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Ultraviolet complete Lorentz-invariant theory with superluminal signal propagation

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We describe a UV complete asymptotically fragile Lorentz-invariant theory exhibiting superluminal signal propagation. Its low energy effective action contains "wrong" sign higher dimensional operators. Nevertheless, the theory gives rise to an S matrix, which is defined at all energies. As expected for a nonlocal theory, the corresponding scattering amplitudes are not exponentially bounded on the physical sheet, but otherwise are healthy. We study some of the physical consequences of this S matrix.

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I. INTRODUCTION

A number of interesting apparently consistent low energy effective theories giving rise to long distance modifications of gravity has been constructed over the past decade [1–5]. Explaining the observed acceleration of the Universe following this path, which was one of the major motivations for these developments, remains a long shot. Nevertheless, these models are of definite interest from both theoretical and phenomenological points of view. In particular, they give rise to a number of striking observational signatures, including anomalous precession of the Moon perihelion [6,7], a strong monochromatic gravitational line from a massive graviton [8], hairy supermassive black holes [9], an exotic cosmic microwave background B-mode spectrum [10], or a violation of the equivalence principle for compact astrophysical objects [11].

On the theoretical side, all these models are effective field theories with a very low cutoff scale and it remains to be seen whether any of them can be UV completed into a microscopic theory with acceptable physical properties. There is a common underlying physical reason why finding a UV completion for these models is hard. One of the prices (or gains, depending on a viewpoint) to pay for affecting gravity at long distance scales is the possibility of superluminal signal propagation. This results in a tension between locality and causality making it impossible to construct a conventional UV completion [12]. The tension is especially severe in long distance modifications of gravity possessing a Poincaré invariant ground state, as required for applying the arguments of [12]. A somewhat related tension in Lorentz-violating scenarios arises in the presence of black hole horizons [13].

The simplest toy model, where the arguments of [12] apply, describes a single Goldstone boson X with an effective Lagrangian of the form

$$\mathcal{L} = \frac{1}{2}(\partial X)^2 + \frac{c}{\Lambda^4}(\partial X)^4 + \cdots. \quad (1)$$

It is straightforward to construct a UV completion for this model if the constant c is positive (we work in the mostly "−" signature). On the other hand, despite its benign appearance, this simple effective field theory cannot be UV completed into a consistent local microscopic quantum theory for negative values of c. The physical reason for this is the existence of classical backgrounds (such as $X \propto t$) exhibiting superluminal propagation of small perturbations.

At this stage two viewpoints are possible. The conservative conclusion is that this problem signals an incurable pathology and effective field theories with wrong sign operators should be discarded. However, an adventurous person may argue that superluminality simply indicates that one should look for a UV completion beyond the realm of conventional quantum field theories. Of course, to justify the second viewpoint one needs to construct an example of such a UV completion, so that it becomes possible to judge whether its physical properties are acceptable. The goal of this paper is to provide a first example of this kind and to study its physical properties.

A disclaimer is in order. All of our construction operates in two space-time dimensions. Two-dimensional theories are special in many respects. In particular, unlike in higher dimensional theories, in two dimensions superluminality does not allow one to construct smooth backgrounds exhibiting closed timelike curves. Somewhat related to this, in two dimensions it is possible to introduce an alternative causal structure, which is compatible with Lorentz symmetry, yet allows for superluminal (in fact, instantaneous) signal propagation. Based on this observation examples of UV complete Lorentz-invariant superluminal theories have already been constructed in the past [14,15]. However, the setup considered here relies on peculiarities of two-dimensional physics to a much lesser extent and the possibility of its extension to higher dimensional models appears more likely.

II. DESCRIPTION OF THE SETUP

Our construction is based on recent progress achieved in understanding the dynamics of long strings [16,17].

One starts with a classical Nambu-Goto action, describing $(D-2)$ scalar fields X^i (transverse perturbations of a string),

$$S_{\rm NG} = -\ell_s^{-2}\int d^2\sigma\sqrt{-\det\left(\eta_{ab}-\ell_s^2\partial_a X^i\partial_b X^i\right)}, \quad (2)$$

where ℓ_s^{-2} is the string tension. This action is the result of gauge fixing the covariant form of Nambu-Goto in a unitary gauge, to avoid the complications that arise when defining an S matrix for theories with constraints such as in conformal gauge. At the quantum level this may be considered as an effective theory similar to the Goldstone Lagrangian (1). This theory is naively nonrenormalizable. Famously, however, it can be consistently quantized when the number of flavors is equal to 24, giving rise to a critical bosonic string [18].

