

New self-dualities and duality cascades

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New self-dualities involving two index tensors are derived. These new self-dualities are used to build various duality cascades. Both vectorlike and chiral cascades are presented. Aside from ending in confinement, these duality cascades can also end in interacting conformal field theories, free field theories, and metastable supersymmetry breaking. Higgsing effects are built into the self-duality so that when the gauge groups are small enough, supersymmetry is broken through the rank condition. Dynamical supersymmetry restoration occurs far from the SUSY breaking vacuum resulting in a long-lived metastable vacuum. It is found that Coulomb branches are critical in the stabilization of runaways and dynamical supersymmetry restoration.

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I. INTRODUCTION

The Klebanov-Strassler (KS) cascade [1–3] has been fruitful ground for better understanding ADS/CFT [4]. The gauge theory description has a $SU(N) \times SU(N+M)$ gauge theory undergoing a duality cascade. As the gauge groups become strongly coupled, Seiberg duality [5] is applied resulting in an identical theory but with smaller gauge groups. The UV has ever increasing numbers of degrees of freedom, while the IR is a gapped theory. The gravity description is a deformed conifold where the singularity is screened, geometrizing confinement.

Cascades are useful for supersymmetry (SUSY) breaking scenarios. Cascades involving SUSY breaking can be used for model building in extra dimensions [6] or for building nonsupersymmetric string vacua [7]. Many attempts have been made to obtain cascades which break supersymmetry. Some of these involve deforming the KS cascade by adding anti-D3 branes [8] yielding a meta stable SUSY breaking vacuum. Others use different backgrounds finding SUSY breaking in the form of runaways [9–13] which in some cases can be stabilized by hand at the end of the cascade [14]. Both of these attempts have their drawbacks, whether it be runaway behavior rather than SUSY breaking [15], singular geometries [16,17], the lack of a field theory description, or the lack of a controllable gravity dual.

Rather than deforming a cascade to break supersymmetry, this paper takes a different approach. We build the SUSY breaking mechanism into the renormalization group cascade itself in the form of Higgsing effects. Higgsing is introduced into a known duality by adding a term linear in the meson. In the IR, the superpotential becomes

$$W = \text{Tr}M + qM\tilde{q}, \quad (1.1)$$

where q and \tilde{q} are the dual quarks with a gauge group of size \tilde{N}_c and N_f flavors. The rank of $q\tilde{q}$ is determined by

$\min(\tilde{N}_c, N_f)$. If $\tilde{N}_c > N_f$ the quarks obtain a vacuum expectation value (vev) that Higgses the gauge group reducing it from \tilde{N}_c to $\tilde{N}_c - N_f$. If $\tilde{N}_c < N_f$, it is impossible to set $q\tilde{q} \sim \mathbb{1}_{N_f \times N_f}$ and supersymmetry is broken by the rank condition [18]. As in the original ISS situation, supersymmetry is dynamically restored by gaugino condensation at large field values.

The rank condition can be built into a cascade if self-dualities are found where Higgsing effects are crucial. In the Klebanov-Strassler cascade, the cascade proceeds as $SU(N) \times SU(N-M) \rightarrow SU(N-2M) \times SU(N-M) \rightarrow SU(N-2M) \times SU(N-3M) \rightarrow \dots$ until one of the gauge group goes negative. A negative gauge group indicates that Seiberg duality has been incorrectly applied and that instead there is an ADS superpotential. However, if Higgsing is important to each step of these dualities, then the gauge group becoming negative is instead an indication that the rank condition is no longer satisfied so that a metastable supersymmetric vacuum is present.

In the large N limit, the gauge group is always large enough so that the quarks obtain a supersymmetric vev. At the bottom of the cascade, the gauge groups are small so that the rank condition is not satisfied and supersymmetry is broken. This approach gives a dynamical reason why the SUSY breaking vacuum is present in the IR. This paper constructs several cascades which exhibit this feature.

Another interesting feature found is the importance of branches of moduli space where the gauge symmetry is broken into subgroups. A runaway is found which is not stabilized in the magnetic theory, but is stabilized in the electric theory on other branches with smaller gauge groups. A similar effect is found in dynamical supersymmetry restoration, where the supersymmetric vacuum is caused by gaugino condensation of a subgroup of the entire gauge symmetry.

The paper is organized as follows. Section II discusses several new gauge theory self-dualities where Higgsing is

important with a more exhaustive list presented in Appendix A. Section III presents a cascade involving adjoints which behaves very similarly to the Klebanov-Strassler cascade. Section IV presents a chiral cascade where the IR features metastable supersymmetry breaking. Section V presents a cascade where the self-dual point is a free field theory. Finally Sec. VI concludes with future directions.

II. SELF-DUAL THEORIES WITH TWO INDEX TENSORS

In this section, we present several self-dualities where the relevant gauge theories have two index tensors. The

approach used to derive these new self-dualities is Higgsing effects. As a result, when these self-dualities are incorporated into a duality cascade, the Higgsing effects generate metastable supersymmetry breaking through the rank condition. This section presents self-dualities for SU gauge groups with either an adjoint or both a symmetric and antisymmetric tensor. Additional self-dualities are presented in Appendix A.

A. $SU(N_c)$ with a single adjoint

A duality for SU gauge groups with an adjoint and N_f flavors was found in Refs. [19,20] and is summarized below. The electric theory is

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$	
Q	\square	\square		1	$1 - \frac{2N_c}{(k+1)N_f}$	(2.1)
\tilde{Q}	$\bar{\square}$		$\bar{\square}$	-1	$1 - \frac{2N_c}{(k+1)N_f}$	
X	Adj				$\frac{2}{k+1}$	

$$W = \frac{\lambda}{k+1} \text{Tr} X^{k+1}. \quad (2.2)$$

This theory is dual to

	$SU(\tilde{N}_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$	
q	\square	$\bar{\square}$		$\frac{N_c}{kN_f - N_c}$	$1 - \frac{2}{k+1} \frac{kN_f - N_c}{N_f}$	(2.3)
\tilde{q}	$\bar{\square}$		\square	$-\frac{N_c}{kN_f - N_c}$	$1 - \frac{2}{k+1} \frac{kN_f - N_c}{N_f}$	
x	Adj				$\frac{2}{k+1}$	
$M_j = QX^j\tilde{Q}$		\square	$\bar{\square}$		$2 - \frac{4N_c}{(k+1)N_f} + \frac{2j}{k+1}$	

$$W = -\frac{\lambda}{k+1} \text{Tr} x^{k+1} + \frac{\lambda}{\mu^2} \sum_{j=0}^{k-1} M_{k-j-1} q x^j \tilde{q},$$

where the index j can run from 0 to $k-1$ and $\tilde{N}_c = kN_f - N_c$. The auxiliary scale μ is present for dimensional reasons. The theory has a runaway if $N_f < \frac{N_c}{k}$ [20]. The argument for a runaway is to study the electric theory with the superpotential

$$W = \sum_{i=0}^k \lambda_i \text{Tr} X^{i+1}. \quad (2.4)$$

Adding in the subleading terms does not affect the large vev behavior. If there is a runaway for the superpotential shown in Eq. (2.4), then there is a runaway for the superpotential

shown in Eq. (2.2). The potential resulting from Eq. (2.4) has k solutions. Consider giving an expectation value to X . These expectation values are labeled by integers i_l which label how many of its eigenvalues are in the l th solution and obey $\sum_{l=1}^k i_l = N_c$. The gauge group is broken down to $SU(N_c) \rightarrow SU(i_1) \times SU(i_2) \times \cdots \times SU(i_k) \times U(1)^{k-1}$. The adjoint is massive and integrated out while each gauge group has N_f flavors. An ADS superpotential [21] is avoided if $i_l \leq N_f$ so that the theory does not have a runaway when $N_f \geq \frac{N_c}{k}$. The development of a runaway coincides with when the dual gauge group (\tilde{N}_c) runs negative.

