Matter creation and cosmic acceleration

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We investigate the creation of cold dark matter (CCDM) cosmology as an alternative to explain the cosmic acceleration. Particular attention is given to the evolution of density perturbations and constraints coming from recent observations. By assuming negligible effective sound speed we compare CCDM predictions with redshift-space-distortion based $f(z)\sigma_8(z)$ measurements. We identify a subtle issue associated with which contribution in the density contrast should be used in this test and then show that the CCDM results are the same as those obtained with ΛCDM. These results are then contrasted with the ones obtained at the background level. For the background tests we have used type Ia supernovae data (Union 2.1 compilation) in combination with baryonic acoustic oscillations and cosmic microwave background observations and also measurements of the Hubble parameter at different redshifts. As a consequence of the studies we have performed at both the background and perturbation levels, we explicitly show that CCDM is observationally degenerate with respect to ΛCDM (dark degeneracy). The need to overcome the lack of a fundamental microscopic basis for the CCDM is the major challenge for this kind of model.

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I. INTRODUCTION

Explaining the recent cosmic acceleration, believed to be related to some form of dark energy, and supported by the observations of high redshift supernovae and by other independent observational data, such as the results coming from cosmic microwave background radiation (CMBR) and with baryonic acoustic oscillation (BAO), is one of the present day challenges in cosmology. Among the many possible proposals (for recent reviews, see, e.g., Ref. [1] and also references therein), possibly a cosmological constant (Λ) is the simplest answer to explain the late-time cosmic acceleration. Of course, this also brings some theoretical difficulties, which are how to explain its origin, right magnitude and why it comes to dominate just now.

Just like the early-time cosmic acceleration associated with inflation, a negative pressure can be seen as a possible driving mechanism for the late-time accelerated expansion of the Universe as well. One of the earliest alternatives that could provide a mechanism producing such accelerating phase of the Universe is through a negative pressure produced by viscous or particle production effects. For instance, one of the first works relating particle production, in particular as a result of a nonstationary gravitational field and that can be described phenomenologically by means of a negative pressure, is due to Zeldovich [2]. This is much similar to the idea put forward by Murphy [3] and also later

by Hu [4], that particle production might also be described equivalently in terms of a bulk viscous pressure in the cosmological fluid. In this context, since a bulk viscous pressure is a negative pressure contribution in the energy-momentum stress tensor [5], it has lead to an extensive literature on applications related to bulk viscous cosmology (for a partial sample of the earliest works on bulk viscous cosmologies, see for example Refs. [3,4,6,7]). In addition, more recently, there has also been a surge of interest in exploring the effects of the bulk pressure as the origin of the present accelerated expansion of the Universe (see, e.g., Ref. [8]). A closed related scenario to the bulk viscous cosmology is that of the so-called adiabatic matter creation, which makes use of ideas of the thermodynamics of open systems in the context of cosmology and initiated by Prigogine and collaborators [9]. A covariant formalism approach has later been formulated in Ref. [10].

Despite the fact that bulk viscous and matter creation cosmologies apparently look similar, they have some fundamental differences. Bulk viscous cosmologies are associated with a generalization of the hydrodynamics of ideal fluids for the case of nonideal ones, with constitutive equations describing the viscous pressures built as additional correction terms to the equilibrium energy-momentum stress tensor [5]. As such, the viscous pressure contributions can be seen as small nonequilibrium contributions for the energy-momentum tensor for nonideal fluids. It happens, however, that most of the effects of a bulk viscous pressure to cosmology, as for example when it is used as a mechanism for inflation, it typically requires an extrapolation beyond the limit of validity for these theories [11] (see, however,

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Ref. [7]). In the context of matter creation, even though also a negative effective pressure can be associated with it, there is in principle no such limitation as with a negative bulk viscous pressure.

Particle creation models [9,10,12–18], as the one treated in this work, should also not be confused with other cosmological scenarios where particle production is present, like, e.g., in warm inflation [19]. In warm inflation models the inflationary evolution can be strongly influenced by relativistic (radiation) particle production. In these models there can also be negative pressure effects as a result of a bulk viscous pressure from the radiation bath, but these effects are in general small in the inflationary context [20].

Many authors have explored scenarios of matter creation in cosmology, but here we are particularly interested in the gravitationally induced particle creation scenario denominated "creation of cold dark matter" (CCDM) [15–18] in which a special choice of the particle production rate produces a cosmology that, at the background level, is indistinguishable from the standard Λ cold dark matter (ΛCDM) model. However, as we are going to discuss in this paper, at the perturbative level this degeneracy is more subtle and care should be taken when contrasting CCDM predictions with those obtained in the standard ΛCDM cosmology. Furthermore, perturbations in the case of CCDM cosmology have mostly been studied in the context of the so-called neo-Newtonian formalism [21]. Following Ref. [22], here we will show that this formalism for studying density perturbations has limitations and a fully relativistic one (like that, for example, of Ref. [23]) is required in the case in which the effective speed of sound cannot be neglected.

The rest of this paper is organized as follows. In Sec. II we briefly review the thermodynamics for matter creation cosmology. In Sec. III we discuss the background equations and their solutions. In Sec. IV both the neo-Newtonian and relativistic approaches are discussed and the differences between the two are given. In Sec. V we analyze the observational constraints on the CCDM model we have considered here. Finally, our conclusions and final remarks are given in Sec. VI.

II. THERMODYNAMICS OF MATTER CREATION IN A SIMPLE FLUID

Let us briefly review here the thermodynamics of matter creation. For simplicity, we will restrict to the case of a single fluid, but it can easily be generalized to multiple coupled fluids as well. To describe the thermodynamic states of a relativistic simple fluid we use the following macroscopic variables: the energy-momentum tensor $T^{\alpha\beta}$; the particle flux vector N^{α}; and the entropy flux vector s^{α}. The energy-momentum tensor satisfies the conservation law, $T^{\alpha\beta}{}_{;\beta} = 0$, and here we consider situations in which it has the perfect-fluid form:

$$T^{\alpha\beta} = (\rho + P)u^{\alpha}u^{\beta} - Pg^{\alpha\beta}. \tag{2.1}$$

In the above equation ρ is the energy density, P is the isotropic dynamical pressure, $g^{\alpha\beta}$ is the metric tensor and u^{α} is the fluid four-velocity (with normalization $u^{\alpha}u_{\alpha} = 1$).

