Subtleties in the BaBar measurement of time-reversal violation

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A first measurement of time-reversal (T) asymmetries that are not also CP asymmetries has been recently achieved by the *BaBar* collaboration. We analyze the measured asymmetries in the presence of direct CP violation, CPT violation, wrong strangeness decays and wrong sign semileptonic decays. We note that the commonly used $S_{\psi K}$ and $C_{\psi K}$ parameters are CP-odd, but have a T-odd CPT-even part and a T-even CPT-odd part. We introduce parameters that have well-defined transformation properties under CP, T and CPT. We identify contributions to the measured asymmetries that are T conserving. We explain why, in order that the measured asymmetries would be purely odd under time-reversal, there is no need to assume the absence of direct CP violation. Instead, one needs to assume (i) the absence of CPT violation in strangeness changing decays, and (ii) the absence of wrong sign decays.

I. INTRODUCTION

The *BABAR* collaboration has recently announced the first direct observation of time-reversal violation in the neutral B meson system [1]. The basic idea is to compare the time-dependent rates of two processes that differ by exchange of initial and final states. The measurement makes use of the Einstein-Podolsky-Rosen (EPR) effect in the entangled B mesons produced in $\Upsilon(4S)$ decays [2–6]. For example, one rate, $\Gamma_{(\psi K_L)_\perp, \ell^- X}$, involves the decay of one of the neutral B's into a ψK_L state, and, after time t, the decay of the other B into $\ell^- X$. The other rate, $\Gamma_{(\ell^+ X)_\perp, \psi K_S}$, involves the decay of one of the neutral B's into $\ell^+ X$, and, after time t, the decay of the other B into ψK_S. Under certain assumptions, to be spelled out below, this is a comparison between the rates of $B_- \to \bar{B}^0$ and $\bar{B}^0 \to B_-$, where $\bar{B}^0$ has a well-defined flavor content $(b\bar{d})$ and B_- is a CP-odd state. The asymmetry is defined as follows:

$$A_T \equiv \frac{\Gamma_{(\psi K_L)_\perp, \ell^- X} - \Gamma_{(\ell^+ X)_\perp, \psi K_S}}{\Gamma_{(\psi K_L)_\perp, \ell^- X} + \Gamma_{(\ell^+ X)_\perp, \psi K_S}}. \quad (1)$$

BABAR take the time dependence of the asymmetry to be of the form

$$A_T = \frac{\Delta S_T^+}{2} \sin(\Delta m t) + \frac{\Delta C_T^+}{2} \cos(\Delta m t). \quad (2)$$

(A more accurate description of the *BABAR* analysis is given in Sec. III.) They obtain

$$\Delta S_T^+ = -1.37 \pm 0.15, \qquad \Delta C_T^+ = +0.10 \pm 0.16. \quad (3)$$

The fact that $\Delta S_T^+ \neq 0$ constitutes their direct demonstration of time reversal violation.

Time reversal violation had been observed earlier in the neutral K meson system by the CPLEAR collaboration [7]. The measurement involves the processes $p\bar{p} \to K^- \pi^+ K^0$ and $p\bar{p} \to K^+ \pi^- \bar{K}^0$. Again, one aims to compare rates of processes that are related by exchange of initial and final states. One rate, Γ_{K^-, e^-}, involves a production of K^- and a neutral K that after time t decay into $e^- \pi^+ \bar{\nu}$. The other rate, Γ_{K^+, ℓ^+}, involves the production of K^+ and a neutral $\bar{K}$ that after time t decay into $e^+ \pi^- \nu$. Under certain assumptions, this is a comparison between the rates of $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$ [8]. The time dependent asymmetry is defined as follows:

$$A_{T,K} \equiv \frac{\Gamma_{K^+, e^+} - \Gamma_{K^-, e^-}}{\Gamma_{K^+, e^+} + \Gamma_{K^-, e^-}}. \quad (4)$$

CPLEAR integrate the rates over times $\tau_S \leq t \leq 20\tau_S$ (τ_S is the lifetime of K_S), and obtain

$$\langle A_{T,K} \rangle_{(1-20)\tau_S} = (6.6 \pm 1.6) \times 10^{-3}. \quad (5)$$

The CPLEAR asymmetry is also a CP asymmetry, since the initial and final states are CP-conjugate. In contrast, the *BABAR* asymmetry is not a CP asymmetry.

In this work, we examine the analysis of A_T, with the aim of answering the following two related questions:
  (i) Would it vanish in the T-symmetry limit?
  (ii) Is the initial state in each of the two processes the T-conjugate of the final state in the other?

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To answer these questions, it is helpful to use only parameters that have well-defined transformation properties under all three relevant symmetries: *CP*, T and *CPT*. In most of the related literature, *CPT* conservation is assumed, and then parameters that are *CP*-odd or *CP*-even are used. For example, the parameters ΔS_T^+ and ΔC_T^+ of Eq. (2) are both *CP*-odd. However, when aiming to demonstrate that time-reversal is violated, one needs to allow for *CPT* violation [9,10]. (Otherwise, T violation follows from *CP* violation.) In this case, most of the parameters used in the literature do not have well-defined transformation under T and *CPT*. In particular, ΔS_T^+ and ΔC_T^+ have, apart from a T-odd *CPT*-even part, also a T-even *CPT*-odd part.

In Sec. II we present our formalism and, in particular, introduce parameters with well-defined transformation properties under *CP*, T and *CPT*. In Sec. III we derive expressions for the asymmetries measured by *BABAR* in terms of these parameters. The results allow us to answer the first question and to clearly formulate the assumptions one needs to make in order to identify the asymmetries measured by *BABAR* with time reversal violation. In Sec. III C we comment on the time-dependence of the asymmetry under various approximations. As concerns the second question, for two processes to be related by time-reversal, the initial state in each of them should be the time-reversal conjugate of the final state in the other. The subtleties related to this requirement, in the context of the *BABAR* measurements, are clarified in Sec. IV. We conclude in Sec. V. In Appendix A we compile relevant experimental constraints on *CP* violation in mixing and in decay, on *CPT* violation, and on wrong-strangeness and wrong-sign *B* decays. In Appendix B we present combinations of measurements that allow us to isolate various parameters of interest. Appendix C contains the full expressions for all the asymmetries measured at *BABAR*. In Appendix D we define "theoretical asymmetries" (or, equivalently, gedanken experiments) involving inverse decays, which provide another way to shed light on the subtleties of experimentally demonstrating time reversal violation. In Appendix E we show how *CPT* violation affects the EPR entanglement of the initial *B* meson state.

II. DEFINITIONS AND NOTATIONS

We use a formalism that allows for *CPT* violation. Similar to Ref. [11], we use *in* and *out* states defined by the reciprocal basis [12]. Translating our notations to those of Ref. [13] is straightforward.

A. $B^0 - \bar{B}^0$ mixing

The effective Hamiltonian $H = M - i\Gamma/2$ describing the evolution of the oscillating system is non-Hermitian. It is thus diagonalized by a general similarity transformation,

$$X^{-1}HX = \text{diag}(\omega_H, \omega_L), \qquad (6)$$

where

$$X = \begin{pmatrix} p\sqrt{1+z} & p\sqrt{1-z} \\ -q\sqrt{1-z} & q\sqrt{1+z} \end{pmatrix}, \qquad (7)$$

with $|p|^2 + |q|^2 = 1$. The complex parameter $z \equiv z^R + iz^I$ represents *CP* and *CPT* violation in mixing. We define a real parameter, R_M, representing T and *CP* violation in mixing:

$$R_M \equiv \frac{1 - |q/p|^2}{1 + |q/p|^2}. \qquad (8)$$

In Eq. (7) we omitted normalization factors that deviate from unity only in $\mathcal{O}(R_M z, z^2)$.