A self-dual point similar to SQCD exists for this theory. Requiring that the dual has the same gauge group gives $N_f = \frac{2N_c}{k}$. The mesons are removed by the superpotential $W = X^{k+1} + \sum_{i+j=k-1} QX^i \tilde{Q} QX^j \tilde{Q}$. As before, all of the

terms in the superpotential are exactly marginal. Unfortunately, this self-duality happens to be difficult to incorporate into a duality cascade.

A new self-dual point is realized for

	$SU(N)$	$SU(N - \delta)$	
Q	\square	\square	(2.5)
\tilde{Q}	$\bar{\square}$	$\bar{\square}$	
X	Adj		

$$W = \lambda_1 \text{Tr} X^4 + \lambda_2 \text{Tr} Q X^2 \tilde{Q} + \lambda_3 \text{Tr} (Q \tilde{Q})^2.$$

Applying the above duality, we arrive at the theory

	$SU(2N - 3\delta)$	$SU(N - \delta)$	
q	\square	$\bar{\square}$	(2.6)
\tilde{q}	$\bar{\square}$	\square	
x	Adj		
M_0		$1 + \text{Adj}$	
M_1		$1 + \text{Adj}$	
M_2		$1 + \text{Adj}$	

$$W = -\lambda_1 \text{Tr} x^4 + \lambda_2 \text{Tr} M_2 + \lambda_3 \text{Tr} M_0^2 + 4 \frac{\lambda_1}{\mu^2} (M_2 q \tilde{q} + M_1 q x \tilde{q} + M_0 q x^2 \tilde{q}).$$

The F term for M_2 results in a vev $\langle q \tilde{q} \rangle = \mathbb{1}_{N-\delta \times N-\delta}$. The vev Higgses the gauge group while the remaining terms give mass to the various fields. After integrating out matter, the dual theory is

	$SU(N - 2\delta)$	$SU(N - \delta)$	
q	\square	$\bar{\square}$	(2.7)
\tilde{q}	$\bar{\square}$	\square	
x	Adj		

$$W = -\lambda_1 \text{Tr} x^4 - 4\lambda_1 \text{Tr} q x^2 \tilde{q} - \left(2\lambda_1 + \frac{\lambda_2^2}{4\lambda_3} \right) \text{Tr} (q \tilde{q})^2.$$

There is a self-dual point for $\delta = 0$ where the R charge of all fields is $1/2$. While the couplings do have a $1/\lambda_3$ component, it is not as simple as a coupling becoming its inverse.

Another self-dual fixed point is

	$SU(N)$	$SU(2N - \delta)$	
Q	\square	\square	(2.8)
\tilde{Q}	$\bar{\square}$	$\bar{\square}$	
X	Adj		

$$W = \lambda_1 \text{Tr} X^3 + \lambda_2 \text{Tr} Q X \tilde{Q}.$$

Exactly at the self-dual point, the theory is connected to a free field theory. Deforming epsilonically away from the free field theory, the theory flows to a Banks-Zaks fixed point. The superpotential terms are relevant and the theory becomes strongly coupled. Applying the above duality and integrating out matter, this theory is dual to

	$SU(N - \delta)$	$SU(2N - \delta)$	
q	\square	$\bar{\square}$	(2.9)
\tilde{q}	$\bar{\square}$	\square	
x	Adj		

$$W = -\lambda_1 \text{Tr} x^3 - 3\lambda_1 \text{Tr} q x \tilde{q}.$$

This theory is IR free and after redefining the phase of x , there is a self-dual point for $\delta = 0$ and $\lambda_2 = 3\lambda_1$.

B. A chiral self-duality

In this subsection, we present a chiral self-duality. A duality for SU gauge groups with an antisymmetric tensor and a symmetric tensor was found in Ref. [22]. The electric theory is

	$SU(N_c)$	$SU(N_f + 8)$	$SU(N_f)$	
Q	\square	\square		(2.10)
\tilde{Q}	$\bar{\square}$		\square	
A	\square			
\tilde{S}	$\bar{\square}$			

$$W = \text{Tr} (A \tilde{S})^{2(k+1)}.$$

Anomaly cancellation requires the additional eight extra flavors of fundamental quarks. This theory was demonstrated to be dual to

	$SU(\tilde{N}_c)$	$SU(N_f + 8)$	$SU(N_f)$
q	\square	$\bar{\square}$	
\tilde{q}	$\bar{\square}$		$\bar{\square}$
a	\square		
\tilde{s}	$\bar{\square}$		
$M_j = Q(\tilde{S}A)^j \tilde{Q}$		\square	\square
$P_{r(\text{even})} = Q(\tilde{A}S)^r \tilde{S}Q$		\square	
$P_{r(\text{odd})} = Q(\tilde{A}S)^r \tilde{S}Q$		\square	
$\tilde{P}_{r(\text{even})} = \tilde{Q}A(\tilde{S}A)^r \tilde{Q}$			\square
$\tilde{P}_{r(\text{odd})} = \tilde{Q}A(\tilde{S}A)^r \tilde{Q}$			\square

(2.11)

$$W = \text{Tr}(a\tilde{s})^{2(k+1)} + \sum_{j=0}^{2k+1} M_{2k+1-j} q(\tilde{s}a)^j \tilde{q} + \sum_{r=0}^{2k} P_{2k-r} q(\tilde{s}a)^r \tilde{s}q + \tilde{P}_{2k-r} \tilde{q}a(\tilde{s}a)^r \tilde{q},$$

where the indices j runs from 0 to $2k + 1$ and r runs from 0 to $2k$ and $\tilde{N}_c = (4k + 3)(N_f + 4) - N_c$. P_r (\tilde{P}_r) are symmetric tensors when r is even (odd) and are antisymmetric tensors when r is odd (even).

There are a couple of self-dualities that can be obtained using the above duality. The one most relevant for constructing cascades is

	$SU(N)$	$SO(N - 6 - \delta)$	$SO(8)$
Q	\square	\square	
Q'	\square		\square
\tilde{Q}	$\bar{\square}$	\square	
A	\square		
\tilde{S}	$\bar{\square}$		

(2.12)

$$W_1 = \text{Tr}(A\tilde{S})^2 + (Q\tilde{Q})^2 + (Q'\tilde{Q})^2 + (Q\tilde{S}A\tilde{Q})^2 + (Q'\tilde{S}Q)^2 + (Q'\tilde{S}Q')^2,$$

which is dual to itself with gauge group $SU(N - 2\delta)$.

This self-duality is different from the vectorlike self-dualities of Sec. II A and Appendix A. At the self-dual point, the R charges of the fields are not 1/2. Instead, a -maximization can be used to calculate their R charges [23]. By tracking the flow as various couplings become relevant and part of the conformal field theory (CFT) [24], or by applying a -maximization subject to inequalities [25], one arrives at the following R charges for the theory when $\delta = 0$:

$$R_Q = R'_Q = \frac{6N + 1}{18N}, \quad R_{\tilde{Q}} = \frac{12N - 1}{18N},$$

$$R_A = \frac{6N + 1}{9N}, \quad R_S = \frac{3N - 1}{9N}. \quad (2.13)$$

Notice that these R charges are not 1/2 and do not go to it as N goes to infinity.

At infinite N , a new fixed point emerges. For infinite N , the maximum is when all of the R charges are 1/2. From the point of view of a -maximization, in R charge space, as N increases a new maximum approaches the surface defined by the superpotential and anomaly free constraints. Only at infinite N does this new maximum satisfy the constraints. The previous maximum ceases to be a maximum when this new maximum appears.

