PHYSICAL REVIEW D **89,** 076002 (2014)

Chiral phase transition with mixing between scalar quarkonium and tetraquark

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(Received 14 November 2013; published 31 March 2014)

In the framework of the two-flavor extended linear sigma model with mixing between scalar quarkonium and tetraquark, we investigate the role of the tetraquark in the chiral phase transition. We explore various scenarios depending on the value of various parameters in our model. The physical mass spectrum of mesons put a tight constraint on the parameter set of our model. We find that a sufficiently strong cubic self interaction of the tetraquark field can drive the chiral phase transition to the first order even at zero quark chemical potential. Weak or absence of the cubic self-interaction term of the tetraquark field make the chiral phase transition crossover at the vanishing density.

DOI: 10.1103/PhysRevD.89.076002 PACS numbers: 11.30.Rd, 11.10.Wx, 12.39.Mk

I. INTRODUCTION

The role of chiral condensate is well known and well studied in the context of chiral phase transition. Recently, the possible role of tetraquark condensate in connection to the chiral phase transition has also been considered [1,2]. The reason behind such consideration stems from the unsettled nature of the lightest scalar $f_0(600)$ ($f_0(500)$ in [3]) or σ meson. This issue is part of the unresolved nature of the scalar mesons below 2 GeV. There are about 19 scalar resonances found below 2 GeV that cannot be explained by the naive quark model. Their mass spectrum and decay patterns are also quite contrary to what is expected from the quark model. An intense effort is going on to understand the nature and properties of these mesons (see Refs. [4–7] and references therein).

Theoretical understanding of the lightest scalar $\sigma/f_0(600)$ is important as it is believed to be the Higgs boson of QCD and plays an important role in chiral symmetry breaking. Though its existence has been confirmed from the $\pi\pi$ scattering process [5,8], the consensus on its nature is still elusive. Conventionally, $f_0(600)$ is regarded as composed of quark-antiquark. But, in order to solve the mass hierarchy problem for scalar mesons below 1 GeV, Jaffe [9] in 1977 proposed to consider the scalar mesons below 1 GeV as tetraquark states and those above 1 GeV to be quarkonium states. Thus, in this picture $f_0(600)$ is predominantly a tetraquark state, whereas $f_0(1370)$ is the lightest quarkonium state made up of quark-antiquark. The sizable tetraquark component has also been demonstrated in a recent lattice simulation study [10]. However, there are other suggestions as well, for example, recent data from $\pi\pi$ and $\gamma\gamma$ scattering [11,12], the K-matrix analysis [13], suggests that it has sizable glueball content.

The role of chiral condensate as an order parameter for chiral phase transition is well established. But the role of tetraquark condensate is not understood and work in this direction has recently been started [1,2].

In [1] the implications of mixing between tetraquark and quarkonium fields on chiral phase transition are studied for zero baryon chemical potential. The authors favor the scenario where $f_0(600)$ is tetraquark dominated and the heavy $f_0(1370)$ is quarkonium dominated. According to their study, the order of the phase transition is strongly correlated with the extent of mixing between the two fields. For a weak coupling constant for the mixing term, a soft first order phase transition is obtained. On the other hand, a strong coupling constant for the mixing term gives rise to a crossover transition. Moreover, the most important and interesting result coming out of their study is that beyond a certain maximum temperature the nature of the heavy and lighter mesons is exchanged. The heavy $f_0(1370)$ becomes tetraquark dominated and the lighter $f_0(600)$ turns quarkonium dominated and becomes degenerate with the pion after the chiral symmetry restoring phase transition. Whereas in [2] an alternate breaking of chiral symmetry in dense matter was proposed.

Using Ginzburg-Landau effective potential consisting of two and four quark states they show that in dense matter a possible phase may arise where chiral symmetry is spontaneously broken but its center symmetry remains unbroken. In this phase conventional chiral condensate vanishes and the chiral symmetry breaking is due to the presence of quartic condensate. Finally, chiral symmetry is restored as quartic condensate also vanishes. Existence of a tricritical point is also predicted between the broken and unbroken center symmetric phase. Thus, in this scenario, restoration of chiral symmetry occurs in two steps.

These studies warrant us to study the effect of mixing between the quarkonium and tetraquark condensates on the

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chiral phase transition in detail. Here, in this work, we study the two-flavor chiral phase transition within the framework of the extended linear sigma model taking into account both quarkonium and tetraquark effective fields. We fix parameters from the physical meson masses. Depending on the possible values of the various parameters, the resulting phase diagram is discussed.

The paper is organized as follows: in the next section we discuss the model we are going to consider and how the various parameters in the model are fixed. In Sec. III we present our result, and, finally, we summarize and conclude in the last section.

II. THE MODEL

We are going to investigate the effect of quarkonium and tetraquark mixing on the chiral phase transition in the framework of the quark-meson model. In this model, quarks propagate in the background potential of mesonic fields and interact with the vacuum expectation values of the scalar mean fields via Yukawa coupling. The generic form of the Lagrangian consist of a fermionic part ($\mathcal{L}_\mathrm{q}$) and a mesonic field part ($\mathcal{L}_\mathrm{m}$) and can be written as

$$\mathcal{L} = \mathcal{L}_\mathrm{q} + \mathcal{L}_\mathrm{m} = \bar{q}(i\gamma^\mu\partial_\mu - g_3\Phi - g_4\Phi')q + \mathcal{L}_\mathrm{m}, \quad (1)$$

with the mesonic part of the Lagrangian being

$$\begin{aligned}\mathcal{L}_\mathrm{m} = {}& \mathrm{Tr}(\partial_\mu\Phi\partial^\mu\Phi^\dagger) + \mathrm{Tr}(\partial_\mu\Phi'\partial^\mu\Phi^{\dagger\prime}) - m_\Phi{}^2\mathrm{Tr}(\Phi^\dagger\Phi) \\ & - m_{\Phi'}{}^2\mathrm{Tr}(\Phi^{\dagger\prime}\Phi') + \frac{\lambda_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi\Phi^\dagger\Phi) \\ & + \frac{\lambda_2}{2}\mathrm{Tr}(\Phi^{\dagger\prime}\Phi'\Phi^{\dagger\prime}\Phi') + g_2\mathrm{Tr}(\Phi'\Phi'\Phi') \\ & - g_1\mathrm{Tr}(\Phi')\mathrm{Tr}(\Phi)\mathrm{Tr}(\Phi) + k[\mathrm{Det}(\Phi) + \mathrm{H.c.}] \\ & - h[\mathrm{Tr}(\Phi) + \mathrm{H.c.}], \end{aligned} \quad (2)$$

