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New localization mechanism of fermions on braneworlds

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It is known that by introducing the Yukawa coupling between the fermion and the background scalar field, a bulk spin-half fermion can be localized on general Randall-Sundrum braneworlds generated by a kinklike background scalar. However, this localization mechanism does not work anymore for Randall-Sundrum braneworlds generated by a scalar whose configuration is an even function of the extra dimension. In this paper, we present a new localization mechanism for spin-half fermions for such a class of braneworld models, in which extra dimension has the topology S^1/Z_2. By two examples, it is shown that the new localization mechanism produces interesting results. In the first model with the brane generated by two scalars, the zero mode of the left-handed fermion is localized on the brane and there is a mass gap between the fermion zero mode and excited KK modes. In the second model with the brane generated by a dilaton scalar, the zero mode of the left- or right-chiral fermion can be localized on the brane and there is no mass gap.

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I. INTRODUCTION

Braneworld scenarios [1–5], which were motivated from string/M theory, have been attracting constant interest in recent years, because they not only show a new viewpoint of spacetime but also provide new approaches to address a large number of outstanding issues such as the hierarchy problem, the cosmological problem, the nature of dark matter and dark energy, black hole production at future colliders as a window on quantum gravity, production of electroweak symmetry breaking without a Higgs boson, and so on. In these scenarios, our $(3+1)$-dimensional spacetime is a submanifold (the brane) embedded in a fundamental higher-dimensional spacetime (the bulk).

An important issue in braneworld theories is the mechanism by which extra dimensions are hidden and ordinary matters are confined on the brane, so that the spacetime is effectively four dimensional, at least at low energy. This can be ensured for those brane models with compact extra dimensions [1–4]. But in other models, extra dimensions can be infinite [5,6]. For these brane models, it is interesting and important to give the mechanism of confinement of ordinary matters on the brane.

With simple field-theoretic models, in which branes are generated naturally by background scalar fields with some potential, one can investigate the localization of ordinary matters on branes. In this paper, we focus on fermions. In braneworld models, the extra dimension is usually supposed to possess Z_2 symmetry; hence, the background scalar fields would have odd or even parity. If the scalar ϕ is an odd function of the extra dimension, the well-known localization mechanism for a fermion is to introduce the Yukawa coupling between the fermion and the background scalar field, i.e. $-\eta\bar{\Psi}\phi\Psi$. There are a lot of works on this localization mechanism (see Refs. [7–16] and references therein). However, if the scalar is an even function of the extra dimension, this mechanism does not work anymore, and we need to introduce a new localization mechanism. This is the goal of this paper.

II. LOCALIZATION MECHANISM

In order to investigate the localization of spin-half fermions on the branes generated by even and/or odd background scalar fields, we introduce a new kink of coupling between the fermions and the background scalars. The five-dimensional action for a massless Dirac fermion coupled to the background real scalar fields reads

$$S_{1/2} = \int d^5x\sqrt{-g}[\bar{\Psi}\Gamma^M(\partial_M+\omega_M)\Psi + \eta\bar{\Psi}\Gamma^M\partial_M F(\phi,\chi,\cdots)\gamma^5\Psi], \quad (1)$$

where $F(\phi,\chi,\cdots)$ is a function of the real scalar fields $\phi,\chi,\cdots,\eta$ is the coupling constant, and $\omega_M = \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}}$ is the spin connection. In five dimensions, Dirac fermions are four-component spinors and their gamma structure is determined by the gamma matrices in curved spacetime: $\{\Gamma^M,\Gamma^N\} = 2g^{MN}$, where $\Gamma^M = E^M_{\bar{M}}\Gamma^{\bar{M}}$ with $E^M_{\bar{M}}$ the vielbein and $\gamma^{\bar{M}}$ the gamma matrices in flat spacetime. The indices of five-dimensional spacetime coordinates and the local Lorentz indices are labeled with capital latin letters $M,N,\cdots$ and $\bar{M},\bar{N},\cdots$, respectively.

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The metric describing a Minkowski brane embedded in a five-dimensional spacetime is assumed as [4,5]

$$ds_5^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{2}$$

which can also be transformed to the conformally flat one,

$$ds_5^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2), \tag{3}$$

by the coordinate transformation $dz = e^{-A(y)}dy$, where e^{2A} is the warp factor, y or z denotes the extra dimension coordinate, and $\eta_{\mu\nu}$ is the induced metric on the brane. The background scalars are only functions of the extra dimension.