The dynamical pressure P is decomposed as

$$P = p + \Pi, \tag{2.2}$$

where p is the equilibrium (thermostatic) pressure and Π is a term present in scalar dissipative processes. Usually, it is associated with the so-called bulk pressure [5]. In the cosmological context, besides this meaning, Π can also be relevant when particle number is not conserved [9]. In this case, $\Pi \equiv p_c$ is called the "creation pressure." It is important to mention that the bulk pressure, as already mentioned in the Introduction, can be seen as a correction to the thermostatic pressure when near to equilibrium, thus, it should be always smaller than the thermostatic pressure, $|\Pi| < p$. This restriction, however, does not apply for the creation pressure. So, when we have matter creation, the total pressure P may become negative and, in principle, drive an accelerated expansion.

The particle flux vector is assumed to have the following form:

$$N^{\alpha} = nu^{\alpha}, \tag{2.3}$$

where n is the particle number density. N^{α} satisfies the balance equation $N^{\alpha}{}_{;\alpha} = n\Gamma$, where Γ is the particle production rate. If $\Gamma > 0$, we have particle creation, particle destruction occurs when $\Gamma < 0$ and if $\Gamma = 0$ particle number is conserved.

The entropy flux vector is given by

$$s^{\alpha} = n\sigma u^{\alpha}, \tag{2.4}$$

where σ is the specific (per particle) entropy. Note that the entropy must satisfy the second law of thermodynamics $s^{\alpha}{}_{;\alpha} \geq 0$. Here we consider adiabatic matter creation; that is, we analyze situations in which σ is constant. With this condition, by using the Gibbs relation, it follows that the creation pressure is related to Γ by [9,10]

$$p_c = -\frac{\rho + p}{3H}\Gamma, \tag{2.5}$$

where $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor of the Friedmann-Robertson-Walker (FRW) metric [see Eq. (3.1) below] and the overdot means differentiation with respect to the cosmic time. It is also straightforward to show that, if σ is constant, the second law of thermodynamics implies that $\Gamma \geq 0$ and, as a consequence, particle destruction ($\Gamma < 0$) is thermodynamically forbidden [9,10]. Since $\Gamma \geq 0$, it follows from Eq. (2.5) that, in an expanding

universe ($H > 0$), the creation pressure p_c cannot be positive.

III. COSMOLOGICAL MODELS WITH PARTICLE CREATION

Before we discuss the evolution of linear perturbations in cosmological models with matter creation, we first consider their background equations. By assuming spatial homogeneity and isotropy, which is a good approximation at large scales, we are lead to the FRW line element,

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]. \quad (3.1)$$

Here $k = 0, \pm1$ characterizes the curvature of the spatial sections of space-time and we are assuming $c = 1$, as usual. For the sake of simplicity, from now on we also assume flat space ($k = 0$), which is in good agreement with CMBR observations. In this paper we are mainly interested in processes that occurred after radiation domination. Therefore, as a first approximation, we neglect radiation and, for the sake of simplicity, we also neglect baryons considering only the presence (and creation) of pressureless ($p = 0$) dark matter particles.

The Einstein equations for the models we consider can be expressed simply as

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \quad (3.2)$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3p_c). \quad (3.3)$$

To the above equations we add Eq. (2.5) (with $p = 0$) to get

$$\dot{\rho} + 3H\rho = \rho\Gamma. \quad (3.4)$$

In order to integrate the above equations, it is necessary to assume a special form for Γ. Several models that have previously been studied in the literature can all be generalized by the following expression for the particles production rate [13]:

$$\Gamma = 3\beta H_0\left(\frac{H}{H_0}\right)^\alpha, \quad (3.5)$$

where α and β are $\mathcal{O}(1)$ dimensionless constants and H_0 is the present value of the Hubble parameter. Throughout this paper we use the subscript "0" to denote the present value of quantities. From the above equations, we get the following differential equation for H:

$$\frac{dH}{dz}(1 + z) = \frac{3}{2}H_0\left[\frac{H}{H_0} - \beta\left(\frac{H}{H_0}\right)^\alpha\right], \quad (3.6)$$

where $z = 1/a - 1$ is the redshift. The above equation can easily be integrated, leading to the result [13]

$$H = \begin{cases} H_0[\beta + (1 - \beta)(1 + z)^{\frac{3}{2}(1-\alpha)}]^{\frac{1}{1-\alpha}}, & \text{if } \alpha \neq 1, \\ H_0(1 + z)^{\frac{3}{2}(1-\beta)}, & \text{if } \alpha = 1. \end{cases} \quad (3.7)$$

From now on we focus on the particular case $\alpha = -1$ in Eq. (3.5). Following Ref. [18], we refer to this model as creation of cold dark matter (CCDM). With $\alpha = -1$, from Eq. (3.7), we obtain

$$\frac{H^2}{H_0^2} = (1 - \beta)(1 + z)^3 + \beta. \quad (3.8)$$

The above equation indicates that the expansion rate H in CCDM has the same exact form as in flat ΛCDM models with β playing the role of the cosmological constant density parameter at present time [13,14], $\Omega_{\Lambda 0}$. Notice that, by using Eq. (3.2), the expression for the particle production rate Eq. (3.5) can be written as [15]

$$\Gamma = \frac{3\beta H_0^2}{H} = 3\beta\left(\frac{\rho_{c0}}{\rho}\right)H, \quad (3.9)$$

where $\rho_{c0} \equiv 3H_0^2/(8\pi G)$ is the critical density at present time, which, in our flat-space and simple-fluid approximation, is equal to the value of dark matter energy density at present. Notice also that for $\alpha = -1$, the creation pressure, $p_c = -\beta\rho_{c0}$, is constant and by using Eq. (3.2), the dark matter energy density can be written as

$$\rho = \rho_{c0}[(1 - \beta)(1 + z)^3 + \beta]. \quad (3.10)$$

As remarked above, the CCDM model mimics exactly the ΛCDM background expansion history, so we should expect good accordance of this model with kinematic cosmological tests like from supernovae type Ia (SNIa) and BAO, that essentially depend only on distances and, thus, does not depend on the perturbation results. Does this mimicry remains at the perturbation level? Answering this question is somewhat subtle and we will discuss it in Sec. V.