III. A $SU(N) \times SU(N)$ CASCADE

Using these new self-dualities, we can build up several cascades involving two index tensors. Many of the cascades are similar and only a few cascades will be presented—each highlighting different features. The first cascade we consider is a cascade involving adjoints. We take the UV gauge theory

	$SU(N)$	$SU(N - \delta)$
Q	\square	\square
\tilde{Q}	$\bar{\square}$	$\bar{\square}$
X_1	Adj	
X_2		Adj

(3.1)

$$W = X_1^4 + X_2^4 + QX_1^2\tilde{Q} + QX_2^2\tilde{Q} + (Q\tilde{Q})^2. \quad (3.2)$$

There are many different UV fixed points and the flows that follow depend on the relative sizes of the different coefficients. We describe one which behaves like the typical duality cascade. For the starting UV fixed point, we take the self-dual $\delta = 0$ fixed point. We have the beta functions

$$\beta_{SU(N)} \sim \delta, \quad \beta_{SU(N-\delta)} \sim -\delta, \quad (3.3)$$

where we have used that at the self-dual point the R charges are all $1/2$. Since the beta functions have opposite signs, we see that $SU(N - \delta)$ becomes more weakly coupled while $SU(N)$ becomes more strongly coupled. Thus we arrive at the fixed point associated with the $SU(N)$ gauge group. Using a -maximization, we find that the most relevant quartic operator is the X_1^4 quartic. Thus the theory flows near the fixed point presented in Sec. II A.

At the $SU(N)$ fixed point, the $SU(N - \delta)$ gauge coupling is irrelevant while the quartic QX_1^2Q is relevant. When this quartic coupling becomes large, the duality of Sec. II A shows us that this theory undergoes Higgsing effects. Using the duality of Sec. II A, we see that this theory becomes a $SU(N - 2\delta) \times SU(N - \delta)$ gauge theory. Now the $SU(N - 2\delta)$ gauge coupling is irrelevant while the $SU(N - \delta)$ gauge coupling is relevant leading to a cascade. We are at the starting point with $N' = N - \delta$. This cascade is much like the original duality cascade of KS except that the bottom of the cascade has new interesting physics possibilities.

This cascade highlights additional constraints that appear when building duality cascades. Two self-dual points are not enough. Dualizing one gauge group effects the superpotential terms that are used to build the other self-duality. For example, the self-duality of Sec. II A has an effect on all of the terms in the superpotential shown in Eq. (3.2). The reason is that after duality, there exists an additional term $X_2^2 M_0$. After M_0 gets integrated out, the superpotential terms X_2^4 , $qX_2^2\tilde{q}$, and $(q\tilde{q})^2$ are all generated. If these terms were not part of the self-duality of the $SU(N - \delta)$ then the duality cascade would not proceed.

A. IR dynamics

We now consider the bottom of the cascade. Take $N = m\delta + \epsilon$ with $m \in \mathbb{Z}$. There are many cases and for simplicity they will be summarized here. If $\epsilon = 0$, we have a goldstone mode and a free field in the IR. If $0 < \epsilon < \frac{\delta}{2}$ the theory has a runaway that is stabilized by the superpotential and the theory obtains a mass gap. If $\frac{\delta}{2} < \epsilon \leq \frac{2\delta}{3}$, there appears to be a long-lived metastable SUSY breaking vacuum, though there is a singlet whose mass is sensitive to unknown Kahler potential corrections¹ and thus cannot be unambiguously shown to be non-tachyonic. For $\frac{\delta}{2} < \epsilon < \delta$, a gauge symmetry breaking supersymmetric vacuum is found far away in field space from the origin.

1. $\epsilon = 0$: Free fields in the IR

If $\epsilon = 0$, the cascade ends with the gauge theory $SU(\delta) \times SU(2\delta)$.

¹This incalculability of the SUSY breaking vacuum is not present in the chiral cascade presented in Sec. IV or a vectorlike cascade presented in Appendix B.

	$SU(\delta)$	$SU(2\delta)$	
Q	\square	\square	(3.4)
\tilde{Q}	$\bar{\square}$	$\bar{\square}$	
X_1	Adj		
X_2		Adj	

$$W = X_1^4 + X_2^4 + QX_1^2\tilde{Q} + QX_2^2\tilde{Q} + (Q\tilde{Q})^2.$$

Dualizing the $SU(2\delta)$ gauge theory yields a $SU(\delta) \times SU(\delta)$ gauge theory. The dual quark vevs now break $U(1)_B$ yielding a goldstone mode. Additionally the singlet meson M_1 remains massless. The remaining modes mass up and the remaining $SU(\delta)$ gauge group is left with just an adjoint which confines. Thus the IR theory is simply the goldstone mode and the singlet meson M_1 .

2. $0 < \epsilon < \frac{\delta}{2}$: Confinement

For $0 < \epsilon < \frac{\delta}{2}$, the cascade ends in the theory shown below

	$SU(\epsilon)$	$SU(\epsilon + \delta)$	
Q	\square	\square	(3.5)
\tilde{Q}	$\bar{\square}$	$\bar{\square}$	
X_1	Adj		
X_2		Adj	

$$W = X_1^4 + X_2^4 + QX_1^2\tilde{Q} + QX_2^2\tilde{Q} + (Q\tilde{Q})^2.$$

Given these choice of parameters, the $SU(\epsilon + \delta)$ theory has a runaway as described in Sec. II A. We give an explicit demonstration of this effect when $\epsilon = \frac{\delta-2}{2}$.

To find an explicit form of the runaway, we start with the situation where $SU(\epsilon + \delta)$ confines with a superpotential and integrate out a flavor. The confining case was worked out in Ref. [26] and the confining superpotential is shown in Eq. (3.13). Integrating out a flavor, one arrives at the superpotential

$$W_{\text{dyn}} \sim \frac{1}{(\det M_2)^{\frac{9}{2}}} ((\det M_2)^2 (M_0 \text{cof} M_2)) + (\det M_2) (M_1 \text{cof} M_2^2), \quad (3.6)$$

$$\text{cof} M = \frac{\partial \det M}{\partial M_{ij}}. \quad (3.7)$$

The runaway is stabilized by the two superpotential terms M_2 and M_0^2 and the theory obtains a mass gap. If we have $\epsilon = \frac{\delta}{2}$, presumably the $SU(\epsilon + \delta)$ confines with a quantum moduli space, though the details have not yet been worked out.

3. $\frac{\delta+1}{2} \leq \epsilon < \delta$: SUSY breaking and restoration

Next we consider the case when $\frac{\delta+1}{2} \leq \epsilon < \delta$. As before, the last step of the cascade is shown in Eq. (3.5). The $SU(\epsilon)$ gauge coupling is irrelevant while the $SU(\epsilon + \delta)$ gauge coupling is relevant. We will show that a long-lived metastable vacuum potentially² exists when $\Lambda_{SU(\epsilon)} \gg \Lambda_{SU(\epsilon+\delta)}$. Before the $SU(\epsilon + \delta)$'s gauge coupling becomes strong, the quartic interactions are all irrelevant. After $SU(\epsilon + \delta)$ becomes strongly coupled, a -maximization can be used to show that the quartic operators are relevant with X_2^4 being the most relevant operator. We dualize the $SU(\epsilon + \delta)$ gauge group and obtain the IR theory

	$SU(\epsilon)$	$SU(2\epsilon - \delta)$	
q	$\bar{\square}$	\square	(3.8)
\tilde{q}	\square	$\bar{\square}$	
X_1	Adj		
x_2		Adj	
M_0	$1 + \text{Adj}$		
M_1	$1 + \text{Adj}$		
M_2	$1 + \text{Adj}$		

$$W = X_1^4 + x_2^4 + X_1^2 M_0 + M_2 + M_0^2 + M_2 q \tilde{q} + M_1 q x_2 \tilde{q} + M_0 q x_2^2 \tilde{q}.$$

The F term for M_2 requires that $q \tilde{q} \sim \mathbb{1}_{\epsilon \times \epsilon}$; however, the rank of $\text{Rank}(q \tilde{q}) = \min(\epsilon, 2\epsilon - \delta) = 2\epsilon - \delta < \epsilon$. So it is impossible to satisfy the F term for M_2 . Thus we see that we have a candidate SUSY breaking vacuum.

