Full pseudoscalar mesonic chiral Lagrangian at p^6 order under the unitary group

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We construct the full p^6 order chiral Lagrangians for the unitary group and special unitary groups, including n_f-, three- and two-flavor cases, all bilinear currents (scalar, pseudoscalar, vector, axial-vector, and tensor currents), and θ parameter. The number of independent operators are 1391, 1326, and 969 for each of the flavor unitary groups. From these results, we find one extra linear relation among the traditional p^4 order low-energy constants under the U(3) group, and some more linear relations with tensor sources for the p^6 order low-energy constants in the special unitary groups. We develop a scheme to obtain the relations for the dependent operators in terms of independent operators.

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I. INTRODUCTION

In low-energy QCD, chiral perturbation theory (ChPT) is a powerful tool in treating hadron physics. With the help of ChPT, we can describe the low-energy pseudoscalar mesons (π, K, η, η') up to a certain degree of precision. In the last three decades, ChPT has matured and can specify next-to-next-to-leading order (NNLO) processes. The first step in the ChPT is to obtain the chiral Lagrangian, where most of the difficulties and discussions arise. Conventionally, one expands the chiral Lagrangian in terms of powers of momentum (p). For the special unitary (SU) group, ChPT for the pseudoscalar meson has been improved from the leading (p^2) order [1], to the next-to-leading (p^4) order (NLO) [2,3], and the NNLO (p^6 order) [4–10]. At present, almost all NNLO chiral Lagrangian have been obtained, including two- and three-flavor quarks, the normal and anomalous parts, and all bilinear light-quark currents (scalar, pseudoscalar, vector, axial-vector, and tensor currents), except for the θ parameter related terms. For the unitary (U) groups, the NLO results were also obtained [11]. Furthermore, one can expand the chiral Lagrangian in terms of p and $1/N_c$ simultaneously. As for $O(p^2)$, the results to order $O(\delta)$, have been obtained [12–14].

At present, under the SU group, the NNLO chiral Lagrangian seems sufficient. The chiral Lagrangian at any higher order is much more complicated and without much physical interest. Even though it can be obtained, the work would be very tedious and long, because of the complexity, and the advantages of such a chiral Lagrangian would vanish. Furthermore, comparisons between theory and experiments have not reached the precision where higher-order computations are needed. Even in the NNLO, some mistakes may appear through the tedium of the calculations, raising doubts about the credibility of the evaluations. When the NNLO chiral Lagrangian was first obtained [4,9], some linear relations among low-energy constants (LECs) had been missed [5–8,10]. This poses the question: Do other hidden relations exit? We need a definite answer. If the existing LECs are not independent, they are not unique, and ChPT cannot work well. In the NNLO, there exists another problem. When one wants all LECs [10,15,16] or discusses some processes, such as $\pi-\pi$ scattering [17,18], $\pi-K$ scattering [19], two-point functions [20], scalar form-factors [21], and $\eta \to 3\pi$ process [22], one may meet some operators which are linear dependent on the independent terms in the chiral Lagrangian. One needs to obtain the linear relations among these operators and the independent terms. Almost all relations are simple and they should be obtained easily. Nevertheless, some of them are quite long (see for example [6] and Appendix B in [5]). The problem is how are these complex calculations to be done, and computations made more reliable.

Besides more higher-order terms, or remaining with the NNLO under the SU groups, one may extend the calculation to the NNLO under the U group. For the conventional SU chiral Lagrangian at the NNLO, there exists no singlet vector sources, singlet axial-vector sources, or θ parameter, but we may need to consider them in some special problems. In some of these problems, these circumstances may need to be considered at even NLO, although

