

# Searches for $CP$ -violating dimension-6 electroweak gauge boson operators

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We correct a number of earlier calculations of the loop contribution of a dimension-6,  $CP$ -violating operator involving  $W^+W^-\gamma$  to the neutron electric dipole moment, showing that measurements imply a very strong bound on the operator. We also quantify the link between this operator and a companion operator involving  $W^+W^-Z$ , which has been suggested as a target for new physics searches at the Large Hadron Collider, showing that even strongly coupled new physics could only be observable in the proposed searches if it appeared at a scale below  $\sim 170$  GeV.

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## I. INTRODUCTION

Despite its laudable performance, the first run of the Large Hadron Collider (LHC) saw no evidence for physics beyond the Standard Model, and neither have many other experiments, putting the naturalness paradigm under severe pressure. This has the twofold effect of pushing the bounds on the new physics scale higher and making theorists' rhetoric about what we should look for less convincing. In light of this, it makes increasing sense for experiments to frame their searches in terms of effective Lagrangians, in which new physics is parameterized by higher dimension operators built out of the Standard Model degrees of freedom. Even if no new physics is found, this approach ensures that the LHC leaves a useful legacy in its wake, in the form of optimal, model-independent constraints on possible new physics.

Of particular interest (independent of the naturalness issue) are higher dimensional operators violating  $CP$ , which could generate the baryon asymmetry in the Universe. In this work, we examine two such dimension-6 operators [1], namely,  $\mathcal{O}_\gamma \equiv W_\nu^{+\mu} W_\lambda^{-\nu} \tilde{F}_\mu^\lambda$  and  $\mathcal{O}_Z \equiv W_\nu^{+\mu} W_\lambda^{-\nu} \tilde{Z}_\mu^\lambda$ , where  $W_\nu^\pm$  is the usual field strength tensor for  $W^\pm$ ,  $\tilde{F}_{\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$  is the dual field strength tensor for the photon, and  $\tilde{Z}_{\mu\nu}$  the dual field strength tensor for the  $Z$ .

$\mathcal{O}_\gamma \equiv W_\nu^{+\mu} W_\lambda^{-\nu} \tilde{F}_\mu^\lambda$  contributes to the electric dipole moment (EDM) of the neutron via the 1-loop diagrams in Fig. 1. We have found five independent computations in the literature (see Table I) of the diagrams of Fig. 1; no two sets of authors agree on the result, with one set [5] suggesting a suppression by a factor of  $\sim 10^{-10}$  compared to a naïve estimate. As we explain in Sec. II, most of these discrepancies arise because of an inappropriate choice of regulator. Dimensional regularization is the way forward, and using this, we correct the result of [5] and confirm an earlier result of [3], when interpreted correctly. There is no

suppression of the EDM, and so there is little point in searching for  $\mathcal{O}_\gamma$  at the LHC.

By contrast, LHC searches for  $\mathcal{O}_Z$  have been suggested more than once (although there are still contributions to the neutron EDM at two loops, which we do not consider here) [7–9]. However, as we explain in Sec. III, any attempt to increase the coefficient,  $\alpha_Z$ , of  $\mathcal{O}_Z$  without increasing  $\alpha_\gamma$  of  $\mathcal{O}_\gamma$  to preserve the  $SU(2) \times U(1)$  relation  $c_W \alpha_\gamma = s_W \alpha_Z$  necessarily lowers the cutoff of the EFT. This is most easily seen in an  $SU(2) \times U(1)$ -invariant formalism, where deviations from the relation  $c_W \alpha_\gamma = s_W \alpha_Z$  arise from operators of dimension eight or higher, but it can also be seen easily enough in the original formulation in terms of operators  $\mathcal{O}_\gamma$  and  $\mathcal{O}_Z$ . Moreover, it is evident (in either formulation) that the cutoff is lowered by further factors of gauge couplings. As a result, we conclude that visible effects of the operator  $\mathcal{O}_Z$  at the LHC require a new physics scale around 170 GeV. If there did exist strongly coupled physics at such a low scale, effects would appear all over the place at the LHC.