The traditional treatment of string dynamics focuses on the properties of short (open or closed) strings, which is natural if the goal is to provide a target space-time interpretation. However, for the purpose of this paper we are mainly interested in a purely two-dimensional interpretation of (2). From this perspective the most natural and simplest set of observables to consider are the S matrix elements for the scattering of world sheet perturbations of an infinitely long string. Geometrically these represent wiggles (or phonons) propagating along the string. As explained in [17], in this language the critical string can be *defined* by its exact S matrix. The theory is integrable and reflectionless, so that the two-particle scattering is purely elastic and is characterized by the phase shift of the form

$$e^{2i\delta(s)} = e^{is\ell_s^2/4}, \quad (3)$$

where s is the Mandelstam variable. Phase shifts for multiparticle processes are obtained by summing pairwise the phase shifts of all the colliding particles.

Despite its simplicity, the phase shift (3) exhibits a number of remarkable properties, and corresponds to an integrable theory of gravity rather than to a conventional field theory. In particular, the UV asymptotics of this theory are not controlled by a conventional fixed point. This is clear from the form of the phase shift (3) which retains a nontrivial dependence on the microscopic length scale ℓ_s at arbitrarily high energies. Clearly, this is incompatible with scale invariant behavior in the UV. This gives rise to a number of unconventional features, and this type of asymptotic behavior was called "asymptotic fragility."

An asymptotically fragile phase shift (3) satisfies all the properties expected from a healthy S matrix. In particular, it is analytic and polynomially bounded everywhere *on the physical sheet*, which is usually taken as a definition of locality in the S-matrix language. On the other hand, it exhibits an essential singularity at $s\to\infty$, which prevents one from defining local off-shell observables, corresponding to (3). This is one of the reasons why this theory is gravitational, rather than a conventional field theory. Another gravitational property of the model is the universal time delay proportional to the center of mass energy of the collision,

$$\delta t_{\rm del} = \frac{1}{2}E\ell_s^2,$$

which may be considered as an integrable precursor of black hole formation and evaporation.

Perhaps the most direct proof that the phase shift (3) indeed defines a critical string comes from calculating the finite volume spectrum of the theory using the thermodynamic Bethe ansatz (TBA) technique [19,20]. This way one exactly reproduces the spectrum of a critical bosonic string.

As a further check, it is straightforward to see that the phase shift (3) agrees with the perturbative tree-level and one-loop amplitudes following from the Nambu-Goto action (2) at $D=26$. However, the action (2) is nonrenormalizable and presumably at higher loop order has to be supplemented with an infinite set of scheme-dependent counterterms to reproduce the phase shift (3).

Note, that the phase shift (3) defines a consistent relativistic two-dimensional theory for an arbitrary number of flavors. However, for $D\neq 26$ the one-loop amplitude following from the Nambu-Goto action differs[1] from (3) by a rational annihilation term (the Polchinski-Strominger interaction [22]). To reproduce (3) for $D\neq 26$ the action (2) needs to be supplemented with a counterterm, which cancels this effect. This counterterm is perfectly consistent from two-dimensional perspective, but incompatible with nonlinearly realized target space Poincaré symmetry.

A further discussion of this family of integrable models can be found in [16,17]. Here, instead, let us move directly to the main point and describe the superluminal setup.

The basic idea is very simple and motivated by the following observation. Note that, unlike what one may be used to in more sophisticated quantum theories, the perturbative low energy expansion for the phase shift (3) is not asymptotic. It has an infinite radius of convergence and the phase shift is given simply by the sum of all perturbative terms. This suggests that flipping the sign of the coupling constant, ℓ_s^2, may also result in a well-defined theory.

