Semileptonic $B^- \to f_0(1710,1500,1370)e^-\bar{\nu}_e$ decays

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We study the semileptonic decays of $B^- \to f_0(1710, 1500, 1370)e^-\bar{\nu}_e$, in which the three f_0 states mix with glueball, $\bar{s}s$, and $(\bar{u}u + \bar{d}d)/\sqrt{2}$ states, respectively. By averaging the mixings fitted in the literature, we find that the branching ratios of $B^- \to f_0 e^-\bar{\nu}_e$ are $O(10^{-6})$, $O(10^{-6})$, and $O(10^{-5})$, respectively, which can be simultaneously observed in experiments at B factories. The large predicted branching rate for $B^- \to f_0(1370)e^-\bar{\nu}_e$ would provide a clean mode to directly observe the $f_0(1370)$ state.

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It is believed that some exotic states with nonstandard internal structures, such as the four-quark and two-gluon bound states [1], have been seen already. For example, the isovector $a_0(980)$ and the isodoublet $K_0^*(800)$ can be identified as $a_0(980) \equiv \bar{d}u\bar{s}s$ and $K_0^*(800) \equiv \bar{s}u(\bar{u}u + \bar{d}d)$ in the tetraquark (four-quark) picture, instead of $a_0(980) \equiv \bar{d}u$ and $K_0^*(800) \equiv \bar{s}u$ in the standard $\bar{q}q$ picture. In addition, since only two of the three isoscalars of $f_0(1710)$, $f_0(1500)$, and $f_0(1370)$ can be simultaneously fitted into the nonet, a glueball (G) as a multigluon bound state can be a solution. Note that the lattice QCD (LQCD) calculations predict that the lightest glueball of $J^{PC} = 0^{++}$ is composed of two gluons with the mass in the range of 1.5–1.7 GeV [2,3]. These three f_0 states clearly mix with the glueball and quark-antiquark states.

Although $f_0(1710)$ or $f_0(1500)$ is taken to be mainly a glueball state [4–9], the radiative $J/\psi \to f_0(1370)\gamma$ decay via a gluon-rich process has not been observed yet, whereas the other two decays of $J/\psi \to f_0(1710, 1500)\gamma$ are clearly established [10]. This can be understood from the destructive G-$\bar{q}q$ interference [4,7] or simply the weak couplings [11] for the resonant $f_0(1370) \to K\bar{K}$ $(\pi\pi)$ in $J/\psi \to K\bar{K}\gamma$ $(J/\psi \to \pi\pi\gamma)$. Nonetheless, it accords with the doubt of having seen the $f_0(1370)$ state with direct observations [12,13]. We note that a resonant scalar state, once identified as $f_0(1370)$ [14,15] in the $\pi\pi$ spectrum of $\bar{B}_s^0 \to J/\psi\pi^+\pi^-$, was reexamined to be more like $f_0(1500)$ [13], while only $f_0(1500)$ is found [16] in the analysis of $B^- \to K^+K^-K^-$. In addition, in the $\pi\pi$ spectrum of $D_s^+ \to \pi^+\pi^-\pi^+$, no peak around 1370 MeV is found in the recent investigation [17] and it is not conclusive for $f_0(1370)$ in the $\pi\pi$ spectrum of $J/\psi \to \phi(1020)\pi\pi$ [18] either. As a result, a concrete direct measurement for $f_0(1370)$ is urgently needed.

In this study, we propose to use the semileptonic $B^- \to f_0(1370)e^-\bar{\nu}_e$ decay, arising from $b \to u\ell\bar{\nu}_\ell$ at quark level, to search for $f_0(1370)$. It is interesting to note that, in contrast with the partial observations in the aforementioned weak decays, all three $B^- \to f_0 e^-\bar{\nu}_e$ decays can be measured, providing a new way to simultaneously examine $f_0(1710)$, $f_0(1500)$, and $f_0(1370)$. According to the measured branching ratios of $B \to M(\bar{n}n)e^-\bar{\nu}_e$ [10] with $M(\bar{n}n) = \pi^0, \eta^{(\prime)}, \omega, \rho$ and $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$, $\mathcal{B}(B^- \to f_0 e^-\bar{\nu}_e)$ are expected to be of order 10^{-6}–10^{-5}, which are accessible to the B factories. In this report, we average the mixings fitted in the literature [6–9] for the three f_0 states to explicitly evaluate the branching ratios of $B^- \to f_0 e^-\bar{\nu}_e$.