With the conformally flat metric (3), the spin connection ω_M reads as $\omega_\mu = \frac{1}{2}(\partial_z A(z))\gamma_\mu\gamma_5$ and $\omega_5 = 0$, and the equation of motion for the five-dimensional Dirac fermion is

$$[\gamma^\mu\partial_\mu + \gamma^5(\partial_z + 2\partial_z A(z)) + \eta\partial_z F]\Psi = 0. \tag{4}$$

Note that for the usual Yukawa coupling $-\eta F(\phi,\chi,\cdots)\bar{\Psi}\Psi$, the corresponding Dirac equation is given by [7,8,10–14,17–19],

$$[\gamma^\mu\partial_\mu + \gamma^5(\partial_z + 2\partial_z A(z)) + \eta e^{A(z)}F]\Psi = 0. \tag{5}$$

In order to investigate the above Dirac equation (4), we make the general chiral decomposition in terms of four-dimensional effective Dirac fields,

$$\Psi(x,z) = \sum_n \psi_{Ln}(x)f_{Ln}(z)e^{-2A(z)} + \sum_n \psi_{Rn}(x)f_{Rn}(z)e^{-2A(z)}, \tag{6}$$

where $\psi_{Ln}(x) = -\gamma^5\psi_{Ln}(x)$ and $\psi_{Rn}(x) = \gamma^5\psi_{Rn}(x)$ are the left- and right-chiral components of the four-dimensional effective Dirac fermion field, respectively, and they satisfy the four-dimensional massive Dirac equations $\gamma^\mu\partial_\mu\psi_{Ln}(x) = \mu_n\psi_{Rn}(x)$ and $\gamma^\mu\partial_\mu\psi_{Rn}(x) = \mu_n\psi_{Ln}(x)$. Note that the summation indices are not necessarily the same for the left- and right-chiral KK modes. In the following, we mainly focus on the left- and right-chiral KK modes $f_{Ln}(z)$ and $f_{Rn}(z)$ of the five-dimensional Dirac field, which satisfy the following coupled equations,

$$[\partial_z - \eta\partial_z F]f_{Ln}(z) = +\mu_n f_{Rn}(z), \tag{7a}$$

$$[\partial_z + \eta\partial_z F]f_{Rn}(z) = -\mu_n f_{Ln}(z), \tag{7b}$$

which can be recast into

$$U^\dagger U f_{Ln}(z) = \mu_n^2 f_{Ln}(z), \tag{8a}$$

$$U U^\dagger f_{Rn}(z) = \mu_n^2 f_{Rn}(z), \tag{8b}$$

with the operator U defined as $U = \partial_z - \eta\partial_z F(\phi)$. The above equations can also be rewritten as the Schrödinger-like equations

$$[-\partial_z^2 + V_L(z)]f_L(z) = \mu_n^2 f_L(z), \tag{9a}$$

$$[-\partial_z^2 + V_R(z)]f_R(z) = \mu_n^2 f_R(z), \tag{9b}$$

where the effective potentials for the KK modes $f_{L,R}$ are

$$V_{L,R}(z) = (\eta\partial_z F)^2 \pm \partial_z(\eta\partial_z F). \tag{10}$$

The form of Eq. (8) and the supersymmetric partner potentials (10) show that there are no tachyon fermion KK modes with negative mass square, and they also indicate that we may obtain a chiral massless fermion on the brane for the new coupling. For the usual Yukawa coupling $-\eta F(\phi,\chi,\cdots)\bar{\Psi}\Psi$, the corresponding effective potentials read [7,8,10–14,17–19]

$$V_{L,R}(z) = (\eta e^A F)^2 \pm \partial_z(\eta e^A F). \tag{11}$$

On the other hand, in order to derive the effective action on the brane for the four-dimensional massless and massive Dirac fermions, we need the following orthonormality conditions for the KK modes $f_{Ln}(z)$ and $f_{Rn}(z)$,

$$\int f_{Lm}f_{Ln}dz = \int f_{Rm}f_{Rn}dz = \delta_{mn}, \int f_{Lm}f_{Rn}dz = 0, \tag{12}$$

which are the same as the case of the usual Yukawa coupling.

The zero modes of the left- and right-chiral fermions turn out to be

$$f_{L0,R0}(z) \propto \exp\left[\pm\int_0^z d\bar{z}\eta\partial_{\bar{z}}F\right] = e^{\pm\eta F}. \tag{13}$$

The normalization condition is

$$\int |f_{L0,R0}(z)|^2 dz \propto \int e^{\pm 2\eta F}dz < \infty. \tag{14}$$

So, there is at most one of the left- and right-chiral fermion zero modes that can be localized on the brane if the extra dimension in the conformal coordinate z is infinite. Without loss of generality, we mainly focus on $\eta > 0$ in this paper.