So, inspired by the success of this approach for a world sheet theory of a bosonic string, let us try to define a new integrable theory by its exact scattering phase shift δ_n of the form

$$e^{2i\delta_n(s)} = e^{-is\ell_s^2/4}, \quad (4)$$

where, as before, we assume $\ell_s^2>0$. When expanded at low energies this scattering amplitude violates the positivity

[1] $D=3$ is another interesting exceptional case, cf. [21].

condition of [12]. It can be reproduced from the action (2), with $\ell_s^2 \to -\ell_s^2$, and supplemented with the same set of scheme-dependent counterterms as required to reproduce the conventional phase shift (3). Geometrically this action describes an infinitely long string in a space-time with $(D-1)$ time coordinates and a single spatial coordinate. The target space interpretation of such a system is highly obscure, but for our purposes we do not need it and will consider the system from a purely two-dimensional point of view.

In agreement with general arguments of [12] this theory is even more nonlocal than the world sheet theory of a conventional string (3). Namely, not only does the phase shift (4) exhibit an essential singularity at the infinity, it is also polynomially unbounded on the physical sheet. This represents an interesting step forward compared to the usual situation with superluminal effective theories. Conventionally, these theories are nonrenormalizable, and one simply concludes that superluminality indicates the absence of the UV completion. Here, the theory is nonrenormalizable, but we managed to construct finite on-shell scattering amplitudes. We still are not able to determine local off-shell observables, but the same situation holds for the conventional string world sheet theory, which is known nevertheless to be an interesting and healthy physical system. So an interesting question arises to compare the two models and to characterize in what sense the "wrong" sign theory is pathological as compared to its "right" sign cousin, if this is really the case.

In the rest of the paper we will make several steps in this direction, by probing some of the physics following from the superluminal phase shift (4). At the technical level all of our calculations are close counterparts of the corresponding steps in the analysis of the "right" sign theory (3). Essentially, they can be performed by flipping the sign of ℓ_s^2 in the formulas of [17]. Nevertheless, as we will see, the resulting physics turns out to be quite different.

III. CLASSICAL TIME ADVANCE

Let us start by illustrating superluminal properties of the phase shift (4) as seen in the classical limit. For simplicity, let us consider a single flavor case, so that the corresponding classical action is

$$S = \ell_s^{-2} \int d\tau d\sigma \sqrt{1+\ell_s^2(\partial X)^2}. \tag{5}$$

In agreement with the discussion in the Introduction, superluminality is manifest when considering a classical background of the form

$$X_{\rm cl} = \frac{v\tau}{\ell_s}.$$

Namely, the quadratic action for small perturbations π around this background takes the form

$$S_2 = \int d\tau d\sigma \left(\frac{1}{2(1+v^2)^{1/2}}(\partial\pi)^2 - \frac{v^2}{2(1+v^2)^{3/2}}(\partial_\tau \pi)^2 \right).$$

This gives rise to a linear dispersion relation

$$\omega = c_s k$$

with a superluminal velocity

$$c_s = \sqrt{1+v^2}.$$

Related to this superluminality the phase shift (4) gives rise to a time *advance*

$$\delta t_{\rm ad} = \frac{1}{2} E \ell_s^2 \tag{6}$$

for scattering processes around the trivial background $X=0$. It is instructive to see how this time advance comes out at the classical level. For this purpose let us consider a purely left-moving field configuration, which is always a classical solution for the action (5),

$$X_{\rm cl} = X(\tau+\sigma).$$

Understanding the scattering of a small right-moving perturbation off this background amounts to studying the null geodesics propagating in the metric induced by the classical solution

$$\begin{aligned} ds^2 &= (\eta_{ab} + \ell_s^2 \partial_a X \partial_b X) d\sigma^a d\sigma^b \\ &= (1+\ell_s^2 X'^2) d\tau^2 + 2\ell_s^2 X'^2 d\sigma d\tau - (1-\ell_s^2 X'^2) d\sigma^2. \end{aligned}$$

The null geodesic equation is given by

$$\frac{d\tau}{d\sigma} = \frac{-\ell_s^2 X'^2 + 1}{\ell_s^2 X'^2 \pm 1},$$

where the upper sign corresponds to a right-moving excitation which experiences a nontrivial time shift. We see this is a time advance since $\tau' < 1$. Note that τ' can even become negative for large enough X'. In this case the right mover moves "back in time" at intermediate times, which simply indicates that τ is not a good Cauchy time for such a background. The time advance is given by

$$\begin{aligned} \delta t_{\rm ad} &= \int_{-\infty}^{\infty} d\sigma (1-\tau') \\ &= \int_{-\infty}^{\infty} d\sigma \frac{2\ell_s^2 X'^2}{\ell_s^2 X'^2 + 1} \\ &= \int_{-\infty}^{\infty} d\sigma_+ \ell_s^2 X'^2(\sigma_+) = \ell_s^2 \Delta E, \end{aligned} \tag{7}$$

where ΔE is the energy of the classical solution relative to the vacuum. This classical time advance exactly agrees with

(6), following from the exact quantum S matrix, after one takes into account that the expression (6) calculates the time shift in the rest frame of the colliding wave packets, as opposed to the "lab frame" time advance (7).