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of course, this may not be very accurate compared with NNLO. The extra $U(1)$ symmetry is related to the η' particle, which is the relevant singlet on the diagonal of the meson matrix when one is dealing with the properties of η' or with the interaction that includes η' [23], especially $\eta-\eta'$ mixing [24–28]. One needs to mix the singlet and octet first to separate η and η', and then to study their other properties. Parameter θ comes from the strong CP violation, which induces baryon electric dipole moments [29,30]. One can estimate θ or baryon electric dipole moments with a θ-dependent term. Nevertheless certain phenomena are related to singlet vector or axial-vector sources, for instance, the electromagnetic interactions corrections to the mesons [31–33]. In these situations, the θ parameter, singlet vector, and axial-vector sources cannot be ignored. For its convenience, the $1/N_c$ expansion is a useful scheme to obtain the approximate analytic solution. In ChPT, the θ parameter and η' mass cannot be ignored and should also be included in the N_c power expansion [12,14,34]. Sometimes, tensor sources need to be considered; as for example, for the magnetic susceptibility of the quark condensate defined by the vacuum expectation value of the tensor current [35–37]. The purpose of our study is to obtain the full U group chiral Lagrangian up to NNLO, including all bilinear quark currents and θ parameter terms. With this chiral Lagrangian and techniques developed, all the above-mentioned problems can be solved more accurately and concomitantly.

This paper is organized as follows: In Sec. II, we review the two kinds of basis to construct the chiral Lagrangian and provide definitions of symbols. In Sec. III, we collect all possible linear relations for the chiral Lagrangian. Section IV is an introduction to our method of generating the chiral Lagrangian. In Sec. V, we list our results for the U and SU groups up to order p^6. Section VI concludes with a summary.

II. CHIRAL TRANSFORMATIONS AND CHIRAL BASIS

In QCD, the original Lagrangian is denoted $\mathcal{L}^0_{\rm QCD}$. We usually introduce some external sources and the topological charge operator to obtain the corresponding Green's functions. The complete bilinear coupling external sources contain scalar (s), pseudoscalar (p), vector (v^μ), axial-vector (a^μ), and tensor ($\bar{t}^{\mu\nu}$) currents. The topological charge operator is $G_{\mu\nu}\tilde{G}^{\mu\nu}$, where $G^{\mu\nu}$ is the gluon field strength matrix, and $\tilde{G}^{\mu\nu}=\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}$. The QCD Lagrangian is expanded as

$$\mathcal{L}=\mathcal{L}^0_{\rm QCD}+\bar{q}(\not{v}(x)+\not{a}(x)\gamma_5-s(x)+ip(x)\gamma_5+\sigma_{\mu\nu}\bar{t}^{\mu\nu}(x))q-\frac{1}{16\pi^2}\theta(x)\,{\rm tr}_c(G_{\mu\nu}\tilde{G}^{\mu\nu}), \tag{1}$$

where q denotes the light quark fields. For low-energy QCD, we mainly focus on the nonet of the pseudoscalar mesons (π,K,η,η'). As in [2,3], we collect the nonet in the $U(x)$ matrix as

$$U(x)=e^{i\phi(x)/F_0} \tag{2}$$

$$\phi(x)\overset{n_f=3}{=\!=\!=}\sum_{a=0}^{8}\lambda_a\phi_a(x)=\begin{pmatrix}\pi^0+\frac{1}{\sqrt{3}}\eta^8+\sqrt{\frac{2}{3}}\eta^0 & \sqrt{2}\pi^+ & \sqrt{2}K^+\\ \sqrt{2}\pi^- & -\pi^0+\frac{1}{\sqrt{3}}\eta^8+\sqrt{\frac{2}{3}}\eta^0 & \sqrt{2}K^0\\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta^8+\sqrt{\frac{2}{3}}\eta^0\end{pmatrix}. \tag{3}$$

Where F_0 is the pseudoscalar meson decay constant in the chiral limit, and $\lambda_i(i=1\ldots8)$ are the Gell-Mann matrices, and $\lambda_0=\sqrt{2/n_f}{\rm I}_{n_f\times n_f}$. We usually choose $n_f=2$ or 3. If the light quark are assumed massless, $\mathcal{L}$ is invariant under the $U_L(n_f)\times U_R(n_f)$ group. For an effective field theory, we often focus on the meson fields U. Under the $U_L(n_f)\times U_R(n_f)$ transformations, U transforms as