Another point to be stressed here is that, although in CCDM we have a kind of unification of the dark sector, it does not solve or alleviate the so-called cosmological constant problems. For instance, the old cosmological constant problem is not solved since, like in quintessence, Λ is assumed to be zero from the beginning. Of course we hope that this problem will be resolved in the context of quantum field theory and not by cosmology. However, the fine-tuning and the cosmic coincidence problems are essentially the same as in ΛCDM. To better understand this, we now write the total dark matter energy density ρ as

$$\rho = \rho_{\text{conserved}} + \rho_{\text{created}}, \quad (3.11)$$

where $\rho_{\text{conserved}} = \rho_{c0}(1 - \beta)(1 + z)^3$ is the conserved part of the dark matter energy density and $\rho_{\text{created}} = \rho_{c0}\beta$ is the created one. The cosmological problems can now be cast as follows: Why was the created (and constant) part so small (as compared to the energy densities of other fields) and finely adjusted in the beginning of the Universe evolution? Why only at recent times are the conserved and the created (and constant) dark matter energy densities comparable? Therefore, CCDM model has essentially the same conceptual difficulties as ΛCDM. Indeed, from the theoretical point of view the situation is even worse in CCDM. Although some authors (see, e.g., Refs. [15,17,18] and references therein) try to motivate the CCDM scenario in terms of gravitational particle production in an expanding universe, currently there is no fundamental basis for the chosen particle production rate and we can only treat CCDM as a phenomenological model. In this context, we adopt a more pragmatic approach and, in the following sections, we discuss if observations that depend on the growth of perturbations can distinguish CCDM from ΛCDM. If yes, the CCDM model can be tested. If it produces results that are not compatible with the observations, then it can be discarded from the beginning. If the results are better than the ones produced with the ΛCDM, then we can pursue further and look more closely for the microscopic motivations for the model. However, if the CCDM and ΛCDM are observationally degenerated with each other, then we must resort to the Occam's Razor principle to guide us. Accordingly, the simplest model (i.e., ΛCDM) becomes preferable unless further theoretical developments change the current situation.

IV. EVOLUTION OF LINEAR DENSITY PERTURBATIONS: NEO-NEWTONIAN VERSUS RELATIVISTIC APPROACH

We now turn our attention to the growth of linear perturbation in matter creation models. Following Refs. [15,17], we first consider it in the neo-Newtonian context. The idea of a Newtonian expanding universe was developed by Milne [24] and also by McCrea and Milne [25] in the 1930's. By considering a pressureless fluid and assuming Newtonian dynamics and gravitation, it was shown that the governing Newtonian differential equations are identical in form to the relativistic ones. This approach, known as Newtonian cosmology (NC), is quite helpful in giving insight into the physical significance of an expanding universe. The NC equations were generalized to include uniform pressure by McCrea [26] in a paper in which the hypothesis of continuous creation of matter was investigated. The same equations were reobtained later in Ref. [27] in a different context. However, as pointed out in Ref. [28], although the Newtonian background evolution equations with pressure are identical in form to the relativistic ones (assuming zero spatial curvature), at the perturbative level they are only equivalent when pressure is

zero. To circumvent this difficulty, in Ref. [21] it was suggested a modification of the continuity equation. This formulation, known as neo-Newtonian approach, has also limitations, as pointed out in Ref. [22], as we now discuss.

The basic equations that describe the neo-Newtonian formulation are [21,26,27]

$$\nabla_r^2 \phi = 4\pi G(\rho + 3P), \quad (4.1)$$

$$\left(\frac{\partial \mathbf{u}}{\partial t}\right)_r + (\mathbf{u} \cdot \nabla_{\mathbf{r}})\mathbf{u} = -\nabla_{\mathbf{r}}\phi - (\rho + P)^{-1}\nabla_{\mathbf{r}}P, \quad (4.2)$$

$$\left(\frac{\partial \rho}{\partial t}\right)_r + \nabla_{\mathbf{r}} \cdot (\rho\mathbf{u}) + P\nabla_{\mathbf{r}} \cdot \mathbf{u} = 0. \quad (4.3)$$

Equations (4.1), (4.2) and (4.3) are, respectively, the modified Poisson, Euler and energy conservation equations, where relativistic effects of pressure were included. In the above equations, $\mathbf{u}$ is the velocity field and ϕ is the gravitational potential of the cosmic fluid.

As usual in perturbation theory [23], we assume small perturbations around the homogeneous background solution in the form $\rho = \tilde{\rho} + \delta\rho = \tilde{\rho}(1 + \delta)$, $P = \tilde{P} + \delta P$, $\phi = \tilde{\phi} + \varphi$, and $\mathbf{u} = H\mathbf{r} + \mathbf{v}$. We use a tilde to denote background quantities. Introducing comoving coordinates $\mathbf{x} = \mathbf{r}/a$, neglecting shear and vorticity and taking into account the background equations, after some algebra we can derive the following differential equation for the density contrast [22]:

$$\ddot{\delta} - [3(2w - c_s^2 - c_{\text{eff}}^2) - 2]H\dot{\delta}$$
$$+ 3H^2 \left\{ \left[\frac{3}{2}w^2 - 4w - \frac{1}{2} + 3c_s^2\right] \right.$$
$$\left. + c_{\text{eff}}^2(3c_s^2 - 6w - 1) + \frac{(c_{\text{eff}}^2)^{\cdot}}{H} + \frac{k^2}{a^2}\frac{c_{\text{eff}}^2}{3H^2} \right\}\delta = 0, \quad (4.4)$$

where $w = \tilde{P}/\tilde{\rho}$, $c_{\text{eff}}^2 = \delta P/\delta\rho$, $c_s^2 = \dot{\tilde{P}}/\dot{\tilde{\rho}} = w - \dot{w}/[3H(1 + w)]$ and k is the comoving wave number. We are looking for solutions of the form $\delta(\mathbf{x}, t) = \sum_k \delta_k(t)e^{i\mathbf{k}\cdot\mathbf{x}}$ and for the sake of simplicity, we have dropped the index k from δ in Eq. (4.4). We have also assumed that c_{eff}^2 is a function of time only.

Let us now consider the evolution of density perturbations in a general-relativistic framework. In this case, following standard calculations [23], assuming zero anisotropic pressure perturbations, besides flat space, we obtain

$$\ddot{\Delta} - [3(2w - c_s^2) - 2]H\dot{\Delta}$$
$$+ 3H^2 \left\{ \left[\frac{3}{2}w^2 - 4w - \frac{1}{2} + 3c_s^2\right] + \frac{k^2}{a^2}\frac{c_s^2}{3H^2} \right\}\Delta$$
$$= -\frac{k^2}{a^2}w\hat{\Gamma}, \quad (4.5)$$

where

$$\hat{\Gamma} \equiv \frac{\delta P}{\tilde{P}} - c_s^2 \frac{\delta}{w} = \frac{(c_{\text{eff}}^2 - c_s^2)}{w} \Delta \qquad (4.6)$$

is the gauge-invariant entropy perturbation, $c_{\text{eff}}^2 \equiv \frac{\delta P}{\delta \rho}\big|_{\text{rest}}$ is the effective sound speed (defined in the matter rest frame) [29] and the gauge-invariant quantity Δ represents the matter density contrast in the slicing such that the matter four-velocity is orthogonal to constant time hypersurfaces [23],

$$\Delta = \delta + 3(1 + w)H\frac{a}{k}(v - B), \qquad (4.7)$$

where $v - B$ is associated with the deviation of the matter four-velocity from the vector normal to the constant time hypersurfaces.