IV. THERMODYNAMICS AND FINITE VOLUME SPECTRUM

An important piece of physical information, which becomes accessible when the exact S matrix is known, is the finite temperature equation of state of the system, i.e. the free energy density as a function of temperature, $f(T)$. For an integrable theory it can be extracted using the (ground state) TBA [19]. Moreover, for a Lorentz-invariant theory, free energy determines also the vacuum Casimir energy $E_0(R)$ on a circle of circumference R through

$$E_0(R) = Rf(R^{-1}).$$

The derivation of the ground state TBA equations for a superluminal phase shift (4) is completely parallel to the one presented in [17] for a conventional string. As expected, the result is different only by a flip of a sign in front of ℓ_s^2. Namely, the free energy density is given by

$$f(R^{-1}) = -\ell_s^{-2} + \frac{1}{2\pi R}\sum_{j=1}^{D-2}\int_0^\infty dp' \ln(1 - e^{-R\epsilon_L^j(p')}) + \frac{1}{2\pi R}\sum_{j=1}^{D-2}\int_0^\infty dp' \ln(1 - e^{-R\epsilon_R^j(p')}),$$

where the "pseudoenergies" $\epsilon_{L,R}$ are determined from a system of integral equations of the form

$$\epsilon_L^i(p) = p\left(1 - \frac{\ell_s^2}{2\pi R}\sum_{j=1}^{D-2}\int_0^\infty dp' \ln(1 - e^{-R\epsilon_R^j(p')})\right)$$
$$\epsilon_R^i(p) = p\left(1 - \frac{\ell_s^2}{2\pi R}\sum_{j=1}^{D-2}\int_0^\infty dp' \ln(1 - e^{-R\epsilon_L^j(p')})\right).$$

Just like for a conventional string, these equations are straightforward to solve analytically. By picking up the solution which approaches the free theory in the limit $\ell_s^2 \to 0$, we obtain

$$E_0(R) = Rf(R^{-1}) = -\sqrt{\frac{R^2}{l_s^4} + \frac{4\pi}{l_s^2}\frac{D-2}{12}}.$$

Not surprisingly, this expression closely resembles the ground state energy of a bosonic string. One major difference however, is that the free energy is real at all temperatures. This means that the Hagedorn behavior is not present in this theory. In a sense, the thermodynamics of a superluminal theory is less pathological than that of a conventional string.

Related to this the dependence of the energy density on the pressure,

$$\rho = \frac{p}{1 + l_s^2 p}, \tag{8}$$

does not exhibit a singularity, which was present for a conventional string.

The absence of the Hagedorn behavior is straightforward to understand. From a two-dimensional perspective the Hagedorn behavior arises as a consequence of a large binding energy following from the phase shift (3). This results in the fast growth of the density of states implying the divergence of the heat capacity in the thermodynamic limit of the theory. By flipping the sign of ℓ_s^2 we replaced attraction with repulsion, which eliminated the Hagedorn behavior.

To confirm this interpretation let us take a look at the spectrum of the excited states of the superluminal theory in a finite volume. Following the derivation in [17] we obtain

$$E(N, \tilde{N}) = -\left(\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{l_s^4} - \frac{4\pi}{l_s^2} \times\left(N + \tilde{N} - \frac{D-2}{12}\right)\right)^{1/2}, \tag{9}$$

where the positive integers $N, \tilde{N}$ count the total left- and right-moving Kaluza-Klein momentum of a state (in units of $2\pi/R$). For a conventional bosonic string these would be called the levels of a state. It is straightforward to see now that the two-particle binding energy is positive

$$\Delta E = E(1,1) - E(0,0) - 2(E(1,0) - E(0,0)) > 0$$

i.e., the interaction is repulsive.