We first study the candidate SUSY breaking vacuum. In the UV, all of the perturbations are irrelevant so the deformations are all small in value and that perturbation theory is reliable. We decompose our theory around the vacuum

$$q = \begin{pmatrix} \mathbb{1} + \chi_+ + \chi_- \\ \rho_+ + \rho_- \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \mathbb{1} + \chi_+ - \chi_- \\ \rho_+ - \rho_- \end{pmatrix}, \quad (3.9)$$

$$X_1 = \begin{pmatrix} \alpha & \beta \\ \tilde{\beta} & \gamma \end{pmatrix}, \quad M_{0,1,2} = \begin{pmatrix} Y_{0,1,2} & Z_{0,1,2} \\ \tilde{Z}_{0,1,2} & \Phi_{0,1,2} \end{pmatrix}. \quad (3.10)$$

²The mass of one of the singlets is incalculable. This problem is not present in cascades presented in Sec. IV or Appendix B.

As in the ISS scenario, the fields inside of the q , \tilde{q} , and M_2 fields obtain either a tree-level mass, are eaten by gauge bosons, or obtain a positive 1-loop mass. M_0 , Y_1 , and x_2 have a supersymmetric tree-level mass. All fields in M_1 , M_2 , X_1 , x_2 charged under either gauge group receive a positive 2-loop mass from gauge mediation. The only matter field which is not stabilized at this point is the singlet part of M_1 . Being coupled via an irrelevant operator, its mass cannot reliably be determined [27]. Incalculable Kahler potential corrections to the mass of M_1 are of the form

$$V \sim \frac{qq^\dagger M_1^2}{\Lambda^2}. \quad (3.11)$$

The loop level corrections to its mass from the superpotential are

$$m_{\text{superpotential}} \sim \frac{|\langle q \rangle|^2}{(16\pi^2)^L \Lambda^2}, \quad (3.12)$$

where L is the loop level. The incalculable Kahler potential contributions to the mass of M_1 are larger than the calculable loop level corrections. Thus while it appears that there is likely a SUSY breaking vacuum, its stability cannot be reliably determined.

In Ref. [18], a supersymmetric vacuum was found at large field values. For $\epsilon = \frac{\delta+1}{2}$, $SU(\epsilon + \delta)$ confines with the superpotential [26]

$$W_{\text{dyn}} \sim \tilde{q} M_2 q + (\det M_2)^2 (M_0 \text{cof} M_2) + (\det M_2) (M_1 \text{cof} M_2)^2. \quad (3.13)$$

One can explicitly show that this dynamical superpotential allows for the existence of a supersymmetric vacuum at large field values. When $\epsilon > \frac{\delta+1}{2}$, gaugino condensation at large field values allows for the existence of a supersymmetric vacuum. Consider large field values of the singlet in M_2 and giving the adjoint the vev³

$$x_2 = x \begin{pmatrix} \mathbb{1}_{2\epsilon - \delta - 1 \times 2\epsilon - \delta - 1} & 0 \\ 0 & -2\epsilon + \delta + 1 \end{pmatrix}. \quad (3.14)$$

The vev in the adjoint breaks the gauge group down to $SU(2\epsilon - \delta - 1) \times U(1)$ while M_2 lifts all of the matter for this unbroken gauge group. The IR matter content of $SU(2\epsilon - \delta - 1)$ consists of a single adjoint. This adjoint obtains a mass from the quartic so that the final dynamical scale corresponding to gaugino condensation is

$$\Lambda_{SU(2\epsilon - \delta - 1)}^{6\epsilon - 3\delta - 3} \sim M_2^\epsilon \Lambda_{SU(2\epsilon - \delta)}^{\epsilon - \delta + 1} x^{4\epsilon - 2\delta - 4}. \quad (3.15)$$

³Appendix C shows how taking symmetry breaking vevs can be used to rederive the well-known ADS superpotential.

Finally, the linear and quartic term in conjunction with gaugino condensation has a superpotential

$$W \supset (M_2^\epsilon \Lambda_{SU(2\epsilon-\delta)}^{\epsilon-\delta+1} x^{4\epsilon-2\delta-4})^{1/(2\epsilon-\delta-1)} + \mu^2 M_2 + \frac{x^4}{\Lambda_{SU(2\epsilon-\delta)}}. \quad (3.16)$$

The linear term was irrelevant in the UV so that we have $\mu \sim \frac{\Lambda_{SU(\epsilon+\delta)}^2}{\Lambda_{SU(\epsilon)}} \ll \Lambda_{SU(2\epsilon-\delta)} \sim \Lambda_{SU(\epsilon+\delta)}$, given our initial assumption that the original dynamical scales were far apart. We see that we have a supersymmetric vacuum at

$$M_2 \sim \Lambda_{SU(2\epsilon-\delta)} \left(\frac{\mu}{\Lambda_{SU(2\epsilon-\delta)}} \right)^{\frac{4\epsilon-2\delta}{\delta}} \sim \mu \left(\frac{\mu}{\Lambda_{SU(2\epsilon-\delta)}} \right)^{\frac{4\epsilon-3\delta}{\delta}}, \quad (3.17)$$

$$x \sim \Lambda_{SU(2\epsilon-\delta)} \left(\frac{\mu}{\Lambda_{SU(2\epsilon-\delta)}} \right)^{\frac{\epsilon}{\delta}} \sim \mu \left(\frac{\mu}{\Lambda_{SU(2\epsilon-\delta)}} \right)^{\frac{\epsilon-\delta}{\delta}}. \quad (3.18)$$

We had assumed that the vev of x_2 broke the gauge group down to $SU(2\epsilon - \delta - 1)$, but the scaling of these solutions are independent of which gauge group we broke $SU(2\epsilon - \delta)$ down to. We are studying the range $\frac{\delta+1}{2} \leq \epsilon < \delta$. In this range the supersymmetric vacuum always obeys $M_2, x \ll \Lambda_{SU(2\epsilon-\delta)}$. Thus our supersymmetric vacuum is under control. For it to be long lived, we need M_2 or $x \gg \mu$. We find that $x \gg \mu$ for all values in the range of interest while $M_2 \gg \mu$ if $\epsilon < \frac{3\delta}{4}$. As this vacuum goes to infinity as $\epsilon \rightarrow 0$, our metastable SUSY breaking vacuum, if it exists, is long lived.

If $\frac{\delta}{2} < \epsilon \leq \frac{2\delta}{3}$, $SU(2\epsilon - \delta)$ is IR free while if $\frac{2\delta}{3} < \epsilon < \delta$, $SU(2\epsilon - \delta)$ is a CFT. If the gauge group is a CFT, then the perturbative analysis conducted before around the SUSY breaking vacuum is not reliable and conformal perturbation theory would need to be applied. The SUSY breaking vacuum is always far away in field space from the origin.