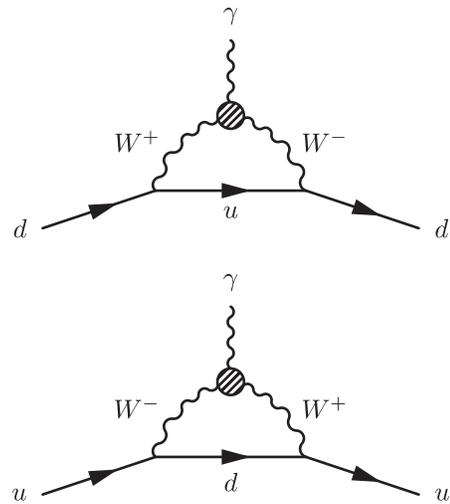


FIG. 1. One-loop EDMs of  $\mathcal{O}_\gamma$ : 1-loop contributions of  $\mathcal{O}_\gamma = W_\nu^{+\mu} W_\lambda^{-\nu} \tilde{F}_\mu^\lambda$  (shaded blob) to the neutron EDM. Custodial symmetry implies that the diagrams change sign under  $m_u \leftrightarrow m_d$ .

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Dedicated searches of the type advocated would be superfluous.

## II. THE 1-LOOP CONTRIBUTION TO THE NEUTRON EDM

Let us first then consider the contribution to the neutron EDM. To facilitate comparison with the existing literature we consider not  $\mathcal{O}_\gamma$  (and  $\mathcal{O}_Z$ ), but rather the  $SU(2) \times U(1)$ -invariant  $CP$ -odd operator  $\mathcal{O}_W = W^{+\mu}_\nu W^{-\nu}_\lambda \tilde{W}^{3\lambda}_\mu$  [and the  $SU(2) \times U(1)$ -noninvariant operator  $\mathcal{O}_B \equiv W^{+\mu}_\nu W^{-\nu}_\lambda \tilde{B}^\lambda_\mu$ ] [10,11].

We list the results of the five calculations in Table I. One can see from the table that there is disagreement not only on the result but also on whether the 1-loop diagrams are finite or not. Boudjema *et al.* [3] did the computation with a number of different regulators and obtained a number of different results, showing that any result can be obtained by varying the regulator; we quote only the result obtained using dimensional regularization in the table.

Most of these discrepancies are resolved by noting that unlike for the case of renormalizable quantum field theory (where many regulators lead to identical results) in non-renormalizable EFT one cannot use a dimensionful cutoff. Indeed, EFT works because tree-level contributions to scattering from the infinity of higher dimension operators are suppressed by powers of the small external momenta. But these same operators are, with a generic regulator, unsuppressed in loop diagrams, where energies up to the cutoff are allowed (see, e.g., [13]). The only way to maintain predictivity, i.e., to maintain a finite number of loop diagrams with sizable contributions, is to use a dimensionless regulator, such as dimensional regularization with  $\overline{\text{MS}}$ . Here the only mass scales that can appear in the numerators of diagrams correspond to light masses or momenta, with the renormalization scale appearing only in logarithms.

This still leaves us with the problem that two of the results in the table are done using  $\overline{\text{MS}}$  but nevertheless disagree, with that of Novales-Sánchez and Toscano [5]

TABLE I. EDM calculations: the effective operator  $-\frac{1}{2}d_f\bar{\psi}\sigma^{\mu\nu}\psi\tilde{F}_{\mu\nu}$  for a down quark  $\psi$  of mass  $m_f$  due to a 1-loop diagram including the operator  $-i\alpha_\gamma W^{+\mu}_\nu W^{-\nu}_\lambda \tilde{F}^\lambda_\mu$ . The sign of the result is reversed for an up quark.

Authors	Regularization	$d_f$
Atwood <i>et al.</i> [2]	cutoff $\Lambda$	$m_f\alpha_\gamma \frac{g_W^2}{64\pi^2} \left[ \ln\left(\frac{\Lambda^2}{m_f^2}\right) + \mathcal{O}(1) \right]$
Boudjema <i>et al.</i> [3]	$\overline{\text{MS}}$	$m_f\alpha_\gamma \frac{g_W^2}{64\pi^2}$
Hoogeveen [4]	cutoff $\Lambda$	0
Novales-Sánchez and Toscano [5]	$\overline{\text{MS}}$	$m_f\alpha_\gamma \frac{g_W^2}{64\pi^2} \cdot \frac{2m_f^2}{3m_W^2}$
de Rújula <i>et al.</i> [6]	cutoff $\Lambda$	$m_f\alpha_\gamma \frac{g_W^2}{64\pi^2} \cdot \frac{2}{s_W^2} \ln\left(\frac{\Lambda^2}{m_f^2}\right)$