$$U\to g_R U g_L^\dagger,\qquad g_L\in U_L,\qquad g_R\in U_R. \tag{4}$$

Simultaneously, the external sources also need transforming. For convenience, we usually collect the sources as

$$\begin{aligned} r^\mu&=v^\mu+a^\mu,\\ l^\mu&=v^\mu-a^\mu,\\ \chi&=2B_0(s+ip),\\ \hat{\theta}&=i\theta,\\ X&=\langle\ln U\rangle+\hat{\theta},\\ t^{\mu\nu}&=\frac{1}{2}\bar{t}^{\mu\nu}-\frac{i}{4}\epsilon^{\mu\nu\lambda\rho}\bar{t}_{\lambda\rho}. \end{aligned} \tag{5}$$

The notation $\langle\cdots\rangle$ is the trace over flavor indices. B_0 is a constant related to the quark condensate. Under

$U_L(n_f) \times U_R(n_f)$ rotations (chiral rotations), these sources transform as

$$\begin{aligned} l^\mu &\to g_L l_\mu g_L^\dagger + i g_L \partial^\mu g_L^\dagger \\ r^\mu &\to g_R r_\mu g_R^\dagger + i g_R \partial^\mu g_R^\dagger \\ \chi &\to g_R \chi g_L^\dagger \\ t^{\mu\nu} &\to g_R t^{\mu\nu} g_L^\dagger \\ X &\to X. \end{aligned} \tag{6}$$

For $\hat{\theta}$, one usually introduces its covariant derivative

$$\nabla^\mu \hat{\theta} = \partial^\mu \hat{\theta} + 2i\langle a^\mu \rangle, \tag{7}$$

which is invariant under the chiral rotations. It is more convenient to introduce the field strengths $F_L^{\mu\nu}$ and $F_R^{\mu\nu}$ associated with l^μ, r^μ,

$$\begin{aligned} F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu], \\ F_L^{\mu\nu} &= \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu]. \end{aligned} \tag{8}$$

Under $U_L(n_f) \times U_R(n_f)$ rotations, they transform as

$$F_L^{\mu\nu} \to g_L F_L^{\mu\nu} g_L^\dagger, \qquad F_R^{\mu\nu} \to g_R F_R^{\mu\nu} g_R^\dagger. \tag{9}$$

We can now construct the chiral Lagrangian, with $U, U^\dagger, F_L^{\mu\nu}, F_R^{\mu\nu}, \chi, \chi^\dagger, t^{\mu\nu}, t^{\dagger\mu\nu}, \nabla^\mu\hat{\theta}$, and X, as well as their covariant derivatives; we call this set the L(eft)R(ight)-basis. These covariant derivations combine with l^μ and r^μ, retaining the same transformations as the fields on which they act. Different transformations have different covariant derivations. They can be obtained from (6). Following [4], we express them as

$$\begin{aligned} A \to g_R A g_L^\dagger &: D^\mu A = \partial^\mu A - i r^\mu A + i A l^\mu, \\ B \to g_L B g_R^\dagger &: D^\mu B = \partial^\mu B - i l^\mu B + i B r^\mu, \\ C \to g_R C g_R^\dagger &: D^\mu C = \partial^\mu C - i r^\mu C + i C r^\mu, \\ D \to g_L D g_L^\dagger &: D^\mu D = \partial^\mu D - i l^\mu D + i D l^\mu, \\ E \to E &: D^\mu E = \partial^\mu E. \end{aligned} \tag{10}$$

In principle, the conditions constraining the chiral Lagrangian relate only to its symmetries, which include the Lorentz symmetry, local chiral symmetry, parity symmetry, and charge conjugation symmetry. We list these properties on for the LR-basis in Table I. Because the chiral Lagrangian needs to be real, we also list the Hermiticity of the LR-basis. The operators in Table I with covariant derivations have the same properties.

At present, we can construct the chiral Lagrangian with the LR-basis. However, different bases have different chiral rotation properties, particularly, in (10), the covariant derivatives relations are complex. Such differences can yield among the covariant derivatives quite complex properties for some identities, for instance the strength tensor relations and the Bianchi identity, which will be introduced later in (17) and (22), respectively, in regard to another basis. In using the LR-basis some linear relations may get lost [4]. Hence, we usually use another basis, which we refer to as building blocks. We will until the very end use these building blocks to construct the chiral Lagrangian, but will return to the LR-basis for the contact terms.