where, for two light flavors the quark field "q" can be represented as $q = (u, d)$ and g_3, g_4 are the Yuakwa coupling constants for the quarkonium and tetraquark fields, respectively. The mesonic Lagrangian part has two effective fields: a 2×2 matrix field Φ, which denotes the bare quarkonium field, and a 2×2 matrix field Φ', which denotes the bare tetraquark field. Following the convention of the linear sigma model, we express the quarkonium and the tetraquark fields as

$$\Phi = \frac{1}{2}(\sigma_b + \eta_b) + \frac{1}{2}(\vec{\alpha}_b + i\vec{\pi}_b).\vec{\tau}, \quad (3)$$

$$\Phi' = \frac{1}{2}(\sigma'_b + \eta'_b) + \frac{1}{2}(\vec{\alpha}'_b + i\vec{\pi}'_b).\vec{\tau}, \quad (4)$$

with τ_i ($i = 1, 2, 3$) representing the 2×2 Pauli matrix. The transformation properties of these fields under $U(2)_L \times U(2)_R$ symmetry are defined as follows:

$$\Phi \to U_L \Phi U^\dagger{}_R, \quad (5)$$

$$\Phi' \to U_L \Phi' U^\dagger{}_R, \quad (6)$$

where $U_{L,R}$ are group elements of the $U(2)_L \times U(2)_R$ symmetry.

The mesonic spectra consist of sixteen physical mesons: a pair of scalar isoscalars $\{f_0(600), f_0(1370)\}$, a pair of pseudoscalar isoscalars $\{\eta_p, \eta'_p\}$, a pair of scalar isovectors $\{\vec{\alpha}_p, \vec{\alpha}_p{}'\}$, and a pair of pseudoscalar isovectors $\{\vec{\pi}_p, \vec{\pi}_p{}'\}$. Here, pseudoscalar isoscalar η_p and η'_p mesons are composed of u and d quarks only. The bare quarkonium and tetraquark fields mixed with each other to give rise to physical mesonic fields, one of them being quarkonium dominated and the other tetraquark dominated mesons.

In mesonic part of the Lagrangian [see Eq. (2)], the cubic term for the tetraquark meson with the coupling constant g_2, the mixing term between quarkonium and tetraquark with the coupling constant g_1, and the last term mimicking the finite quark mass for the quarkonium with the coupling constant h explicitly breaks the $U(2)_L \times U(2)_R$ symmetry, whereas the instanton determinant term explicitly breaks the axial $U(1)_A$ symmetry. The other terms in the potential part of the Lagrangian are invariant under $U(2)_L \times U(2)_R$ symmetry. However, we spontaneously break the $SU(2)_A$ part of the symmetry of these terms as well by assuming vacuum expectation values for the σ_b and σ'_b fields. The mass and the quartic interaction terms are the standard terms used in the linear σ model. An explicit symmetry breaking term to account for the finite quark mass and an instanton determinant term for the field Φ are also used. The explicit chiral symmetry breaking terms for the field Φ render π_b and η_b massive. The instanton determinant term is responsible for the splitting of masses between π_b and η_b. The choice of the cubic term is motivated from the study in Ref. [2].

We will investigate the chiral phase transition at the mean field level. Following the standard procedure, we expand the fields around the vacuum expectation values: $\sigma_b = \sigma + \sigma_f$ and $\sigma'_b = \chi + \sigma'_f$, where σ and χ are the vacuum expectation values of the corresponding fields. Keeping only the mean fields and integrating out the fermionic fields, we obtain (neglecting the ultraviolet divergent vacuum energy term [14,15]) the expression for the thermodynamical potential at temperature T and chemical potential μ as

$$\begin{aligned}\Omega = {}& U(\sigma,\chi) - 2TN_cN_f\int\frac{d^3q}{(2\pi)^3} \\ & \times[\ln(1 + e^{-(E_q-\mu)/T}) + \ln(1 + e^{-(E_q+\mu)/T})], \end{aligned} \quad (7)$$

where

$$\begin{aligned}U(\sigma,\chi) = {}& -\frac{1}{2}m_\Phi{}^2\sigma^2 - \frac{1}{2}m_{\Phi'}{}^2\chi^2 + \frac{1}{16}\lambda_1\sigma^4 + \frac{1}{16}\lambda_2\chi^4 \\ & + \frac{1}{4}g_2\chi^3 - g_1\sigma^2\chi + \frac{1}{2}k\sigma^2 - 2h\sigma. \end{aligned} \quad (8)$$

The single particle energy is given by $E_q = \sqrt{p^2 + m_q{}^2}$ and the constituent quark mass (m_q) is given by $m_q = g_3\sigma + g_4\chi$. The number of colors N_c and flavors N_f of quark used in this paper are 3 and 2, respectively.

From the extremum condition of the thermodynamic potential we obtain equation of motions for σ and χ:

$$\frac{\partial\Omega}{\partial\sigma} = 0, \qquad \frac{\partial\Omega}{\partial\chi} = 0. \tag{9}$$

We will solve this set of coupled equation of motions, Eq. (9), self consistently at each value of temperature T and chemical potential μ to determine the behavior of σ and χ as a function of temperature and chemical potential and analyze the effect of quarkonium-tetraquark mixing on the chiral phase transition.