For the mass and width eigenvalues, we use

$$\omega_{H,L} = m_{H,L} - \frac{i}{2}\Gamma_{H,L}, \quad m = \frac{m_L + m_H}{2}, \quad \Gamma = \frac{\Gamma_L + \Gamma_H}{2},$$

$$x = \frac{m_H - m_L}{\Gamma}, \quad y = \frac{\Gamma_H - \Gamma_L}{2\Gamma}. \qquad (9)$$

The incoming mass eigenstates are

$$|B_H^{\text{in}}\rangle = p\sqrt{1+z}|B^0\rangle - q\sqrt{1-z}|\bar{B}^0\rangle,$$

$$|B_L^{\text{in}}\rangle = p\sqrt{1-z}|B^0\rangle + q\sqrt{1+z}|\bar{B}^0\rangle. \qquad (10)$$

Their time evolution is given by

$$|B_{H,L}^{\text{in}}(t)\rangle = e^{-i\omega_{H,L}t}|B_{H,L}^{\text{in}}\rangle. \qquad (11)$$

The outgoing states are

$$\langle B_H^{\text{out}}| = \frac{1}{2pq}(q\sqrt{1+z}\langle B^0| - p\sqrt{1-z}\langle \bar{B}^0|),$$

$$\langle B_L^{\text{out}}| = \frac{1}{2pq}(q\sqrt{1-z}\langle B^0| + p\sqrt{1+z}\langle \bar{B}^0|). \qquad (12)$$

B. *B* decay amplitudes

We define decay amplitudes,

$$A_f \equiv A(B^0 \to f) = \langle f|T|B^0\rangle,$$

$$\bar{A}_f \equiv A(\bar{B}^0 \to f) = \langle f|T|\bar{B}^0\rangle, \qquad (13)$$

and the inverse-decay amplitudes,

$$A_f^{\text{ID}} \equiv A(f^T \to B^0) = \langle B^0|T|f^T\rangle,$$

$$\bar{A}_f^{\text{ID}} \equiv A(f^T \to \bar{B}^0) = \langle \bar{B}^0|T|f^T\rangle, \qquad (14)$$

where f^T is T-conjugate (i.e. reversed spins and momenta) to f. *CP* violation in decay and in inverse decay is manifest in the direct asymmetries

$$\mathcal{A}_f \equiv \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|\bar{A}_{\bar{f}}|^2 + |A_f|^2}, \qquad \mathcal{A}_f^{\text{ID}} \equiv \frac{|A_f^{\text{ID}}|^2 - |\bar{A}_{\bar{f}}^{\text{ID}}|^2}{|A_f^{\text{ID}}|^2 + |\bar{A}_{\bar{f}}^{\text{ID}}|^2}, \quad (15)$$

where $\bar{f}$ is CP-conjugate to f.

We define complex parameters, θ_f, representing CPT violation in the decay:

$$\theta_f = \theta_f^R + i\theta_f^I \equiv \frac{A_{\bar{f}}^{\text{ID}}/\bar{A}_{\bar{f}}^{\text{ID}} - \bar{A}_f/A_f}{A_{\bar{f}}^{\text{ID}}/\bar{A}_{\bar{f}}^{\text{ID}} + \bar{A}_f/A_f}. \qquad (16)$$

Under CP, $\theta_f \to -\theta_{\bar{f}}$, while under T, $\theta_f \to \theta_{\bar{f}}$. Thus, for final CP eigenstates, $\theta_f \neq 0$ breaks CPT and CP, but not T.

We further define the phase convention independent combination of amplitudes and mixing parameters,

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f}\sqrt{\frac{1 + \theta_f}{1 - \theta_f}} = \frac{q}{p}\frac{A_{\bar{f}}^{\text{ID}}}{\bar{A}_{\bar{f}}^{\text{ID}}}\sqrt{\frac{1 - \theta_f}{1 + \theta_f}}. \qquad (17)$$

In the CPT limit $z = \theta_f = 0$ and the standard definition of λ_f is recovered. It is convenient to introduce the following functions of λ_f:

$$C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f \equiv \frac{2\mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2}, \quad G_f \equiv \frac{2\mathcal{R}e(\lambda_f)}{1 + |\lambda_f|^2}, \quad (18)$$

with $C_f^2 + G_f^2 + S_f^2 = 1$.

We emphasize that the definition of λ_f in Eq. (17) and, consequently, the definitions of S_f, C_f and G_f in Eq. (18) differ from the standard definitions of these parameters in the literature. Our definition lends itself straightforwardly to the theoretical analysis that we are doing. The standard definition lends itself straightforwardly to the description of the experimentally measured rates. The relation between the two will be further clarified in the next subsection. The distinction between the two is crucial for understanding the subtleties in the interpretation of the *BABAR* measurements. Of course, in the absence of CPT violation, the two sets of definitions coincide.

C. T, CP and CPT transformations

The transformation rules for the parameters defined in the previous subsection under T, CP and CPT, are summarized in Table I.

As explained above, it is very convenient for our purposes to use parameters that are either even or odd under all three transformations. Using superscript $+$ for T-even, and $-$ for T-odd, we define the following combinations:

TABLE I. Transformation rules of the various parameters under T, CP and CPT.

Parameter	T	CP	CPT
R_M	$-R_M$	$-R_M$	R_M
z	z	$-z$	$-z$
λ_f	$1/\lambda_{\bar{f}}$	$1/\lambda_{\bar{f}}$	λ_f
S_f	$-S_{\bar{f}}$	$-S_{\bar{f}}$	S_f
C_f	$-C_{\bar{f}}$	$-C_{\bar{f}}$	C_f
G_f	$G_{\bar{f}}$	$G_{\bar{f}}$	G_f
$\mathcal{A}_f$	$-\mathcal{A}_f^{\text{ID}}$	$-\mathcal{A}_f$	$\mathcal{A}_f^{\text{ID}}$
θ_f	$\theta_{\bar{f}}$	$-\theta_{\bar{f}}$	$-\theta_f$

$$C_f^- = \frac{1}{2}(C_f + C_{\bar{f}}), \qquad C_f^+ = \frac{1}{2}(C_f - C_{\bar{f}}),$$

$$S_f^- = \frac{1}{2}(S_f + S_{\bar{f}}), \qquad S_f^+ = \frac{1}{2}(S_f - S_{\bar{f}}),$$

$$G_f^- = \frac{1}{2}(G_f - G_{\bar{f}}), \qquad G_f^+ = \frac{1}{2}(G_f + G_{\bar{f}}),$$

$$\theta_f^- = \frac{1}{2}(\theta_f - \theta_{\bar{f}}), \qquad \theta_f^+ = \frac{1}{2}(\theta_f + \theta_{\bar{f}}). \qquad (19)$$

We further define

$$\mathcal{A}_f^- = \frac{\mathcal{A}_f + \mathcal{A}_f^{\text{ID}}}{2} = R_M - C_{f_{CP}}, \quad \mathcal{A}_f^+ = \frac{\mathcal{A}_f - \mathcal{A}_f^{\text{ID}}}{2} = -\theta_{f_{CP}}^{+,R},$$

$$(20)$$

where the last step in each equation is relevant only for CP eigenstates and we expand to linear order in $C_{f_{CP}}$, R_M and $\theta_{f_{CP}}$. A summary of the transformation properties of these parameters is provided in Table II. For final CP eigenstates, such as $f = \psi K_{S,L}$, S_f and C_f are T-odd, while G_f and θ_f are T-even, so that $S_f^+ = C_f^+ = G_f^- = \theta_f^- = 0$. We summarize the transformation properties for final CP eigenstates also in Table II.

In practice, inverse decays are not accessible to the experiments. In particular, experiments are not sensitive to λ_f, as defined in Eq. (17), but to the related observable λ_f^e, defined via

TABLE II. Eigenvalues of the various parameters under T, CP and CPT.