TABLE I. Chiral rotations (R), P, C and Hermiticity of the LR-basis (O).

O	R	P	C	H.c.
U	$g_R U g_L^\dagger$	$U^\dagger$	U^T	$U^\dagger$
χ	$g_R \chi g_L^\dagger$	$\chi^\dagger$	χ^T	$\chi^\dagger$
$F_L^{\mu\nu}$	$g_L F_L^{\mu\nu} g_L^\dagger$	$F_R^{\mu\nu}$	$-(F_R^{\mu\nu})^T$	$F_L^{\mu\nu}$
$F_R^{\mu\nu}$	$g_R F_R^{\mu\nu} g_R^\dagger$	$F_L^{\mu\nu}$	$-(F_L^{\mu\nu})^T$	$F_R^{\mu\nu}$
$t^{\mu\nu}$	$g_R t^{\mu\nu} g_L^\dagger$	$t^{\dagger\mu\nu}$	$-(t^{\mu\nu})^T$	$t^{\dagger\mu\nu}$
X	X	$-X$	X	$-X$
$\nabla^\mu\hat{\theta}$	$\nabla^\mu\hat{\theta}$	$-\nabla^\mu\hat{\theta}$	$\nabla^\mu\hat{\theta}$	$-\nabla^\mu\hat{\theta}$

Following [5,8,9], we define the Goldstone boson field u, $U = u^2$, and introduce a compensator field h. The field u transforms as

$$u \to u = g_R u h^\dagger = h u g_L^\dagger. \tag{11}$$

Hence, under chiral rotations, all the building blocks O are invariant or transform as

$$O \to h O h^\dagger. \tag{12}$$

These building blocks and their relations to the LR-basis are

$$\begin{aligned} u^\mu &= i\{u^\dagger(\partial^\mu - i r^\mu)u - u(\partial^\mu - i l^\mu)u^\dagger\}, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ h^{\mu\nu} &= \nabla^\mu u^\nu + \nabla^\nu u^\mu, \\ f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \\ t_\pm^{\mu\nu} &= u^\dagger t^{\mu\nu} u^\dagger \pm u t^{\mu\nu\dagger} u, \\ \nabla^\mu\hat{\theta} &= \partial^\mu\hat{\theta} + 2i\langle a^\mu \rangle, \\ X &= \langle \ln U \rangle + \hat{\theta}. \end{aligned} \tag{13}$$

We can see that almost all the operators on the left-hand side of (13) transform as (12); the exceptions $\nabla^\mu\hat{\theta}$ and X are invariant.

Furthermore, we also need to define the covariant derivative for all operators. For the operators transforming as (12), the covariant derivatives are

TABLE II. P, C, and Hermiticity of the building blocks.

O	P	C	H.c.
u^μ	$-u_\mu$	$(u^\mu)^T$	u^μ
$h^{\mu\nu}$	$-h_{\mu\nu}$	$(h^{\mu\nu})^T$	$h^{\mu\nu}$
$\chi_\pm$	$\pm\chi_\pm$	$(\chi_\pm)^T$	$\pm\chi_\pm$
$f_\pm^{\mu\nu}$	$\pm f_{\pm\mu\nu}$	$\mp(f_\pm^{\mu\nu})^T$	$f_\pm^{\mu\nu}$
$t_\pm^{\mu\nu}$	$\pm t_{\pm\mu\nu}$	$-(t_\pm^{\mu\nu})^T$	$\pm t_\pm^{\mu\nu}$
$\nabla^\mu\theta$	$-\nabla^\mu\theta$	$\nabla^\mu\theta$	$-\nabla^\mu\theta$
X	$-X$	X	$-X$

$$\nabla^\mu O = \partial^\mu O + [\Gamma^\mu, O], \tag{14}$$

where the chiral connection is

$$\Gamma^\mu = \frac{1}{2}\{u^\dagger(\partial^\mu - ir^\mu)u + u(\partial^\mu - il^\mu)u^\dagger\}. \tag{15}$$