III. PARAMETER FIXING IN THE VACUUM

There are a total of 12 parameters in our model: $m_\Phi{}^2$, $m^2_{\Phi'}$, λ_1, λ_2, g_1, g_2, k, h, g_3, g_4, and zero temperature values of σ, χ. Out of these 12 parameters, the coupling constants g_3, g_4 are from the fermionic part ($\mathcal{L}_{\mathrm{q}}$) of the Lagrangian and the other 10 are from the mesonic part ($\mathcal{L}_{\mathrm{m}}$). The values of the 10 parameters in the mesonic part of the Lagrangian are determined from the physical meson masses, the pion decay constant ($f_\pi = 92.4$ MeV), and two extremum conditions for the mesonic potential:

$$\frac{\partial U(\sigma,\chi)}{\partial\sigma} = 0, \qquad \frac{\partial U(\sigma,\chi)}{\partial\chi} = 0. \tag{10}$$

Values of the parameters so fixed are kept constant for the whole range of temperature and chemical potentials. The physical meson masses are so chosen that for each kind of meson, one of the masses is below 1 GeV and the other is above 1 GeV. One of them is the likely choice for the quarkonium dominated meson and the other is tetraquark dominated, as found by other studies [16,17].

The physical meson masses are obtained by diagonalizing the bare meson mass matrices. The expression for those bare matrices as a function of the quarkonium, the tetraquark fields, and the coupling constants are noted below:

For the sigma mesons, we have

$$(M^2_{f_0}) = \begin{bmatrix} \frac{1}{2}\lambda_1\sigma^2 + 2\frac{h}{\sigma} & -2g_1\sigma \\ -2g_1\sigma & \frac{1}{2}\lambda_2\chi^2 + \frac{3}{4}g_2\chi + g_1\frac{\sigma^2}{\chi} \end{bmatrix}. \tag{11}$$

For pions, we have

$$(M^2_\pi) = \begin{bmatrix} 2g_1\chi + 2\frac{h}{\sigma} & 0 \\ 0 & g_1\frac{\sigma^2}{\chi} - \frac{9}{4}g_2\chi \end{bmatrix}. \tag{12}$$

For eta, we have

$$(M^2_\eta) = \begin{bmatrix} 4g_1\chi - 2k + 2\frac{h}{\sigma} & 2g_1\sigma \\ 2g_1\sigma & -\frac{9}{4}g_2\chi + g_1\frac{\sigma^2}{\chi} \end{bmatrix}. \tag{13}$$

Last, the bare mass matrix for the α_p meson reads

$$(M^2_\alpha) = \begin{bmatrix} \frac{1}{2}\lambda_1\sigma^2 + 2\frac{h}{\sigma} + 2g_1\chi - 2k & 0 \\ 0 & \frac{1}{2}\lambda_2\chi^2 + \frac{3}{4}g_2\chi + g_1\frac{\sigma^2}{\chi} \end{bmatrix}. \tag{14}$$

From the mass matrices, we find that there is no mixing for the pion and α mesons. We choose the lightest pion as a quarkonium meson and the heavier counterpart as the tetraquark meson in its quark content. This is in agreement with our current understanding. Since there is no mixing for pion, we define the zero temperature value of σ equal to the pion decay constant ($\sigma = f_\pi$). From the expression of the pion mass we see that the symmetry breaking terms contribute to its mass and that the absence of those terms in our Lagrangian would make the pion massless. The same statement also holds for the eta mesons, although in this case there is mixing between quarkonium and tetraquark fields. In the absence of mixing, $(M^2_\eta)_{11}$ would represent the physical eta meson mass and, comparing it with conventional pion mass $(M^2_\pi)_{11}$, we find the difference between their masses is coming from the instanton term, which is in line with our expectation.

In the following, we discuss three sets of parameters, which will be used for our analysis of the chiral phase transition in Sec. IV.

A. Case I: λ_2, g_2, k, $h = 0$

Here, in the simplest version of the model, we want to explore the scenarios where the lowest scalar is either a quarkonium dominated or tetraquark dominated meson. We find that within the limit of physical meson masses (including the experimental uncertainty of $m_{\pi'}$: 1.2–1.4 GeV, $m_{f_0(600)}$: 0.4–1.2 GeV, and $m_{f_0(1370)}$: 1.2–1.5 GeV), the scenario where the lightest scalar isoscalar is tetraquark dominated meson, cannot be realized. For this, in this case, we take the value of the physical meson masses to be slightly different from their real world values. To make the comparison between the two scenarios meaningful, we keep the physical meson masses for both the scenarios as close as possible (see Table I). In this case, the value of the parameters λ_1, g_1 and the zero temperature value of χ are calculated using the physical masses of $m_{f_0(600)}$, $m_{f_0(1370)}$; $m_{\pi'_p}$ mesons and the values of m^2_Φ, $m^2_{\Phi'}$ are calculated using the extremum conditions for the mesonic potential [see Eq. (10)]. Please note that even if $h = 0$ in this case, the π_p meson is still massive because of the interaction term between quarkonium and tetraquark fields, which breaks the chiral symmetry explicitly. Since the value of the m_π mass is not used in the parameter fixing, its mass, when calculated with the obtained parameter values, is comparatively higher than the real world pion mass. Since here we are only interested in qualitative comparison of the two scenarios and more elaborate studies are considered in the other cases (see case II and case III), we keep this high pion mass.

Utilizing the expressions for the π'_p mass together with the relations

$$\mathrm{Tr}[(M^2_{f_0})] = m^2_{f_0(600)} + m^2_{f_0(1370)}, \tag{15}$$

$$\mathrm{Det}[(M^2_{f_0})] = m^2_{f_0(600)} \times m^2_{f_0(1370)}, \tag{16}$$

we get the expressions for g_1, χ, λ_1 as

$$g_1 = \frac{1}{2f_\pi}\sqrt{(m^2_{f_0(600)} + m^2_{f_0(1370)} - m^2_{\pi'_p})m^2_{\pi'_p} - m^2_{f_0(600)} \times m^2_{f_0(1370)}}, \tag{17}$$

$$\chi = g_1 \frac{\sigma^2}{m^2_{\pi'_p}}, \tag{18}$$

$$\lambda_1 = \frac{2}{\sigma^2}[m^2_{f_0(600)} + m^2_{f_0(1370)} - m^2_{\pi'_p}]. \tag{19}$$

Using Eq. (10), we can calculate the values for m^2_Φ and $m^2_{\Phi'}$ from the expressions

$$m^2_\Phi = -\left(-\frac{1}{4}\lambda_1\sigma^2 + 2g_1\chi\right), \tag{20}$$

$$m^2_{\Phi'} = -g_1\frac{\sigma^2}{\chi}. \tag{21}$$

The value of the physical meson masses used is given in Table I. Depending on what value we choose for the mass of $m_{\pi'_p}$, we get two scenarios:

Scenario 1: The values of the parameters are such that the lowest scalar $f_0(600)$ is a quarkonium dominated meson, whereas the heavier one, $f_0(1370)$, is tetraquark dominated. The values of the parameters are given in Table II.

