Transverse single spin asymmetries at small x and the anomalous magnetic moment

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(Received 30 August 2013; published 29 April 2014)

We show that in the McLerran-Venugopalan model an axial asymmetrical valence quark distributions in the transverse plane of a transversely polarized proton can give rise to a spin-dependent odderon. Such polarized odderon is responsible for the transverse single spin asymmetries for jet production in the backward region of pp collisions and open charm production in the semi-inclusive deep inelastic scattering process.

DOI: 10.1103/PhysRevD.89.074050 PACS numbers: 24.85.+p, 12.38.Bx, 12.39.St, 13.88.+e

I. INTRODUCTION

The exploration of transverse single spin asymmetries (SSAs) in high-energy scattering experiments has a long history, starting from the mid 1970s [1]. The large size of the observed SSAs for single inclusive hadron production came as a big surprise and, *a priori*, posed a challenge for QCD, as the naive parton model predicts the asymmetries are proportional to the quark mass [2,3] and thus very small. During the past few decades, the remarkable theoretical progress has been achieved by going beyond the naive parton model and following mainly two approaches: one approach is based on transverse momentum dependent (TMD) factorization [4,5] and the other on collinear twist-3 factorization [6–10]. In TMD factorization, naive time reversal odd TMD distributions and the fragmentation function, known as the quark/gluon Sivers functions [4] and the Collins fragmentation function [5], can account for the large SSAs, while in the collinear twist-3 approach, the SSAs arise from twist-3 quark gluon correlator so-called Efremov-Teryaev-Qiu-Sterman function (ETQS) [6,7], tri-gluon correlation functions [8,9], and twist-3 collinear fragmentation functions [10].

Beyond TMD factorization and collinear twist-3 formalism, some alternative mechanisms underlying the large SSAs have been proposed, such as the soft coherent dynamics [11], QCD instanton mechanism [12,13], and QCD odderon interaction [14,15]. The authors of Refs. [14] investigated the odderon's contribution to SSAs in the context of a heavy fermion model. A later calculation formulated in the saturation/color glass condensate (CGC) framework suggests that SSAs can be generated by the interaction of spin-dependent light-cone wave function of the projectile with the target gluon field via C-odd odderon exchange [15]. The odderon excitation considered in Ref. [15] comes from the unpolarized proton/nucleus and is spin independent. In this paper, however, we focus on studying SSAs generated by a spin-dependent odderon that comes from the transversely polarized proton. Such a polarized odderon is responsible for SSAs for jet production in the backward region of a polarized proton in pp collisions and open charm production in the semi-inclusive deep inelastic scattering (SIDIS) process at small x.

In recent years, the interplay between spin physics and saturation physics has been forming into an active field of research. The early work includes the study of small x evolution of spin-dependent structure function g_1 [16]. It was pointed out that the spin asymmetries could also be generated by the pomeron-odderon interference effect [17]. SSAs at forward rapidity in pA collisions were investigated in Refs. [15,18]. More recently, the quark/gluon Boer-Mulders distributions inside a large nucleus were studied in Refs. [19].

In this paper, we explore SSA phenomena at small x and identify a spin-dependent odderon as the main source of SSAs. The paper is organized as follows. In Sec. II, we show that, in the Mclerran-Venugopalan (MV) model, a nonvanishing contribution to the spin-dependent odderon amplitude arises from the left-right asymmetrical color source distribution in the transverse plane of a transversely polarized proton. In Sec. III, we present compact expressions for SSAs in jet production in the backward region of pp collisions and open charm production in SIDIS at small x. Both asymmetries are generated by the polarized odderon. We summarize our paper in Sec. IV.

II. SPIN-DEPENDENT CLASSICAL ODDERON

In perturbative QCD, the odderon is a color-singlet exchange and can be formed by three gluons in a symmetric color state. It has negative C-parity and therefore dominates the differences between particle-particle and particle-antiparticle scatterings at high energy. The energy dependence of the odderon exchange is described by the Bartels-Kwiecinski-Praszalowicz (BKP) equation [20]. Within the CGC formalism, one can identify the following operator as the dipole odderon operator [21],

$$\hat{O}(R_\perp, r_\perp) = \frac{1}{2i}[\hat{D}(R_\perp, r_\perp) - \hat{D}(R_\perp, -r_\perp)], \qquad (1)$$

where

$$\hat{D}(R_\perp, r_\perp) = \frac{1}{N_c}\mathrm{Tr}\left[U\left(R_\perp + \frac{r_\perp}{2}\right)U^\dagger\left(R_\perp - \frac{r_\perp}{2}\right)\right], \quad (2)$$

with the Wilson line being defined as

$$U(x_\perp) = \mathrm{P}e^{ig\int_{-\infty}^{+\infty} dx^- A_+(x^-, x_\perp)}. \quad (3)$$

