/\psi \to Pe^+e^-$: (a) $\eta' \to \gamma\pi^+\pi^-$, (b) $\eta' \to \pi^+\pi^-\eta$ ($\eta \to \gamma\gamma$), (c) $\eta \to \pi^+\pi^-\pi^0$, (d) $\eta \to \gamma\gamma$, and (e) $\pi^0 \to \gamma\gamma$. The (black) dots with error bars are data, the (red) dashed lines represent the signal, the (green) dotted-dashed curves show the nonpeaking background shapes, and the (yellow) shaded components are the shapes of the peaking backgrounds from the $J/\psi \to P\gamma$ decays. Total fits are shown as the (blue) solid lines.

represented by the signal MC shape convoluted with a Gaussian function, with parameters determined from the fit to the data. The Gaussian function is to describe the MC-data difference due to resolution. The shape for the nonpeaking background is described by a first- or secondorder Chebychev polynomial, and the background yield and its PDF parameters are allowed to float in the fit. The dominant peaking background from the γ-conversion events in the $J/\psi \to P\gamma$ decay is obtained from the MC-simulated shape with the number fixed to the normalized value. The fitting ranges for the η' , η , and π^0 modes are $0.85-1.05 \text{ GeV}/c^2$, $0.45-0.65 \text{ GeV}/c^2$, and $0.08-0.20 \text{ GeV}/c^2$, respectively. As discussed in Sec. III, the estimated numbers of peaking background events are subtracted from the fitted yields. The net signal yields for all modes are summarized in Table I.

To further demonstrate the high quality of signal events, the candidate events within $\pm 3\sigma$ of the pseudoscalar meson mass region for each mode are projected to the $M_{e^+e^-}$ mass distribution in the region of $[0.0,0.1]~{\rm GeV}/c^2$ as shown in Fig. 4. The signal MC events are generated based on the

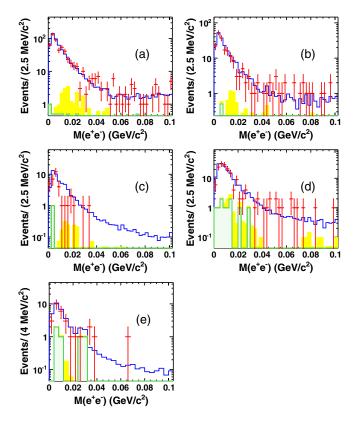


FIG. 4 (color online). The $M_{e^+e^-}$ mass distributions in $J/\psi \to Pe^+e^-$: (a) $\eta' \to \gamma \pi^+\pi^-$, (b) $\eta' \to \pi^+\pi^-\eta$ ($\eta \to \gamma \gamma$), (c) $\eta \to \pi^+\pi^-\pi^0$, (d) $\eta \to \gamma \gamma$, and (e) $\pi^0 \to \gamma \gamma$. The (red) dots with error bars are data, the (yellow) shaded components are from the γ -conversion backgrounds in the $J/\psi \to P\gamma$ decays, and the (green) light-shaded histograms are from nonpeaking backgrounds estimated from the sidebands on the pseudoscalar mass spectra. The (blue) histograms represent the sum of backgrounds and MC-simulated signals.

amplitude squared in Eq. (3) of Ref. [16] for each mode, normalized to the fitted yield. The number of the peaking backgrounds from γ -conversion events is fixed to the expected value, and the nonpeaking backgrounds are estimated by using the sidebands of the pseudoscalar mass spectra. The consistency of the data shapes with signal MC events indicates clear signals in all modes for the EM Dalitz decays $J/\psi \rightarrow Pe^+e^-$.

IV. SYSTEMATIC UNCERTAINTIES

Table III compiles all sources of systematic uncertainties in the measurement of the branching fractions. Most systematic uncertainties are determined from comparisons of clean, high statistics test samples with results from MC simulations.

The MDC tracking efficiency of the charged pion is studied using the control samples of $\psi' \to \pi^+\pi^- J/\psi$, $J/\psi \to l^+ l^ (l = e, \mu)$, and $J/\psi \to \pi^+ \pi^- \pi^0$ [29]. The difference between data and MC simulation is 1.0% for each charged pion. The tracking efficiency for the electron or positron is obtained with the control sample of radiative Bhabha scattering $e^+e^- \rightarrow \gamma e^+e^-$ (including $J/\psi \rightarrow$ $\gamma e^+ e^-$) at the J/ψ energy point. The tracking efficiency is calculated with $\epsilon_{\rm electron} = N_{\rm full}/N_{\rm all}$, where $N_{\rm full}$ indicates the number of γe^+e^- events with all final tracks reconstructed successfully, and $N_{\rm all}$ indicates the number of events with one or both charged lepton particles successfully reconstructed in addition to the radiative photon. The difference in tracking efficiency between data and MC simulation is calculated bin by bin over the distribution of transverse momentum vs the polar angle of the lepton tracks. The uncertainty is determined to be 1.0% per electron. Tracking uncertainties are treated as fully correlated and thus added linearly.