Scenario 2: In this case the nature of the scalar isoscalar mesons is just opposite to that of scenario 1. But for that we have to take the input value for $m_{\pi'_p}$ as slightly less than its range of possible values 1.2–1.4 GeV. Here, the lowest scalar, $f_0(600)$, is a tetraquark dominated meson, whereas the heavier one, $f_0(1370)$, is quarkonium dominated. The values of the parameters so obtained are given in Table II.

TABLE I. Values of physical meson masses used in case I.

Mesons	$m_{f_0(600)}$ (GeV)	$m_{f_0(1370)}$ (GeV)	m_{π_p} (GeV)	$m_{\pi'_p}$ (GeV)
Scenario 1	0.8	1.5	0.42	1.3
Scenario 2	0.8	1.5	0.49	1.1

B. Case II: λ_2, k, $h \neq 0$ but $g_2 = 0$

To discuss how the parameter set for $g_2 = 0$ is obtained, we first present the equations we are going to use to determine the values of g_1, χ, h, and k,

$$m^2_{\pi_p} = 2g_1\chi + 2\frac{h}{\sigma}, \tag{22}$$

$$m^2_{\pi'_p} = g_1\frac{\sigma^2}{\chi}, \tag{23}$$

$$\mathrm{Tr}[(M^2_\eta)] = m^2_{\eta_p} + m^2_{\eta'_p}, \tag{24}$$

$$\mathrm{Det}[(M^2_\eta)] = m^2_{\eta_p} \times m^2_{\eta'_p}. \tag{25}$$

Now, utilizing equations (23), (24), and (25), we get the equation for g_1 as

$$g_1 = \frac{1}{2\sigma}\sqrt{(m^2_{\eta_p} + m^2_{\eta'_p} - m^2_{\pi'_p})m^2_{\pi'_p} - m^2_{\eta_p} \times m^2_{\eta'_p}}. \tag{26}$$

From (23), we get the vacuum expectation values of the tetraquark field as

$$\chi = g_1\frac{\sigma^2}{m^2_{\pi'_p}}. \tag{27}$$

Using (22), (26), and (27), we can determine the value of h from the following equation:

$$h = \frac{\sigma}{2}[m^2_{\pi_p} - 2g_1\chi]. \tag{28}$$

According to the convention followed in this paper, the vacuum expectation values σ and χ are positive. To make

TABLE II. Parameter set for case I.

Parameters	σ (GeV)	χ (GeV)	$m_\Phi{}^2$ (GeV^2)	$m_{\Phi'}{}^2$ (GeV^2)	λ_1	g_1 (GeV)
Scenario 1	92.4×10^{-3}	2.1×10^{-2}	4.26×10^{-1}	−1.69	281.1	4.15
Scenario 2	92.4×10^{-3}	2.9×10^{-2}	5.95×10^{-1}	−1.21	393.55	4.17

TABLE III. Values of the physical meson masses used for case II.

Fields	$m_{f_0(600)}$	$m_{f_0(1370)}$	m_{π_p}	$m_{\pi'_p}$	m_{η_p}	$m_{\eta'_p}$
Mass (GeV)	0.6	1.35	0.14	1.29	0.55	1.3

TABLE IV. Parameter set for case II.

Parameters	σ (GeV)	χ (GeV)	$m_\Phi{}^2$ (GeV^2)	$m_{\Phi'}{}^2$ (GeV^2)	λ_1	λ_2	g_1 (GeV)	h (GeV^3)	k (GeV^2)
Value	92.4×10^{-3}	5.23×10^{-3}	1.9×10^{-2}	-1.6	87.99	9103.07	1.02	4.2×10^{-4}	-1.49×10^{-1}

sure we have the minimum of the mesonic potential lying in the quadrant where both σ and χ are positive, we should have $g_1 > 0$ and $h > 0$. Apart from that, we should also have positive λ_1, λ_2 in order to make our mesonic potential bounded from below. There is a large uncertainty in the value of $m_{\pi'_p}$: (1.2–1.4) GeV. But, if we impose the constraints $g_1 > 0$ and $h > 0$ and fix the values of m_{η_p} and $m_{\eta'_p}$ at 0.55 and 1.3 GeV, respectively, we find that the only allowed value is $m_{\pi'_p} = 1.29$ GeV (up to two significant digits after the decimal). Mass values higher than that would make $g_1 < 0$ and for a mass less than 1.29 GeV the value of h becomes negative.

The value of k can be determined from (22), (23), and (24):

$$k = \frac{1}{2}[2g_1\chi - (m^2_{\eta_p} + m^2_{\eta'_p} - m^2_{\pi_p'} - m^2_{\pi_p})]. \tag{29}$$

Then the mass matrix of f_0 mesons can be utilized to determine the values of λ_1 and λ_2. The relevant equations here are

$$\mathrm{Tr}[(M^2_{f_0})] = m^2_{f_0(600)} + m^2_{f_0(1370)}, \tag{30}$$

$$\mathrm{Det}[(M^2_{f_0})] = m^2_{f_0(600)} \times m^2_{f_0(1370)}. \tag{31}$$

Like the value of $m_{\pi'_p}$, there are also large uncertainties in the values of $m_{f_0(600)}$: (0.4–1.2) GeV and $m_{f_0(1370)}$: (1.2–1.5) GeV. Here, we have used $m_{f_0(600)} = 0.6$ GeV and $m_{f_0(1370)} = 1.35$ GeV. Two sets of values can be obtained from equations (30) and (31). But only one of them satisfies the condition $\lambda_1, \lambda_2 > 0$ and is considered in this work. We have checked for other values of $m_{f_0(600)}$ in the range (0.4–1.2 GeV) and $m_{f_0(1370)}$ in the range (1.2–1.5) GeV and one of the solutions for λ_2 remains always negative for the entire mass range.