being suppressed by a factor  $m_f^2/m_W^2$ , where  $m_f$  is a light quark mass. Two purported explanations for the suppression are given in [5]. The first is custodial symmetry. The operator  $\mathcal{O}_W$  is indeed invariant under a  $SU(2)_L \times SU(2)_R$  symmetry under which the  $W$  boson transforms as a  $(3,1)$ , but this cannot explain the suppression. This is easily seen in the following way. In the limit  $m_u = m_d$ , custodial symmetry becomes an *exact* symmetry of all of the interactions appearing in the diagrams of Fig. 1. If custodial symmetry suppresses the EDM, then the result quoted in [5] should vanish in this same limit, but it does not. In fact, custodial symmetry does not imply a constraint on either diagram; rather it relates the two diagrams, which sum to zero in the limit  $m_u = m_d$ . [To see this, consider the element of  $SU(2)_L \times SU(2)_R$  given in the fundamental representation by  $L = R = e^{\frac{i\sigma^1}{2}} = i\sigma^1$ . Up to an overall phase, this effects the transformation  $W^1 \rightarrow W^1$ ,  $W^2 \rightarrow -W^2$ ,  $W^3 \rightarrow -W^3$ ,  $u_L \rightarrow d_L$ ,  $d_L \rightarrow u_L$ . The upshot is that one of the charged current vertices picks up a minus sign when transforming from top to bottom in Fig. 1.] The second explanation invokes the decoupling theorem [14], which, applied to the situation at hand, states that all effects of  $W$  bosons on low-energy physics (such as the neutron EDM) should decouple in the limit  $m_W \rightarrow \infty$ . While this is quite true, one cannot take the limit  $m_W \rightarrow \infty$  within an EFT without simultaneously taking  $1/\sqrt{\alpha_\gamma} \equiv \Lambda_\gamma \rightarrow \infty$ . Thus, the  $\alpha_\gamma$  in the EDM result guarantees that the decoupling theorem is obeyed, without the need for an extra factor of  $m_W^2$ . In fact, it turns out that the calculation in [5] is erroneous: we have repeated the computation independently, and we obtain the same result as Ref. [3] does using  $\overline{\text{MS}}$ , namely

$$d_f = m_f\alpha_\gamma \frac{g_W^2}{64\pi^2}. \quad (1)$$

This may be translated into a bound on  $\alpha_\gamma$ . Expressing a fermion  $f$ 's EDM operator as  $-\frac{1}{2}id_f\bar{\psi}_f\sigma^{\mu\nu}\psi_f\tilde{F}_{\mu\nu}$ , experiment gives  $|d_n| < 2.9 \times 10^{-26} e \text{ cm}$  at 90% C.L. for the neutron [15]. We use the form factors of [16] to convert this into quark EDM bounds:  $d_n \sim 1.77d_d - 0.48d_u$ . The result (1) gives  $|\alpha_\gamma| \lesssim 6 \times 10^{-8} \text{ GeV}^{-2}$  [17]. Evidently, barring implausible cancellations against contributions coming from other higher dimension operators, we cannot hope to see an effect from  $\mathcal{O}_W$  alone in searches at the LHC.

## III. CUTOFFS IN AN EFT WITH $\mathcal{O}_B$

Having discounted  $\mathcal{O}_W$  alone as a means of getting a visible  $WWZ$  effect at the LHC, we now consider the operator  $\mathcal{O}_B = W^{+\mu}_\nu W^{-\nu}_\lambda \tilde{B}^\lambda_\mu$ . This latter operator does not respect the  $SU(2) \times U(1)$  gauge symmetry of the renormalizable standard model (SM). Now, this invariance is clearly not a *sine qua non*—it is violated, for example, in the mass terms of the  $W$  and  $Z$  bosons in a Higgsless model. However, then, as now, we must ask what the cutoff of the

theory is. One can see that the cutoff must be lowered in the case at hand by considering the contributions to a scattering amplitude of operators with different powers of momenta: when these are equal, the momentum expansion of EFT breaks down, and predictivity is lost.