The photon detection efficiency and its uncertainty are studied using three different methods described in Ref. [29]. On average, the efficiency difference between data and MC simulation is less than 1.0% per photon [29]. The uncertainty from π^0 reconstruction is determined to be

1.0% per π^0 from the control sample $J/\psi \to \pi^+\pi^-\pi^0$ [30], and that for η reconstruction is 1.0% from the control sample $J/\psi \to p\bar{p}\eta$ [30].

The uncertainty on electron identification is studied with the control sample of radiative Bhabha scattering $e^+e^- \rightarrow \gamma e^+e^-$ (including $J/\psi \rightarrow \gamma e^+e^-$); samples with backgrounds less than 1.0% are obtained [31]. The efficiency difference for electron identification between the data and MC simulation of about 1.0% is taken as our uncertainty.

In this analysis, the peaking background from the γ -conversion events in $J/\psi \to P\gamma$ decay is suppressed by requiring δ_{xy} < 2 cm. The uncertainty due to this requirement is studied using a sample of $J/\psi \rightarrow$ $\pi^+\pi^-\pi^0, \pi^0 \to \gamma e^+e^-$, which includes both the π^0 Dalitz decay and $\pi^0 \rightarrow \gamma \gamma$ decay with one of the photons converted to an electron-positron pair. Figures 5(a) and 5(c) show the π^0 mass distributions without and with the requirement, and the purity of the sample is better than 99%. The mass distributions of the electron-positron pair are shown in Figs. 5(b) and 5(d) for the events without and with the requirement of δ_{xy} < 2.0 cm, respectively. For comparison, the shape of the MC-generated signal is also plotted. To generate signal events, for the decay $\pi^0 \to \gamma e^+ e^-$, the form factor is modeled by the simple pole approximation as

$$|F(q^2)| = 1 + \alpha q^2 / m_{\pi^0}^2, \tag{3}$$

where q is the total 4-momentum of the electron-positron pair, m_{π^0} is the nominal π^0 mass, and $\alpha=0.032\pm0.004$ is the slope parameter [23]. Extended ML fits to the $M_{e^+e^-}$ distributions are performed to obtain the signal yields of the $J/\psi \to \pi^+\pi^-\pi^0(\pi^0 \to \gamma e^+e^-)$ events as shown in Figs. 5(b) and 5(d). The data-MC difference of 1.0% is considered as the systematic uncertainty for our γ -conversion veto requiring $\delta_{xy} < 2.0$ cm.

The uncertainty from the kinematic fit comes from the inconsistency between the data and MC simulation of the track helix parameters; inaccuracies in our MC simulation

TABLE III. Summary of systematic uncertainties (%). The terms with asterisks are correlated systematic uncertainties between $\eta' \to \gamma \pi^+ \pi^-$ and $\eta' \to \pi^+ \pi^- \eta$ ($\eta \to \pi^+ \pi^- \pi^0$ and $\eta \to \gamma \gamma$).

	$\eta' o \gamma \pi^+ \pi^-$	$\eta' o \pi^+\pi^-\eta$	$\eta \to \pi^+\pi^-\pi^0$	$\eta \to \gamma \gamma$	$\pi^0 \to \gamma \gamma$
MDC tracking*	4.0	4.0	4.0	2.0	2.0
Photon detection *	1.0	2.0	2.0	2.0	2.0
$\pi^0(\eta)$ reconstruction	_	1.0	1.0	1.0	1.0
Electron identification*	2.0	2.0	2.0	2.0	2.0
Veto of the γ conversion*	1.0	1.0	1.0	1.0	1.0
4C kinematic fit	1.0	1.0	1.0	1.0	1.0
Form factor	1.0	1.1	1.1	2.2	3.1
Signal shape	0.9	0.5	0.8	0.1	1.0
Background shape	0.9	1.0	1.0	2.7	4.0
Cited branching fractions	2.0	1.7	1.2	0.5	0.0
Number of J/ψ^*	1.2	1.2	1.2	1.2	1.2
Total	5.6	5.8	5.7	5.4	6.6