Finally, the values of m^2_Φ and $m^2_{\Phi'}$ can be determined from the extremum condition mentioned in Eq. (10). The explicit expression for them is given below:

$$m^2_\Phi = -\left(-\frac{1}{4}\lambda_1\sigma^2 + 2g_1\chi - k + 2\frac{h}{\sigma}\right), \tag{32}$$

$$m^2_{\Phi'} = -\left(-\frac{1}{4}\lambda_2\chi^2 + g_1\frac{\sigma^2}{\chi}\right). \tag{33}$$

The values of the input physical meson masses and the resultant output parameter set are given in Tables III and IV, respectively.

C. Case III: λ_2, g_2, k, $h \neq 0$

To fix the parameters for $g_2 \neq 0$, we follow the same procedure as mentioned above. Here, in this case we have one more parameter, g_2. For this, we made an assumption that $\chi < \sigma$, which is consistent with all previous studies. Now the constraints, $g_1 > 0$, $h > 0$, $\lambda_1 > 0$, $\lambda_2 > 0$, restrict the value of χ to a certain range. We assume $\chi = \sigma/n$ and calculate the parameter set for small and large possible values of "n" ($n = 10$ and 18), respecting all the constraints.

The expressions for g_1, h, k remain the same as mentioned in Eqs. (26), (28), and (29). The expression for g_2 in this case reads as follows:

$$g_2 = \frac{4}{9\chi}\left(g_1\frac{\sigma^2}{\chi} - m^2_{\pi'_p}\right). \tag{34}$$

Here, for $n = 18$, i.e., if χ is small, we get the sign of g_2 to be positive, while for $n = 10$, corresponding to a comparatively large value of χ, the sign of g_2 becomes negative. The values of λ_1 and λ_2 are calculated using Eqs. (30) and (31).

Finally, m^2_Φ and $m^2_{\Phi'}$ are calculated from the following expressions using Eq. (10):

$$m^2_\Phi = -\left(-\frac{1}{4}\lambda_1\sigma^2 + 2g_1\chi - k + 2\frac{h}{\sigma}\right) \tag{35}$$

$$m^2_{\Phi'} = -\left(-\frac{1}{4}\lambda_2\chi^2 + g_1\frac{\sigma^2}{\chi} - \frac{3}{4}g_2\chi\right). \tag{36}$$

As in the case for $g_2 = 0$, here also, we only find one set of solutions that respects the constraint $\lambda_1 > 0$ and $\lambda_2 > 0$. The values of the input physical meson masses used here are the same as in the previous section (see Table III) and the output parameters obtained are

TABLE V. Parameter set for $g_2 \neq 0$. In this set, $\chi = \sigma/n$, where $n = 18$ is used.

Parameters	σ (GeV)	χ (GeV)	$m_\Phi{}^2$ (GeV^2)	$m_{\Phi'}{}^2$ (GeV^2)	λ_1	λ_2	g_1 (GeV)	g_2 (GeV)	h (GeV^3)	k (GeV^2)
Value	92.4×10^{-3}	5.13×10^{-3}	1.9×10^{-2}	-1.63	87.93	7527.9	10.16×10^{-1}	2.25	4.2×10^{-4}	-1.49×10^{-1}

TABLE VI. Parameter set for $g_2 \neq 0$. In this set, $\chi = \sigma/n$, where $n = 10$ is used.

Parameters	σ (GeV)	χ (GeV)	$m_\Phi{}^2$ (GeV^2)	$m_{\Phi'}{}^2$ (GeV^2)	λ_1	λ_2	g_1 (GeV)	g_2 (GeV)	h (GeV^3)	k (GeV^2)
Value	92.4×10^{-3}	9.24×10^{-3}	2.7×10^{-2}	-0.63	89.88	25785.1	1.02	-34.88	3.8×10^{-5}	-1.45×10^{-1}

given in Tables V and VI, corresponding to positive and negative g_2, respectively.

IV. RESULTS FOR PHASE TRANSITIONS

There are two more parameters in our model that have not been discussed yet. They are the Yukawa coupling constants g_3 and g_4. Their values are fixed from the given value of the constituent quark mass. Since we have one condition and two undetermined coupling constants, we assume $g_4 = g_3/n_\chi$, where $n_\chi > 1$. For a particular value of g_3, if we change the value of g_4 by changing n_χ then there is no qualitative change in the behavior of σ, χ. However, the nature of the phase transition is affected if we change the value of g_3. This can be seen from Fig. 1. If we increase the value of g_3, then we can get the first order transition even at zero chemical potential. This dependence of the order of the phase transition on the values of the model parameters in the mean field approximation of the linear sigma model/quark-meson model is not new and is already noted in [1,15,18]. In this work, we have used $g_3 = 3.0$ and $g_4 = g_3/10$ corresponding to the vacuum constituent quark mass of 0.28 GeV. The values of g_3 and g_4 are so chosen to make the chiral phase transition crossover at zero chemical potential, as found by the lattice simulation study [19].

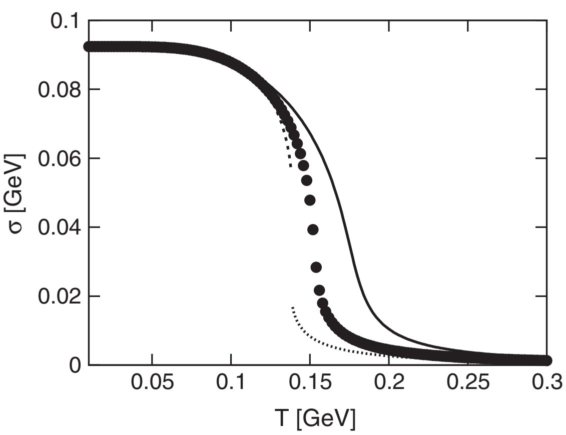

FIG. 1. Variation of σ with temperature at zero chemical potential for different values of g_3. From left to right, the values of g_3 are 3.5, 3.0, and 2.5. Parameters are corresponding to the scenario $g_2 = 0$, i.e., Table IV.