To this end, consider the diagrams of Fig. 2, which represent contributions to  $W^+W^-B$  scattering at different orders in the momentum expansion. For simplicity, we assume that the gauge couplings and hence the masses and mixings are small: this suffices us to derive the functional dependence of the cutoff. The first diagram, which is at leading order in the momentum expansion, arises in the SM from the three-point Yang-Mills vertex with an insertion of the  $W^3$ - $B$  mixing operator  $\frac{1}{2}gg'v^2W_\mu^3B^\mu$ , where  $v = 246$  GeV. It has size  $gg'v^2 \cdot \frac{1}{p^2} \cdot gp$ , whereas the second diagram involves  $\mathcal{O}_B$ , arises at the next order in the momentum expansion, and has value  $\alpha_B p^3$ . The momentum expansion breaks down roughly at a scale  $\Lambda$  where these terms become of equal size, namely when

$$\Lambda \sim (g^2g'v^2\Lambda_B^2)^{\frac{1}{4}}, \quad (2)$$

using  $\Lambda_B \equiv 1/\sqrt{\alpha_B}$ . It is thus clear that the cutoff that follows from the presence of  $\mathcal{O}_B$  is not the naive  $\Lambda_B$  but is suppressed. The suppression comes not just from the ratio  $v/\Lambda_B$ , but also from factors of the gauge couplings. Thus, for a given size of  $\alpha_B$  (which sets the size of new physics effects of  $\mathcal{O}_B$  at the LHC and elsewhere), we find a cutoff that is rather lower than the naive one.

Our result can be obtained in a more perspicuous fashion by using a formalism that is completely equivalent but in which  $SU(2) \times U(1)$  is manifest, albeit nonlinearly realized [19,20]. To do so, we define the sigma model field  $\Sigma = e^{\frac{i\pi^a\sigma^a}{v\sqrt{2}}}$ , where  $\pi^a$  are three Goldstone boson fields and  $\sigma^a$  are the Pauli matrices. Under an  $SU(2) \times U(1)$  transformation,  $\Sigma \rightarrow U_L \Sigma U_Y^\dagger \equiv e^{i\alpha_L \frac{\sigma^a}{2}} \Sigma e^{-i\alpha_Y \frac{\sigma^3}{2}}$ . The terms

$$+m_W^2 W^{+\mu} W_\mu^- + \frac{1}{2}m_Z^2 Z^\mu Z_\mu - i\frac{1}{\Lambda_B^2} W^{+\mu} W_\nu^- \tilde{B}_\mu^\lambda \quad (3)$$

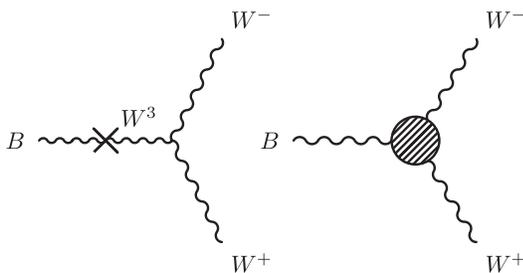


FIG. 2.  $W^+W^-B$  scattering: contributions to  $W^+W^-B$  scattering in the SM with the operator  $\mathcal{O}_B = W^{+\mu}W_\nu^- \tilde{B}_\mu^\lambda$  (denoted by a shaded blob) added. The  $\times$  denotes the mixing between  $W^3$  and  $B$  in the SM.

in our original EFT Lagrangian can then be rewritten as

$$\frac{v^2}{2} \text{Tr}((D_\mu \Sigma)^\dagger (D^\mu \Sigma)) - \frac{2}{\Lambda_B^2} \text{Tr}(\Sigma^\dagger W_\nu^\mu W_\lambda^\nu \Sigma \tilde{B}_\mu^\lambda), \quad (4)$$

where  $D_\mu \equiv \partial_\mu + igW_\mu^a \frac{\sigma^a}{2} + ig'B_\mu \frac{\sigma^3}{2}$ . To see that the two EFTs are equivalent, it suffices to fix the gauge  $\Sigma = 1$  in (4).

As has been emphasized, e.g., in [21], the formulation (4) is far more convenient for the purposes of extracting the EFT cutoff  $\Lambda$ . Indeed, the cutoff is finite because scattering amplitudes involving longitudinal gauge boson polarizations grow with the energy, but at high energies, we may replace the longitudinal gauge bosons by Goldstone bosons, and the cutoff can be extracted from the latter via the Goldstone boson equivalence theorem. The result [22,23] is that if we write the EFT as

$$\mathcal{L} = \Lambda^2 v^2 F\left(\frac{\partial}{\Lambda}, \frac{gA}{\Lambda}, \Sigma\right) \quad (5)$$

(where we have generically indicated a gauge field and its coupling by  $A$  and  $g$ , respectively), then the theory has cutoff  $\Lambda \lesssim 4\pi v$  [24]. This result immediately tells us that the coefficient  $\alpha_B$  in (4) is given by

$$\alpha_B \sim \frac{g^2g'v^2}{\Lambda^4}, \quad (6)$$

where  $\Lambda$  is the true cutoff of the theory [25]. Once again, we see that the cutoff is not  $\Lambda_B \equiv 1/\sqrt{\alpha_B}$ .