The small x evolution equation of this odderon operator was constructed using the dipole model [22] and the general JalilianMarian-Iancu-McLerran-Weigert-Leonidov-Kovner (JIMWLK) equation [21]. In the low parton densities region, the Bartels-Lipatov-Vacca (BLV) solution [23] to the BKP equation can be recovered from both the dipole model calculation and the JIMWLK equation with C-odd initial conditions.

The odderon is absent in the original MV model [24] in which the distribution of the large x color source is assumed to take a Gaussian form. However, it has been shown that a classical odderon can be generated by an additional cubic term in the modified weight function $W[\rho]$, which is given by Ref. [25],

$$W[\rho] = \exp\left\{-\int d^2x_\perp\left[\frac{\rho_a(x_\perp)\rho_a(x_\perp)}{2\mu(x_\perp)} - \frac{g d_{abc}\rho^a(x_\perp)\rho^b(x_\perp)\rho^c(x_\perp)}{4N_c\mu^2(x_\perp)}\right]\right\}, \quad (4)$$

where $\mu(x_\perp)$ is the density of color sources per unit transverse area and related to the valence quark distribution in the transverse plane, $\int dx_q f_q(x_q, x_\perp) = 6\mu(x_\perp)/g^2$, with x_q being the longitudinal momentum fraction carried by valence quark, and d^{bca} is the symmetric structure constant of the color SU(3) group. Using the above weight function to compute the expectation value of the odderon operator, one obtains [25]

$$\begin{aligned}\int d^2R_\perp\theta(R_0 - |R_\perp|)\langle\hat{O}(R_\perp, r_\perp)\rangle &= c_0\alpha_s^3\int d^2R_\perp\theta(R_0 - |R_\perp|)\\ &\times\int d^2z_\perp \ln^3\frac{|R_\perp + r_\perp/2 - z_\perp|}{|R_\perp - r_\perp/2 - z_\perp|}e^{-\frac{1}{4}r_\perp^2 Q_s^2}\\ &\times\frac{1}{3}\int dx_q f_q(x_q, z_\perp),\end{aligned} \quad (5)$$

where R_0 is the radius of the proton and the transverse center of the parton longitudinal momentum is chosen to be as the origin. The color coefficient c_0 is defined as $c_0 = \frac{(N_c^2-1)(N_c^2-4)}{4N_c^3}$. $Q_s^2 = \alpha_s C_F \mu(R_\perp)\ \ln\frac{1}{r_\perp^2\Lambda_{\mathrm{QCD}}^2}$ is the quark saturation momentum. Here, we insert a theta function because we have assumed that the dipole must hit the proton directly in order to be able to interact with quarks inside of it.

To proceed further, we first neglect the dependence of Q_s^2 on $R_\perp$ and $r_\perp$. To integrate out $R_\perp$, we use a mathematical trick introduced in Ref. [15]. One notices that

$$\int d^2R_\perp \ln^3\frac{|R_\perp + r_\perp/2 - z_\perp|}{|R_\perp - r_\perp/2 - z_\perp|} = 0 \quad (6)$$

if the integration carries over the whole transverse plane. This result implies

$$\begin{aligned}&\int d^2R_\perp \ln^3\frac{|R_\perp + r_\perp/2 - z_\perp|}{|R_\perp - r_\perp/2 - z_\perp|}\theta(R_0 - |R_\perp|)\\ &= -\int d^2R_\perp \ln^3\frac{|R_\perp + r_\perp/2 - z_\perp|}{|R_\perp - r_\perp/2 - z_\perp|}\theta(|R_\perp| - R_0).\end{aligned} \quad (7)$$

The fact that $|r_\perp|$ is much smaller than R_0 for a perturbative dipole allows us to expand the integrand on the right-hand side of the above equation in powers of $|r_\perp|/|2R_\perp|$ as well as $|z_\perp|/|R_\perp|$. To the first nontrivial order, one has

$$\begin{aligned}&\int d^2R_\perp \ln^3\frac{|R_\perp + r_\perp/2 - z_\perp|}{|R_\perp - r_\perp/2 - z_\perp|}\theta(R_0 - |R_\perp|)\\ &\approx -\frac{3\pi}{4R_0^2}r_\perp^2(r_\perp\cdot z_\perp).\end{aligned} \quad (8)$$