Before presenting our result, let us first discuss the nature of the mesonic potential $U(\sigma,\chi)$ in a vacuum. As can be seen from the parameter set presented in Tables IV, V, and VI, the sign of $m_{\Phi'}{}^2$ is opposite to that of $m_\Phi{}^2$. Its sign indicates that it has the opposite sign to what is required for spontaneous breaking. This can be seen from Fig. 2 (the right one) where the potential in the χ direction (for constant $\sigma = 92.4 \times 10^{-3}$ GeV) is plotted. There is only one minimum and the minimum of the potential is slightly tilted in the $\chi > 0$ direction because of the explicit symmetry breaking terms. On the other hand, because of the negative sign of $m_\Phi{}^2$, the potential in the σ direction exhibits the kind of pattern expected for spontaneous symmetry breaking. The minimum in the $\sigma > 0$ direction is lower than that in the opposite direction (see the left-hand side of Fig. 2, here $\chi = 5.23 \times 10^{-3}$ GeV) because $h > 0$. This indicates, the origin of σ and χ condensates have different reasons in this work. Explicit symmetry breaking is the origin for χ, whereas for σ it is the spontaneous breaking.

For the values of parameters presented in Tables IV, V, and VI, we find, irrespective of the scalar or pseudoscalar nature of the mesons, that mesons with lower mass are always quarkonium dominated and the mesons above 1 GeV are tetraquark dominated. The mixing angles for the f_0 meson for parameter sets presented in Tables IV, V, and VI are −7.51, −7.44, and −7.45 (in degrees), respectively. For η mesons, the mixing angles for the above-mentioned parameter sets are 7.88, 7.84, and 7.86 (in degrees), respectively. Like pions, there is no mixing for the α meson. The masses of α_p, α'_p mesons are 0.83 and 1.34 GeV, respectively, for all three parameter sets. Since there is no mixing, the lower mass α_p meson is purely quarkonium and the heavier counterpart is purely tetraquark in nature.

To characterize the phase transition and to find the transition temperature, we have used the susceptibilities of the order parameters. The susceptibility matrix is defined as [2,20]

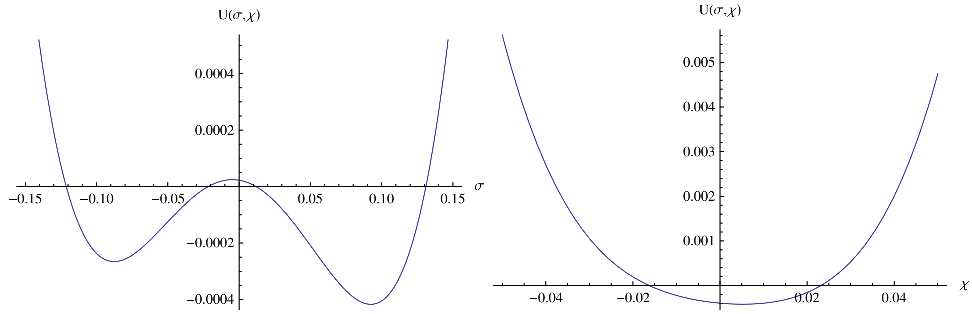

FIG. 2 (color online). Nature of the mesonic potential $U(\sigma,\chi)$ in a vacuum. Parameters are corresponding to Table IV. See text for details.

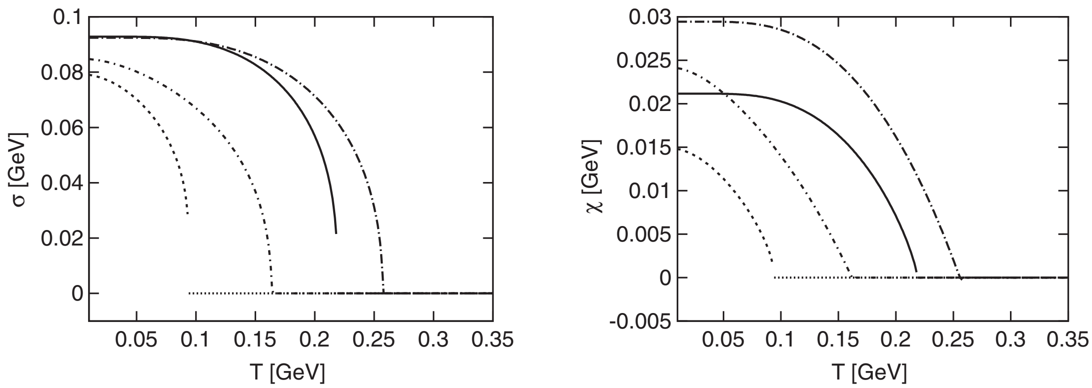

FIG. 3. Variation of σ and χ, with temperature for different values of chemical potential corresponding to the parameter set of case I. The solid ($\mu = 0$ GeV) and dotted ($\mu = 0.36$ GeV) lines are for scenario 1, whereas the long ($\mu = 0.0$ GeV) and short dashed-dot ($\mu = 0.36$ GeV) lines are for scenario 2.

$$\hat{\chi} = \frac{1}{C_{\chi\chi}C_{\sigma\sigma} - C_{\sigma\chi}^2}\begin{bmatrix} C_{\chi\chi} & C_{\sigma\chi} \\ C_{\sigma\chi} & C_{\sigma\sigma} \end{bmatrix}, \tag{37}$$

where C_{xx} ($x = \sigma, \chi$) are the second derivatives of the thermodynamic potential with respect to x:

$$C_{xx} = \frac{\partial^2\Omega}{\partial x^2}. \tag{38}$$

Susceptibility of σ is defined as $\chi_{2Q} = \hat{\chi}_{11}$ and that of χ is given by $\chi_{4Q} = \hat{\chi}_{22}$. We determine the transition temperature from the peak position of the respective susceptibilities. For the critical point, $C_{\chi\chi}C_{\sigma\sigma} - C_{\sigma\chi}^2$ becomes zero, corresponding to the zero curvature of the thermodynamic potential.

