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Symmetries of mesons after unbreaking of chiral symmetry and their string interpretation

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Using the chirally invariant overlap Dirac operator, we remove its lowest-lying quasizero modes from the valence quark propagators and study evolution of isovector mesons with $J = 1$. At the truncation level, about 50 MeV $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries get restored. However, we observe a degeneracy not only within the chiral and $U(1)_A$ multiplets, but also a degeneracy of all possible chiral multiplets, i.e., the observed quantum levels have a symmetry larger than $U(2)_L \times U(2)_R$ and their energy does not depend on the spin orientation of quarks and their parities. We offer a possible interpretation of these energy levels as the quantum levels of the dynamical QCD string. The structure of the radial $J = 1$ spectrum is compatible with $E = (n_r + 1)\hbar\omega$ with $\hbar\omega = 900 \pm 70$ MeV.

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I. INTRODUCTION

A consistent and systematic picture of hadrons made of light quarks is missing. There is a general understanding that both confinement and spontaneous breaking of chiral symmetry (SBχS) are important for hadronic mass generation. A large degeneracy is seen in the highly excited mesons [1,2], which is, however, absent in the observed spectrum of mesons with masses below 1.7–1.8 GeV. The physics of the low-lying hadrons should be affected by the SBχS that might obscure the primary confinement picture. We want to disentangle the primary physics that is responsible for a genesis of hadrons from the SBχS.

In order to address this issue we artificially remove the chiral symmetry breaking dynamics from the valence quark propagators keeping at the same time the gluonic gauge configurations intact [3–5]. It has been known for a long time that the quark condensate of the vacuum is directly related to the density of the close-to-zero modes of the Dirac operator [6]. The quark propagator can be written in terms of projectors onto the eigenmodes of the Dirac operator. We subtract from the full propagator its lowest-lying modes that represent only a tiny part of the full amount of modes,

$$S_{RD(k)} = S_{\mathrm{Full}} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|, \tag{1}$$

where λ_i and $|\lambda_i\rangle$ are the eigenvalues and the corresponding eigenvectors of the Dirac operator. This way we artificially restore ("unbreak") the chiral symmetry.

In Ref. [4] we have noticed that after unbreaking the chiral symmetry a degeneracy of hadrons develops that is higher than $SU(2)_L \times SU(2)_R$. Such a degeneracy should reflect the underlying symmetry and dynamics. Still, the quality of our data that support a degeneracy was not too high.

A reason for lacking a high quality was that in Refs. [3–5] a not chirally symmetric lattice Dirac operator of the Wilson type was used. An eigenvalue decomposition in terms of the Hermitian Dirac operator $\gamma_5 D$ was adopted. It causes ambiguity as there is no direct bijective relation between the low-lying modes of $\gamma_5 D$ and of the Dirac operator D; exact zero modes could not be exactly identified. In the present work we use the manifestly chirally invariant overlap Dirac operator [7] where such problems are absent.

From the truncated quark propagators, we construct hadron propagators. When we observe an exponential decay signal of the corresponding correlation function, we interpret it as a physical state and extract its mass. Our task is to see what will happen with hadrons and their masses when we subtract the lowest-lying modes of the Dirac operator.

II. LATTICE TECHNOLOGY

We adopt 100 gauge field configurations generated with $n_f = 2$ dynamical overlap fermions on a $16^3 \times 32$ lattice with the spacing $a \sim 0.12$ fm [8,9]. The pion mass in this ensemble is $M_\pi = 289(2)$ MeV [10]. For each hadron we use a set of up to eight interpolators with different Dirac structures (see Table I) and different exponential smearings of sources and sinks with several smearing widths. We determine correlators for all polarizations. The quark propagators were generously provided by the JLQCD collaboration and were computed by the combining of the exact 100 low modes with stochastic estimates for the higher modes [9].