Parameter	T	CP	CPT
R_M, S_f^-, C_f^-, G_f^-, $\mathcal{A}_f^-$	$-$	$-$	$+$
z, θ_f^+, $\mathcal{A}_f^+$	$+$	$-$	$-$
θ_f^-	$-$	$+$	$-$
S_f^+, C_f^+, G_f^+	$+$	$+$	$+$
$S_{f_{CP}}$, $C_{f_{CP}}$	$-$	$-$	$+$
$\theta_{f_{CP}}$	$+$	$-$	$-$
$G_{f_{CP}}$	$+$	$+$	$+$

$$\lambda_f^e \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} = \lambda_f(1 - \theta_f), \qquad (21)$$

where the second equation holds to first order in θ_f. Accordingly, the experiments are sensitive to the following parameters:

$$C_f^e = C_f + (1 - C_f^2)\theta_f^R,$$
$$S_f^e = S_f(1 - C_f\theta_f^R) - G_f\theta_f^I,$$
$$G_f^e = G_f(1 - C_f\theta_f^R) + S_f\theta_f^I. \qquad (22)$$

D. Wrong-strangeness and wrong-sign decays

Among the final CP eigenstates, we focus on decays into the final $\psi K_{S,L}$ states (neglecting effects of ϵ_K). We distinguish between the right strangeness decays,

$$A_{K^0} = A(B^0 \to K^0), \qquad \bar{A}_{\bar{K}^0} = A(\bar{B}^0 \to \bar{K}^0), \qquad (23)$$

and the wrong strangeness decays,

$$\bar{A}_{K^0} = A(\bar{B}^0 \to K^0), \qquad A_{\bar{K}^0} = A(B^0 \to \bar{K}^0). \qquad (24)$$

We define

$$\hat{C}_{\psi K} \equiv \frac{1}{2}(C_{\psi K_S} + C_{\psi K_L}), \qquad \Delta C_{\psi K} \equiv \frac{1}{2}(C_{\psi K_S} - C_{\psi K_L}),$$

$$\hat{S}_{\psi K} \equiv \frac{1}{2}(S_{\psi K_S} - S_{\psi K_L}), \qquad \Delta S_{\psi K} \equiv \frac{1}{2}(S_{\psi K_S} + S_{\psi K_L}),$$

$$\hat{G}_{\psi K} \equiv \frac{1}{2}(G_{\psi K_S} - G_{\psi K_L}), \qquad \Delta G_{\psi K} \equiv \frac{1}{2}(G_{\psi K_S} + G_{\psi K_L}), \qquad (25)$$

and

$$\hat{\theta}_{\psi K} \equiv \frac{1}{2}(\theta_{\psi K_L} + \theta_{\psi K_S}), \qquad \Delta\theta_{\psi K} \equiv \frac{1}{2}(\theta_{\psi K_S} - \theta_{\psi K_L}). \qquad (26)$$

In the limit of no wrong strangeness decay, $\lambda_{\psi K_S} = -\lambda_{\psi K_L}$ [14] (Ref. [14] assumes CPT conservation, but this is not a

necessary assumption) and, consequently, $\Delta C_{\psi K}$, $\Delta G_{\psi K}$, $\Delta S_{\psi K}$ and $\Delta\theta_{\psi K}$ vanish.

Among the flavor specific final states, we focus on decays into final $\ell^\pm X$ states. Here we distinguish between the right sign decays,

$$A_{\ell^+} = A(B^0 \to \ell^+ X), \qquad \bar{A}_{\ell^-} = A(\bar{B}^0 \to \ell^- X), \qquad (27)$$

and the wrong sign decays,

$$A_{\ell^-} = A(B^0 \to \ell^- X), \qquad \bar{A}_{\ell^+} = A(\bar{B}^0 \to \ell^+ X). \qquad (28)$$

We define $C_\ell^\pm$, $S_\ell^\pm$ and $G_\ell^\pm$ according to Eq. (19), with $f = \ell^+$, and a super-index $+$ ($-$) denoting a T conserving (violating) combination. Taking the wrong sign decays to be much smaller in magnitude than the right sign decays, we have $|\lambda_{\ell^+}| \ll 1$ and $|\lambda_{\ell^-}^{-1}| \ll 1$. We will work to first order in $|\lambda_{\ell^+}|$ and in $|\lambda_{\ell^-}^{-1}|$, which means that we set $C_\ell^+ = 1$ and $C_\ell^- = 0$. On the other hand, the four other relevant parameters are linear in $|\lambda_{\ell^+}|$ and in $|\lambda_{\ell^-}^{-1}|$:

$$S_\ell^\pm \simeq \mathcal{I}m(\lambda_{\ell^+} \pm \lambda_{\ell^-}^{-1}), \qquad G_\ell^\pm \simeq \mathcal{R}e(\lambda_{\ell^+} \pm \lambda_{\ell^-}^{-1}). \qquad (29)$$

III. TIME-REVERSAL ASYMMETRIES

A. The master formula

Consider a pair of B-mesons produced in $\Upsilon(4S)$ decay, where one of the B-mesons decays at time t_1 to a final state f_1, and the other B-meson decays at a later time, $t_2 = t_1 + t$, to a final state f_2. The time dependent rate for this process, to linear order in R_M, z and θ, is given by

$$\Gamma_{(f_1)_\perp,f_2} = N_{(1)_\perp,2}e^{-\Gamma(t_1+t_2)}[\kappa_{(1)_\perp,2}\cosh(y\Gamma t) + \sigma_{(1)_\perp,2}\sinh(y\Gamma t) + C_{(1)_\perp,2}\cos(x\Gamma t) + S_{(1)_\perp,2}\sin(x\Gamma t)]. \qquad (30)$$

where

$$N_{(1)_\perp,2} = [1 + (C_1 + C_2)(R_M - z^R)]\mathcal{N}_1\mathcal{N}_2,$$

$$\kappa_{(1)_\perp,2} = [1 - G_1G_2 + (C_1 + C_2 + C_2G_1 + C_1G_2 - C_2G_2G_1 - C_1G_2G_1)z^R - (S_1 + S_2)z^I + G_1G_2(C_2\theta_2^R + C_1\theta_1^R)$$
$$- G_1S_2\theta_2^I - G_2S_1\theta_1^I],$$

$$\sigma_{(1)_\perp,2} = [G_2 - G_1 - (C_2 - C_1 - C_2G_2 - C_1G_2 + C_2G_1 + C_1G_1)z^R - (G_2S_1 - G_1S_2)z^I - C_2G_2\theta_2^R + S_2\theta_2^I + C_1G_1\theta_1^R - S_1\theta_1^I],$$

$$C_{(1)_\perp,2} = -[C_2C_1 + S_2S_1 + (C_2^2C_1 + C_2C_1^2 + C_1G_2 + C_2G_1 + C_2S_2S_1 + C_1S_2S_1)z^R$$
$$- (S_2 + S_1)z^I - G_2S_1\theta_2^I + (C_1 - C_2^2C_1 - C_2S_2S_1)\theta_2^R - G_1S_2\theta_1^I + (C_2 - C_2C_1^2 - C_1S_2S_1)\theta_1^R],$$

$$S_{(1)_\perp,2} = [C_1S_2 - C_2S_1 + (C_2C_1S_2 + C_1^2S_2 + G_1S_2 - C_2^2S_1 - C_2C_1S_1 - G_2S_1)z^R$$
$$+ (C_2 - C_1)z^I - C_1G_2\theta_2^I + (C_2^2S_1 - C_2C_1S_2 - S_1)\theta_2^R + C_2G_1\theta_1^I - (C_1^2S_2 - C_2C_1S_1 - S_2)\theta_1^R], \qquad (31)$$

and, for the sake of brevity, we denote $X_i \equiv X_{f_i}$ for $X = S$, C, G, θ and $\mathcal{N}_i = (|A_{f_i}|^2 + |\bar{A}_{f_i}|^2)$. Eq. (30) is our "master formula," and serves as the starting point for all our calculations. Note that the decomposition into $e^{-\Gamma(t_1+t_2)} \times f(t)$ for the $\Upsilon(4S)$ decay products holds even in the presence of CPT violation. (See Appendix E.)

B. The *BABAR* T-asymmetry

In what follows we set $y = 0$, $C_\ell^+ = 1$, $C_\ell^- = 0$ and $\epsilon_K = 0$. We do so for the sake of simplicity: All these effects can, in principle, be accounted for. (Care is needed when accounting for the effect of Kaon mixing [15–20].) We work to linear order in the following small parameters:

$$R_M, z^R, z^I, \theta_f^R, \theta_f^I, \hat{C}_{\psi K}, \Delta C_{\psi K}, \Delta S_{\psi K}, \Delta G_{\psi K}, S_\ell^\pm, G_\ell^\pm. \quad (32)$$