A. Phase diagram for case I

The behavior of the order parameters along with the resultant phase diagram corresponding to the parameter set for case I are summarized in Figs. 3 and 4.

As mentioned in the last section, here we have two scenarios depending on the mass of $m_{\pi'}$. For $m_{\pi'} = 1.3$ GeV corresponding to scenario 1, the lowest isoscalar is quarkonium dominated, whereas, for scenario 2 (we consider $m_{\pi'} = 1.1$ GeV, which is slightly less than the value quoted in the particle data group, 1.2–1.4 GeV), we have the lowest isoscalar as a tetraquark dominated meson.

We find, for both the cases, that for all values of the chemical potential, the transition temperatures calculated from the susceptibilities χ_{2Q} and χ_{4Q} are the same. This can also be seen from the behavior of the order parameters

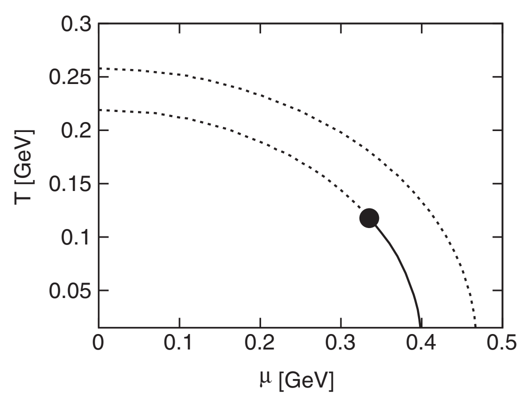

FIG. 4. Phase diagram for case I. The dotted line represents the second order phase transition and the solid line stands for the first order phase transition. The upper phase boundary line corresponds to scenario 2 and the lower one corresponds to scenario 1.

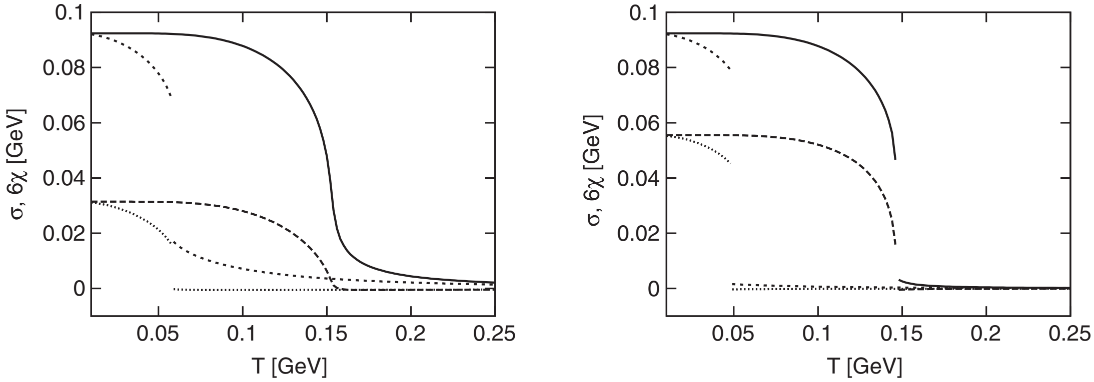

FIG. 5. Variation of σ and χ with temperature for different values of chemical potential. The figure in the left panel is for $g_2 = 0$ and the right one is for $g_2 = -34.88$. The solid ($\mu = 0$ GeV) and short dashed ($\mu = 0.27$ GeV) lines are for the variation of σ. Variation of χ is represented by long dashed ($\mu = 0$ GeV) lines and points ($\mu = 0.27$ GeV) for both the figures.

presented in Fig. 3. In Fig. 3, the temperature variation of σ and χ is presented for low and high values of chemical potential. From the figure, we see that σ and χ varies more slowly with temperature in the case for scenario 2 than in scenario 1. Consequently, the transition temperature in scenario 2 is always higher than in scenario 1. For both the scenarios, both σ and χ go to zero after the phase transition because $h = 0$. But, for scenario 1, there is a jump in the case of σ after a certain temperature and this gap in the order parameter increases slowly with the chemical potential. For χ, this gap is vanishingly small at low chemical potential and slowly increases with the chemical potential.

If we compare the phase diagrams shown in Fig. 4, we see that, for scenario 2, the order of the phase transition is second order for both low as well as high values of the chemical potential. But, for scenario 1, the second order phase transition changes to the weak first order phase transition above some critical value of the chemical potential, thus indicating the presence of a critical point. We consider the values of the chemical potential and the temperature at which the curvature of the thermodynamic potential becomes greater than 10^{-4} as the location of the critical point. Using this condition, we find that the critical point for scenario 1 is located at $T_c = 117.7$ MeV and $\mu_c = 335$ MeV. The departure from the zero curvature of the thermodynamical potential together with the gap in the order parameter are taken as the indication of a weak first order phase transition. We are calling it weak first order because curvature of the thermodynamic potential remains very small ($\sim 10^{-3}$) for $\mu > 335$ MeV.

B. Phase diagram for case II and case III

The nature of the phase transition corresponding to cases II and III is summarized in Figs. 5 and 6.