In brane models, it is usually assumed that the extra dimension has Z_2 symmetry, so the effective potentials for the fermion KK modes should be symmetric with respect to the extra dimension. In order to ensure the potentials $V_{L,R}(z)$ are even functions of z, F should be even and odd functions of z according to Eqs. (10) and (11), respectively. This can be ensured for odd background scalars. However, if the scalars are even, the usual Yukawa coupling $-\eta F(\phi,\chi,\cdots)\bar{\Psi}\Psi$ does not work for the fermion localization, and we need the new one introduced in the action (1).

III. LOCALIZATION OF FERMIONS ON BRANES

Next, we will investigate localization of fermions on various branes by using the new localization mechanism presented here. We will give two examples.

We first investigate the localization of fermions on a two-field-thick brane. The system is described by the action including two interacting scalars ϕ and π,

$$S=\int d^5x\sqrt{-g}\left[\frac{R}{2\kappa_5^2}-\frac{1}{2}(\partial\phi)^2-\frac{1}{2}(\partial\pi)^2-V(\phi,\pi)\right], \quad (15)$$

where R is the scalar curvature. We set $\kappa_5^2=8\pi G_5=1$ with G_5 the five-dimensional Newton constant. The line element is also assumed as (3) for a static Minkowski brane. The solution for this system can be found via the superpotential method [20] with $V=\frac{1}{2}e^{-2\sqrt{1/3}\pi}[(\frac{dW(\phi)}{d\phi})^2-W(\phi)^2]$ and $W(\phi)=va\phi(1-\frac{\phi^2}{3v^2})$ [18,21],

$$\phi(z)=v\tanh(az), \quad (16a)$$

$$A(z)=-\frac{v^2}{9}\left[\ln\cosh^2(az)+\frac{1}{2}\tanh^2(az)\right], \quad (16b)$$

$$\pi(z)=\sqrt{3}A(z), \quad (16c)$$

where both v and a are positive constants. It can be seen that the solution for ϕ is a kink (odd) and $\pi=\sqrt{3b}A$ is a dilaton field (even).

In order to localize fermions on the brane, we need to consider the coupling between fermions and the background scalars. If we consider the usual Yukawa coupling between fermions and the kink ϕ, i.e., $-\eta\bar{\Psi}\phi\Psi$, then we will find that fermion zero modes cannot localized on the brane [18].

Here, we ask an interesting question: can fermions be localized on the brane if they couple with the even background scalar, i.e., the dilaton field π? In order to answer this question, we apply the localization mechanism developed in this paper and consider the simplest case $F=\pi$, i.e., with the coupling of $\eta\bar{\Psi}\Gamma^M(\partial_M\pi)\gamma^5\Psi$, for which the potentials (10) are given by

$$V_{L,R}(z)=\frac{1}{54}a^2v^2\eta[2v^2\eta(1+\mathrm{sech}^2(az))^2\tanh^2(az) \pm 3\sqrt{3}(\cosh(2az)-5)\mathrm{sech}^4(az)]. \quad (17)$$

The values of $V_L(z)$ and $V_R(z)$ at $z=0$ and $z\to\pm\infty$ are given by

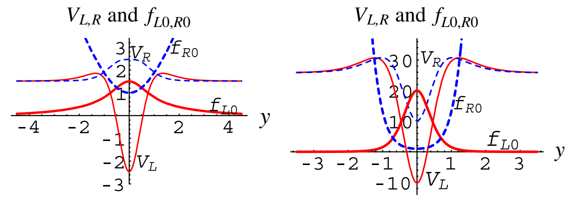

FIG. 1 (color online). The shapes of the potentials $V_L(z)$ (red thin curve) and $V_R(z)$ (blue thin dashed curves) as well as the corresponding fermion zero modes $f_{L0}(z)$ (red thick curve) and $f_{R0}(z)$ (blue thick dashed curves) for the two-field brane. The parameters are set to $a=1$, $v=2$, and $\eta=1.6$ (left) and $\eta=6.6$ (right).

$$V_{L,R}(0)=\mp\frac{2}{3\sqrt{3}}a^2v^2\eta,$$
$$V_{L,R}(\pm\infty)=\frac{1}{27}a^2v^4\eta^2. \quad (18)$$