For the analysis of the cross-correlation matrices we employ the variational method and solve the generalized

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TABLE I. Interpolators for the $\rho(1^{--})$, $a_1(1^{++})$ and $b_1(1^{+-})$ mesons ($i, j = 1, 2, 3$) with seven different smearing widths at the sink/source.

Channel	Interpolator types	Number of interpolators
ρ	$\bar{q}\gamma_i\vec{\tau}q$, $\bar{q}\gamma_i\gamma_t\vec{\tau}q$	8
a_1	$\bar{q}\gamma_i\gamma_5\vec{\tau}q$	5
b_1	$\bar{q}\gamma_i\gamma_j\vec{\tau}q$	6

TABLE II. Masses of the ρ, ρ', a_1, b_1 mesons extracted at the specific truncation levels k.

	$k = 10$		$k = 20$	
am	$n_r = 0$	$n_r = 1$	$n_r = 0$	$n_r = 1$
ρ	0.573(10)	1.096(57)	0.625(7)	1.107(58)
ρ'	0.589(13)	1.108(48)	0.614(8)	1.139(48)
a_1	0.576(10)	1.051(62)	0.615(8)	1.062(61)
b_1	0.568(14)	1.077(51)	0.607(8)	1.106(51)

eigenvalue problem [11]. In the figures we show effective energy values (nearest neighbor slopes) to demonstrate plateau behavior. The horizontal bars on the effective energy plots indicate the fit range. The one-exponential fits are to the eigenvalues of the correlation matrices in the range shown in the effective mass plots. The error bars are determined by the single-elimination jackknife method. Table II gives the resulting numbers for two stages of truncation, $k = 10, 20$.

III. OBSERVATIONS

We have studied the ground state and the excited states of all possible $\bar{q}q$ isovector $J = 1$ mesons, i.e., $\rho(1^{--})$, $a_1(1^{++})$, $b_1(1^{+-})$. Typical results for the eigenvalues of the correlation matrices and the effective mass plots are shown in Fig. 1. Upon removal of the lowest eigenmodes of the Dirac operator from the valence quark propagators, we observe an improving signal for clean exponential decay. The evolution of masses of the ground and excited states is shown in Fig. 2. Around a truncation energy of 40–65 MeV (approximately 10 eigenmodes in our ensemble), an onset of a degeneracy of four states, ρ, ρ', a_1, b_1, as well as a degeneracy of their excited states is seen [12]. The degeneracy indicates a yet unknown symmetry.

Before discussing the symmetry issue, we want to exclude a possibility that the observed energy levels are levels of a free quark-antiquark pair in a box. For a system of a free quark and antiquark, the lowest level would represent the energy of the quark and antiquark with the $p = 0$ momentum, which is compatible with the $S-$ and incompatible with the P-wave of relative motion. The next level would correspond to $p = 2\pi/L$ (L is the spatial box size) for each quark and is compatible with both S- and P-waves. Consequently, a system of a free quark and antiquark in a box cannot produce degenerate ground states of opposite parities. We do observe levels of a nontrivially bound (confined) quark-antiquark system.

IV. SYMMETRIES

Which symmetries does the observed degeneracy represent? All possible $SU(2)_L \times SU(2)_R$ multiplets of the $J = 1$ mesons are listed in Table III [1,2].

When chiral symmetry is restored but the states still exist, we expect that all mesons will fall into multiplets of the chiral group and within each independent chiral multiplet the mesons will be degenerate. In the chirally symmetric world there must be two independent ρ-mesons. One of them is a member of the $(0,1) \oplus (1,0)$ multiplet and can be created only by an operator with the same chiral structure, e.g., by the vector current, and that should be degenerate with its chiral partner a_1. Another ρ-meson forms together with its chiral partner h_1 the $(1/2, 1/2)_b$ representation and is coupled only to an operator of the type $\bar{q}\sigma^{0i}\vec{\tau}q$.