Because $\nabla^\mu\hat{\theta}$ and X are invariant under the chiral rotations, their covariant derivatives are the ordinary partial derivative. For convenience, we will use below ∇ instead of ∂, because the commutator is zero in (14). With this covariant derivative, $f_-^{\mu\nu}$ can be simplified as

$$f_-^{\mu\nu} = -\nabla^\mu u^\nu + \nabla^\nu u^\mu. \tag{16}$$

The field strength tensor $\Gamma_{\mu\nu}$ associated with this covariant derivative is

$$[\nabla^\mu, \nabla^\nu]O = [\Gamma^{\mu\nu}, O], \tag{17}$$

$$\Gamma^{\mu\nu} = \nabla^\mu\Gamma^\nu - \nabla^\nu\Gamma^\mu - [\Gamma^\mu, \Gamma^\nu] = \frac{1}{4}[u^\mu, u^\nu] - \frac{i}{2}f_+^{\mu\nu}. \tag{18}$$

The chiral Lagrangian can be constructed by the building blocks in (13) and their covariant derivatives.[1] As in [5], we choose for $\chi_\pm$ a more convenience form for its covariant derivatives;

$$\chi_\pm^\mu = u^\dagger D^\mu\chi u^\dagger \pm uD^\mu\chi^\dagger u = \nabla^\mu\chi_\pm - \frac{i}{2}\{\chi_\mp, u^\mu\}, \tag{19}$$

which has a more simple relationship in the LR-basis. With regard to the LR-basis, we list the properties of the building blocks in Table II.

III. FROM BUILDING BLOCKS TO CHIRAL LAGRANGIAN

Given the properties of the building blocks in Table II, we can construct the chiral Lagrangian by permuting and combining these building blocks. We shall focus mainly on the full $U(n_f)\times U(n_f)$ group, including all external sources and θ parameter $(v^\mu, a^\mu, s, p, \bar{t}^{\mu\nu}, \theta)$. Nevertheless, for orders above $O(p^4)$, the numbers of terms are very large. The higher the order, the much larger the number of terms becomes. Hence listing all terms is tedious. However, these terms are not linearly independent, we only need to find all linear conditions and list only those independent terms. All linear conditions come from the following relations.

(i) Partial integration: Except for a total derivative term, we can remove one covariant derivative with respect to another operator in the following manner

$$\langle\nabla^\mu AB\cdots\rangle\langle CD\cdots\rangle\cdots = -\langle A\nabla^\mu B\cdots\rangle\langle CD\cdots\rangle\cdots - \cdots - \langle AB\cdots\rangle\langle\nabla^\mu CD\cdots\rangle\cdots - \langle AB\cdots\rangle\langle C\nabla^\mu D\cdots\rangle\cdots - \cdots, \tag{20}$$

where "$\cdots$" represent one or more operators.

(ii) Equations of motion: From the chiral Lagrangian and Euler-Lagrange equation, the lowest-order equations of motion (EOM) in the LR-basis for the chiral Lagrangian are given in Eq. (22) of Ref. [11]. In the building-blocks basis, we have

$$\begin{aligned}\nabla_\mu u^\mu = {} & \frac{i}{2}\frac{W_0'}{W_1} - \frac{i}{2}\frac{W_1'}{W_1}\langle u^\mu u_\mu\rangle + i\frac{W_1'}{W_1}(\langle u_\mu\rangle + i\nabla_\mu\hat{\theta})u^\mu + \frac{i}{2}\frac{W_2}{W_1}\chi_- - \frac{1}{2}\frac{W_3}{W_1}\chi_+ \\ & - \frac{i}{2}\frac{W_2'}{W_1}\langle\chi_+\rangle + \frac{1}{2}\frac{W_3'}{W_1}\langle\chi_-\rangle + \left(\frac{i}{2}\frac{W_5'}{W_1} - \frac{i}{2}\frac{W_6'}{W_1}\right)(\nabla_\mu\hat{\theta})(\nabla^\mu\hat{\theta}) + \frac{i}{2}\frac{W_5}{W_1}\nabla_\mu\nabla^\mu\hat{\theta}.\end{aligned} \tag{21}$$

[1]The ∇^μ in $\nabla^\mu\hat{\theta}$ is not this covariant derivative. However, we believe no confusion arises because they are always combinative. Higher-order covariant derivatives in $\nabla^\mu\hat{\theta}$ are equal to the ordinary derivatives or (14). Therefore, we persist with this form for the sake of convenience.