Here, we only consider the positive coupling constant η, for which we have $V_L(0)<0$ and $V_R(0)>0$. Since the value of the potential $V_L(z)$ is positive at the boundary along the extra dimension, there is a mass gap, and those left-chiral fermion KK modes (including the zero mode) with $m_n^2<\frac{1}{27}a^2v^4\eta^2$ belong to a discrete spectrum and those with $m_n^2>\frac{1}{27}a^2v^4\eta^2$ belong to a continuous one. For right-chiral fermion KK modes, the zero mode cannot be localized on the brane, and the spectrum is decided by the value of the coupling constant η. For $0<\eta<6\sqrt{3}/v^2$, $V_R(0)>V_R(\pm\infty)$, there are no bound right-chiral fermion KK modes; namely, no right-chiral fermions can be localized on the brane. If $\eta>6\sqrt{3}/v^2$, there may exist a finite number of bound right-chiral fermion KK modes, whose masses also satisfy $m_n^2<\frac{1}{27}a^2v^4\eta^2$. The typical shapes of $V_L(z)$ and $V_R(z)$ are shown in Fig. 1 for $0<\eta<6\sqrt{3}/v^2$ and $\eta>6\sqrt{3}/v^2$.

The left-chiral fermion zero mode reads

$$f_{L0}(z)\propto\cosh^{-\frac{\eta v^2}{6\sqrt{3}}}(az)\exp\left(-\frac{\eta v^2}{6\sqrt{3}}\tanh^2(az)\right). \quad (19)$$

It is easy to show that the zero mode (19) is normalizable for any positive η, so it is localized on the brane, while the right-chiral fermion zero mode is divergent (for positive coupling η) at the boundary of the extra dimension and cannot be localized. The shapes of left- and right-chiral fermion zero modes are plotted in Fig. 1.

The massive KK modes can be solved numerically, but we do not discuss them here.

Next, we turn to another brane scenario—the scalar-tensor brane.

In the RS1 model [4], there are two 3-branes located at the boundaries of a compact extra dimension with the topology S^1/Z_2. In order to solve the gauge hierarchy problem by the exponential warp factor e^{-ky}, our Universe should be located on the negative tension brane. However, this would give a "wrong-signed" Friedmann-like equation, which leads to a severe cosmological problem [22,23]. In the RS2 model, the cosmological problem has been solved, but the gauge hierarchy problem is left.

Recently, a simple generation of the RS1 model in the scalar-tensor gravity was given in Ref. [24]. In this model, our world is moved to the positive tension brane but the hierarchy problem is also solved. The action for the scalar-tensor gravity is given by [24]

$$S_5 = \frac{M_5}{2}\int d^5x\sqrt{|g|}\mathrm{e}^{k\phi}[R-(3+4k)(\partial\phi)^2], \qquad (20)$$

where M_5 is the five-dimensional scale of gravity, and k is a coupling constant. The braneworld is generated by the scalar ϕ. For the special case of $k=-1$, the above action is just the standard bosonic part of the effective string action involving only the metric and the dilaton.

The line element is given by (3) for a static Minkowski brane. The conformal coordinate $z\in[-z_b, z_b]$ denotes an S^1/Z_2 orbifold extra dimension. This system has two sets of brane solutions. The first one is given by [24]

$$\mathrm{e}^{A(z)} = (1+\beta|z|)^{\frac{1}{3+2k}}, \qquad (21a)$$

$$\phi(z) = \frac{2}{3+2k}\ln(1+\beta|z|), \qquad (21b)$$

where $\beta>0$ and $k<-3/2$. The second solution reads [24]

$$\mathrm{e}^{A(z)} = (1+\beta|z|)^{\frac{3+4k}{9+6k}}, \qquad (22a)$$

$$\phi(z) = -\frac{2}{3+2k}\ln(1+\beta|z|), \qquad (22b)$$

where $\beta>0$ and $-3/2<k<-3/4$.

Both solutions describe the same braneworld picture: there are a positive tension brane at the origin and a negative one at the boundary z_b, which is similar to the RS1 model. However, the massless graviton for both solutions here is localized on the negative tension brane, which is opposite to the case of the RS1 model. Then it can be shown that, if we suppose that the Standard Model fields are confined on the positive tension brane localized at $z=0$, which is crucial to overcome the severe cosmological problem of the RS1 model, the gauge hierarchy problem can be solved in this brane model [24].