To compare the relativistic and neo-Newtonian differential equations for the density contrast, we go to the rest gauge [29], where $\Delta = \delta$, and write Eq. (4.5) as

$$\ddot{\delta} - [3(2w - c_s^2) - 2]H\dot{\delta}$$
$$+ 3H^2 \left\{ \left[\frac{3}{2}w^2 - 4w - \frac{1}{2} + 3c_s^2 \right] + \frac{k^2}{a^2} \frac{c_{\text{eff}}^2}{3H^2} \right\} \delta = 0.$$
$$(4.8)$$

Therefore, by simple inspection, we see that even for time-independent c_{eff}^2, Eqs. (4.4) and (4.8) are only identical when the effective sound speed c_{eff}^2 is equal to zero. It should also be remarked that in the more general case, in which $c_{\text{eff}}^2 \neq 0$, the last term inside the braces in Eq. (4.8) can only be neglected in the long-wavelength limit ($k = 0$), in which case the Newtonian approximation is not valid. Therefore, using Eq. (4.4), assuming $c_{\text{eff}}^2 \neq 0$ and neglecting the last term inside the braces is not a correct procedure (as adopted for example in Ref. [18]), first because Eq. (4.4) is not valid for $c_{\text{eff}}^2 \neq 0$, and second because the Newtonian approximation is also not valid in the long-wavelength limit.

V. THE CCDM MODEL: THEORY VERSUS OBSERVATIONS

Let us now consider the observational constraints on the CCDM model from the linear growth of energy density perturbation data. For this model, the background creation pressure $\tilde{p}_c = -\beta\rho_{c0}$ is constant and, therefore, $c_s^2 = 0$. We first assume $c_{\text{eff}}^2 = 0$, such that the neo-Newtonian and the general-relativistic approaches are equivalent. Notice that this corresponds to adiabatic perturbations ($\hat{\Gamma} = 0$), since c_{eff}^2 and c_s^2 are equal. By changing the variable from the cosmic time t to the scale factor a, recalling Eq. (3.8) and that for a constant creation pressure, as we are considering here, we have $H_0^2 = -wH^2/\beta$, we then obtain that Eq. (4.8) can be written as

$$\delta'' + \frac{3}{2a}(1 - 5w)\delta' + \frac{3}{2a^2}(3w^2 - 8w - 1)\delta = 0, \quad (5.1)$$

where the prime denotes derivative with respect to the scale factor a and the equation of state parameter is given by $w(a) = -\beta/[\beta + (1 - \beta)a^{-3}]$. Observe that for $\beta = 0$ ($w = 0$), there is no matter creation and the model reduces to the Einstein–de Sitter model. In the opposite limit, $\beta = 1$ ($w = -1$), there is no conserved dark matter and the de Sitter model is recovered. To integrate Eq. (5.1), we introduce a new variable $x = -a^3\beta/(1 - \beta)$ and write the density contrast as $\delta(x) = a/(1 - x)G(x)$. With these definitions we rewrite Eq. (5.1) as

$$x(1 - x)G''(x) + \left(\frac{11}{6} - \frac{7}{3}x \right)G'(x) - \frac{1}{3}G(x) = 0. \quad (5.2)$$

The exact solution of the above equation can be expressed in terms of hypergeometric functions $_2F_1(a, b; c; x)$ as

$$G(x) = C_1 {}_2F_1\left(\frac{1}{3}, 1; \frac{11}{6}; x \right) + C_2 x^{-\frac{5}{6}} {}_2F_1\left(-\frac{1}{2}, \frac{1}{6}; \frac{1}{6}; x \right),$$
$$(5.3)$$

where C_1 and C_2 are arbitrary constants. The first term on the right-hand side of Eq. (5.3), by looking at the asymptotic behavior of the hypergeometric function, can be identified with the growing mode, while the second term is a decaying mode. Neglecting the decaying mode, we write δ as

$$\delta(a, \beta) = \frac{a}{1 + \frac{a^3\beta}{1-\beta}} {}_2F_1\left(\frac{1}{3}, 1; \frac{11}{6}; -\frac{a^3\beta}{1-\beta} \right), \qquad (5.4)$$

where the density contrast is normalized such that for $a \ll 1$ we have $\delta = a$, since at high redshifts the CCDM behaves like the Einstein–de Sitter model. In Fig. 1 we show the density contrast δ for dark matter in CCDM as a function of the scale factor for several values of β.

The density contrast for dark matter ($\delta_m = \frac{\delta\rho_m}{\rho_m}$) in a flat ΛCDM model, such that $\Omega_{m0} = 1 - \beta$, can expressed as [30]

$$\delta_m(a, \beta) = a {}_2F_1\left(\frac{1}{3}, 1; \frac{11}{6}; -\frac{a^3\beta}{1-\beta} \right). \qquad (5.5)$$

In Fig. 2 we show the density contrast δ_m as a function of the scale factor for several values of β. By comparing Fig. 1 with Fig. 2, it is clear the density contrast suppression in CCDM, as we increase β, when compared to the ΛCDM case. We remark that this suppression is stronger than the one obtained by the authors in Ref. [31], who have used a different approach, and is in accordance with Fig. 1 of Ref. [17]. But we are then left with the question, what is the origin of this suppression? To answer this question note that
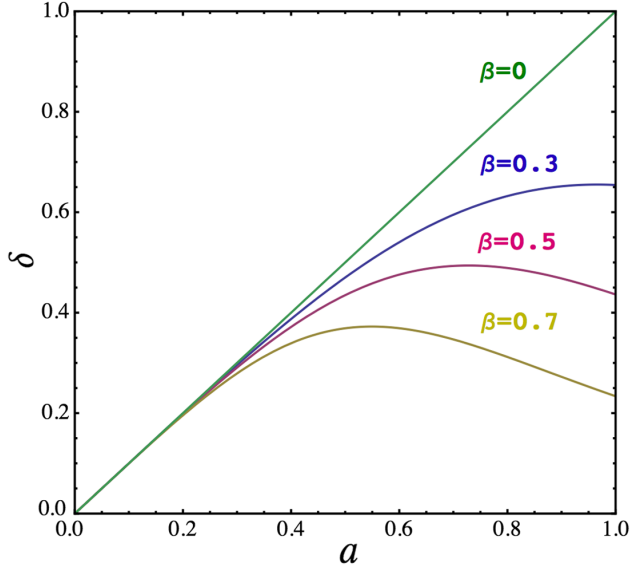
FIG. 1 (color online). The density contrast δ in CCDM as a function of the scale factor a, for different values of β.