A degeneracy of the $(0,1) \oplus (1,0)$ ρ-meson with the a_1-meson is a clear signal of the chiral $SU(2)_L \times SU(2)_R$ restoration. We have not yet studied the isoscalar mesons, since they contain disconnected graphs and their observation represents a challenge for the simulation. An observation of the degeneracy within one of the multiplets is sufficient to confirm chiral restoration, however. Consequently, a similar degeneracy should be seen in other chiral pairs.

The $U(1)_A$ symmetry connects the $(1/2, 1/2)_b$ ρ-meson with the b_1-meson [1,2]. At a truncation energy around 50 MeV the onset of this degeneracy is also seen. We conclude that simultaneously both $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries get restored [13]. Both chiral and $U(1)_A$ breaking are produced by the same low-lying modes of the Dirac operator, which is consistent with the instanton-induced mechanism [14–16].

The restored $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry requires a degeneracy of four mesons that belong to $(1/2, 1/2)_a$ and $(1/2, 1/2)_b$ chiral multiplets [1,2]; see Table III. This symmetry does not require, however, a degeneracy of these four states with other mesons, in particular with a_1 and its chiral partner ρ. We clearly see the latter degeneracy. This implies that there is some higher symmetry, that includes $U(2)_L \times U(2)_R$ as a subgroup. While a degeneracy of all isovector mesons is sufficient to claim this, this higher symmetry also requires a degeneracy of all mesons in Table III, which is a task for future studies.

One could ask whether this situation is similar or not to the high temperature deconfining regime, where chiral and $U(1)_A$ symmetries are also restored. In the latter case correlators of the chiral partners also become identical. This