The W coefficients will be defined in (53). Some minus signs in (21) are required to match the coefficients in [11]. The conclusion of [5,38] was that up to $O(p^6)$, the lowest-order EOM are satisfied. If we want to evaluate a higher-order chiral Lagrangian, we need to develop the EOM to a higher order, which only adds terms on the right-hand side of (21). If we are only wanting the independent terms, we can also eliminate the factor $\nabla_\mu u^\mu$ without any difficulties.

(iii) Bianchi identity: From Eqs. (18), we can get

$$\nabla^\mu \Gamma^{\nu\lambda} + \nabla^\nu \Gamma^{\lambda\mu} + \nabla^\lambda \Gamma^{\mu\nu} = 0, \tag{22}$$

which gives a relation between the covariant derivatives of $\Gamma^{\mu\nu}(f_+^{\mu\nu})$.

(iv) Schouten identity: When constructing the odd parity terms, we need to use the odd-intrinsic-parity factor $\epsilon^{\mu\nu\lambda\rho}$, as for example in the Schouten identity,

$$\epsilon^{\mu\nu\lambda\rho}A^\sigma - \epsilon^{\sigma\nu\lambda\rho}A^\mu - \epsilon^{\mu\sigma\lambda\rho}A^\nu - \epsilon^{\mu\nu\sigma\rho}A^\lambda - \epsilon^{\mu\nu\lambda\sigma}A^\rho = 0. \tag{23}$$

(v) Tensor relations: For tensor sources, using (5) and (13), one can obtain two relations between the odd-intrinsic-parity tensor $t_{-\mu\nu}$ and the even-intrinsic-parity tensor $t_{+\mu\nu}$ [9];

$$\epsilon_{\mu\nu\lambda\rho}t_\pm^{\lambda\rho} = 2it_{\mp\mu\nu}. \tag{24}$$

With these relations we can obtain the following transformations

$$O_1 t_+ O_2 t_- O_3 \leftrightarrow O_1 t_- O_2 t_+ O_3, \text{etc.} \tag{25}$$

and develop relations between the two $t_\pm^{\mu\nu}$'s.

(iv) Contact terms: In the chiral Lagrangian, we usually focus on the mesons fields, and separate the pure external sources terms and θ terms, called contact terms, leaving only meson dependent terms.

TABLE III. Power counting of each base.

Order	LR-basis	Building blocks
$O(p^0)$	$U, \hat{\theta}, X$	$u, \hat{\theta}, X$
$O(p^1)$	a^μ, v^μ	u^μ
$O(p^2)$	$\chi, t^{\mu\nu}$	$\chi_\pm, f_\pm^{\mu\nu}, t_\pm^{\mu\nu}$

(vii) Cayley-Hamilton relations: Usually, one develops the chiral Lagrangian for mesons composed of the lightest two or three quarks. In these situations, the operators are 2×2 or 3×3 matrices. The Cayley-Hamilton theorem states that every $n\times n$ matrix A satisfies its own characteristic equation, $p(\lambda)$,

$$p(\lambda) = \det(A - \lambda I_n) = 0, \qquad p(A) = 0, \tag{26}$$

and hence gives a relation between the trace of an operator and its determinant. For 2×2 and 3×3 matrices,

$$\begin{aligned} 2\times 2&: \ \det(A) = \frac{1}{2}(\langle A\rangle^2 - \langle A^2\rangle), \\ 3\times 3&: \ \det(A) = \frac{1}{6}(\langle A\rangle^3 - 3\langle A\rangle\langle A^2\rangle + 2\langle A^3\rangle). \end{aligned} \tag{27}$$