Substituting Eq. (8) back into Eq. (5), we obtain

$$\begin{aligned}&\int d^2R_\perp\theta(R_0 - |R_\perp|)\langle\hat{O}(R_\perp, r_\perp)\rangle\\ &\approx -\frac{c_0\alpha_s^3\pi}{4R_0^2}r_\perp^2 e^{-\frac{1}{4}r_\perp^2 Q_s^2}\int dx_q d^2z_\perp(r_\perp\cdot z_\perp)f_q(x_q, z_\perp).\end{aligned} \quad (9)$$

For a transversely polarized proton, impact-parameter-dependent valence quark distribution can be parameterized as [26]

$$f_q(x_q, z_\perp) = \sum_{u,d}\left\{\mathcal{H}(x_q, z_\perp^2) - \frac{1}{2M}\epsilon_\perp^{ij}S_{\perp i}\frac{\partial\mathcal{E}(x_q, z_\perp^2)}{\partial z_\perp^j}\right\}, \quad (10)$$

where $S_\perp$ is the proton transverse spin vector and M is the proton mass. The generalized parton distributions (GPDs) $\mathcal{H}(x_q, z_\perp^2)$ and $\mathcal{E}(x_q, z_\perp^2)$ are the Fourier transformed GPDs H and E with zero skewedness, respectively. Inserting Eq. (10) into Eq. (9), one immediately obtains

$$\int d^2R_\perp \theta(R_0 - |R_\perp|)\langle \hat{O}(R_\perp, r_\perp)\rangle$$
$$= -\frac{c_0\alpha_s^3\pi}{8MR_0^2} e^{-\frac{1}{4}r_\perp^2 Q_s^2} r_\perp^2 \epsilon_\perp^{ij} S_{\perp i} r_{\perp j} \int dx_q d^2z_\perp \sum_{u,d} \mathcal{E}(x_q, z_\perp^2)$$
$$= -\frac{c_0\alpha_s^3\pi}{8MR_0^2} e^{-\frac{1}{4}r_\perp^2 Q_s^2} r_\perp^2 \epsilon_\perp^{ij} S_{\perp i} r_{\perp j}(\kappa_p^u + \kappa_p^d), \quad (11)$$

where κ_p^u and κ_p^d are the contributions from up and down quarks to the anomalous magnetic moment of the proton, respectively. An earlier attempt to connect SSA phenomena to GPD E was made in paper [27].

A few comments are in order on the above analytic result:

(i) First, the odderon exchange under our consideration is clearly spin dependent.

(ii) The polarized odderon originates from the transverse distortion of the impact-parameter-dependent parton distribution function inside a transversely polarized proton. Such transverse distortion has been clearly seen in a lattice QCD calculation [28].

(iii) Given the nucleon's magnetic moment $\kappa_p = 1.793$ and $\kappa_n = -1.913$, κ_p^u and κ_p^d can be roughly estimated as $\kappa_p^u = 1.673$ and $\kappa_p^d = -2.033$ by using the isospin symmetry. Obviously, the contributions from u and d quarks to the polarized odderon largely cancel out.

We conclude this section by making a final remark on our result. The MV model is expected to work better for a large nucleus. However, its application to a proton target turns out to be quite successful phenomenologically [29]. Therefore, qualitatively speaking, our analysis presented here might be relevant in phenomenological studies as well.

III. OBSERVABLES

As discussed in the introduction, the spin-dependent odderon is responsible for the transverse single spin asymmetry for jet production in the backward region of the polarized proton in pp collisions. The leading-order result for jet production was first derived in Ref. [30]. Recently, the next-to-leading-order correction to the cross section was also calculated in Refs. [31].