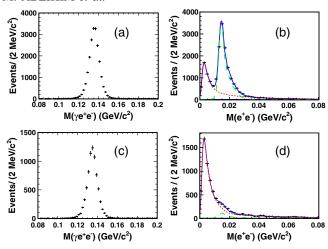


FIG. 5 (color online). Data of $J/\psi \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma e^+e^-$. The distributions of π^0 masses are in (a) and (c); the distributions of the $M_{e^+e^-}$ are in (b) and (d). The upper two plots [(a) and (b)] are for events without the requirement of $\delta_{xy} < 2$ cm; the lower two plots [(c) and (d)] are for events with the requirement. The dots with error bars are data. In (b) and (d), the (red) dashed curves are the MC-simulated signals, and the (green) dot-dashed curves are the MC-simulated shapes from $J/\psi \to \pi^+\pi^-\pi^0(\gamma\gamma)$ in which one of the photons converts to an electron-positron pair. Total fits are shown as the (blue) solid lines.

of photons have previously been shown to be much smaller [32]. Following the procedure described in Refs. [32,33], we take the difference between the efficiencies with and without helix parameter corrections as the systematic uncertainty, which is 1.0% in each mode.

In the analysis, the form factor is parametrized by the simple pole approximation as shown in Eq. (2) with the pole mass $\Lambda = m_{w'} = 3.686 \text{ GeV}/c^2$ in the signal MC generator. Direct information on the pole mass is obtained by studying the efficiency-corrected signal yields for each given $M_{e^+e^-}$ bin i for the decay $J/\psi \to \eta' e^+ e^- (\eta' \to \gamma \pi^+ \pi^-)$, which is the channel with the highest statistics in this analysis. The resolution in $M_{e^+e^-}$ is found to be about 5 MeV in the MC simulation. This is much smaller than a statistically reasonable bin width, chosen as $0.1 \text{ GeV}/c^2$, and hence no unfolding is necessary. The signal yields are background subtracted bin by bin and then efficiency corrected. By using Eq. (1), the value of the $|F_{J/\psi\eta'}|^2$ is extracted for each given bin i as shown in Fig. 6. Fitting this extracted $|F_{J/\psi\eta'}|^2$ vs $M_{e^+e^-}$ data, the pole mass in Eq. (2) is determined to be $\Lambda = (3.1 \pm 1.0) \text{ GeV}/c^2$. To estimate the uncertainty on the signal efficiency originating from the choice of the pole mass, the signal events are generated with $\Lambda = 3.0 \text{ GeV}/c^2$ and $\Lambda = 4.0 \text{ GeV}/c^2$ for each signal mode, respectively. The relative difference of the detection efficiency in each signal mode is taken as the systematic uncertainty, as listed in Table III.

In the fits to the mass distributions of the pseudoscalar mesons, the signal shapes are described by the MC signal shape convoluted with a Gaussian function. Alternative fits

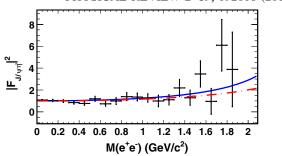


FIG. 6 (color online). Form factor for $J/\psi \to \eta' e^+ e^- \times (\eta' \to \gamma \pi^+ \pi^-)$. The crosses are data, the (red) dotted-dashed curve is the prediction of the simple pole model with the pole mass $\Lambda = m_{\psi'} = 3.686 \; {\rm GeV}/c^2$, and the fit is shown as the (blue) solid curve.

are performed by fixing the signal shape to the MC simulation, and the systematic uncertainties are set based on the changes observed in the yields. The uncertainty due to the nonpeaking background shape is estimated by varying the PDF shape and fitting range in the ML fit for each mode. The changes in yields for these variations give systematic uncertainties due to these backgrounds. The numbers of the expected peaking backgrounds from the photon conversion in radiative decay $J/\psi \rightarrow P\gamma$ are summarized in Table II; the errors are negligible for each mode.

The branching fractions for the decay of π^0 , η , and η' are taken from the world averages [23]. The corresponding uncertainties on the branching fractions are taken as the systematic uncertainties. The uncertainty in the number of J/ψ decays in our data sample is 1.24% [17], which is taken as a systematic uncertainty.

Assuming all systematic uncertainties in Table III are independent, the total systematic uncertainty is obtained by adding them in quadrature. Totals for the five modes range from 5.4% to 6.6%.