A further inspection of the spectrum (9) reveals, however, that in spite of the absence of the Hagedorn behavior for the ground state, this spectrum exhibits an even more bizarre property. Namely, for *any* value of R there are infinitely many states with imaginary energies. We illustrated this behavior in Fig. 1, where we have shown the region of positive and negative E^2 in the $(N, \tilde{N})$ plane for several values of D and R/ℓ_s. This property indicates that the superluminal theory cannot be put in a finite volume, at least in a conventional way.

Pathological at first sight, this behavior is straightforward to understand. Indeed, as a consequence of the time advance (7) two points at the same constant time surface become causally connected if the "string" between them is excited to sufficiently high energy. Identification of these two points is clearly a bad idea then, because it results in a closed timelike curve. From (7) we can estimate that this happen when

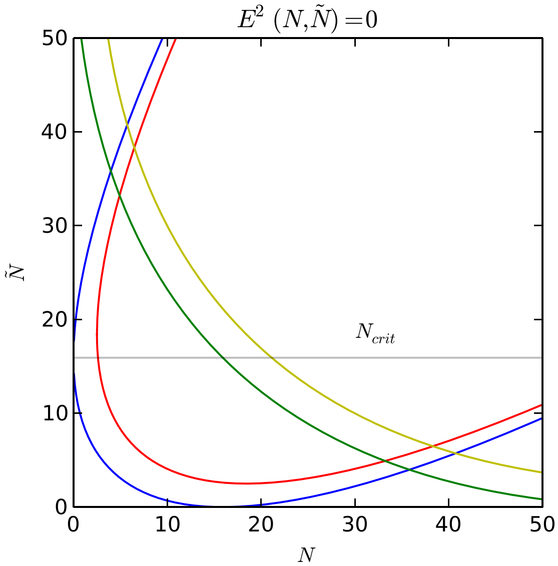

FIG. 1 (color online). Above is a plot of the region where energy becomes imaginary. Blue and green are at $D = 2$, $R/l_s = 10$ and 20 respectively. Red and Yellow are at $D = 62$, $R/l_s = 10$ and 20 respectively. The allowed region is the left of each curve. The lines are blue, red, green, yellow as you move away from the origin.

$$\ell_s^2 \Delta E \simeq \frac{2\pi \ell_s^2 N}{R}$$

becomes of order R. This nicely matches what we observe in Fig. 1, namely the $E^2 = 0$ curve with $D = 2$ (which corresponds to the classical answer for the spectrum) touches the N axis at

$$N_{\rm crit} = \frac{R^2}{2\pi \ell_s^2}.$$

We see that the proliferation of states with imaginary energies in the finite volume cannot be considered as a pathology of the theory, but rather is a sign that we attempted to perform an illegal action. It would be interesting to find whether the model can nevertheless be consistently put in a finite volume. Classically, this would require identification of points on Cauchy slices, which are background dependent and in general do not correspond to constant time surfaces. TBA, at least in its standard form, is not an appropriate tool in such a situation, so currently we do not know how to proceed in the quantum theory. It is also puzzling that the TBA result exhibits an island of apparently healthy $E^2 > 0$ states with arbitrary large $N > N_{\rm crit}$ (see Fig. 1). Classically these contain a closed timelike curve, so that their physical interpretation is obscure.

V. COSMOLOGY

Another interesting property of the conventional world sheet theory is the presence of cosmological backgrounds [17]. Let us see what these look like in the superluminal theory. For simplicity, we restrict to a single field case. In a $(1+1)$-dimensional world isotropy is not a constraint, so one is looking for homogeneous solutions. Imposing an invariance under coordinate translations, $\sigma \to \sigma + \text{const}$ leaves one with only trivial vacuum solutions. Instead, following [17], one can look for homogeneous solutions where the isometry is generated by boosts. In fact, there is a larger set of solutions, labeled by a continuous parameter γ, such that the isometry is generated by the combination of a boost and a shift of the field of the form

$$X(\sigma^+, \sigma^-) \to X((1+\epsilon)\sigma^+, (1+\epsilon)^{-1}\sigma^-) - \epsilon\gamma, \qquad (10)$$

where $\sigma^\pm = \tau \pm \sigma$. A general field configuration invariant under (10) can be presented in the form

$$\ell_s X(\sigma^+\sigma^-) = f(\sigma^+\sigma^-) + \frac{\gamma}{2}\log\frac{\sigma^+}{\sigma^-}. \qquad (11)$$