We consider the two processes that are relevant to the experimentally measured asymmetry (1). Using the master formulas (30), and implementing our approximations, we obtain the following time-dependent decay rates:

$$\Gamma_{(\psi K_L)_\perp, \ell^- X}$$
$$= [1 - R_M + z^R] \mathcal{N}_{\psi K_L} \mathcal{N}_{\ell^- X}$$
$$\times \{ [1 - G_{\psi K_L} G_{\ell^-} - (1 + G_{\psi K_L}) z^R - S_{\psi K_L} z^I]$$
$$- [-C_{\psi K_L} + S_\ell - S_{\psi K_L} - G_{\psi K_L} z^R - S_{\psi K_L} z^I - \theta_{\psi K_L}^R] \cos(x\Gamma t)$$
$$+ [S_{\psi K_L} - S_{\psi K_L} z^R - z^I - G_{\psi K_L} \theta_{\psi K_L}^I] \sin(x\Gamma t) \}, \quad (33)$$

$$\Gamma_{(\ell^+ X)_\perp, \psi K_S}$$
$$= [1 + R_M - z^R] \mathcal{N}_{\ell^+ X} \mathcal{N}_{\psi K_S}$$
$$\times \{ [1 - G_{\ell^+} G_{\psi K_S} + (1 + G_{\psi K_S}) z^R - S_{\psi K_S} z^I]$$
$$- [C_{\psi K_S} + S_{\psi K_S} S_{\ell^+} + G_{\psi K_S} z^R - S_{\psi K_S} z^I + \theta_{\psi K_S}^R] \cos(x\Gamma t)$$
$$+ [S_{\psi K_S} + S_{\psi K_S} z^R - z^I - G_{\psi K_S} \theta_{\psi K_S}^I] \sin(x\Gamma t) \}, \quad (34)$$

where the overall decay exponential factor is omitted. The analysis performed by *BABAR*, as described in Ref. [1], is as follows. The time dependent decay rates are measured and fitted to time-dependence of the form (30), approximating (as we do) $y = 0$. The quantities ΔS_T^+ and ΔC_T^+, to which the *BABAR* results of (3) apply, correspond to the following combinations:

$$\Delta S_T^+ = \frac{S_{(\psi K_L)_\perp, \ell^- X}}{\kappa_{(\psi K_L)_\perp, \ell^- X}} - \frac{S_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}} \equiv S_{\ell^-, K_L}^- - S_{\ell^+, K_S}^+,$$

$$\Delta C_T^+ = \frac{C_{(\psi K_L)_\perp, \ell^- X}}{\kappa_{(\psi K_L)_\perp, \ell^- X}} - \frac{C_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}} \equiv C_{\ell^-, K_L}^- - C_{\ell^+, K_S}^+. \quad (35)$$

The last identity relates our definitions in Eq. (30) with those of *BABAR* in Ref. [1]. In the latter, a super-index $+$ refers to the case the leptonic tag occurs before the CP tag, as in $\Gamma_{(\ell^+ X)_\perp, \psi K_S}$, while a super-index $-$ refers to the case that the CP tag occurs before the leptonic tag, as in

$\Gamma_{(\psi K_L)_\perp, \ell^- X}$. We note that the normalization of Eqs. (35) removes the dependence on the total production rates and effects such as direct CP violation in leptonic decays.

We obtain the following expressions for these observables:

$$\Delta S_T^+ = -2[(\hat{S}_{\psi K} - \hat{G}_{\psi K} \hat{\theta}_{\psi K}^I) + \hat{S}_{\psi K} \hat{G}_{\psi K} (G_\ell^- - z^R)],$$
$$\Delta C_T^+ = 2[(\hat{C}_{\psi K} + \hat{\theta}_{\psi K}^R) + \hat{S}_{\psi K} (S_\ell^- - z^I)]. \quad (36)$$

If we switch off all the T-odd parameters, we are left with the following T conserving (TC) contributions:

$$\Delta S_T^+(\text{T-odd parameters} = 0) = 2\hat{G}_{\psi K} \hat{\theta}_{\psi K}^I,$$
$$\Delta C_T^+(\text{T-odd parameters} = 0) = 2\hat{\theta}_{\psi K}^R. \quad (37)$$

These contributions are CPT violating. Yet, since $\hat{\theta}_{\psi K}$ involves inverse decays, we are not aware of any way to experimentally bound it, and to exclude the possibility that it generates the measured value of ΔS_T^+. We would like to emphasize, however, the following three points.

  (i) The appearance of the T conserving, CPT violating effects should come as no surprise. As explained in the discussion of Eq. (22), experiments can only probe $S_{\psi K}^e$ and $C_{\psi K}^e$, which include these terms.
 (ii) While we are not aware of any way to constrain $\hat{\theta}_{\psi K}$ from tree-level processes, it may contribute to measurable effects, such as CPT violation in mixing, at the loop level. In the absence of a rigorous framework that incorporates CPT violation, it is impossible to calculate such effects.
(iii) It would of course be extremely exciting if the *BABAR* measurement is affected by CPT violating effects.

An additional interesting feature of Eqs. (36) is the appearance of terms that are quadratic in T-odd parameters,

$$\Delta S_T^+(\text{quadratic in T-odd parameters}) = -2\hat{G}_{\psi K} \hat{S}_{\psi K} G_\ell^-,$$
$$\Delta C_T^+(\text{quadratic in T-odd parameters}) = 2\hat{S}_{\psi K} S_\ell^-. \quad (38)$$

While these terms would vanish if we take all T-odd parameters to zero, they are still T-conserving. Note that since we expand to linear order in all T-odd parameters, except for $\hat{S}_{\psi K_S}$, there are additional T conserving, $\hat{S}_{\psi K}$-independent, contributions that are quadratic in T-odd parameters that are not presented in Eqs. (38). Since $\hat{G}_{\psi K}^2 + \hat{S}_{\psi K}^2 \leq 1$, the maximal absolute value of the term on the right-hand side of Eq. (38) for ΔS_T^+ is 1, $|2\hat{G}_{\psi K} \hat{S}_{\psi K} G_\ell^-| \leq 1$. Thus, if experiments establish $|\Delta S_T^+| > 1$, such a result cannot be explained by this term alone.

We are now also able to formulate the conditions under which the *BABAR* measurement would demonstrate T violation unambiguously:

$$\hat{\theta}_{\psi K} = G_\ell^- = S_\ell^- = 0. \tag{39}$$

In words, the necessary conditions are the following:
  (i) The absence of *CPT* violation in strangeness changing decays.
  (ii) The absence of wrong sign decays or, if wrong sign decays occur, the absence of direct *CP* violation in semileptonic decays.

C. The time dependence of A_T

It is often assumed that the time-dependence of the time-reversal asymmetry A_T is of the form of Eq. (2). Equations (33) and (34) reveal, however, that even when taking $y = 0$ and expanding to linear order in the various small parameters, the time dependence is more complicated:

$$A_T = R_T + C_T \cos(x\Gamma t) + S_T \sin(x\Gamma t) + B_T \sin^2(x\Gamma t)$$
$$+ D_T \sin(x\Gamma t) \cos(x\Gamma t). \tag{40}$$

Even when we neglect *CPT* violation, wrong sign decays, and wrong strangeness decays, the time dependence is, in general, more complicated than Eq. (2). If, in addition, one neglects *CP* violation in decay, then the only source of T violating effects is in mixing and should therefore vanish at $t = 0$:

$$A_T = -R_M[1 - \cos(x\Gamma t)] - \hat{S}_{\psi K} \sin(x\Gamma t) + R_M \hat{S}_{\psi K}^2 \sin^2(x\Gamma t). \tag{41}$$

One may argue that R_M is experimentally constrained to be very small. But then one should also take into account the fact that $\hat{C}_{\psi K}$ is experimentally constrained to be very small, and the asymmetry has the simpler well-known form

$$A_T = -\hat{S}_{\psi K} \sin(x\Gamma t). \tag{42}$$

Moreover, the $\hat{S}_{\psi K}$ parameter is measured and known. The whole point of the *BABAR* measurement is not to measure the values of the parameters, but to demonstrate time-reversal violation in processes that are not *CP*-conjugate. It is perhaps more appropriate not to take experimental information from previous measurements.

IV. *CP* VIOLATION IN RIGHT-STRANGENESS DECAYS

A priori, one would expect that direct *CP* violation in right-strangeness decays is enough to allow for $A_T \neq 0$ even in the T-symmetry limit. However, in the previous section we found that this is not the case. In this section we first explain the reasoning behind the naive expectation, and, second, obtain the conditions under which the two processes measured by *BABAR* are not T conjugate.