V. RESULTS

The branching fractions of the EM Dalitz decays $J/\psi \to Pe^+e^-$, where P stands for η' , η , and π^0 , are calculated with the formula

$$\mathcal{B}(J/\psi \to Pe^+e^-) = \frac{N_S}{N_{J/\psi} \cdot \mathcal{B}(P \to F) \cdot \epsilon},$$
 (4)

where N_S and ϵ are the number of signal events and the detection efficiency for each mode, respectively, listed in Table I. Here, $N_{J/\psi} = (225.3 \pm 2.8) \times 10^6$ is the number of J/ψ events, and $\mathcal{B}(P \to F)$ is the product of the branching fraction of the pseudoscalar decays into the final states F, taken from the PDG [23]. The calculated branching fractions are summarized in Table IV.

The branching fractions of $J/\psi \to \eta' e^+ e^-$ and $J/\psi \to \eta e^+ e^-$ measured in different decay modes are consistent with each other within the statistical and uncorrelated systematic

TABLE IV. Summary of the measurements of the branching fractions, where the first uncertainties are statistical and the second ones are systematic. The theoretical prediction [16] for the branching fractions are listed in the last column.

Mode	Branching fraction	Combined result	Theoretical prediction
$ \frac{J/\psi \to \eta' e^+ e^- (\eta' \to \gamma \pi^+ \pi^-)}{J/\psi \to \eta' e^+ e^- (\eta' \to \pi^+ \pi^- \eta)} $	$(6.01 \pm 0.20 \pm 0.34) \times 10^{-5}$ $(5.51 \pm 0.29 \pm 0.32) \times 10^{-5}$	$(5.81 \pm 0.16 \pm 0.31) \times 10^{-5}$	$(5.66 \pm 0.16) \times 10^{-5}$
$J/\psi \to \eta e^+ e^- (\eta \to \pi^+ \pi^- \pi^0)$ $J/\psi \to \eta e^+ e^- (\eta \to \gamma \gamma)$	$(1.12 \pm 0.13 \pm 0.06) \times 10^{-5}$ $(1.17 \pm 0.08 \pm 0.06) \times 10^{-5}$	$(1.16 \pm 0.07 \pm 0.06) \times 10^{-5}$	$(1.21 \pm 0.04) \times 10^{-5}$
$J/\psi \to \pi^0 e^+ e^- (\pi^0 \to \gamma \gamma)$	$(7.56 \pm 1.32 \pm 0.50) \times 10^{-7}$	$(7.56 \pm 1.32 \pm 0.50) \times 10^{-7}$	$(3.89^{+0.37}_{-0.33}) \times 10^{-7}$

uncertainties. In Table III, the items with asterisks denote the correlated systematic errors, while the others are uncorrelated. The measurements from different modes are therefore combined with the approach in Ref. [34], which uses a standard weighted least-squares procedure, taking into consideration the correlations between the measurements. For $J/\psi \to \eta' e^+ e^-$, the correlation coefficient between $\eta' \to \gamma \pi^+ \pi^-$ and $\eta' \to \pi^+ \pi^- \eta$ is $\rho(1,2) = 0.46$; for $J/\psi \to \eta e^+ e^-$, it is $\rho(1,2) = 0.13$. The weighted averages of the BESIII measurements are listed in Table IV.

VI. SUMMARY

In summary, with a sample of $(225.3 \pm 2.8) \times 10^6 J/\psi$ events in the BESIII detector, the EM Dalitz decays $J/\psi \to Pe^+e^-$, where P stands for η' , η , and π^0 , have been observed for the first time. The branching fractions of $J/\psi \rightarrow \eta' e^+ e^-$, $J/\psi \rightarrow \eta e^+ e^-$, and $J/\psi \rightarrow \pi^0 e^+ e^-$ are measured to be $\mathcal{B}(J/\psi \to \eta' e^+ e^-) = (5.81 \pm 0.16 \pm 0.31) \times 10^{-5}$, $\mathcal{B}(J/\psi \to \eta e^+ e^-) = (1.16 \pm 0.07 \pm 0.06) \times 10^{-5}$, and $\mathcal{B}(J/\psi \to \pi^0 e^+ e^-) = (7.56 \pm 1.32 \pm 0.50) \times 10^{-7}$, respectively. The measurements for $J/\psi \rightarrow \eta' e^+ e^-$ and $J/\psi \to \eta e^+ e^-$ decay modes are consistent with the theoretical prediction in Ref. [16]. However, the theoretical prediction for the decay rate of $J/\psi \to \pi^0 e^+ e^-$ based on the VMD model is $(3.89^{+0.37}_{-0.33}) \times 10^{-7}$, about 2.5 standard deviations from the measurement in this analysis, which may indicate that further improvements of the QCD radiative and relativistic corrections are needed.

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