Now, we would like to investigate the localization of fermions on the scalar-tensor brane. We first note that, since the scalar field $\phi(z)$ in this brane model is an even function of z, we cannot use the Yukawa coupling with the form $-\eta\bar{\Psi}F(\phi)\Psi$, which would lead to an odd effective potential. We would like to show that, if we apply the new localization mechanism presented in this paper, fermions can be localized on the positive tension brane even the extra dimension is extended to $-\infty<z<\infty$. Here, we consider the simple case $F(\phi)=\phi^q$ with q a positive integer and ϕ given by the first solution (21b), for which the potentials (10) are given by

$$V_{L,R}(z) = \frac{4q\beta^2\phi^{q-2}(z)}{(3+2k)^2(1+\beta|z|)^2}[q\phi^q(z)\eta^2 \pm (q-1-\ln(1+\beta|z|))\eta] \pm \frac{4\beta\eta\delta_{q,1}\delta(z)}{(3+2k)}. \qquad (23)$$

The values of $V_{L,R}(z)$ at $z\to 0$ and $z\to\pm\infty$ are given by

$$V_{L,R}(z\to 0)\to \begin{cases} \frac{2\beta^2[2\eta^2\mp(3+2k)\eta]}{(3+2k)^2} \pm \frac{4\beta\eta\delta(z)}{(3+2k)}, & q=1 \\ \pm\frac{8\beta^2\eta}{(3+2k)^2}, & q=2, \\ 0, & q\geq 3 \end{cases}$$
$$V_{L,R}(z\to\pm\infty)\to 0. \qquad (24)$$

From Eqs. (23)–(24), we know that there are two potential barriers around the positive tension brane, and the potential barriers trend to vanish at the boundaries of the extra dimension. For the case $q=1$ and $\eta>0$, the potentials for left- and right-chiral fermion KK modes have a negative and positive $\delta(z)$ potential wells, respectively. For odd $q>1$, the potential $V_L(z)$ around the positive tension brane located at $z=0$ is negative, which leads to a bound left-chiral fermion zero mode. For even $q>1$, the potential $V_R(z)$ around the positive tension brane is negative, which results in a bound right-chiral fermion zero mode.

The fermion zero modes are

$$f_{L0,R0}(z)\propto\exp\left[\pm\eta\left(\frac{2}{3+2k}\ln(1+\beta|z|)\right)^q\right]. \qquad (25)$$

The normalization conditions are given by

$$\int_{-\infty}^{\infty} f^2_{L0,R0}(z)dz<\infty. \qquad (26)$$

For $q=1$, if $\eta>-\frac{3+2k}{4}(>0)$, the above integral for $f_L(z)$ is finite, so the zero mode of the left-chiral fermion can be localized on the positive tension brane. For odd $q>1$ and even $q>1$, with any positive coupling η, the zero modes of the left- and right-chiral fermions are localized on the brane, respectively.

For the solution (22b), the results are almost the same except that the localization condition of left-chiral fermion zero mode is $\eta>\frac{3+2k}{4}(>0)$ for the case $q=1$.

When $z_b<\infty$, the localizations of the left- and right-chiral fermion zero modes are opposite; namely, one is localized on the positive tension brane and another is localized on the negative tension one.

IV. CONCLUSION

In this paper, we presented a new localization mechanism for fermions in a class of braneworld models, in which the extra dimension has the topology S^1/Z_2. This new localization mechanism is necessary for those braneworlds generated only by a dilaton scalar. In such braneworld models, the background scalar is an even function of the extra dimension. Therefore, the usual localization mechanism, by introducing the Yukawa coupling between the fermion and the background scalar, cannot work anymore because the effective potentials for fermion KK modes are not even functions of the extra dimension, while the new localization mechanism introduced in this paper will give good results.

We illustrated this with two examples. The first example is about a brane generated by two scalar fields with an interaction potential, one is the usual kink scalar and another is the dilaton. For this model, our new localization mechanism gave a very good and interesting result: the zero mode of the left-handed fermion is localized on the brane, there is a mass gap between the fermion zero mode and excited KK modes, and there are some bound discrete fermion KK modes and a series of continue fermion KK modes.

In the second example, we considered a brane generated by a dilaton scalar. This model is a simple generation of the RS1 model in the scalar-tensor gravity. In this model, our world is moved to the positive tension brane, and the hierarchy problem and cosmological problem can be solved synchronously [24]. In order to localize fermions on the positive tension brane, we considered the new coupling introduced in this paper with $F(\phi) = \phi^q$ and positive coupling constant. It was found that the zero modes of the left- and right-chiral fermions can be localized on the brane for odd and even positive integer q, respectively. There are a series of gapless continuous massive fermion KK modes, which cannot be localized on the brane.

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