Using (27), with some tricks (see [4,9]), for arbitrary 2×2 matrices A and B, we can get 0

$$AB+BA-A\langle B\rangle - B\langle A\rangle - \langle AB\rangle + \langle A\rangle\langle B\rangle = 0. \tag{28}$$

For arbitrary 3×3 matrices A, B, and C, we can get

$$\begin{aligned} 0 = {} & ABC + ACB + BAC + BCA + CAB + CBA - AB\langle C\rangle - AC\langle B\rangle - BA\langle C\rangle - BC\langle A\rangle - CA\langle B\rangle \\ & - CB\langle A\rangle - A\langle BC\rangle - B\langle AC\rangle - C\langle AB\rangle - \langle ABC\rangle - \langle ACB\rangle + A\langle B\rangle\langle C\rangle + B\langle A\rangle\langle C\rangle + C\langle A\rangle\langle B\rangle \\ & + \langle A\rangle\langle BC\rangle + \langle B\rangle\langle AC\rangle + \langle C\rangle\langle AB\rangle - \langle A\rangle\langle B\rangle\langle C\rangle. \end{aligned} \tag{29}$$

Hence, when developing the two- or three-flavor chiral Lagrangian, the Cayley-Hamilton relations cannot be ignored.

The above conditions lead to all linear relations for the chiral Lagrangian. No other linear relations exist. Hence, if we can use all conditions systematically, the chiral Lagrangian can be obtained. The next section introduces our method in detail.

IV. CONSTRUCTION OF THE CHIRAL LAGRANGIAN

In this section, we work through the above situations one by one to construct the chiral Lagrangian.

A. Power counting

In this paper, we only adopt the conventional power counting rule given in [2,3], expand the chiral Lagrangian in powers of momentum. We show the power counting scheme in Table III. Each covariant derivative counts as $O(p^1)$. For tensor fields, we use the power counting rule in [9], and consider tensor fields as combined (axial-)vector fields. Hence the chiral Lagrangian has only even orders in the expansion.

In Table II and III, one can see that, X is of order $O(p^0)$, hence in the chiral Lagrangian it can exist in a functional form. X also has odd parity, so a Lagrangian without X can

TABLE IV. Example of operator and index numbering. No significance is given to the specific numbers. These can be big or small, or even personal preferences.

Operator	$\langle$	$\rangle$	∇	u	h	χ_+	χ_-	f_-	Γ_+	t_+	t_-	$\hat{\theta}$
Number	82	83	1004	1011	1058	1021	1141	1071	1007	2604	2605	1201
Index	μ	ν	λ	ρ	σ							
Number	1	2	3	4	5							

have both even or odd parity terms. The general form of the chiral Lagrangian to any given order is

$$\mathcal{L}^{2n} = \sum_m f_m(X) O_m, \tag{30}$$

where the $f_m(X)$ are any functions of X, and O_m are operators up to order $O(p^{2n})$. To construct the chiral Lagrangian of given order is to find all O_m up to that order.

B. Reduced building blocks

The building blocks in Table II are not very suitable for constructing the chiral Lagrangian directly, because some building blocks, such as $f_+^{\mu\nu}$, couple to different sources. Although the calculation is a little complex, we can break it up into smaller steps. First, before the calculations, we reduce the building blocks to more basic ones, each containing a given number of external sources. We call these the reduced building blocks, which we use when searching linear relations. At the end of the calculation, we revert to the original building blocks.

From (18), we can see $f_+^{\mu\nu}$ is related to $\Gamma^{\mu\nu}$, with some redundant u^μ. If we have only the linear independent terms, the choice of $\Gamma^{\mu\nu}$ is much more convenient, because it is directly relevant in the Bianchi identity(22). Ignoring the redundant u^μ makes no sense when we are only wanting the independent terms. Their effects can only be compensated in contributions of u_μ. For the same reason, for (19), we choose $\nabla^\mu\chi_\pm$ instead of $\chi_\pm^\mu$. We call those building blocks in Table II along with the substitutions

$$f_+^{\mu\nu} \leftrightarrow i\Gamma^{\mu\nu}, \qquad \chi_\pm^\mu \leftrightarrow \nabla^\mu\chi_\pm \tag{31}$$

the reduced building blocks. The P, C, and Hermiticity properties between the original building blocks and the reduced building blocks are the same.