We start with the effective Hamiltonian at quark level, given by

$$\mathcal{H}(b \to u\ell\bar{\nu}) = \frac{G_F V_{ub}}{\sqrt{2}} \bar{u}\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma^\mu(1-\gamma_5)\nu, \quad (1)$$

for the $b \to u$ transition with the recoiled W boson to the lepton pair $\ell\bar{\nu}$. The amplitude for $B^- \to f_0^i e^-\bar{\nu}_e$ can be simply factorized as

$$\mathcal{A}(B^- \to f_0^i e^-\bar{\nu}_e) = \frac{G_F V_{ub}}{\sqrt{2}} \alpha_3^i \langle \bar{n}n|\bar{u}\gamma_\mu(1-\gamma_5)b|B^-\rangle \times \bar{e}\gamma^\mu(1-\gamma_5)\nu_e, \quad (2)$$

where α_3^i is the coefficient of the mixing state of $\bar{n}n$ defined in Eq. (6). The matrix element for the $B^- \to \bar{n}n$ transition is given by

$$\langle \bar{n}n|\bar{u}\gamma_\mu(1-\gamma_5)b|B^-\rangle = i\left[\left(p_\mu - \frac{m_B^2 - m_{f(\bar{n}n)}^2}{q^2} q_\mu\right)F_1(q^2) + \frac{m_B^2 - m_{f(\bar{n}n)}^2}{q^2} q_\mu F_0(q^2)\right], \quad (3)$$

with $p = p_B - q$ and $q = p_B - p_{\bar{n}n} = p_e + p_{\bar{\nu}_e}$, where the momentum dependences for the form factors $F_{0,1}$ are parametrized in the form of

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)}. \quad (4)$$

Subsequently, the differential decay width is given by

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$$d\Gamma = \frac{1}{(2\pi)^3}\frac{|\bar{\mathcal{A}}|^2}{32M_B^3}dm_{12}^2 dm_{23}^2, \quad (5)$$

with $m_{12} = p_{f_0} + p_e$, $m_{23} = p_e + p_{\bar{\nu}_e}$, and $|\bar{\mathcal{A}}|^2$ standing for the amplitude squared derived from Eqs. (2), (3), and (4) with the bar denoting the summation over lepton spins.

In our numerical analysis, we adopt the PDG [10] to have $|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3}$ and $(m_{f_0(1710)}, m_{f_0(1500)}, m_{f_0(1370)}) = (1720, 1505, 1350)$ MeV, while $m_{\bar{n}n} = 1470$ MeV is from Refs. [6,7]. The parameters for $F_{0,1}$ shown in Table I are calculated in the light-front QCD approach [19], where we have used the constituent quark masses of $m_{u,d} = 0.26 \pm 0.04$ and $m_b = 4.62^{+0.18}_{-0.12}$ GeV and the meson decay constants of f_B and f_π from the PDG [10]. We note that our results in Table I are in agreement with those in the perturbative QCD approach [20].

Now, we define

$$|f_0^i\rangle = \alpha_j^i |f_j\rangle, \quad (6)$$

TABLE I. The form factors of $B^- \to \bar{n}n$ at $q^2 = 0$.

$F_{0,1}$	$F(0)$	a	b
F_0	0.20 ± 0.03	$0.65^{+0.15}_{-0.05}$	$0.29^{+0.17}_{-0.01}$
F_1	0.20 ± 0.03	$1.32^{+0.08}_{-0.02}$	$0.64^{+0.11}_{-0.08}$

where f_0^i ($i = 1, 2, 3$) stand for $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$, f_j ($j = 1, 2, 3$) represent G, $\bar{s}s$, and $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$, and α_j^i ($i, j = 1, 2, 3$) are the mixings of a $3 \otimes 3$ matrix [4–7].