$$\frac{\delta}{\delta_m} = \frac{1}{1 + \frac{a^3 \beta}{1-\beta}} = \frac{(1-\beta)a^{-3}}{(1-\beta)a^{-3} + \beta} = \frac{\rho_m}{\rho}, \qquad (5.6)$$

where $\rho_m = \rho_{c0}\Omega_{m0}a^{-3}$ is the energy density of dark matter in flat ΛCDM and ρ, given by Eq. (3.10), is the total CDM energy density in CCDM. Notice that ρ_m is also equal to $\rho_{cl} = \rho_{c0}(1-\beta)a^{-3}$, the CDM *clustered part* in CCDM. From Eq. (5.6), we get that $\delta\rho = \delta\rho_m$ and, therefore, the mentioned suppression in the density contrast appears, when the constant, *nonclustered and created part* of the CCDM energy density starts to become non-negligible. It is important to keep in mind that matter in CCDM clusters
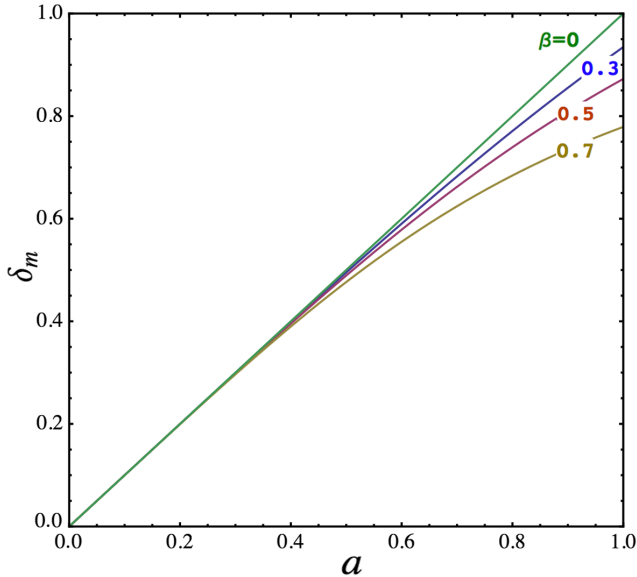

FIG. 2 (color online).   The matter density contrast δ_m in ΛCDM as a function of the scale factor a, for different values of $\beta = 1 - \Omega_{m0}$.

exactly in the same manner as it does in ΛCDM, since the gravitational potential is the same. Furthermore, light also follows the same geodesics and, since we have assumed $c_{\text{eff}}^2 = 0$, we cannot observationally distinguish CCDM from ΛCDM. This property is related to the dark degeneracy [32] and remounts to the discussion on the ΛCDM limit of the generalized Chaplygin gas model [33,34].

The above consideration is particularly relevant when one wants to compare CCDM model predictions with observations that depend on how linear perturbations grow. Consider, for instance, the $f(z)\sigma_8(z)$ test [35], where $f(z)$ is the linear growth rate and $\sigma_8(z)$ is the redshift-dependent root-mean-square mass fluctuation in spheres with radius $8h^{-1}$ Mpc. In CCDM, which of the two quantities, δ or δ_m, should we use in this test? Unlike in Ref. [18], in this work we use $\delta_m = \delta_{\text{cl}} \equiv \frac{\delta\rho}{\rho_{\text{cl}}}$ instead of $\delta = \frac{\delta\rho}{\rho}$. The justification for this choice is based on the fact that for the $f(z)\sigma_8(z)$ test only *clustered matter* is important.

To compare CCDM model predictions with observations we use the redshift-space-distortion based $f(z)\sigma_8(z)$ measurements [35], which are displayed in Table I. The data were obtained by the following surveys: 6dFGRS [36], 2dFGRS [37], WiggleZ [38], SDSS LRG [39], BOSS CMASS [40] and VIPERS [41].

Here $f(z)$ is the linear growth rate given by

$$f(z) \equiv \frac{d \ln \delta_{\text{cl}}}{d \ln a} = -(1+z)\frac{d \ln \delta_{\text{cl}}}{dz}, \qquad (5.7)$$

and

$$\sigma_8(z) = \sigma_{80}\frac{\delta_{\text{cl}}(z)}{\delta_{\text{cl}}(z=0)} \qquad (5.8)$$

is the redshift-dependent root-mean-square mass fluctuation in spheres with radius $8h^{-1}$ Mpc.

For the $f\sigma_8$ test we use the following χ^2 statistics:

$$\chi_{f\sigma_8}^2 = \sum_{i=1}^{10} \frac{[f\sigma_8^{\text{obs}}(z_i) - f(z_i,\beta)\sigma_8(z_i,\sigma_{80},\beta)]^2}{\sigma_{f\sigma_8}^2(z_i)}. \qquad (5.9)$$

TABLE I.   Observational data for redshift-space-distortion based $f(z)\sigma_8(z)$ and the sources from where we have obtained them.