In $\Gamma_{(\psi K_L)_\perp, \ell^- X}$, the initial *B*-meson state is orthogonal to the one that decays to ψK_L. In $\Gamma_{(\ell^+ X)_\perp, \psi K_S}$, the final state is the one that decays into ψK_S. Are these two states identical? They would be if the state that does not decay to ψK_L, $|B_{(\to \psi K_L)_\perp}\rangle$ and the state that does not decay into ψK_S, $|B_{(\to \psi K_S)_\perp}\rangle$ were orthogonal to each other. Using

$$|B^{\mathrm{in}}_{(\to \psi K_L)_\perp}\rangle = N_L[\bar{A}_{\psi K_L}|B^0\rangle - A_{\psi K_L}|\bar{B}^0\rangle],$$
$$|B^{\mathrm{in}}_{(\to \psi K_S)_\perp}\rangle = N_S[\bar{A}_{\psi K_S}|B^0\rangle - A_{\psi K_S}|\bar{B}^0\rangle], \tag{43}$$

where $N_{S,L}$ are normalization coefficients, and neglecting wrong strangeness decays, we obtain

$$\langle B_{(\to \psi K_S)_\perp}|B_{(\to \psi K_L)_\perp}\rangle \propto \mathcal{A}_{\psi K}, \tag{44}$$

where $\mathcal{A}_f$ is the direct asymmetry defined in Eq. (15). (The same orthogonality condition can be obtained by using $\langle B_{\to \psi K_S}|B_{\to \psi K_L}\rangle$.) The conclusion is that the state that is orthogonal to the one that decays to ψK_L and the state that decays to ψK_S are not the same state, unless there is no direct *CP* violation in the $B \to \psi K$ decays. This is presumably the reason why the theoretical paper [4] and the experimental paper [1] explicitly state that they neglect direct *CP* violation.

However, as we learned from the analysis in Sec. III, the correct question is not whether the state that does not decay to ψK_L is the same as the state that decays to ψK_S. Instead, the question is whether it is the same as the state generated in the inverse decay of ψK_S. (The orthogonality of the two *B*-mesons at t_1 is guaranteed by the EPR entanglement, allowing one to label the initial *B* state at t_1.) This would be the case if $|B_{(\to \psi K_L)_\perp}\rangle$ and $|B_{(\psi K_S \to)_\perp}\rangle$ were orthogonal. Using

$$|B^{\mathrm{in}}_{\psi K_S \to}\rangle = N_S^{\mathrm{ID}}[A_{\psi K_S}^{\mathrm{ID}}|B^0\rangle + \bar{A}_{\psi K_S}^{\mathrm{ID}}|\bar{B}^0\rangle],$$
$$|B^{\mathrm{in}}_{(\psi K_S \to)_\perp}\rangle = N_S^{\mathrm{ID}}[\bar{A}_{\psi K_S}^{\mathrm{ID}*}|B^0\rangle - A_{\psi K_S}^{\mathrm{ID}*}|\bar{B}^0\rangle], \tag{45}$$

we obtain

$$\langle B_{(\psi K_S \to)_\perp}|B_{(\to \psi K_L)_\perp}\rangle \propto \lambda_{\psi K_S} + \lambda_{\psi K_L} + (\lambda_{\psi K_S} - \lambda_{\psi K_L})\hat{\theta}_{\psi K}$$
$$\propto \Delta C_{\psi K} - \frac{\Delta G_{\psi K} + i\Delta S_{\psi K}}{\hat{G}_{\psi K} + i\hat{S}_{\psi K}} - \hat{\theta}_{\psi K}. \tag{46}$$

Similarly,

$$\langle B_{(\psi K_L \to)_\perp}|B_{(\to \psi K_S)_\perp}\rangle \propto \lambda_{\psi K_L} + \lambda_{\psi K_S} + (\lambda_{\psi K_L} - \lambda_{\psi K_S})\hat{\theta}_{\psi K}$$
$$\propto \Delta C_{\psi K} - \frac{\Delta G_{\psi K} + i\Delta S_{\psi K}}{\hat{G}_{\psi K} + i\hat{S}_{\psi K}} + \hat{\theta}_{\psi K}. \tag{47}$$

We thus learn that the state that is orthogonal to the one that decays to $\psi K_L(\psi K_S)$ is the same as the state that is generated in the inverse decay of $\psi K_S(\psi K_L)$, unless there are wrong strangeness decays or CPT violation in strangeness changing decays. In the absence of these phenomena, the two processes are related by exchange of the initial and final CP-tagged states, as required for time-reversal conjugate processes.

One can repeat an analogous analysis for the lepton-tagged states. The question to be asked is whether the state that does not decay to $\ell^+ X$ is orthogonal to the state that is not generated in the inverse decay of $\ell^- X$. For the overlap between these two states, we obtain:

$$\langle B_{(\ell^- X \to)_\perp} | B_{(\to \ell^+ X)_\perp} \rangle \propto \lambda_{\ell^+}. \tag{48}$$

If $\lambda_{\ell^+} = 0$, the two states are orthogonal. We thus learn that the state that is orthogonal to the one that decays into $\ell^+ X$ is the same as the state that is generated in the inverse decay of $\ell^- X$, unless there are wrong sign decays and wrong sign inverse decays. If wrong sign decays can be neglected, then the two processes are related by exchange of the initial and final lepton-tagged states, as required for time-reversal conjugate processes. (For a related asymmetry, involving $\Gamma_{(\psi K_L)_\perp, \ell^+ X}$ and $\Gamma_{(\ell^- X)_\perp, \psi K_S}$, the corresponding condition is $1/\lambda_{\ell^-} = 0$.)

If Eq. (46) and Eq. (48) are both zero, then $\Gamma_{(\psi K_L)_\perp, \ell^- X} = \Gamma_{(\psi K_S)^T, \ell^- X}$ and $\Gamma_{(\ell^+ X)_\perp, \psi K_S} = \Gamma_{(\ell^- X)^T, \psi K_S}$ (under proper normalization such that the number of initial B's are equal). In this case, A_T as defined in Eq. (1) is indeed a T-asymmetry. We conclude that if wrong strangeness decays, wrong sign decays and CPT violation in strangeness changing decays can be neglected, then the two processes measured by *BABAR* represent two T-conjugate processes, and then there should be no T conserving contributions to ΔS_T^+ and ΔS_T^-. In particular, one need not assume the absence of direct CP violation.

V. CONCLUSIONS

The *BABAR* collaboration has measured time-reversal asymmetries in B decays. Two main ingredients—the EPR effect between the two B-mesons produced in $\Upsilon(4S)$ decays and the availability of both lepton-tagging and CP-tagging—allow the experimenters to approximately realize the main principle of time-reversal conjugate processes: exchanging initial and final states.

A precise exchange is impossible. The final state is identified by the B-meson decay, and the T-conjugate process requires, therefore, that a B-meson is produced in the corresponding inverse decay. Instead, the experimenters measure a process where the initial B-meson is identified by the decay of the other, entangled B-meson. We showed, however, that the initial B-meson prepared by lepton tagging, and the one that would be produced in the

appropriate inverse decay are not identical only if there are wrong-sign decays. The initial B-meson prepared by CP tagging, and the one that would be produced in the appropriate inverse decay, are not identical only if there are wrong-strangeness contributions, or in the presence of CPT violation in decays.

The effect of CPT violation in decay has gained very little attention in the literature. One reason is that it can only be probed by measuring both decay rates and inverse decay rates, but the latter are practically inaccessible to experiments. For precisely this reason, there are no bounds on these effects. In principle, they could play a significant role in the asymmetries measured by *BABAR*, in spite of the fact that they are T conserving.

As concerns the questions posed in the Introduction, we find the following answers:

  (i) The A_T measurement by *BABAR* would vanish in the T-symmetry limit provided that CPT is conserved in strangeness changing decays [see Eq. (37)].
  (ii) The initial state in each of the two processes would be the T-conjugate of the final state in the other, provided that there are neither wrong strangeness decays nor wrong sign decays nor CPT violation in strangeness changing decays. In particular, there is no need to assume the absence of direct CP violation [see Eqs. (46)–(48)].

Both wrong-sign and wrong-strangeness effects are expected to be very small. Yet, the existing experimental upper bounds on these effects are rather weak. The same set of measurements used for the time-reversal asymmetries can be used (in different combinations) to constrain also the wrong-sign and wrong-strangeness contributions.

While in this work we concentrated on very specific measurements in B decays, our results are more general. They apply straightforwardly, with minor changes, to other meson systems. The main ideas also apply to neutrino oscillation experiments. Observation of $P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha)$ has been advocated as a way to establish T-violation. Such a result can arise, however, also from nonstandard interactions in the production or the detection processes [21–23].