At leading order, for the quark-initiated subprocess, the cross section reads

$$\frac{d\sigma^{pA\longrightarrow qX}}{d^2k_\perp dY} = \sum_f xq_f(x) \int \frac{d^2r_\perp}{(2\pi)^2} e^{-ik_\perp\cdot r_\perp} \times \int d^2R_\perp \langle \hat{D}(R_\perp, r_\perp)\rangle_{x_g}, \quad (12)$$

where $x = \frac{|k_\perp|}{\sqrt{s}} e^{-Y}$ and $x_g = \frac{|k_\perp|}{\sqrt{s}} e^{Y}$, with Y being the rapidity. $q_f(x)$ is the normal integrated quark distribution from the unpolarized proton. Note that we have neglected the elastic scattering contribution to the cross section in the above expression. The dipole S matrix can be decomposed into the even and odd pieces under the exchange of the transverse coordinates

$$\hat{D}(R_\perp, r_\perp) = \hat{S}(R_\perp, r_\perp) + i\hat{O}(R_\perp, r_\perp), \quad (13)$$

with the symmetric part being defined as

$$\hat{S}(R_\perp, r_\perp) = \frac{1}{2}[\hat{D}(R_\perp, r_\perp) + \hat{D}(R_\perp, -r_\perp)], \quad (14)$$

The cross section then can be reexpressed as

$$\frac{d\sigma^{pA\longrightarrow qX}}{d^2k_\perp dY} = \sum_f xq_f(x) \int \frac{d^2r_\perp}{(2\pi)^2} e^{-ik_\perp\cdot r_\perp} \times \int d^2R_\perp \langle \hat{S}(R_\perp, r_\perp) + i\hat{O}(R_\perp, r_\perp)\rangle_{x_g}$$
$$= \sum_f xq_f(x)\left\{F_{x_g}(k_\perp^2) + \frac{1}{M}\epsilon_\perp^{ij} S_{\perp i} k_{\perp j} O_{1T,x_g}^\perp(k_\perp^2)\right\}. \quad (15)$$

Here, we introduce a spin-dependent odderon in momentum space: $O_{1T,x_g}^\perp(k_\perp^2)$. To some extent, $O_{1T,x_g}^\perp(k_\perp^2)$ can be considered as a C-odd partner of the gluon Sivers function. In the MV model, the unpolarized gluon distribution is given by

$$F_{x_g}(k_\perp^2) = \pi R_0^2 \int \frac{d^2r_\perp}{(2\pi)^2} e^{-ik_\perp\cdot r_\perp} e^{-\frac{1}{4}r_\perp^2 Q_s^2}. \quad (16)$$

Using Eq. (11), it is easy to derive

$$O_{1T,x_g}^\perp(k_\perp^2) = \frac{-c_0\alpha_s^3(\kappa_p^u + \kappa_p^d)}{4R_0^4}\left[\frac{\partial}{\partial k_\perp^2}\frac{\partial}{\partial k_\perp^i}\frac{\partial}{\partial k_{\perp i}} F_{x_g}(k_\perp^2)\right]. \quad (17)$$

A few comments are in order on the above analytic result:

(i) We note that $\int d^2k_\perp k_\perp^2 O_{1T,x_g}^\perp(k_\perp^2) = 0$. This relation implies that $O_{1T,x_g}^\perp(k_\perp^2)$ has a node in $k_\perp^2$, and the $k_\perp^i$ weighted cross section $\int d^2k_\perp \langle k_\perp^i d\sigma\rangle$ is zero.

(ii) The single spin asymmetry is determined by the ratio $k_\perp O_{1T,x_g}^\perp / F_{x_g}$. From Eqs. (16) and (17), one finds that this ratio scales as $k_\perp$ at low transverse momentum, while it scales as $1/k_\perp^3$ at high transverse momentum.

(iii) The ratio $k_\perp O_{1T,x_g}^\perp / F_{x_g}$ should drop with decreasing x_g as the power of $(x_g)^{0.3}$ since the leading high-energy odderon intercept is equal to 1 according to the BLV solution.

For the gluon-initiated channel, the cross section reads

$$\frac{d\sigma^{pA\longrightarrow gX}}{d^2k_\perp dY}=xg(x)\int\frac{d^2r_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}\int d^2R_\perp\langle\hat{\tilde{D}}(R_\perp,r_\perp)\rangle_{x_g},\tag{18}$$

where $g(x)$ is the normal integrated gluon distribution from the unpolarized proton and $\hat{\tilde{D}}(r_\perp)$ is given by

$$\hat{\tilde{D}}(R_\perp,r_\perp)=\frac{1}{N_c^2-1}\mathrm{Tr}\left[\tilde{U}\left(R_\perp+\frac{r_\perp}{2}\right)\tilde{U}^\dagger\left(R_\perp-\frac{r_\perp}{2}\right)\right],\tag{19}$$