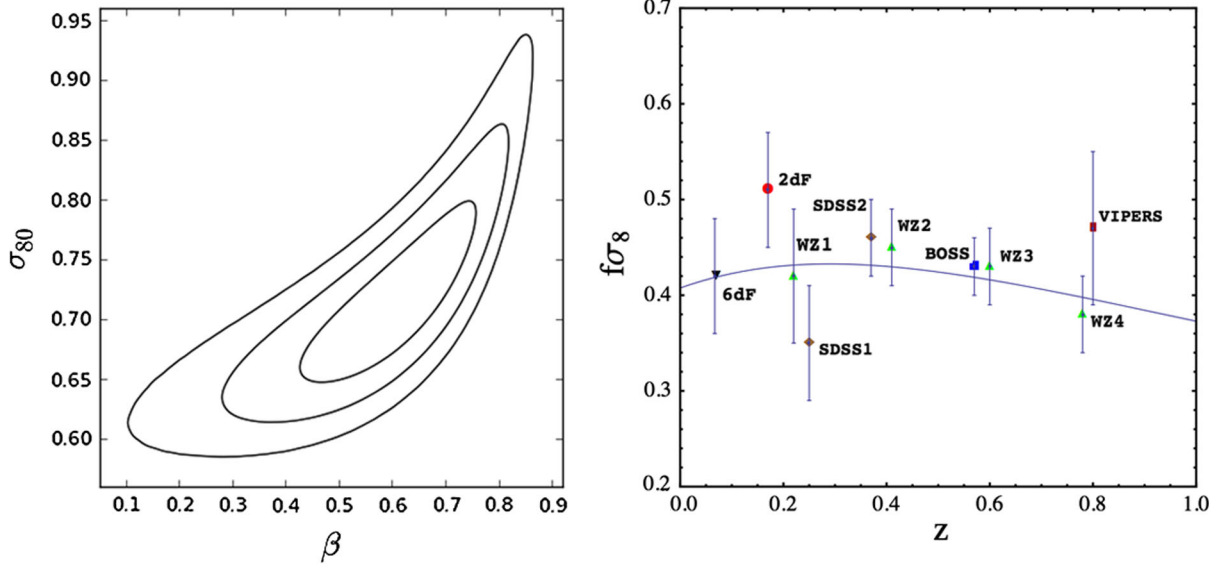| z	$f\sigma_8$	Survey	Ref.
0.07	0.42 ± 0.06	6dFGRS	[36]
0.17	0.51 ± 0.06	2dFGRS	[37]
0.22	0.42 ± 0.07	WiggleZ	[38]
0.25	0.35 ± 0.06	SDSS LRG	[39]
0.37	0.46 ± 0.04	SDSS LRG	[39]
0.41	0.45 ± 0.04	WiggleZ	[38]
0.57	0.43 ± 0.03	BOSS CMASS	[40]
0.60	0.43 ± 0.04	WiggleZ	[38]
0.78	0.38 ± 0.04	WiggleZ	[38]
0.80	0.47 ± 0.08	VIPERS	[41]

FIG. 3 (color online). Results for the $f\sigma_8$ test. Left panel: Confidence regions in the (β, σ_{80}) plane. From the outer to inner curves: Regions of 99.7%, 95.5% and 68.3% C.L. Right panel: The $f\sigma_8$ data points (from Table I) and the best fit model curve $f\sigma_8$, as a function of redshift.
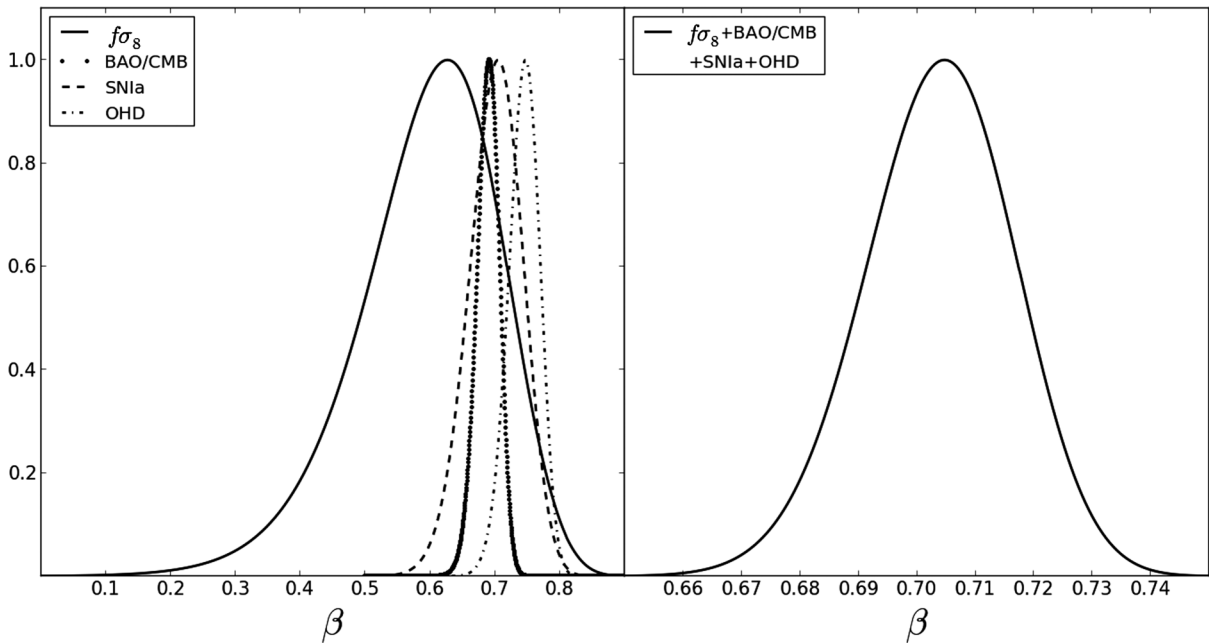

FIG. 4. Left panel: The one-dimensional β PDF for the redshift-space-distortion based $f(z)\sigma_8(z)$ test (solid curve) and for each background test, shown by the dotted, dashed and dash-dotted curves. Right panel: The one-dimensional PDF for β obtained by combining the previous four tests.

To obtain the probability distributions (PDFs) in all the considered tests in this work, the Metropolis-Hasting algorithm has been used [42]. Generally, to obtain the PDFs, 40 chains were generated with 10^6 points for each chain.

The results for the $f\sigma_8$ test are displayed in Fig. 3 (left panel). For this test, we obtain that $\beta = 0.63^{+0.09(0.17)}_{-0.12(0.26)}$, and $\sigma_{80} = 0.70^{+0.05(0.11)}_{-0.04(0.07)}$. In Fig. 3 (right panel) we also show

the $f\sigma_8$ data points we have used, along with their respective error bars, given by the results shown in Table I, and $f\sigma_8$ for the best fit model, as a function of redshift. In Fig. 4, after a flat marginalization with respect to σ_{80}, we show the one-dimensional PDF for β (given by the solid curve).

For the background tests, which involve essentially only distances and, thus, are independent of the perturbation

results, we use the following observables: (i) *The Union 2.1 type Ia supernovae compilation* [43]—this compilation is an update of the Union 2 [44] and include supernovae observed by the Hubble Space Telescope Cluster Survey. This compilation is composed of 580 selected supernovae fitted by the SALT2-1 lightcurve fitter [45]. In our approach we have considered the covariance matrix with systematics errors (available in the site mentioned in [45]), obtaining $\beta = 0.70^{+0.04(0.08)}_{-0.04(0.09)}$. (ii) *The CMB/BAO test*—we followed the procedure described in Sec. 3.2 of Ref. [46], including one new data point from the BOSS survey [47] and new data from WMAP-9 yrs [48]. With this test we get $\beta = 0.69^{+0.02(0.03)}_{-0.02(0.04)}$. (iii) *Measurements of the Hubble parameter at different redshifts*—for this observable we use the same data set and procedure as described in Ref. [49] and we obtain $\beta = 0.75^{+0.02(0.04)}_{-0.02(0.05)}$.