Behavior of the order parameters at low and high values of the chemical potentials are presented in Fig. 5, where the left figure corresponds to $g_2 = 0$ and the right one to $g_2 < 0$. We find for $g_2 = 0$ (see Table IV) and $g_2 > 0$ (see Table VI), the behavior of the order parameters are qualitatively similar. This is expected as the positive cubic interaction coupling constant for the tetraquark field is relatively small and the other parameters are almost of the same values. As can be seen from Fig. 5 (left), for small chemical potential we have a crossover transition that turns into first order transition at a high value of the chemical potential because the finite "h" term for the Φ field makes $\sigma > 0$ even at high temperature. But the absence of such a term for the Φ' field makes χ go to zero at high temperature. However, the nature of the transition is quite different corresponding to the scenario in which $g_2 < 0$ (see Table V). In this case, the strong cubic interaction term makes the transition first order for the whole range of chemical potentials, as can be seen from Fig. 5 (right). Here, a relatively low value of "h" makes σ go to zero at high temperature. However, there is one similarity with respect to the chiral phase transition temperature for various cases considered in this work. Like in case I, we note from both the figures in Fig. 5, that the transitions for σ and χ are

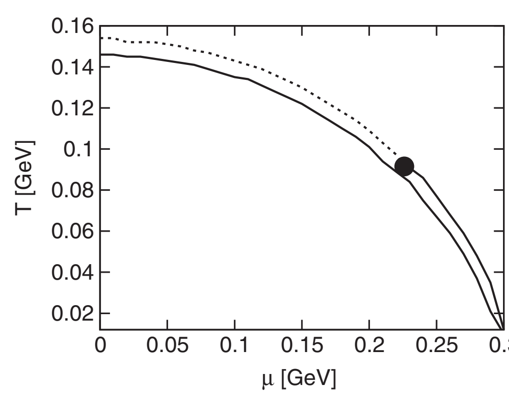

FIG. 6. Phase diagram for case III. The solid line indicates the first order phase transition and the dashed line is for the crossover transition. The upper phase boundary is for $g_2 = 2.25$ and the lower one is for $g_2 = -34.88$. The bold circle indicates the location of the critical end point. See text for details.

occurring at the same temperature that is verified from the peak positions of the respective susceptibilities.

As a result, the resultant phase diagram shown in Fig. 6 is represented with a single phase boundary line for each case. For $g_2 = 0$ and $g_2 > 0$ we have qualitatively the same feature and thus we have included the phase boundary for case III only in Fig. 6. In this case, we have a crossover transition at low chemical potential that turns into first order above some critical value of the chemical potential. Thus, we have a critical end point for $g_2 = 0$ and $g_2 > 0$. The location of the critical end point for $g_2 = 0$ is ($\mu = 0.26$ GeV, $T = 0.069$ GeV) and for $g_2 > 0$ is ($\mu = 0.226$ GeV, $T = 0.0915$ GeV), whereas for $g_2 < 0$ we have only a first order phase transition line owing to the strong cubic interaction term.

V. SUMMARY AND CONCLUSION

In the framework of the two-flavor quark-meson model, we have investigated the effect of mixing between quarkonium and tetraquark fields on the chiral phase transition.

The mixing between the effective fields is introduced through an interaction term that breaks the chiral symmetry explicitly. In addition to the interaction term, we also considered a cubic self-interaction term for the effective tetraquark field, an instanton determinant term, and a term mimicking the effect of the current quark mass. The existence of the heavy tetraquark dominated meson $\pi'(1.2\text{–}1.4\ \text{GeV})$, $\eta'(1.3\ \text{GeV})$ indicates that apart from the finite quark mass term, we need another symmetry breaking term to account for the mass of π' and η'. In our model those extra terms are given by the interaction term and the cubic self-interaction term, which break the chiral symmetry explicitly. Those terms not only give mass to π' but also to π. The extent to which they contribute to the π mass varies depending on the different scenarios. But, nevertheless, they are bounded by the smallness of the π and other meson masses. Thus, the smallness of the pion mass is preserved. As a result, the effect of the cubic interaction term for the quarkonium and tetraquark field as well as the cubic self-interaction term for the tetraquark field don't interfere/override the symmetry breaking pattern. The relevant question arising here is that of the origin of those terms. But, within the setup and context of our present model, this question cannot be answered and will be addressed in our future endeavor.

The parameters of our model are calculated from the masses of the physical mesons, pion decay constants, and the stability conditions of the mesonic potential. We first considered the effect of the mixing term without considering the cubic self-interaction term for the tetraquark field, the term mimicking the current quark mass and the instanton determinant term. Within the allowed experimental range for the masses for $f_0(600)$, $f_0(1370)$, π and π' mesons, we find our lowest scalar $f_0(600)$ meson is quarkonium dominated.

For the scenario where $f_0(600)$ is tetraquark dominated, we find that the chiral phase transition is second order for both low and high values of the quark chemical potential. On the other hand, if we increase the absolute value of the mass of the bare tetraquark field, thereby increasing the value of the π' mass, we can have a weak first order phase transition above some critical value of the chemical potential. Comparing the transition in both cases, we find that the transition temperature is lowered with the increase of the absolute value of the bare tetraquark field mass.

Next, we study the effect of the cubic self-interaction term (with coupling constant g_2) of the tetraquark fields. We find that the physical meson mass spectrum and the vacuum stability conditions put a tight constraint on our parameter set. From the resulting parameter sets, we find that the lowest scalar meson $f_0(600)$ is a quarkonium dominated meson, whereas $f_0(1370)$ is tetraquark dominated. For $g_2 = 0$ (but including the effect of the finite current quark mass and the instanton term) and small but positive g_2, the chiral phase transition is a crossover for small values of the quark chemical potential and then, above some critical value of the chemical potential, the transition becomes first order. Thus, we have a critical end point in this case and the resultant phase diagram matches well with the current consensus regarding the two-flavor phase diagram, but, with a strong and negative g_2, makes not only the transition of χ first order but the transition for σ as well becomes first order irrespective of the low or high value of the quark chemical potential. A strong and negative g_2 also makes the chiral phase transition temperature lower than that for the case of $g_2 = 0$ or positive. For all the various scenarios considered in our study, the common feature among all of them is that the transition for quarkonium and tetraquark happens at the same temperature for all values of the chemical potential.

ACKNOWLEDGMENTS

This work is supported by the NSFC under Grants No. 11250110058 and No. 11275213, the DFG and NSFC (CRC 110), the CAS fellowship for young foreign scientists under Grant No. 2011Y2JB05, the CAS key project KJCX2-EW-N01, and the K. C. Wong Education Foundation and Youth Innovation Promotion Association of CAS.

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