ACKNOWLEDGMENTS

APPENDIX A: RELEVANT DATA

How certain is it that the small parameters listed in Eq. (32) are indeed small? In this appendix, we compile the relevant experimental constraints. We caution the reader that some of these constraints were derived making assumptions that we do not make. For example, CPT symmetry is assumed when deriving the bounds on R_M. Thus, the upper bounds and ranges that we quote below assume no cancelations among different contributions and, even then, should be taken only as rough estimates.

Based on Ref. [24], we find the following range for R_M:

$$R_M = (-0.2 \pm 2.0) \times 10^{-3} \quad \text{from } \Upsilon(4S), \quad \text{(A1)}$$

$$R_M = (+0.3 \pm 0.9) \times 10^{-3} \quad \text{world average.} \quad \text{(A2)}$$

Reference [25] uses the *BABAR* measurement [26] to constrain z:

$$z^R = (-2 \pm 5) \times 10^{-2}, \quad z^I = (+0.8 \pm 0.4) \times 10^{-2}. \quad \text{(A3)}$$

Reference [24] obtains the following ranges for the $S_{\psi K_{S,L}}$ and $C_{\psi K_{S,L}}$ parameters:

$$S_{\psi K_S} = +0.665 \pm 0.024, \quad C_{\psi K_S} = +0.024 \pm 0.026,$$
$$S_{\psi K_L} = -0.663 \pm 0.041, \quad C_{\psi K_L} = -0.023 \pm 0.030.$$
$$\text{(A4)}$$

Note that here we take $S_{\psi K_L} \to -S_{\psi K_L}$ with respect to [24]. Naively combining these ranges, we obtain

$$\Delta C_{\psi K} = +0.02 \pm 0.02, \quad \hat{C}_{\psi K} = +0.00 \pm 0.02, \quad \text{(A5)}$$

$$\Delta S_{\psi K} = +0.00 \pm 0.02, \quad \hat{S}_{\psi K} = +0.66 \pm 0.02. \quad \text{(A6)}$$

Direct bounds on wrong-strangeness B decays are given by the *BABAR* collaboration in Ref. [27]. Reference [25] quotes $|\lambda_{\psi K^{*0}}| < 0.25$ at 95% CL, which is weaker than the above results.

As concerns wrong-sign B decays, we use the individual branching ratios from Ref. [25] to obtain

$$\frac{\text{BR}(B^+ \to \ell^+ \nu X)}{\text{BR}(B^0 \to \ell^+ \nu X)} = 1.06 \pm 0.04. \quad \text{(A7)}$$

In the isospin limit and in the absence of wrong-sign decays, we should have

$$\frac{\text{BR}(B^+ \to \ell^+ \nu X)}{\text{BR}(B^0 \to \ell^+ \nu X)} = \frac{\tau_{B^+}}{\tau_{B^0}} = 1.08 \pm 0.01. \quad \text{(A8)}$$

Comparing Eq. (A7) to (A8) we obtain, at 2σ

$$|\bar{A}_{\ell^+}/A_{\ell^+}| < 0.44. \quad \text{(A9)}$$

Using the exclusive process $B^+ \to \ell^+ \nu \bar{D}^0$ instead of the inclusive one results in a weaker bound. One can also use different tagging techniques in measurements of CP asymmetries to place bounds on wrong sign decays. Assuming CPT to be a good symmetry, we find

$$C_{CP}^{\text{lep}} - C_{CP}^{\text{Kaon}} = \hat{S}_{\psi K_S} S_\ell^+, \quad \text{(A10)}$$

where C_{CP}^{lep} and C_{CP}^{Kaon} are the measured coefficients for the $\cos(x\Gamma t)$ function with lepton and Kaon tagging, respectively. Using the results of [28] and Eq. (A6) we get, at 2σ,

$$|S_\ell^+| < 0.24. \quad \text{(A11)}$$

APPENDIX B: ISOLATING PARAMETERS OF INTEREST

Combinations of observables measured by *BABAR* allow us to constrain various parameters of interest. We here use *BABAR*'s notation. The combinations that correspond to the CP-odd $C_{\psi K}^e$ and $S_{\psi K}^e$, defined in Eq. (22), can be isolated via the following combinations:

$$-\frac{1}{2}(C_{\ell^+,K_S}^{\pm} + C_{\ell^+,K_L}^{\pm}) = \frac{1}{2}(C_{\ell^-,K_S}^{\pm} + C_{\ell^-,K_L}^{\pm})$$
$$= \hat{C}_{\psi K} + \hat{\theta}_{\psi K}^R,$$
$$\frac{\pm 1}{2}(S_{\ell^+,K_S}^{\pm} - S_{\ell^+,K_L}^{\pm}) = \frac{\mp 1}{2}(S_{\ell^-,K_S}^{\pm} - S_{\ell^-,K_L}^{\pm})$$
$$= \hat{S}_{\psi K} - \hat{G}_{\psi K}\hat{\theta}_{\psi K}^I. \quad \text{(B1)}$$

Upper bounds on combinations of the wrong-sign T-odd parameters S_ℓ^- and G_ℓ^-, defined in Eq. (19), and the CPT violating parameter z, defined in Eq. (7), can be obtained as follows:

$$\frac{-1}{2}(C_{\ell^+,K_S}^{\pm} + C_{\ell^-,K_S}^{\pm}) = S_{\psi K_S}(S_\ell^- - z^I),$$
$$\frac{-1}{2}(C_{\ell^+,K_L}^{\pm} + C_{\ell^-,K_L}^{\pm}) = S_{\psi K_L}(S_\ell^- - z^I),$$
$$\frac{\pm 1}{2}(S_{\ell^+,K_S}^{\pm} + S_{\ell^-,K_S}^{\pm}) = G_{\psi K_S}S_{\psi K_S}(G_\ell^- - z^R),$$
$$\frac{\pm 1}{2}(S_{\ell^+,K_L}^{\pm} + S_{\ell^-,K_L}^{\pm}) = G_{\psi K_L}S_{\psi K_L}(G_\ell^- - z^R). \quad \text{(B2)}$$

From the results of [1] we can get

$$\hat{S}_{\psi K} - \hat{G}_{\psi K}\hat{\theta}_{\psi K}^I = 0.69 \pm 0.04 \quad \text{(B3)}$$

and the following bounds can be deduced

$$|\hat{C}_{\psi K} + \hat{\theta}^R_{\psi K}| < 0.07,$$
$$|G_{\psi K_{S,L}} S_{\psi K_{S,L}} (G^-_\ell - z^R)| < 0.10,$$
$$|S_{\psi K_{S,L}} (S^-_\ell - z^I)| < 0.06, \qquad \text{(B4)}$$

at 2σ level. In case we assume no *CPT* violation, naive combination of the above will lead to

$$|S^-_\ell| < 0.10, \qquad |G^-_\ell| < 0.21, \qquad \text{(B5)}$$

at 2σ level.

APPENDIX C: EXPERIMENTAL ASYMMETRIES

In this appendix, we provide the full expressions for the asymmetries measured at *BABAR*. For the relation between our notation (30) and those of Table I of [1], see Eq. (35) and the discussion below it. The asymmetries measured by *BABAR* are the following:

$$\Delta S^+_T = \frac{S_{(\psi K_L)_\perp, \ell^- X}}{\kappa_{(\psi K_L)_\perp, \ell^- X}} - \frac{S_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}}, \qquad \Delta C^+_T = \frac{C_{(\psi K_L)_\perp, \ell^- X}}{\kappa_{(\psi K_L)_\perp, \ell^- X}} - \frac{C_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}},$$

$$\Delta S^-_T = \frac{S_{(\ell^- X)_\perp, \psi K_L}}{\kappa_{(\ell^- X)_\perp, \psi K_L}} - \frac{S_{(\psi K_S)_\perp, \ell^+ X}}{\kappa_{(\psi K_S)_\perp, \ell^+ X}}, \qquad \Delta C^-_T = \frac{C_{(\ell^- X)_\perp, \psi K_L}}{\kappa_{(\ell^- X)_\perp, \psi K_L}} - \frac{C_{(\psi K_S)_\perp, \ell^+ X}}{\kappa_{(\psi K_S)_\perp, \ell^+ X}},$$

$$\Delta S^+_{CP} = \frac{S_{(\ell^- X)_\perp, \psi K_S}}{\kappa_{(\ell^- X)_\perp, \psi K_S}} - \frac{S_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}}, \qquad \Delta C^+_{CP} = \frac{C_{(\ell^- X)_\perp, \psi K_S}}{\kappa_{(\ell^- X)_\perp, \psi K_S}} - \frac{C_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}},$$