In the left panel of Fig. 4, besides the result for the $f\sigma_8$ test (solid curve), we also display the one-dimensional PDF for β for each background test: CMB/BAO (dotted curve), SNeIa (dashed curve) and OHD (dash-dotted curve). We also display in Fig. 4 (right panel) the β one-dimensional PDF for the combined $f\sigma_8$ plus the three background tests, which gives $\beta = 0.71^{+0.01(0.02)}_{-0.01(0.03)}$.

At this point it is important to make the following remark. If we have considered $c^2_{\rm eff} \neq 0$, instead of Eq. (5.1), we would get from Eq. (4.8) the following differential equation:

$$\delta'' + \frac{3}{2a}(1 - 5w)\delta' + \frac{3}{2a^2}(3w^2 - 8w - 1)\delta + \frac{c^2_{\rm eff}k^2}{H^2a^4}\delta = 0.$$

(5.10)

It can be shown that below the Jeans length, $\lambda_J = \sqrt{|c^2_{\rm eff}|\pi/G\rho}$, the k dependence of the last term in the left-hand side of the above equation can cause strong oscillations if $c^2_{\rm eff} > 0$, or exponential growth if $c^2_{\rm eff} < 0$. Only models with $|c^2_{\rm eff}| \ll 1$ are acceptable at linear scales. An interesting question, but that is beyond the scope of this paper, is to estimate upper limits that will be imposed on $c^2_{\rm eff}$ by future surveys like Euclid.

In our approximation, we have not considered the presence of baryons. If we had taken them into account, still assuming $c^2_{\rm eff} = 0$, it can be shown that their density contrast has the same dependence with redshift as clustered dark matter [given by Eq. (5.5)]. Since the dependence with redshift of both energy densities is also the same, it will not be possible to distinguish the CCDM scenario from ΛCDM, by using measurements of the gas mass fraction in clusters [50], as suggested in Ref. [18].

VI. CONCLUSIONS

In this paper, we have studied the CCDM scenario as a possible explanation for the late-time cosmic acceleration.

We have compared the relativistic and neo-Newtonian differential equations for the density contrast for the CCDM model. Both relativistic and neo-Newtonian cases agree with each other only when the effective sound speed $c^2_{\rm eff}$ is equal to zero. We have argued that even in the more general case, in which $c^2_{\rm eff}$ is considered nonvanishing, but the momentum dependent term in the equation for the density contrast is neglected, a somewhat common consideration assumed by some authors, that this is also not a consistent approximation for the density contrast differential equation. This approximation of neglecting the momentum dependent term is only justifiable in the long-wavelength limit ($k = 0$), which is in turn exactly the case where the Newtonian approximation is not valid. Thus, the neo-Newtonian approach is not consistent with the full relativistic equations when $c^2_{\rm eff} \neq 0$, and the Newtonian approximation is not valid in the long-wavelength limit.

Next, we have compared the CCDM predictions at the perturbative level with those obtained from the ΛCDM. We have used for this comparison redshift-space-distortion observational data. We have shown that the CCDM model produces results for the parameter β (that at the background level plays the role of the cosmological constant density parameter) that are fully consistent with the ones expected from ΛCDM. Independent tests were also carried out at the background level. These tests show that the result for β predicted by the CCDM models is also consistent with the result from ΛCDM. We pointed out that this consistency with the ΛCDM can only be achieved when we properly identify the clustering part ($\delta_{\rm cl}$) of the density contrast, as analyzed and argued in Sec. V. This subtle issue concerning the density contrast may be related to the difficulties with the CCDM model found in previous works. For example, the authors of Ref. [31] have found that CCDM models tend to overestimate peculiar velocities of galaxies in the linear regime. They have also found that because the density contrast today, obtained from the ΛCDM model, tends to be higher than the one predicted by the CCDM, that this would result also in an overestimate of the present density of massive galaxies clusters in these alternative models. This result comes as a consequence of the density contrast suppression as we increase β and shown in the previous section. However, this difficult is no longer present when the clustering part $\delta_{\rm cl}$ is used instead. As pointed out in Sec. V, the matter in CCDM clusters exactly in the same manner as it does in ΛCDM, since the gravitational potential is the same.

In summary, we have shown that CCDM models with $c^2_{\rm eff} = 0$ are degenerate with ΛCDM not only at the background level, but also at the linear perturbative level. We can generally expect this degeneracy to remain at higher order. Although ΛCDM has several conceptual problems (smallness of Λ, cosmic coincidence problem, etc.), the CCDM model does not solve any of them either. Therefore, in the absence of a more fundamental microscopic basis for

the particles creation rate and that originates the specific CCDM model treated here, ΛCDM is a simpler alternative to explain observations (Occam's Razor).

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[1] M. Li, X.-D. Li, S. Wang, and Y. Wang, Commun. Theor. Phys. **56**, 525 (2011); S. Tsujikawa, arXiv:1004.1493.

[2] Y. B. Zeldovich, Pis'ma Zh. Eksp. Teor. Fiz. **12**, 443 (1970).

[3] G. L. Murphy, Phys. Rev. D **8**, 4231 (1973).

[4] B. L. Hu, Phys. Lett. **90A**, 375 (1982).

[5] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[6] L. Diosi, B. Keszthelyi, B. Lukacs, and G. Paal, Acta Phys. Pol. B **15**, 909 (1984); I. Waga, R. C. Falcao, and R. Chanda, Phys. Rev. D **33**, 1839 (1986); J. D. Barrow, Nucl. Phys. **B310**, 743 (1988); R. Maartens, Classical Quantum Gravity **12**, 1455 (1995); W. Zimdahl, D. Pavon, and J. Triginer, Helv. Phys. Acta **69**, 225 (1996); W. Zimdahl and D. Pavon, Phys. Rev. D **61**, 108301 (2000).

[7] J. A. S. Lima, R. Portugal, and I. Waga, Phys. Rev. D **37**, 2755 (1988).

[8] M. Giovannini, Classical Quantum Gravity **22**, 5243 (2005); S. del Campo, R. Herrera, and D. Pavon, Phys. Rev. D **75**, 083518 (2007); C. Bogdanos, A. Dimitriadis, and K. Tamvakis, Phys. Rev. D **75**, 087303 (2007); B. Li and J. D. Barrow, Phys. Rev. D **79**, 103521 (2009); J.-S. Gagnon and J. Lesgourgues, J. Cosmol. Astropart. Phys. 09 (2011) 026; I. Brevik, E. Elizalde, S. Nojiri, and S. D. Odintsov, Phys. Rev. D **84**, 103508 (2011); O. F. Piattella, J. C. Fabris, and W. Zimdahl, J. Cosmol. Astropart. Phys. 05 (2011) 029.