To obtain the mixing matrix (α_j^i), there are two scenarios (I and II) in the literature. In scenario I, $f_0(1500)$ is considered to be the glueball candidate, such that $f_0(1500)$ with $m_{f_0(1500)} = 1505$ MeV has a large mixing to G, to match with the glueball state with $m_G \simeq 1500$ MeV in the quenched LQCD calculation [2]. Here, we take the mixing matrices of $(\alpha_j^i)_a$ in scenario I to be

$$(\alpha_j^i)_I = \begin{pmatrix} 0.36 & 0.93 & 0.09 \\ -0.84 & 0.35 & -0.41 \\ 0.40 & -0.07 & -0.91 \end{pmatrix}, \quad \begin{pmatrix} -0.05 & 0.95 & -0.29 \\ 0.80 & -0.14 & -0.59 \\ 0.60 & 0.26 & 0.75 \end{pmatrix}, \quad \begin{pmatrix} -0.83 & -0.45 & -0.33 \\ -0.40 & 0.89 & -0.22 \\ -0.39 & 0.05 & 0.92 \end{pmatrix}, \quad (7)$$

where $a = 1, 2, 3$ correspond to the three fittings in Refs. [6,8,9], respectively. We remark that although $|\alpha_1^2|$ [9] in the third matrix of Eq. (7) related to G is small, it is still reasonable to have the $a = 3$ case in scenario I as m_G be fitted to be 1580 MeV, which is close to the quenched LQCD value. We note that the signs of α_j^i vary due to the different theoretical inputs. In this study, we shall take the absolute values $|\alpha_j^i|$ to represent the magnitudes of the mixings and average them in terms of

$$\bar{\alpha}_j^i = \frac{\Sigma_{a=1}^3 |\alpha_j^i|_a}{3}, \qquad \Delta\bar{\alpha}_j^i = \sqrt{\frac{\Sigma_{a=1}^3 (\bar{\alpha}_j^i - |\alpha_j^i|_a)^2}{3}}, \quad (8)$$

where $\bar{\alpha}_j^i$ is the central value of each averaged absolute mixing and $\Delta\bar{\alpha}_j^i$ reflects the deviation among the fittings. As a result, from Eq. (7) we obtain

$$(\bar{\alpha}_j^i)_I = \begin{pmatrix} 0.41 \pm 0.32 & 0.78 \pm 0.23 & 0.24 \pm 0.10 \\ 0.68 \pm 0.20 & 0.46 \pm 0.32 & 0.41 \pm 0.15 \\ 0.46 \pm 0.10 & 0.13 \pm 0.10 & 0.86 \pm 0.08 \end{pmatrix}. \quad (9)$$

Scenario II prefers $f_0(1710)$ instead of $f_0(1500)$ as a glueball state with $m_G \simeq 1700$ MeV, also predicted by the unquenched LQCD [3]. In this scenario, the fitted values for α_j^i in Refs. [7–9] are given by

$$(\alpha_j^i)_{II} = \begin{pmatrix} 0.93 & 0.18 & 0.32 \\ 0.03 & 0.84 & -0.54 \\ -0.36 & 0.51 & 0.78 \end{pmatrix}, \quad \begin{pmatrix} -0.96 & 0.17 & -0.23 \\ 0 & -0.82 & 0.57 \\ 0.29 & 0.55 & 0.79 \end{pmatrix}, \quad \begin{pmatrix} -0.99 & -0.05 & -0.04 \\ -0.03 & 0.90 & -0.42 \\ -0.05 & 0.41 & 0.90 \end{pmatrix}, \quad (10)$$

respectively. Note that the three $|\alpha_1^1|$ values in Eq. (10) are consistently bigger than 0.9, indicating $f_0(1710)$ to be mainly G. Similarly, from Eq. (10) we get

$$(\bar{\alpha}^i_j)_{II} = \begin{pmatrix} 0.96 \pm 0.02 & 0.13 \pm 0.06 & 0.20 \pm 0.12 \\ 0.02 \pm 0.01 & 0.85 \pm 0.03 & 0.51 \pm 0.06 \\ 0.23 \pm 0.13 & 0.49 \pm 0.06 & 0.82 \pm 0.05 \end{pmatrix}. \tag{11}$$