IV. A CHIRAL CASCADE

One can build a cascade from the chiral self-dual theory presented in Sec. II B. Gauging the $SO(N-6)$ gauge group, we see that at infinite N , the theory is self-dual under both duality transformations of the SU and SO gauge groups. Assume we start with the gauge group $SU(N) \times SO(N-6-\delta)$ with $N, \delta \gg 1$. The gauge theory and superpotential of the starting theory is

	$SU(N)$	$SO(N-6-\delta)$	$SO(8)$
Q	\square	\square	
Q'	\square		\square
\tilde{Q}	$\overline{\square}$	\square	
A	\square		
\tilde{S}	$\overline{\square}$		

(4.1)

$$W = \text{Tr}(A\tilde{S})^2 + (Q\tilde{Q})^2 + (Q'\tilde{Q})^2 + (Q\tilde{S}A\tilde{Q}) + (Q'\tilde{S}Q)^2 + (Q'\tilde{S}Q')^2.$$

Starting near the infinite N fixed point, finite N effects render the SO gauge coupling irrelevant and the SU gauge coupling relevant. Thus we dualize the SU gauge group to obtain the theory $SU(N-2\delta) \times SO(N-6-\delta)$. Now the SO gauge group's gauge coupling becomes stronger while the SU gauge group becomes weaker. Applying Seiberg duality to the SO gauge group [5,28] yields the final theory $SU(N-2\delta) \times SO(N-3\delta+10)$. Thus we see that we have a cascade which exhibits the effect that

$$N' = N - 2\delta, \quad \delta' = \delta - 16. \quad (4.2)$$

Thus the cascade slows down as it approaches the IR. If the cascade does not end before δ runs negative, then repeated applications of duality do not reduce the rank of the gauge groups.

After n steps of the cascade we see that

$$N(n) = N - 2n\delta + 16n(n-1), \quad \delta(n) = \delta - 16n. \quad (4.3)$$

The minimum occurs at

$$n_c = \frac{\delta+8}{16}, \quad N_c = N - \left(\frac{\delta+8}{4} \right)^2. \quad (4.4)$$

If we want the cascade to end, it must do so before repeated dualities start increasing the gauge group. Large δ ensures many applications of the duality operation.

The IR dynamics of this cascade are many and varied. In what follows, only a few examples are taken to highlight the new possibilities. The duality can end with the SO gauge group. As an example take the cascade ending with $N = 3\delta - 10$ and $\delta > 10$. The SU gauge group dualizes to a $SU(\delta-10) \times SO(2\delta-16)$ gauge group. Now we notice that the SO gauge group confines. There are two physically distinct branches, one with an ADS superpotential and one without. We take the branch with no ADS superpotential. The mesons have explicit mass terms and in the IR we are left with the theory

	$SU(\delta - 10)$	$SO(8)$
Q'	\square	\square
A	\square	
\tilde{S}	$\overline{\square}$	

(4.5)

So we have an interacting CFT in the IR.

If instead the SU gauge group ends the cascade, it does so with supersymmetry breaking,

$$W = \text{Tr}(A\tilde{S})^2 + (Q'\tilde{S}Q')^2.$$

	$SU(2N - 6 - 3\delta)$	$SO(N - 6 - \delta)$	$SO(8)$
q	\square	\square	
q'	\square		\square
\tilde{q}	$\overline{\square}$	\square	
a	\square		
\tilde{s}	$\overline{\square}$		
$M_0 = Q\tilde{Q}$		\square^2	
$M'_0 = Q'\tilde{Q}$		\square	\square
$M_1 = Q\tilde{S}A\tilde{Q}$		\square^2	
$M'_1 = Q'\tilde{S}A\tilde{Q}$		\square	\square
$P = Q\tilde{S}Q$		\square	
$P' = Q'\tilde{S}Q$		\square	\square
$P'' = Q'\tilde{S}Q'$			\square
$\tilde{P} = Q\tilde{S}Q$		\square	

(4.6)

$$W = \text{Tr}(a\tilde{s})^2 + M_0^2 + M'^2_0 + M_1 + P'^2 + P''^2 + q\tilde{q}M_1 + q'\tilde{q}M'_1 + q\tilde{s}a\tilde{q}M_0 + q'\tilde{s}a\tilde{q}M'_0 + \tilde{q}a\tilde{q}\tilde{P} + q\tilde{s}qP + q\tilde{s}q'P' + q'\tilde{s}q'P''.$$

Taking $N < 2\delta$, we find that supersymmetry is broken through the rank condition. As before, the F term for $M_1, \langle q\tilde{q} \rangle = \mathbb{1}$ cannot be satisfied as the rank of $\langle q\tilde{q} \rangle$ is not large enough. As in the previous example, all of the bosons have positive 1-loop or 2-loop masses. Two singlets P_1 and P_2 deserve special attention. We decompose

$$P = \begin{pmatrix} P_1 \mathbb{1}_{2N-6-3\delta \times 2N-6-3\delta} & 0 \\ 0 & P_2 \mathbb{1}_{2\delta-N \times 2\delta-N} \end{pmatrix}. \quad (4.7)$$

The singlet which gave problems in the previous example is the sum $P_1 + P_2$. The vevs of q and \tilde{q} result in a mass term between P_1 and \tilde{s} . Thus the remaining singlet is P_2 . Notice that while P_2 is a singlet under the surviving $SO(2N - 6 - 3\delta)$ symmetry, it is not a singlet under the original $SO(N - 6 - \delta)$ gauge symmetry. Thus it receives a 2-loop positive mass from Higgsed gauge mediation [29].

Thus we find that all bosons are stabilized so that we have a bonafide SUSY breaking vacuum.

To find supersymmetric vacua at large field values, we give large vevs to a and \tilde{s} . A D flat direction is

$$\langle a \rangle = \begin{pmatrix} a_1 \sigma_2 \\ \cdot \\ a_n \sigma_2 \end{pmatrix}, \quad \langle \tilde{s} \rangle = \begin{pmatrix} s_1 \mathbb{1} \\ \cdot \\ s_n \mathbb{1} \end{pmatrix}, \quad (4.8)$$

$$|a_i|^2 - |s_i|^2 = \text{constant}. \quad (4.9)$$

For simplicity, we'll explore the direction parametrized by $a_i = s_i = x$. This vev breaks the gauge group down to $U(\lfloor \frac{2N-6-3\delta}{2} \rfloor)$. The quarks decompose into $2N - 4 - 2\delta$ flavors. These flavors can be given a mass by going out in field space for the mesons M_1 and P'' . Now the $U(\lfloor \frac{2N-6-3\delta}{2} \rfloor)$ has no matter and gaugino condensation

occurs. For simplicity, we assume δ is even so that the floor function can be ignored,

$$\Lambda_{SU(N-3-3\delta/2)}^{3N-9-9\delta/2} = M_1^{2N-12-2\delta} \Lambda_{SU(2N-6-3\delta)}^{5N-25-17\delta/2} x^{-4N+20+6\delta} P'^8. \quad (4.10)$$

Thus the superpotential is

$$W \supset (M_1^{2N-12-2\delta} \Lambda_{SU(2N-6-3\delta)}^{5N-25-17\delta/2} x^{-4N+20+6\delta})^{1/(N-3-3\delta/2)} + \frac{x^4}{\Lambda_{SU(2N-6-3\delta)}} + \mu_1^2 M_1 + \mu_2 P'^2. \quad (4.11)$$

For simplicity, we will assume that $\mu_1 \sim \mu_2 = \mu$. By adjusting the ratio of the UV dynamical scales we can live in the region $\mu \ll \Lambda_{SU(2N-6-3\delta)}$. The solution for the SUSY preserving vacuum is then

$$P \sim \Lambda_{SU(2N-6-3\delta)} \left(\frac{\mu}{\Lambda_{SU(2N-6-3\delta)}} \right)^{\frac{2N-5\delta/2-10}{\delta}} \sim \mu \left(\frac{\mu}{\Lambda_{SU(2N-6-3\delta)}} \right)^{\frac{2N-7\delta/2-10}{\delta}}, \quad (4.12)$$

$$M_1 \sim \Lambda_{SU(2N-6-3\delta)} \left(\frac{\mu}{\Lambda_{SU(2N-6-3\delta)}} \right)^{\frac{4N-6\delta-20}{\delta}} \sim \mu \left(\frac{\mu}{\Lambda_{SU(2N-6-3\delta)}} \right)^{\frac{4N-7\delta-20}{\delta}}, \quad (4.13)$$

$$x \sim \Lambda_{SU(2N-6-3\delta)} \left(\frac{\mu}{\Lambda_{SU(2N-6-3\delta)}} \right)^{\frac{N-\delta-5}{\delta}} \sim \mu \left(\frac{\mu}{\Lambda_{SU(2N-6-3\delta)}} \right)^{\frac{N-2\delta-5}{\delta}}. \quad (4.14)$$

We had assumed that the gauge group was broken down to $U(\lfloor \frac{2N-6-3\delta}{2} \rfloor)$. As before, if a different vev was chosen to give a different symmetry breaking pattern, the SUSY vacuum has the same scaling behavior. We are working in the range $\frac{3\delta}{2} + 3 < N < 2\delta$. As long as $2N - 6 - 3\delta > 4$, we have $P, M_1, x \ll \Lambda$ so that our vacuum is under control. If $2N - 6 - 3\delta \leq 4$, the vev of M_1 goes beyond the UV cutoff. We always have $x \gg \mu$ so that we have a long-lived metastable SUSY breaking vacuum.