Thus far, we have avoided referring to the Higgs doublet  $H$ , but it is straightforward to include it in the discussion. If the Higgs is present (and the LHC suggests that it is), then we have one more field that can be included in our EFT [27]. This can be done straightforwardly by the replacement  $\Sigma \rightarrow H$  in (4). The Higgs field unitarizes gauge boson scattering, and so the cutoff of the resulting EFT can be made arbitrarily large. Nevertheless, the operator  $\alpha'_B \text{Tr}(H^\dagger W_\nu^\mu W_\lambda^\nu H \tilde{B}_\mu^\lambda)$  is of dimension eight, and the resulting  $WWZ$  operator has coefficient  $\sim \alpha'_B v^2$ . Yet again, observability of the effects of  $\alpha'_B \neq 0$  implies new physics at a low energy scale.

Now that we have some confidence in our result, we should ask just how low the cutoff must be in order for us to have a chance of seeing the effects of  $\mathcal{O}_Z$  at the LHC. To date, there have been no dedicated ATLAS or CMS searches for such operators [28], and so we content ourselves with reinterpreting the projections of [7] for searches for  $CP$  violation via the operator  $\alpha_Z \mathcal{O}_Z$ , in the light of our results. A  $CP$ -odd observable is constructed using the momenta of the leptonic decay products of a  $W^+W^-$  pair. Using reasonable cuts the authors find, for the SM plus  $\mathcal{O}_Z$ , with  $100 \text{ fb}^{-1}$  of data, the 14-TeV LHC is sensitive at the  $7\sigma$  level to  $|\alpha_Z m_W^2| = 0.1$ . The nonzero contribution to the  $CP$ -odd observable comes from the interference between

SM and  $\mathcal{O}_Z$  amplitudes, a term linear in  $\alpha_Z$ , whereas the statistical fluctuations in the number of events (i.e. the size of a  $\sigma$ ) come predominantly from the constant SM cross section. Hence, for a  $5\sigma$  detection we require  $|\alpha_Z m_W^2| \gtrsim \frac{5}{7} \times 0.1$ , or equivalently  $|\alpha_Z| \gtrsim (300 \text{ GeV})^{-2}$ .

Given our size estimate (6) for  $\alpha_B$  in terms of the true cutoff, we conclude that this maximum of sensitivity corresponds to a theory with cutoff  $\Lambda \sim 170 \text{ GeV}$ . An electroweak sector that becomes strongly coupled at this energy would contain an infinite set of non-SM effective operators, each with  $O(1)$  effects on scattering amplitudes at momenta  $\sim 170 \text{ GeV}$ , and by extension  $O(1)$  contributions to electroweak precision tests. Needless to say, such large effects are absent in existing measurements. We infer

from this absence that the effects of  $\mathcal{O}_Z$ , like those of  $\mathcal{O}_\gamma$ , are unlikely to be seen at the LHC.

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- [11]  $CP$ -odd operators such as  $\mathcal{O}_W$  and  $\mathcal{O}_B$  also contribute to  $CP$ -even electroweak precision observables, via diagrams containing  $\geq 2$  insertions. For example,  $\mathcal{O}_B$  gives contributions to  $g^2 \hat{U}$  and  $g^{-2} m_W^2 \hat{T}$  (defined as in [12]) of order  $\alpha_B^2 \frac{m_W^4}{16\pi^2}$  and  $\alpha_B^2 g^2 \frac{m_W^6}{(16\pi^2)^2}$ , respectively, implying weak bounds of  $\alpha_B \lesssim (100 \text{ GeV})^{-2}$  and  $\alpha_B \lesssim (20 \text{ GeV})^{-2}$ , respectively. Note that the bound from  $\hat{T}$  is unusually poorly constraining as  $\mathcal{O}_B$ 's contribution must be 2-loop (any 1-loop diagram of two  $\mathcal{O}_B$ 's has derivatives on the external legs).  $\mathcal{O}_W$ 's contribution to  $2g^{-2} m_W^{-2} W$  of  $\sim \alpha_W^2 \frac{m_W^2}{16\pi^2}$  gives a similarly loose limit of  $\alpha_W \lesssim (90 \text{ GeV})^{-2}$ .
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