Consequently, from the two scenarios in Eqs. (9) and (11), the branching ratios of $B^- \to f_0(1710, 1500, 1370)e^-\bar{\nu}_e$ can be calculated based on Eqs. (2)–(5). Our results are shown in Table II, where the uncertainties come from $|\alpha^i_3|$, $|V_{ub}|$, and $F_{0,1}$, respectively.

With the mixing matrix elements in Eqs. (9) and (11), we are able to specifically study the productions of the three f_0 states before the measurements. For example, we find that $\mathcal{B}(B^- \to f_0(1370)e^-\bar{\nu}_e)$ is about $2.57(2.33) \times 10^{-5}$ in scenario I (II). Besides, $\mathcal{B}(B^- \to f_0(1710)e^-\bar{\nu}_e)$ and $\mathcal{B}(B^- \to f_0(1500)e^-\bar{\nu}_e)$ in the two scenarios are predicted to be of order 10^{-6}. Since $\mathcal{B}(B^- \to Ge^-\bar{\nu}_e)$ has been demonstrated to be as small as 1.1×10^{-6} [21], where the magnitude of the uncertainty is as large as the central value, its contribution to $\mathcal{B}(B^- \to f_0e^-\bar{\nu}_e)$ can be negligible. The only exception is that, due to the largest $|\alpha^1_1| = 0.96$ for scenario II in Eq. (11), $\mathcal{B}(B^- \to f_0(1710)e^-\bar{\nu}_e) \simeq 1.0 \times 10^{-6}$ from the $B \to G$ transition, which is compatible to $\mathcal{B}(B^- \to f_0(1710)e^-\bar{\nu}_e) \simeq 1.4 \times 10^{-6}$ from the $B \to \bar{n}n$ transition. With the branching ratios to be of order 10^{-6}–10^{-5}, it is possible to measure the three modes simultaneously. This will improve the knowledge of the mixing matrix as well as the glueball.

In sum, by averaging the mixings of $|\alpha^i_j|$, fitted from the most recent studies in the literature, we have found that $\mathcal{B}(B^- \to f_0(1370)e^-\bar{\nu}_e)$ are around 2.6 and 2.3×10^{-5} in scenarios I and II, respectively. This decay mode is promising to be measured in the B factories, which would resolve the doubt for the existence of $f_0(1370)$. In addition, we have also shown that $\mathcal{B}(B^- \to f_0(1710)e^-\bar{\nu}_e)$ and $\mathcal{B}(B^- \to f_0(1500)e^-\bar{\nu}_e)$ are of order 10^{-6}. The measurements of these three modes will provide us with some useful information about the three f_0 states.

TABLE II. The branching ratios of $B^- \to f_0(1710, 1500, 1370)e^-\bar{\nu}_e$ decays with the uncertainties corresponding to those in $|\alpha^i_3|$, $|V_{ub}|$, and $F_{0,1}$, respectively.

Mode	Scenario I
$f_0(1710)$	$(1.96^{+1.97+0.49+0.65}_{-1.29-0.43-0.52}) \times 10^{-6}$
$f_0(1500)$	$(5.89^{+5.09+1.47+1.81}_{-3.52-1.31-1.58}) \times 10^{-6}$
$f_0(1370)$	$(2.57^{+0.50+0.64+0.83}_{-0.45-0.57-0.67}) \times 10^{-5}$
Mode	**Scenario II**
$f_0(1710)$	$(1.36^{+2.12+0.34+0.47}_{-1.14-0.30-0.33}) \times 10^{-6}$
$f_0(1500)$	$(9.11^{+2.27+2.28+2.79}_{-2.02-2.02-2.44}) \times 10^{-6}$
$f_0(1370)$	$(2.33^{+0.29+0.58+0.75}_{-0.28-0.52-0.60}) \times 10^{-5}$

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