When the IR gauge group is ≤ 4 , there are special cases. These correspond to the cases mentioned before where the vev of M_1 goes beyond the UV cutoff. It is expected that a supersymmetric vacuum exists for these cases, but using our present understanding of the chiral gauge theory, it cannot be shown to be the case. For IR gauge group $SU(1)$, we have a confining theory where instantons presumably generate a dynamical superpotential which causes a supersymmetric vacuum to exist. For $SU(2)$ and $SU(3)$, giving a

vev to the two index tensors does not break the gauge group down to a non-Abelian gauge group and instead down to a $U(1)$ gauge theory. Again, we would expect instanton effects to generate a dynamical superpotential that restores supersymmetry.⁴ Finally for a gauge group $SU(4)$, we have the supersymmetric vacuum at $M_1 \sim \Lambda$ so that the vacuum is not under control.

V. A FREE FIELD THEORY BASED CASCADE

We construct a cascade based on a self-dual free fixed point. Rather than being deformed into a weakly coupled Banks-Zaks fixed point, it becomes strongly coupled due to relevant superpotential interactions. The starting point of this next cascade is

	$SU(2N - \delta)$	$SU(N)^4$	
$Q_i + \tilde{Q}_i$	\square	\square	(5.1)
X	Adj		
X_i		Adj	

$$W = X^3 + X_i^3 + Q_i X \tilde{Q}_i + Q_i X_i \tilde{Q}_i,$$

where the notation $SU(N)^4$ indicates four different $SU(N)$ gauge groups all with bifundamentals connecting them to the $SU(2N)$ gauge theory. For simplicity, consider the scenario where all 4 $SU(N)$ gauge groups are related by a \mathbb{Z}_4 symmetry. Applying the cubic duality of Sec. II A, we see that we have the cascade, $SU(2N - \delta) \times SU(N)^4 \rightarrow SU(2N - \delta) \times SU(N - \delta)^4 \rightarrow SU(2N - 3\delta) \times SU(N - \delta)^4$ so that after two applications of the duality, we arrive back at the original situation with $N' = N - \delta$.

As before, up to issues with an incalculable singlet, the theory can have metastable SUSY breaking in the IR. If $N = m\delta + \epsilon$ with $m \in \mathbb{Z}$, for $0 \lesssim \epsilon \lesssim \frac{\delta}{6}$ ($\frac{\delta}{2} \lesssim \epsilon \lesssim \frac{2\delta}{3}$) the $SU(2N - \delta) (SU(N)^4)$ gauge group develops a runaway that is stabilized by the superpotential and the theory develops a mass gap. For $\frac{\delta}{6} \lesssim \epsilon \lesssim \frac{\delta}{2}$ ($\frac{2\delta}{3} \lesssim \epsilon \lesssim \delta$), the $SU(2N - \delta) (SU(N)^4)$ gauge group potentially has metastable SUSY breaking. For simplicity, we have ignored edge cases.

The runaway here is stabilized in a different manner than is typically seen. Assume that we are at the bottom of the cascade and the $SU(N)^4$ gauge group has a runaway [26]. The starting theory is shown in Eq. (5.1). For simplicity, take $N = \frac{2\delta-1}{3}$ so that the low energy theory is

⁴This intuition is from Appendix C where instantons generate the ADS superpotential when the gauge group is broken to $U(1)$.

	$SU(2N - \delta)$	(5.2)
X	Adj	
$M_{0,i} = Q_i \tilde{Q}_i$	1 + Adj	
$M_{1,i} = Q_i X_i \tilde{Q}_i$	1 + Adj	

$$W = X^3 + M_{1,i} + M_{0,i}X + \frac{M_{0,i} \text{cof} M_{1,i}}{(\det M_{1,i})^3}.$$

The F term for the singlet part of $M_{0,i}$ still sets $M_{1,i} \rightarrow \infty$. So one might expect that there is still a runaway. However, from the original electric theory, we can show that there exists another branch where SUSY vacua exists.

Recall the original argument for the runaway in the electric theory. In Sec. II A, a vev was given to X_i and it was observed that obtaining a standard ADS superpotential was unavoidable. However, the new superpotential has the coupling $Q_i X_i \tilde{Q}_i$ so that giving X_i a vev also masses up all of the flavors and gaugino condensation occurs instead of an ADS runaway. As before, give X_i a vev

$$X_i = x_i \begin{pmatrix} \mathbb{1}_{N-1 \times N-1} & 0 \\ 0 & -N+1 \end{pmatrix}. \quad (5.3)$$

This vev breaks the gauge group down to $SU(N-1)$ and masses up both the adjoint and Q. Thus we have a new dynamical scale which is

$$\Lambda_{N-1}^{3N-3} = \Lambda_N^\delta x_i^{3N-\delta-3}. \quad (5.4)$$

Thus after gaugino condensation, the superpotential is

$$W \supset (\Lambda_N^\delta x_i^{3N-\delta-3})^{1/(N-1)} + \lambda x_i^3. \quad (5.5)$$

Solving for the vev x_i , we find that there is the solution

$$x_i \sim \lambda^{\frac{1-N}{\delta}} \Lambda = \lambda^{\frac{2(2-\delta)}{3\delta}} \Lambda. \quad (5.6)$$

So we see that we can trust our vacuum as long as $\lambda \ll 1$ so that $x_i \gg \Lambda$. As SUSY theories do not have phases transitions when varying parameters, we expect that there exists a SUSY vacuum even when $\lambda \gtrsim 1$. Our runaway is cured by a gauge symmetry breaking SUSY vacuum. This vacuum is seen in the electric theory rather than the magnetic theory as the vevs are large.

VI. CONCLUSION

In this article, we presented new self-dualities and several different cascades with interesting physics in the IR. These cascades all involved two index tensors and their IR physics included confinement, CFTs, and metastable supersymmetry breaking. The self-dualities used exhibited Higgsing effects and utilized cubic, quartic, and sextic operators. A chiral cascade was constructed which slowed down in the IR and had its self-dual point at infinite N .

The gravity duals of these cascades would be very interesting. As the field theory is under control, it would be interesting to check what the metastable vacuum corresponds to in the gravity dual. As the gravity dual involves a large N limit where the dynamical scales and quartic couplings are all roughly equal, the metastable SUSY breaking vacuum and the SUSY vacuum would be exponentially close and the metastable vacuum would likely cease to be stable.

The cascades presented in this paper do not have moduli spaces where $\mathcal{N} = 4$ $SU(N)$ gauge groups appear so it is unlikely that these cascades appear as D3 branes at a singularity. Finding different cascades which admit a brane realization as D3 branes at a conifold would prove very enlightening.