with $\tilde{U}$ being the Wilson line in the adjoint representation. In the large N_c limit, the cross section is approximated as

$$\frac{d\sigma^{pA\longrightarrow gX}}{d^2k_\perp dY}\approx xg(x)\int\frac{d^2r_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}\times\int d^2R_\perp\langle\{[\hat{S}(R_\perp,r_\perp)]^2+[\hat{O}(R_\perp,r_\perp)]^2\}\rangle_{x_g},\tag{20}$$

where all antisymmetric interference terms $\hat{S}(R_\perp,r_\perp)\hat{O}(R_\perp,r_\perp)$ completely cancel out. We are only left with the symmetric terms that do not contribute to the spin asymmetry. Since SSA vanishes in the gluon-initiated jet production process, one might expect that the spin asymmetry rises with the increasing $|x_F|$ in the backward region of pp collisions.

Recently, the single spin asymmetry in inclusive jet production has been measured in both forward and backward regions at the AnDY experiment at the RHIC [32]. There is at least one experimental data point in the backward region that is inconsistent with zero within the error bar. The possible two sources of the spin asymmetry in the backward region are the polarized odderon and the gluon Sivers function. However, we have shown that the gluon Sivers function dies out very quickly with decreasing x_g [33]. Therefore, this measurement likely indicates that the polarized odderon indeed exists.

Let us now turn to discuss the SSA in open charm production in the SIDIS process. The differential cross section for this process has been calculated in the dipole model [34] and in the CGC formalism [35]. The next-to-leading order correction to this process is also available in Refs. [36,37]. At leading order, the cross section in momentum space reads

$$\begin{aligned}\frac{d\sigma}{dx_BdzdQ^2dyd^2l_\perp}&=\frac{\alpha_{em}^2e_c^2}{2\pi^4x_BQ^2}\left[1-y+\frac{y^2}{2}\right][z^2+(1-z)^2]\int\frac{d^2p_\perp}{(2\pi)^2}\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k_\perp'}{(2\pi)^2}\frac{(l_\perp-k_\perp)\cdot(l_\perp-k'_\perp)}{[\rho+(l_\perp-k_\perp)^2][\rho+(l_\perp-k'_\perp)^2]}\\&\times\langle\mathrm{Tr}\{[U(k_\perp)U^\dagger(k_\perp-p_\perp)-(2\pi)^4\delta^2(k_\perp)\delta^2(k_\perp-p_\perp)]\\&\times[U(k'_\perp-p_\perp)U^\dagger(k'_\perp)-(2\pi)^4\delta^2(k'_\perp)\delta^2(k'_\perp-p_\perp)]\}\rangle_{x_g}.\end{aligned}\tag{21}$$

Here, the common kinematical variables in the SIDIS process are defined as $Q^2=-q\cdot q$, $x_B=Q^2/2P\cdot q$, $y=q\cdot P/P_e\cdot P$ and $z=l\cdot P/P\cdot q$, where l, P_e, P, and q are momenta for produced charm quark, incoming lepton and proton, and virtual photon, respectively. ρ is defined as $\rho=z(1-z)Q^2$. $U(k_\perp)$ is the Fourier transform of $U(x_\perp)$. For simplicity, we have neglected charm quark mass and only taken into account the transverse polarized virtual photon contribution to the differential cross section. The above formula can be reorganized and expressed in a more compact form,

$$\begin{aligned}&\frac{d\sigma}{dx_BdzdQ^2dyd^2l_\perp}\\&=\frac{\alpha_{em}^2e_c^2}{2\pi^4x_BQ^2}\int\frac{d^2k_\perp}{(2\pi)^2}H(k_\perp,l_\perp,Q^2)\langle\mathrm{Tr}[U(k_\perp)U^\dagger(k_\perp)]\rangle_{x_g}\\&=\frac{\alpha_{em}^2e_c^2N_c}{2\pi^4x_BQ^2}\int d^2k_\perp H(k_\perp,l_\perp,Q^2)\\&\times\left[F_{x_g}(k_\perp^2)+\frac{1}{M}\epsilon_\perp^{ij}S_{\perp i}k_{\perp j}O_{1T,x_g}^\perp(k_\perp^2)\right],\end{aligned}\tag{22}$$