[9] E. Gunzig, J. Geheniau, and I. Prigogine, Nature (London) **330**, 621 (1987); I. Prigogine, J. Geheniau, E. Gunzig, and P. Nardone, Proc. Natl. Acad. Sci. U.S.A. **85**, 7428 (1988); Gen. Relativ. Gravit. **21**, 767 (1989).

[10] M. O. Calvao, J. A. S. Lima, and I. Waga, Phys. Lett. **162A**, 223 (1992).

[11] T. Pacher, J. A. Stein-Schabes, and M. S. Turner, Phys. Rev. D **36**, 1603 (1987).

[12] V. H. Cardenas, Eur. Phys. J. C **72**, 2149 (2012); S. K. Modak and D. Singleton, Phys. Rev. D **86**, 123515 (2012); S. Debnath and A. K. Sanyal, Classical Quantum Gravity **28**, 145015 (2011); G. Steigman, R. C. Santos, and J. A. S. Lima, J. Cosmol. Astropart. Phys. 06 (2009) 033.

[13] M. P. Freaza, R. S. de Souza, and I. Waga, Phys. Rev. D **66**, 103502 (2002).

[14] W. Zimdahl, D. J. Schwarz, A. B. Balakin, and D. Pavon, Phys. Rev. D **64**, 063501 (2001).

[15] J. A. S. Lima, J. F. Jesus, and F. A. Oliveira, J. Cosmol. Astropart. Phys. 11 (2010) 027.

[16] S. Basilakos and J. A. S. Lima, Phys. Rev. D **82**, 023504 (2010).

[17] J. F. Jesus, F. A. Oliveira, S. Basilakos, and J. A. S. Lima, Phys. Rev. D **84**, 063511 (2011).

[18] J. A. S. Lima, S. Basilakos, and F. E. M. Costa, Phys. Rev. D **86**, 103534 (2012).

[19] A. Berera, I. G. Moss, and R. O. Ramos, Rep. Prog. Phys. **72**, 026901 (2009).

[20] M. Bastero-Gil, A. Berera, R. Cerezo, R. O. Ramos, and G. S. Vicente, J. Cosmol. Astropart. Phys. 11 (2012) 042.

[21] J. A. S. Lima, V. Zanchin, and R. H. Brandenberger, Mon. Not. R. Astron. Soc. **291**, L1 (1997).

[22] R. R. R. Reis, Phys. Rev. D **67**, 087301 (2003); **68**, 089901 (E) (2003).

[23] H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78**, 1 (1984).

[24] E. A. Milne, Quarterly J. Math. **5**, 64 (1934); republished in Gen. Relativ. Gravit. **32**, 1939 (2000).

[25] W. H. McCrea and E. A. Milne, Quarterly J. Math. **5**, 73 (1934); republished in Gen. Relativ. Gravit. **32**, 1949 (2000).

[26] W. H. McCrea, Proc. R. Soc. London **206**, 562 (1951).

[27] E. R. Harrison, Ann. Phys. (N.Y.) **35**, 437 (1965).

[28] R. K. Sachs and A. M. Wolfe, Astrophys. J. **147**, 73 (1967); Gen. Relativ. Gravit. **39**, 1929 (2007).

[29] W. Hu, *Summer School Lectures: 2002 Astroparticle Physics and Cosmology*, edited by G. Dvali *et al.* (Abdus Salam ICTP, Trieste, 2003), p. 149; Astrophys. J. **506**, 485 (1998).

[30] V. Silveira and I. Waga, Phys. Rev. D **50**, 4890 (1994).

[31] A. de Roany and J. A. d. F. Pacheco, Gen. Relativ. Gravit. **43**, 61 (2011).

[32] M. Kunz, Phys. Rev. D **80**, 123001 (2009).

[33] J. C. Fabris, S. V. B. Goncalves, and R. S. Ribeiro, Gen. Relativ. Gravit. **36**, 211 (2004).

[34] P. P. Avelino, L. M. G. Beca, J. P. M. de Carvalho, and C. J. A. P. Martins, J. Cosmol. Astropart. Phys. 09 (2003) 002.

[35] Y.-S. Song and W. J. Percival, J. Cosmol. Astropart. Phys. 10 (2009) 004.

[36] F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, G. B. Poole, L. Campbell, Q. Parker, W. Saunders, and F. Watson, Mon. Not. R. Astron. Soc. **423**, 3430 (2012).

[37] W. J. Percival *et al.* (2dFGRS Collaboration), Mon. Not. R. Astron. Soc. **353**, 1201 (2004).

[38] C. Blake *et al.*, Mon. Not. R. Astron. Soc. **415**, 2876 (2011).

[39] L. Samushia, W. J. Percival, and A. Raccanelli, Mon. Not. R. Astron. Soc. **420**, 2102 (2012).

[40] B. A. Reid et al., Mon. Not. R. Astron. Soc. **426**, 2719 (2012).

[41] S. de la Torre et al., Astron. Astrophys. **557**, A54 (2013).

[42] P. C. Gregory, Bayesian Logical Data Analysis for the Physical Sciences: A Comparative Approach with Mathematica Support (Cambridge University Press, Cambridge, UK, 2005); D. Gamerman and H. F. Lopes, Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference (Chapman and Hall/CRC, 2006).

[43] N. Suzuki et al., Astrophys. J. **746**, 85 (2012).

[44] R. Amanullah et al., Astrophys. J. **716**, 712 (2010).

[45] http://supernova.lbl.gov/Union.

[46] R. Giostri, M. V. dos Santos, I. Waga, R. R. R. Reis, M. O. Calvão, and B. L. Lago, J. Cosmol. Astropart. Phys. 03 (2012) 027.

[47] L. Anderson et al., Mon. Not. R. Astron. Soc. **427**, 3435 (2013).

[48] http://lambda.gsfc.nasa.gov/product/map/current/parameters .cfm.

[49] O. Farooq and B. Ratra, Astrophys. J. **766**, L7 (2013).

[50] A. B. Mantz, S. W. Allen, R. G. Morris, D. A. Rapetti, D. E. Applegate, P. L. Kelly, A. von der Linden, and R. W. Schmidt, arXiv:1402.6212 [Mon. Not. R. Astron. Soc. (to be published)].