While cascades other than the original KS construction have been proposed, they have mainly been orientifolds of the original picture. In the confining region, the gravity duals of these new cascades would geometrize confinement in the presence of two-index tensors. It would be interesting to compare with the original solution for any similarities.

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APPENDIX A: MORE SELF-DUALITIES

Many new self-dualities can be rederived using the Higgsing effect. In this appendix, we present additional self-dualities which are useful in building duality cascades.

1. $SU(N)$ with a flavor of symmetries

A duality for SU gauge groups with a flavor of symmetric tensors and N_f flavors was found in Ref. [22]. The electric theory is

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_R$
Q	\square	\square		$1 - \frac{N_c - 2k}{(k+1)N_f}$
\tilde{Q}	$\bar{\square}$		\square	$1 - \frac{N_c - 2k}{(k+1)N_f}$
S	$\square\square$			$\frac{1}{k+1}$
\tilde{S}	$\bar{\square}\bar{\square}$			$\frac{1}{k+1}$

$$W = \text{Tr}(\tilde{S}\tilde{S})^{k+1}.$$

This theory was demonstrated to be dual to

	$SU(\tilde{N}_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_R$
q	\square	$\bar{\square}$		$1 - \frac{\tilde{N}_c - 2k}{(k+1)N_f}$
\tilde{q}	$\bar{\square}$		$\bar{\square}$	$1 - \frac{\tilde{N}_c - 2k}{(k+1)N_f}$
s	$\square\square$			$\frac{1}{k+1}$
\tilde{s}	$\bar{\square}\bar{\square}$			$\frac{1}{k+1}$
$M_j = Q(\tilde{S}S)^j\tilde{Q}$		\square	\square	$2 - \frac{2(N_c - 2k)}{(k+1)N_f} + \frac{2j}{k+1}$
$P_r = Q(\tilde{S}S)^r\tilde{S}Q$		$\square\square$	\square	$2 - \frac{2(N_c - 2k)}{(k+1)N_f} + \frac{2r+1}{k+1}$
$\tilde{P}_r = \tilde{Q}S(\tilde{S}S)^r\tilde{Q}$			$\square\square$	$2 - \frac{2(N_c - 2k)}{(k+1)N_f} + \frac{2r+1}{k+1}$

$$W = \text{Tr}(s\tilde{s})^{k+1} + \sum_{j=0}^k M_{k-j}q(\tilde{s}s)^j\tilde{q} + \sum_{r=0}^{k-1} P_{k-r-q}q(\tilde{s}s)^r\tilde{s}q + \tilde{P}_{k-r-q}\tilde{q}s(\tilde{s}s)^r\tilde{q},$$

where the indices j run from 0 to k and r runs from 0 to $k - 1$ and $\tilde{N}_c = (2k + 1)N_f + 4k - N_c$. Applying the same logic as in Sec. II A, we find that a runaway develops if $N_f < \frac{N_c - 4k}{2k+1}$. As before, this runaway occurs when the dual gauge group (\tilde{N}_c) runs negative.

Using this duality, one can show that the theory

	$SU(N)$	$SU(N - 2 - \delta)$
Q	\square	\square
\tilde{Q}	$\bar{\square}$	\square
S	$\square\square$	
\tilde{S}	$\bar{\square}\bar{\square}$	

$$W = \text{Tr}(\tilde{S}\tilde{S})^2 + (Q\tilde{Q})^2 + \text{Tr}(Q\tilde{S}S\tilde{Q})$$

is dual to itself with new gauge group $SU(N - 2\delta)$. Thus there is a self-duality for $\delta = 0$. At the self-dual point, the R charge of all fields is $1/2$.

2. $SU(N)$ with a flavor of antisymmetric tensors

A duality for SU gauge groups with a flavor of antisymmetric tensors and N_f flavors was found in Ref. [22]. The electric theory is

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_R$
Q	\square	\square		$1 - \frac{N_c + 2k}{(k+1)N_f}$
\tilde{Q}	$\bar{\square}$		\square	$1 - \frac{N_c + 2k}{(k+1)N_f}$
A	$\square\square$			$\frac{1}{k+1}$
\tilde{A}	$\bar{\square}\bar{\square}$			$\frac{1}{k+1}$

$$W = \text{Tr}(A\tilde{A})^{k+1}.$$

There are also two additional $U(1)$ symmetries. This theory was demonstrated to be dual to

	$SU(\tilde{N}_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_R$
q	\square	$\bar{\square}$		$1 - \frac{\tilde{N}_c+2k}{(k+1)N_f}$
\tilde{q}	$\bar{\square}$		$\bar{\square}$	$1 - \frac{\tilde{N}_c+2k}{(k+1)N_f}$
a	\square			$\frac{1}{k+1}$
\tilde{a}	$\bar{\square}$			$\frac{1}{k+1}$
$M_j = Q(\tilde{A}A)^j \tilde{Q}$		\square	\square	$2 - \frac{2(N_c+2k)}{(k+1)N_f} + \frac{2j}{k+1}$
$P_r = Q(\tilde{A}A)^r \tilde{A}Q$		\square		$2 - \frac{2(N_c+2k)}{(k+1)N_f} + \frac{2r+1}{k+1}$
$\tilde{P}_r = \tilde{Q}A(\tilde{A}A)^r \tilde{Q}$			\square	$2 - \frac{2(N_c+2k)}{(k+1)N_f} + \frac{2r+1}{k+1}$

(A5)

$$W = \text{Tr}(a\tilde{a})^{k+1} + \sum_{j=0}^k M_{k-j} q(\tilde{a}a)^j \tilde{q} + \sum_{r=0}^{k-1} P_{k-r-q} q(\tilde{a}a)^r \tilde{a}q + \tilde{P}_{k-r-q} \tilde{q}a(\tilde{a}a)^r \tilde{q},$$

where the indices j run from 0 to k and r runs from 0 to $k-1$ and $\tilde{N}_c = (2k+1)N_f - 4k - N_c$. As before, we can examine when a runaway can develop. We find that the theory develops a runaway when $N_f < \frac{N_c+2k}{2k+1}$. Unlike the previous case, this instability occurs after the dual gauge group has run negative. Thus one expects that there is likely a richer set of confining dynamics in these theories as compared to the previous cases.

Using this duality, one can find the self-duality of the theory

	$SU(N)$	$SU(N+2-\delta)$
Q	\square	\square
\tilde{Q}	$\bar{\square}$	$\bar{\square}$
A	\square	
\tilde{A}	$\bar{\square}$	

(A6)

$$W = \text{Tr}(A\tilde{A})^2 + (Q\tilde{Q})^2 + \text{Tr}(Q\tilde{A}A\tilde{Q}).$$

It is dual to an identical gauge theory with gauge group $SU(N-2\delta)$ so that a self-duality is obtained for $\delta = 0$. For the self-dual gauge theory, the R charges of all fields is $1/2$.

3. $SO(N_c)$ with an antisymmetric

There is duality involving antisymmetrics and SO gauge groups [30] which is summarized as follows. The electric theory is

	$SO(N_c)$	$SU(N_f)$
Q	\square	\square
A	\square	

(A7)

$$W = \text{Tr}A^{2k+2}.$$

This theory is dual to

	$SO(\tilde{N}_c)$	$SU(N_f)$
q	\square	$\bar{\square}$
a	\square	
$M_{j(\text{even})} = Qa^j Q$		\square
$M_{j(\text{odd})} = Qa^j Q$		\square

(A8)

$$W = \text{Tr}a^{2k+2} + \sum_{j=0}^{2k} M_{2k-j} qa^j \tilde{q},$$

where the index j can run from 0 to $2k$ and $\tilde{N}_c = (2k+1)N_f + 4 - N_c$.