Then the field equation becomes

$$\partial_{\alpha^2}\left(\frac{\alpha^2 \partial_{\alpha^2} f}{\sqrt{1 + \alpha^2(\partial_{\alpha^2} f)^2 - \frac{\gamma^2}{4\alpha^2}}}\right) = 0, \qquad (12)$$

where $\alpha^2 = \sigma^+\sigma^-$. The general solution then takes the following form:

$$f(\alpha) = L\log\left(\sqrt{L^2 - 4\alpha^2} + \sqrt{\gamma^2 - 4\alpha^2}\right) + \gamma\log\frac{\alpha}{\gamma\sqrt{L^2 - 4\alpha^2} + L\sqrt{\gamma^2 - 4\alpha^2}} + C, \qquad (13)$$

where C and L are the integration constants. The constant C can be, so that the field X remains real. To get an insight into the physics of these solutions it is instructive to inspect the induced metric

$$g_{ab} = \eta_{ab} + \ell_s^2 \partial_a X \partial_b X,$$

which determines the propagation of small perturbations. One observes that this family of solutions splits into two disconnected physical branches, $\mathcal{A}$ and $\mathcal{B}$, where the field takes real values. The $\mathcal{A}$ branch covers the region

$$4\alpha^2 > L^2, \gamma^2,$$

where one can introduce coordinates (ρ, λ) defined as

$$\sigma^\pm = \rho e^{\pm\lambda}.$$

In these coordinates the induced metric becomes

$$ds^2_{\mathcal{A}} = \frac{16\rho^4 - L^2\gamma^2}{\rho^2(4\rho^2 - L^2)} d\rho^2 + 2\frac{L\gamma\sqrt{4\rho^2 - \gamma^2}}{\rho\sqrt{4\rho^2 - L^2}} d\rho d\lambda - (4\rho^2 - \gamma^2) d\lambda^2.$$

By making a shift $\lambda \to \lambda + f(\rho)$ we can get rid of the off-diagonal term in the metric, so that it takes the Friedmann-Robertson-Walker form

$$ds_{\mathcal{A}}^2 = \frac{16\rho^2 d\rho^2}{4\rho^2 - L^2} - (4\rho^2 - \gamma^2)d\lambda^2 = d\tau^2 - (\tau^2 + L^2 - \gamma^2)d\lambda^2, \quad (14)$$

where at the last step we introduced a new time coordinate $\tau = \sqrt{4\rho^2 - L^2}$.

The $\mathcal{B}$ branch of solutions covers the region

$$4\alpha^2 < L^2, \gamma^2.$$

One can repeat all the same steps here, and obtain the metric of the form

$$ds_{\mathcal{B}}^2 = (\gamma^2 - L^2 + r^2)d\lambda^2 - dr^2,$$

where $r = \sqrt{L^2 - 4\rho^2}$.

We conclude that the $\mathcal{A}$ branch describes a cosmological solution and the $\mathcal{B}$ branch corresponds to a static spacetime. For $\gamma^2 < L^2$ the cosmology is nonsingular, while the static solution exhibits a curvature singularity. For larger values of γ the singularity moves on the cosmological branch. This does not appear very different from what one finds for a conventional string. The solutions take the same form there with the role of time and space interchanged. There, for $\gamma^2 < L^2$ one finds a smooth static geometry and for $\gamma^2 > L^2$ a nonsingular bouncing cosmology.

VI. DISCUSSION

To summarize, it is fair to admit that our results are inconclusive at this point. We did not manage to identify a clean pathology associated with a superluminal sign. Definitely, this theory allows one to calculate a smaller set of observables than a conventional renormalizable quantum field theory, but the same holds also for the conventional string world sheet theory. Nevertheless, the latter definitely gives rise to a rich and healthy system. The verdict for the former is not out yet. Both theories exhibit a number of gravitational features. Gravitational theories are expected to be less predictive than quantum field theories by not allowing one to calculate local off-shell observables. Perhaps the most interesting lesson from the construction presented here, is that it raises the question where is the proper place to stop on a slippery road between conventional UV complete quantum field theories and nonrenormalizable effective theories. The former allow sharp prediction of both on-shell and off-shell observables. The latter do not allow sharp predictions at all in the mathematical sense, but often are quite adequate from the practical point of view. Gravitational theories live in the middle.

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