$$\Delta S^-_{CP} = \frac{S_{(\psi K_S)_\perp, \ell^- X}}{\kappa_{(\psi K_S)_\perp, \ell^- X}} - \frac{S_{(\psi K_S)_\perp, \ell^+ X}}{\kappa_{(\psi K_S)_\perp, \ell^+ X}}, \qquad \Delta C^-_{CP} = \frac{C_{(\psi K_S)_\perp, \ell^- X}}{\kappa_{(\psi K_S)_\perp, \ell^- X}} - \frac{C_{(\psi K_S)_\perp, \ell^+ X}}{\kappa_{(\psi K_S)_\perp, \ell^+ X}},$$

$$\Delta S^+_{CPT} = \frac{S_{(\psi K_L)_\perp, \ell^+ X}}{\kappa_{(\psi K_L)_\perp, \ell^+ X}} - \frac{S_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}}, \qquad \Delta C^+_{CPT} = \frac{C_{(\psi K_L)_\perp, \ell^+ X}}{\kappa_{(\psi K_L)_\perp, \ell^+ X}} - \frac{C_{(\ell^+ X)_\perp, \psi K_S}}{\kappa_{(\ell^+ X)_\perp, \psi K_S}},$$

$$\Delta S^-_{CPT} = \frac{S_{(\ell^+ X)_\perp, \psi K_L}}{\kappa_{(\ell^+ X)_\perp, \psi K_L}} - \frac{S_{(\psi K_S)_\perp, \ell^+ X}}{\kappa_{(\psi K_S)_\perp, \ell^+ X}}, \qquad \Delta C^-_{CPT} = \frac{C_{(\ell^+ X)_\perp, \psi K_L}}{\kappa_{(\ell^+ X)_\perp, \psi K_L}} - \frac{C_{(\psi K_S)_\perp, \ell^+ X}}{\kappa_{(\psi K_S)_\perp, \ell^+ X}}. \qquad \text{(C1)}$$

We find the following expressions for these asymmetries:

$$\Delta S^+_T = -\Delta S^-_T = -2[\hat{S}_{\psi K}(1 + \hat{G}_{\psi K}(G^-_\ell - z^R)) - \hat{G}_{\psi K}\hat{\theta}^I_{\psi K}],$$
$$\Delta C^+_T = \Delta C^-_T = 2[\hat{C}_{\psi K} + \hat{S}_{\psi K}(S^-_\ell - z^I) + \hat{\theta}^R_{\psi K}],$$
$$\Delta S^+_{CP} = -\Delta S^-_{CP} = -2[S_{\psi K_S} - G_{\psi K_S}\theta^I_{\psi K_S} + S_{\psi K_S}G_{\psi K_S}G^+_\ell - z^I(1 - \hat{S}^2_{\psi K})],$$
$$\Delta C^+_{CP} = \Delta C^-_{CP} = 2[C_{\psi K_S} + S_{\psi K_S}S^+_\ell + \theta^R_{\psi K_S} + G_{\psi K_S}z^R],$$
$$\Delta S^+_{CPT} = -\Delta S^-_{CPT} = -2[\Delta S_{\psi K} - z^I(1 - \hat{S}^2_{\psi K}) + \hat{G}_{\psi K}(\hat{S}_{\psi K}G_{\ell^+} - \Delta\theta^I_{\psi K} - \hat{S}_{\psi K}z^R)],$$
$$\Delta C^+_{CPT} = \Delta C^-_{CPT} = 2[\Delta C_{\psi K} + \Delta\theta^R_{\psi K} + \hat{S}_{\psi K}(S_{\ell^+} - z^I) + \hat{G}_{\psi K}z^R]. \qquad \text{(C2)}$$

We notice that not only do the T-asymmetries get T-even contributions, as explained in Sec. III B, but also the *CPT* asymmetries get *CPT*-even contributions. All of these effects vanish if there is neither *CPT* violaton in strangeness changing decays nor wrong strangeness decays nor wrong sign decays.

APPENDIX D: THEORETICAL ASYMMETRIES

It is interesting to define "theoretical" asymmetries where the initial states of the corresponding experimental asymmetries are replaced by the time-conjugate of the final states. Thus, instead of the experimental method, of observing one of two entangled *B*-mesons decaying, thus projecting the other *B*-meson onto the orthogonal state, here we refer to corresponding "gedanken experiments," that start with inverse decays:

$$\Delta S_T^{+(t)} = \frac{S_{(\psi K_S)^T, \ell^- X}}{\kappa_{(\psi K_S)^T, \ell^- X}} - \frac{S_{(\ell^- X)^T, \psi K_S}}{\kappa_{(\ell^- X)^T, \psi K_S}}, \qquad \Delta C_T^{+(t)} = \frac{C_{(\psi K_S)^T, \ell^- X}}{\kappa_{(\psi K_S)^T, \ell^- X}} - \frac{C_{(\ell^- X)^T, \psi K_S}}{\kappa_{(\ell^- X)^T, \psi K_S}},$$

$$\Delta S_T^{-(t)} = \frac{S_{(\ell^+ X)^T, \psi K_L}}{\kappa_{(\ell^+ X)^T, \psi K_L}} - \frac{S_{(\psi K_L)^T, \ell^+ X}}{\kappa_{(\psi K_L)^T, \ell^+ X}}, \qquad \Delta C_T^{-(t)} = \frac{C_{(\ell^+ X)^T, \psi K_L}}{\kappa_{(\ell^+ X)^T, \psi K_L}} - \frac{C_{(\psi K_L)^T, \ell^+ X}}{\kappa_{(\psi K_L)^T, \ell^+ X}},$$

$$\Delta S_{CP}^{+(t)} = \frac{S_{(\ell^+ X)^T, \psi K_S}}{\kappa_{(\ell^+ X)^T, \psi K_S}} - \frac{S_{(\ell^- X)^T, \psi K_S}}{\kappa_{(\ell^- X)^T, \psi K_S}}, \qquad \Delta C_{CP}^{+(t)} = \frac{C_{(\ell^+ X)^T, \psi K_S}}{\kappa_{(\ell^+ X)^T, \psi K_S}} - \frac{C_{(\ell^- X)^T, \psi K_S}}{\kappa_{(\ell^- X)^T, \psi K_S}},$$

$$\Delta S_{CP}^{-(t)} = \frac{S_{(\psi K_L)^T, \ell^- X}}{\kappa_{(\psi K_L)^T, \ell^- X}} - \frac{S_{(\psi K_L)^T, \ell^+ X}}{\kappa_{(\psi K_L)^T, \ell^+ X}}, \qquad \Delta C_{CP}^{-(t)} = \frac{C_{(\psi K_L)^T, \ell^- X}}{\kappa_{(\psi K_L)^T, \ell^- X}} - \frac{C_{(\psi K_L)^T, \ell^+ X}}{\kappa_{(\psi K_L)^T, \ell^+ X}},$$

$$\Delta S_{CPT}^{+(t)} = \frac{S_{(\psi K_S)^T, \ell^+ X}}{\kappa_{(\psi K_S)^T, \ell^+ X}} - \frac{S_{(\ell^- X)^T, \psi K_S}}{\kappa_{(\ell^- X)^T, \psi K_S}}, \qquad \Delta C_{CPT}^{+(t)} = \frac{C_{(\psi K_S)^T, \ell^+ X}}{\kappa_{(\psi K_S)^T, \ell^+ X}} - \frac{C_{(\ell^- X)^T, \psi K_S}}{\kappa_{(\ell^- X)^T, \psi K_S}},$$

$$\Delta S_{CPT}^{-(t)} = \frac{S_{(\ell^- X)^T, \psi K_L}}{\kappa_{(\ell^- X)^T, \psi K_L}} - \frac{S_{(\psi K_L)^T, \ell^+ X}}{\kappa_{(\psi K_L)^T, \ell^+ X}}, \qquad \Delta C_{CPT}^{-(t)} = \frac{C_{(\ell^- X)^T, \psi K_L}}{\kappa_{(\ell^- X)^T, \psi K_L}} - \frac{C_{(\psi K_L)^T, \ell^+ X}}{\kappa_{(\psi K_L)^T, \ell^+ X}}. \tag{D1}$$