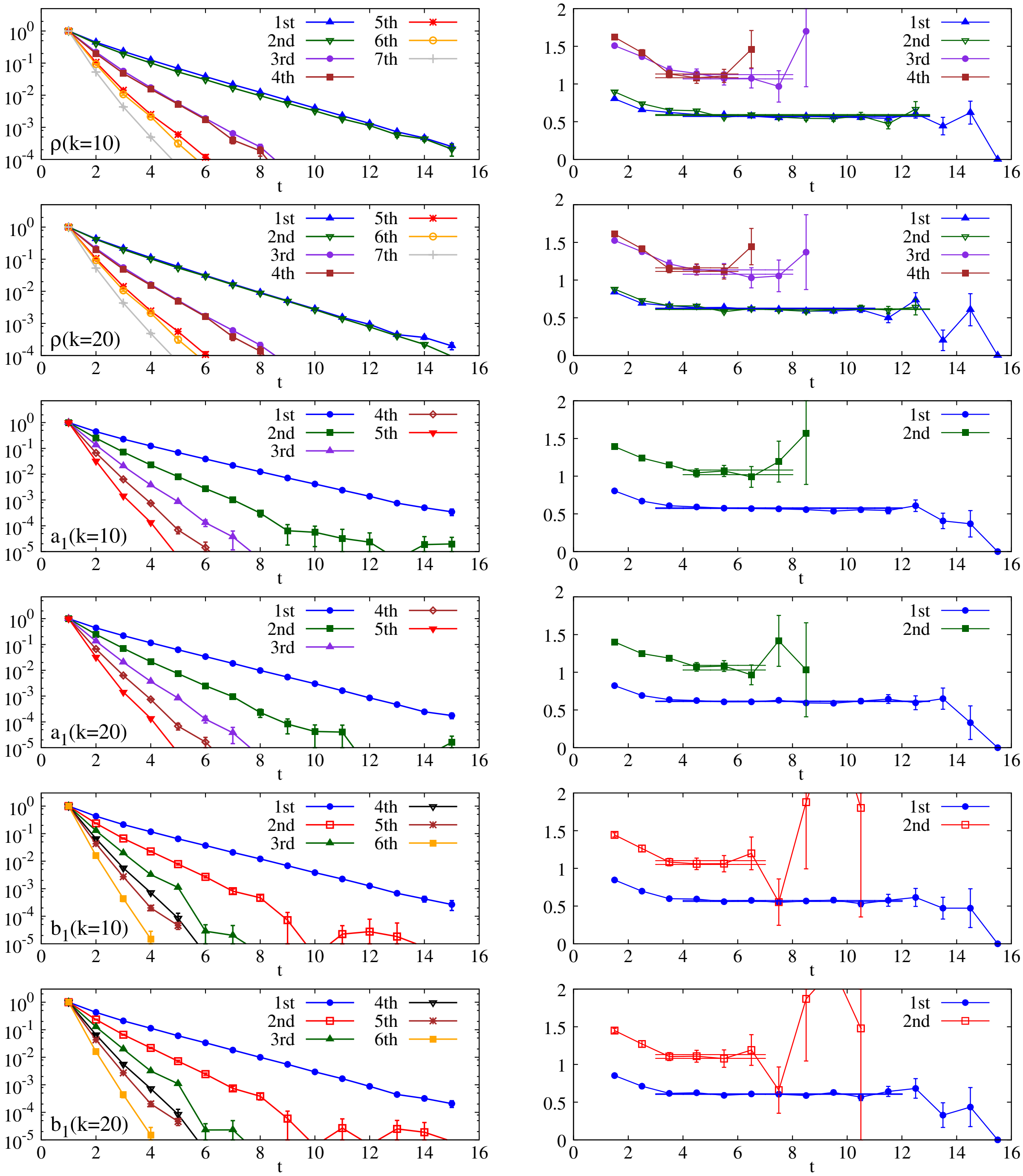

FIG. 1 (color online). Upper panel: ρ-meson ($J^{PC} = 1^{--}$); middle part: a_1-meson ($J^{PC} = 1^{++}$); lower panel: b_1-meson ($J^{PC} = 1^{+-}$). We show eigenvalues of the correlation matrix (the left-hand plots) and effective masses (the right-hand plots) for $k = 10$ and $k = 20$ for each meson, respectively. Please note the degeneracy of the levels of the ρ-meson is different from the untruncated case.

identity does not imply, however, that there is a complete spectrum of the quark-antiquark bound ground and excited states. The high temperature affects the gluodynamics. In our case, in contrast, the gluodynamics are kept intact. The valence quarks have no effect on the Polyakov and Wilson loops and on the potential between the static infinitely heavy quarks.

V. OUR INTERPRETATION. THE QCD DYNAMICAL STRING

It was suspected for a long time and illustrated within the Abelian models that the colored quarks are connected by the color-electric flux tube, which, if long enough, can be approximated by a string. This picture has obtained some support from the lattice simulations with the static quark

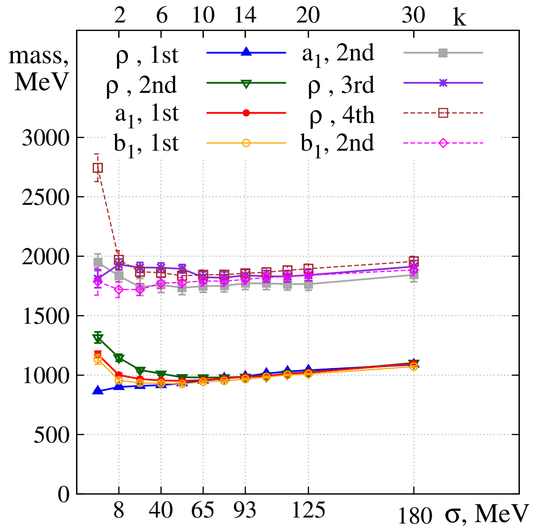

FIG. 2 (color online). Evolution of hadron masses under the low-mode truncation. Both the number k of the removed lowest eigenmodes as well as the corresponding energy gap σ are given.

sources, where a flux tube as well as the linear potential have been observed (for review and references, see [17]). That string is nondynamical (in the sense that its ends are fixed), however, and can only illustrate to some extent physics of hadrons made of heavy quarks. In the light quark sector the fast motion of quarks at the ends of a possible string, the respective chiral symmetry as well as its dynamical breaking, should be of great importance. In this case it is even *a priori* unclear whether the stringy picture has something to do with reality. There is no consistent theory of the QCD string with quarks at the ends.