Using Higgsing, one self-dual fixed point can be reached. The UV is

	$SO(N)$	$SO(N-2-\delta)$
Q	\square	\square
A	\square	

(A9)

$$W = \text{Tr}A^4 + QA^2Q + Q^4.$$

Applying the above duality and integrating out matter, this theory is dual to itself with gauge group $SO(N-2\delta)$.

4. $SO(N_c)$ with a symmetric

There is duality involving symmetric tensors and SO gauge groups [31] which is summarized as follows. The electric theory is

	$SO(N_c)$	$SU(N_f)$	(A10)
Q	\square	\square	
S	\square		

$$W = \text{Tr}S^{k+1}.$$

This theory is dual to

	$SO(\tilde{N}_c)$	$SU(N_f)$	(A11)
q	\square	$\bar{\square}$	
s	\square		
$M_j = QS^jQ$		\square	

$$W = \text{Tr}s^{k+1} + \sum_{j=0}^{k-1} M_{k-j-1} q x^j \tilde{q},$$

where the index j can run from 0 to $k-1$ and $\tilde{N}_c = kN_f + 4k - N_c$.

Using Higgsing, two different self-dual fixed points can be reached. The first is

	$SO(N)$	$SO(2N - 8 - \delta)$	(A12)
Q	\square	\square	
S	\square		

$$W = \text{Tr}S^3 + QSQ.$$

Applying the above duality and integrating out matter, this theory is dual to itself with gauge group $SO(N - \delta)$.

The second self-dual point is realized for

	$SO(N)$	$SO(N - 6 - \delta)$	(A13)
Q	\square	\square	
S	\square		

$$W = \text{Tr}S^4 + QS^2Q + Q^4.$$

Applying the above duality and integrating out matter, this theory is dual itself with gauge group $SO(N - 2\delta)$.

5. $Sp(N_c)$ with an antisymmetric

There is duality involving antisymmetrics and Sp gauge groups [31] which is summarized as follows. The electric theory is

	$Sp(2N_c)$	$SU(2N_f)$	(A14)
Q	\square	\square	
A	\square		

$$W = \text{Tr}A^{k+1}.$$

This theory is dual to

	$Sp(2\tilde{N}_c)$	$SU(2N_f)$	(A15)
q	\square	$\bar{\square}$	
a	\square		
$M_j = QA^jQ$		\square	

$$W = \text{Tr}a^{k+1} + \sum_{j=0}^{k-1} M_{k-j-1} q a^j \tilde{q},$$

where the index j can run from 0 to $k-1$ and $\tilde{N}_c = kN_f - 2k - N_c$.

Using Higgsing, two different self-dual fixed points can be reached. The first is

	$Sp(2N)$	$Sp(4N + 8 - 2\delta)$	(A16)
Q	\square	\square	
A	\square		

$$W = \text{Tr}A^3 + QAQ.$$

Applying the above duality and integrating out matter, this theory is dual to itself with gauge group $Sp(2N - 2\delta)$.

The second self-dual point is realized for

	$Sp(2N)$	$Sp(2N + 6 - 2\delta)$	(A17)
Q	\square	\square	
A	\square		

$$W = \text{Tr}A^4 + QA^2Q + Q^4.$$

Applying the above duality and integrating out matter, this theory is dual to itself with gauge group $Sp(2N - 4\delta)$.

6. $Sp(N_c)$ with a symmetric

There is duality involving symmetric and Sp gauge groups [30] which is summarized as follows. The electric theory is

	$Sp(2N_c)$	$SU(2N_f)$	
Q	\square	\square	(A18)
S	$\square\square$		

$$W = \text{Tr}S^{2k+2}.$$

This theory is dual to

	$Sp(2\tilde{N}_c)$	$SU(2N_f)$	
q	\square	$\bar{\square}$	(A19)
s	$\square\square$		
$M_{j(\text{even})} = Qs^jQ$		\square	
$M_{j(\text{odd})} = Qs^jQ$		$\square\square$	

$$W = \text{Tr}s^{2k+2} + \sum_{j=0}^{2k} M_{2k-j} q s^j \tilde{q},$$

where the index j can run from 0 to $2k$ and $\tilde{N}_c = (2k + 1)N_f - 2 - N_c$.

Using Higgsing, a self-dual fixed point can be reached. It is

	$Sp(2N)$	$Sp(2N + 2 - 2\delta)$	
Q	\square	\square	(A20)
S	$\square\square$		

$$W = \text{Tr}S^4 + QS^2Q + Q^4.$$

Applying the above duality and integrating out matter, this theory is dual to itself with gauge group $Sp(2N - 4\delta)$.

APPENDIX B: A VECTORLIKE CASCADE WITH METASTABLE SUSY BREAKING

We provide another duality cascade which shows that the problem of incalculable singlets can be cured in the context of a vectorlike cascade. The starting gauge theory is

	$SU(N)$	$SO(N + 2 - \delta)$	
Q	\square	\square	(B1)
\tilde{Q}	$\bar{\square}$	\square	
A	$\square\square$		
\tilde{A}	$\bar{\square}\bar{\square}$		

$$W = \text{Tr}(A\tilde{A})^2 + (Q\tilde{Q})^2 + \text{Tr}(Q\tilde{A}A\tilde{Q}).$$

After two applications of duality, the theory goes back to the original theory with $N' = N - 2\delta$. Assume that $2\delta > N > \frac{3\delta-2}{2}$ so that the $SU(N)$ theory ends the cascade through supersymmetry breaking. The dual gauge theory is

	$SU(2N + 2 - 3\delta)$	$SO(N + 2 - \delta)$	
q	\square	\square	(B2)
\tilde{q}	$\bar{\square}$	\square	
a	$\square\square$		
\tilde{a}	$\bar{\square}\bar{\square}$		
M_0		\square^2	
M_1		\square^2	
P		$\square\square$	
\tilde{P}		\square	

$$W = \text{Tr}(a\tilde{a})^2 + M_0^2 + M_1 + M_1q\tilde{q} + M_0q\tilde{a}a\tilde{q} + Pq\tilde{q} + \tilde{P}\tilde{q}a\tilde{q}.$$

As before, supersymmetry is broken through the rank condition by the F term of M_1 . M_0 has a tree-level mass while M_1 and q, \tilde{q} all obtain a positive mass. There is no singlet meson in P so that they all receive positive two-loop masses from gauge mediation. Thus we find that this vectorlike theory has a metastable SUSY breaking vacuum without the singlet problem.

APPENDIX C: GAUGINO CONDENSATION AND ADS SUPERPOTENTIALS

Throughout this paper, we take symmetry breaking vevs and use gaugino condensation to understand the behavior of the theory. In this appendix we show how the ADS superpotential can be derived using the same techniques.

We start with SQCD with $N_f < N_c - 1$. For simplicity we explore the D flat direction

$$\langle Q \rangle = q \begin{pmatrix} \mathbb{1}_{N_f \times N_f} \\ 0_{N_c - N_f \times N_f} \end{pmatrix}, \quad \langle \tilde{Q} \rangle = q \begin{pmatrix} \mathbb{1}_{N_f \times N_f} \\ 0_{N_c - N_f \times N_f} \end{pmatrix}. \quad (\text{C1})$$

The remaining gauge group undergoes gaugino condensation. After using scale matching we find the superpotential

$$W \sim \left(\Lambda_{N_c - N_f}^{3(N_c - N_f)} \right)^{\frac{1}{N_c - N_f}} \sim \left(\frac{\Lambda_{N_c}^{3N_c - N_f}}{q^{2N_f}} \right)^{\frac{1}{N_c - N_f}}. \quad (\text{C2})$$

Using symmetry rotations, the denominator can be expressed as the familiar determinant term. We have rederived the ADS superpotential using gaugino condensation. Notice that this derivation requires that $N_c - N_f \geq 2$. For $N_c = N_f + 1$ instanton calculations are required to show the existence of the superpotential.

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