We use the same approximations as in Sec. III. We find:

$$\Delta S_T^{+(t)} = -2S_{\psi K_S}[1 - G_{\psi K_S}(z^R + G_\ell^+)], \qquad \Delta S_T^{-(t)} = -2S_{\psi K_L}[1 + G_{\psi K_L}(z^R - G_\ell^+)],$$

$$\Delta C_T^{+(t)} = 2[C_{\psi K_S} - S_{\psi K_S}(z^I + S_\ell^+)], \qquad \Delta C_T^{-(t)} = 2[C_{\psi K_L} + S_{\psi K_L}(z^I - S_\ell^+)],$$

$$\Delta S_{CP}^{+(t)} = -2[S_{\psi K_S} - G_{\psi K_S}\theta_{\psi K_S}^I - S_{\psi K_S}G_{\psi K_S}G_\ell^+ - z^I(1 - S_{\psi K_S}^2)],$$

$$\Delta S_{CP}^{-(t)} = -2[S_{\psi K_L} + G_{\psi K_L}\theta_{\psi K_L}^I - S_{\psi K_L}G_{\psi K_L}G_\ell^+ + z^I(1 - S_{\psi K_L}^2)],$$

$$\Delta C_{CP}^{+(t)} = 2[C_{\psi K_S} - S_{\psi K_S}S_\ell^+ + \theta_{\psi K_S}^R + G_{\psi K_S}z^R],$$

$$\Delta C_{CP}^{-(t)} = 2[C_{\psi K_L} - S_{\psi K_L}S_\ell^- - \theta_{\psi K_L}^R - G_{\psi K_L}z^R],$$

$$\Delta S_{CPT}^{+(t)} = 2[z^I(1 - S_{\psi K_S}^2) + G_{\psi K_S}(\theta_{\psi K_S}^I + S_{\psi K_S}z^R)],$$

$$\Delta S_{CPT}^{-(t)} = -2[z^I(1 - S_{\psi K_L}^2) + G_{\psi K_L}(\theta_{\psi K_L}^I + S_{\psi K_L}z^R)],$$

$$\Delta C_{CPT}^{+(t)} = 2[\theta_{\psi K_S}^R - S_{\psi K_S}z^I + G_{\psi K_S}z^R],$$

$$\Delta C_{CPT}^{-(t)} = -2[\theta_{\psi K_L}^R - S_{\psi K_L}z^I + G_{\psi K_L}z^R]. \tag{D2}$$

As expected, the theoretical T asymmetries have only T-odd contributions, the theoretical *CP* asymmetries have only *CP*-odd contributions, and the theoretical *CPT* asymmetries have only *CPT*-odd contributions. Furthermore, in the absence of wrong strangeness decays, wrong sign decays and *CPT* violation in strangeness changing decays, the theoretical asymmetries equal the corresponding experimental asymmetries.

APPENDIX E: EPR ENTANGLEMENT WITH *CPT* VIOLATION

In this appendix we show that the factorization of the decay amplitudes to $e^{-\Gamma(t_1+t_2)} \times f(t_2 - t_1)$ holds in the presence of *CPT* violation. We follow Ch. 9 of [13] with the appropriate modifications for the *CPT* violating case. The initial pair is in a state

$$|\Phi^c\rangle = \frac{1}{\sqrt{2}}[|B(\vec{k})\rangle \otimes |\bar{B}(-\vec{k})\rangle - |\bar{B}(\vec{k})\rangle \otimes |B(-\vec{k})\rangle], \tag{E1}$$

where the relative $(-)$ sign is a result of the C-parity of the $\Upsilon(4S)$. The $\pm\vec{k}$ are the three momenta of the left and right moving meson in the resonance rest frame.

The decay amplitude of the meson with momenta $\vec{k}$ into final state f_1 and of the one with momenta $-\vec{k}$ to final state f_2 at times t_1 and t_2, respectively, is ($t \equiv t_2 - t_1$)

$$\langle f_1, t_1; f_2, t_2 | T | \Phi^c \rangle = \frac{e^{-(\Gamma/2 + im)(t_1 + t_2)}}{\sqrt{2}}$$

$$\times \left[\left(\sqrt{1 - z^2} \left(\frac{p}{q} A_{f_1} A_{f_2} - \frac{q}{p} \bar{A}_{f_1} \bar{A}_{f_2} \right) \right. \right.$$

$$\left. + z(\bar{A}_{f_2} A_{f_1} + \bar{A}_{f_1} A_{f_2}) \right) g_-(t)$$

$$\left. + (\bar{A}_{f_2} A_{f_1} - \bar{A}_{f_1} A_{f_2}) g_+(t) \right], \tag{E2}$$

where

$$g_+(t) = \cos\left(\frac{x\Gamma t}{2}\right)\cosh\left(\frac{y\Gamma t}{2}\right) + i\sin\left(\frac{x\Gamma t}{2}\right)\sinh\left(\frac{y\Gamma t}{2}\right),$$

$$g_-(t) = \cos\left(\frac{x\Gamma t}{2}\right)\sinh\left(\frac{y\Gamma t}{2}\right) + i\sin\left(\frac{x\Gamma t}{2}\right)\cosh\left(\frac{y\Gamma t}{2}\right). \qquad (E3)$$

By squaring the absolute value of the amplitude in Eq. (E2) we get that it factorizes as $e^{-\Gamma(t_1+t_2)} \times f(t)$.

[1] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **109**, 211801 (2012).

[2] M. C. Banuls and J. Bernabeu, Phys. Lett. B **464**, 117 (1999); Nucl. Phys. **B590**, 19 (2000).

[3] L. Wolfenstein, Int. J. Mod. Phys. E **08**, 501 (1999).

[4] J. Bernabeu, F. Martinez-Vidal, and P. Villanueva-Perez, J. High Energy Phys. 08 (2012) 064.

[5] E. Alvarez and A. Szynkman, Mod. Phys. Lett. A **23**, 2085 (2008).

[6] J. Bernabeu, F. J. Botella, and M. Nebot, Phys. Lett. B **728**, 95 (2014).

[7] A. Angelopoulos *et al.* (CPLEAR Collaboration), Phys. Lett. B **444**, 43 (1998).

[8] P. K. Kabir, Phys. Rev. D **2**, 540 (1970).

[9] K. R. Schubert, B. Wolff, J. C. Chollet, J. M. Gaillard, M. R. Jane, T. J. Ratcliffe, and J. P. Repellin, Phys. Lett. B **31**, 662 (1970).

[10] L. Lavoura, Ann. Phys. (N.Y.) **207**, 428 (1991).

[11] L. Alvarez-Gaume, C. Kounnas, S. Lola, and P. Pavlopoulos, Phys. Lett. B **458**, 347 (1999).

[12] J. P. Silva, Phys. Rev. D **62**, 116008 (2000).

[13] G. C. Branco, L. Lavoura, and J. P. Silva, Int. ser. monogr. phys. **103**, 1 (1999).

[14] Y. Grossman, A. L. Kagan, and Z. Ligeti, Phys. Lett. B **538**, 327 (2002).

[15] Y. I. Azimov and A. A. Iogansen, Yad. Fiz. **33**, 388 (1981) [Sov. J. Nucl. Phys.**33**, 205 (1981)].

[16] Y. I. Azimov, Eur. Phys. J. A **4**, 21 (1999).

[17] H. J. Lipkin and Z.-z. Xing, Phys. Lett. B **450**, 405 (1999).

[18] I. I. Bigi and A. I. Sanda, Phys. Lett. B **625**, 47 (2005).

[19] G. Calderon, D. Delepine, and G. L. Castro, Phys. Rev. D **75**, 076001 (2007).

[20] Y. Grossman and Y. Nir, J. High Energy Phys. 04 (2012) 002.

[21] Y. Grossman, Phys. Lett. B **359**, 141 (1995).

[22] M. C. Gonzalez-Garcia, Y. Grossman, A. Gusso, and Y. Nir, Phys. Rev. D **64**, 096006 (2001).

[23] B. Bellazzini, Y. Grossman, I. Nachshon, and P. Paradisi, J. High Energy Phys. 06 (2011) 104.

[24] Y. Amhis *et al.* (Heavy Flavor Averaging Group Collaboration), arXiv:1207.1158.

[25] J. Beringer *et al.* (Particle Data Group Collaboration), Phys. Rev. D **86**, 010001 (2012).

[26] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **70**, 012007 (2004).

[27] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **93**, 081801 (2004).

[28] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **79**, 072009 (2009).