The energy of the string with unbroken chiral symmetry is stored in the gluonic color-electric flux tube that is created by the color charges of quarks at the ends of the string. This energy should not depend on the orientation of the quark spins because the quark spins can interact only with the color-magnetic fields [18]. Consequently, one expects that the energy of the system at a given J should be the same for all possible orientations of the quark spins and their parities.

All eight mesons in Table III can be combined into a reducible chiral representation [19]:

TABLE III. The complete set of $q\bar{q}$ $J=1$ states classified according to the chiral basis. The symbol $\leftrightarrow$ indicates the states belonging to the same representation R of $SU(2)_L \times SU(2)_R$ that must be degenerate in the $SU(2)_L \times SU(2)_R$ symmetric world.

R	Mesons
(0,0)	$\omega(I=0,1^{--}) \leftrightarrow f_1(I=0,1^{++})$
$(1/2,1/2)_a$	$\omega(I=0,1^{--}) \leftrightarrow b_1(I=1,1^{+-})$
$(1/2,1/2)_b$	$h_1(I=0,1^{+-}) \leftrightarrow \rho(I=1,1^{--})$
$(0,1)\oplus(1,0)$	$a_1(I=1,1^{++}) \leftrightarrow \rho(I=1,1^{--})$

$$[(0,1/2)+(1/2,0)]\times[(0,1/2)+(1/2,0)]$$
$$=(0,0)+(1/2,1/2)_a+(1/2,1/2)_b+[(0,1)+(1,0)]. \tag{2}$$

They exhaust all possible chiralities of quarks and antiquarks, i.e., their spin orientations, as well as possible spatial and charge parities for nonexotic mesons. The observed degeneracy of all these mesons suggests that we see the energy levels of the dynamical QCD string that connects the ultrarelativistic quark and antiquark with the total spin $J=1$.

In Fig. 2 we show the two lowest energy levels. We actually also get a signal of the third level, as can be seen from the eigenvalues of the correlation matrices on Fig. 1. The quality of the third level is not sufficient to show this level in Fig. 2.

The observed radial levels at the truncation energy, at which we see the onset of the symmetry, are approximately equidistant and are compatible with the simple relation

$$E_{n_r}=(n_r+1)\hbar\omega, n_r=0,1,\ldots \tag{3}$$

The extracted value of the fundamental string excitation quantum at the truncation energy 65 MeV ($k=10$) amounts to $\hbar\omega=(900\pm70)$ MeV [20]. In order to include the rotational levels in this quantization law, one should study mesons with $J\neq1$, which is planned.

VI. IS THIS STRING OF THE NAMBU-GOTO TYPE?

A principal result for the classical Nambu-Goto open string is that the energy of the string is described in terms of its orbital angular momentum L (which is a conserved quantum number): $M^2\sim L$. For the string with chiral quarks at the ends, the orbital angular momentum L of the relative motion of the quark and antiquark is not a conserved quantum number, though [21]. For instance, two independent ρ-mesons at the same energy level are represented by the mutually orthogonal fixed superpositions of the S- and D-waves.

$$|(0,1)+(1,0);11^{--}\rangle=\sqrt{\frac{2}{3}}|1;{}^3S_1\rangle+\sqrt{\frac{1}{3}}|1;{}^3D_1\rangle,$$
$$|(1/2,1/2)_b;11^{--}\rangle=\sqrt{\frac{1}{3}}|1;{}^3S_1\rangle-\sqrt{\frac{2}{3}}|1;{}^3D_1\rangle.$$

Hence, a description of a dynamical string with chiral quarks at the ends is impossible in terms of a fixed L. The total angular momentum J is a conserved quantum number, of course, as required by Poincare-invariance.

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