where the first term recovers the known unpolarized differential cross section, whereas the second term is the spin-dependent contribution. For anticharm quark production, $U(k_\perp)U^\dagger(k_\perp)$ that appears in the first line of the above equation should be replaced with $U^\dagger(k_\perp)U(k_\perp)$, leading to the exactly opposite SSA as compared to that in charm quark production. The hard part $H(k_\perp,l_\perp,Q^2)$ is given by

$$H(k_\perp,l_\perp,Q^2)=\left[1-y+\frac{y^2}{2}\right][z^2+(1-z)^2]\times\left[\frac{l_\perp-k_\perp}{\rho+(l_\perp-k_\perp)^2}-\frac{l_\perp}{\rho+l_\perp^2}\right]^2.\tag{23}$$

The SSA in open charm production in the SIDIS process has also been calculated using the collinear twist-3 approach [9] (for earlier work, see Ref. [38]). In the framework of the collinear factorization, a C-odd trigluon correlation that gives rise to SSA is defined as [8,9]

$$O^{\alpha\beta\gamma}(x_1,x_2) = -gi^3 \int \frac{dy^- dz^-}{(2\pi)^2 P^+} e^{iy^- x_1 P^+} e^{iz^-(x_2-x_1)P^+} \times \langle pS| d^{bca} F_b^{\beta +}(0) F_c^{\gamma +}(z^-) F_a^{\alpha +}(y^-)|pS\rangle, \tag{24}$$

where we regard all the free Lorentz indices α, β, and γ to be transverse in three dimension. One can also define a C-even trigluon correlation $N^{\alpha\beta\gamma}(x_1,x_2)$ by replacing d^{bca} with the antisymmetric tensor if^{bca} in the above equation [8,9]. Both the C-even and C-odd trigluon correlations contribute to SSAs. However, only the C-odd trigluon correlation is the relevant one at small x as shown in Ref. [33].

It is known that the $k_\perp$ moment of the gluon Sivers function can be related to the gluonic pole C-even trigluon correlation $N^{\alpha\beta\gamma}(x_g,x_g)$. A similar relation between the $k_\perp$ moment of the polarized odderon and the C-odd trigluon correlation can be established. At small x, exponentials that appear in Eq. (24) can be approximated as $e^{iy^- x_1 P^+} \approx 1$ and $e^{iz^-(x_2-x_1)P^+} \approx 1$. In this approximation, one has

$$\int d^2k_\perp k_\perp^\alpha k_\perp^\beta k_\perp^\gamma \frac{1}{M} \epsilon_\perp^{ij} S_{\perp i} k_{\perp j} O_{1T,x_g}^\perp(k_\perp^2) = \frac{-ig^2\pi^2}{2N_c} O^{\alpha\beta\gamma}(x_g), \tag{25}$$

where $O^{\alpha\beta\gamma}(x_g) \equiv O^{\alpha\beta\gamma}(x_1,x_2)$ with x_g being the total momentum transfer carried by gluons, which can be conveniently chosen to be $x_g \equiv \mathrm{Max}\{x_1,x_2\}$. With this derived relation, one is able to compare the full polarized cross section computed in the CGC framework and the collinear twist-3 approach in an overlap kinematical region $l_\perp \ll Q_s$ where both formalisms apply. However, we find that the hard parts calculated in the different approaches differ by a factor 2. The reason for this disagreement is not yet clear. The extra investigation of the hard parts is thus needed.

IV. SUMMARY

In this paper, we have shown that an axial asymmetric color source distribution in the transverse plane of a transversely polarized proton can give rise to a spin-dependent odderon in the MV model. Such a polarized odderon is responsible for SSA in jet production in the backward region of pp collisions and SSA in open charm production in the SIDIS process. As a result, the BLV odderon solution can be tested by studying the x dependence of SSAs. A relation between the $k_\perp$ momentum of the odderon and the collinear twist-3 C-odd trigluon correlation has been established. It is straightforward to extend our formalism to study SSAs in open charm production in pp collisions and in Drell-Yan/direct photon processes, which already have been calculated in the collinear twist-3 framework [39,40] (for earlier work, see Ref. [41]). It is our plan to further explore the possible difference/relation between the CGC formalism and the collinear twist-3 approach in computing SSAs at small x.

ACKNOWLEDGMENTS

I am grateful to Vladimir Braun for bringing my attention to the potential relation between the odderon and the C-odd trigluon correlation. This work has been supported by BMBF (Grant No. 05P12WRFTE).

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