If we obtain the high-order chiral Lagrangian, then their covariant derivatives will appear with the reduced building blocks. To order $O(p^{2n})$, with the partial integration relations, we need $\nabla\nabla\cdots\nabla O$ to order $O(p^n)$ at most, where O is a reduced building block.

Furthermore, for convenience, when we are only determining the independent operators (17) can be simplified to

$$[\nabla_\mu, \nabla_\nu] O \to 0, \tag{32}$$

because the excess terms on the right-hand side of (17) can be constructed by $\Gamma^{\mu\nu}$ and O with two less ∇.

C. Permutation, combination, and primary screening

Permuting all reduced building blocks and their covariant derivatives, and adding appropriate traces, we can get a complete but not linearly independent set of operators. Because of (24), $t_-^{\mu\nu}$ can appear once at most. Nevertheless the number of permutations is very large. A preliminary screening can lead to a simpler calculation.

Most operators can be reduced using the Einstein summation convention $A^\mu A_\mu = A^\nu A_\nu$, and trace relations $\langle AB\rangle = \langle BA\rangle$. Even though the two operators are a little complex, we cannot assess immediately whether they are equal or not. A simple way is to change all operators to a unique form, we call it the *standard form*. We can assess whether two terms are equal directly by comparing their standard forms.[2]

First, we separate all operators and their indices, and assign a number to each operator and each index. Table IV presents an example. For convenience, we consider the brackets ("$\langle$" and "$\rangle$") and covariant derivative (∇) as single operators.

Second, we write down each operator number in order, and then enumerate the indices. We obtain a row vector such as

$$\langle u^\mu u^\nu h_{\mu\nu}\rangle \to (82, 1011, 1011, 1058, 83, 1, 2, 1, 2). \tag{33}$$

We call it the *numerical representation* of an operator. With trace relations, Einstein summation convention and (anti) symmetry index relations, we transform the numerical representation to its smallest possible form.[3] The smallest representation is the *numerical representation*, and the corresponding operator is the original operator in standard form.

With the above procedure, all trivial relations are used, including the Einstein summation convention, trace relations and (anti)symmetry indices relations. Next, we only need to work out one by one the conditions in Sec. III, with the standard form operators. For convenience, all the operators in the following calculations are in standard forms.

[2]This is like a web site verifying passwords in non-case-sensitive form. Changing all characters to upper or lower-case and then assessing them seems much easier.

[3]We introduce a small number on the left-hand side of the numerical representation to denote that it is the smallest form possible.

D. Substitutions and classifications

Because $h^{\mu\nu}$ and $f_{\pm}^{\mu\nu}$ contain covariant derivatives, determining the linear relations is hard with the reduced building blocks. Using (13), (16), and (18), we can clearly reveal the covariant derivatives.

In Sec. III, except for the EOM, each linear relation maintains its number of types of external sources. The relations cannot change scalar sources into vector sources or tensors to pseudoscalars for example.[4] As the EOM is related to $\nabla^{\mu}u_{\mu}$, we only need to ignore terms including this factor. To simplify the problem, we classify all operators by the numbers of types of external sources. One classification contains the same numbers of each external source. If there exists a linear relation, it only affects the operators in the same classification. Moreover, if we ignore the Cayley-Hamilton relations, or in the n_f-flavor case, the other relations cannot change the trace number.[5]

Now we have classified the operator as a big set C_{ij}. Index i indicates the sequence number of the types of the external sources, and j indicates the number of traces. For any given pair (i,j), all permutations of the reduced-building block standard forms $D_{ij,k}$ are known, k indicating the permutation numbers of the C_{ij}; all permutations of the revealable covariant derivatives standard forms $E_{ij,l}$ can be calculated, l indicating the permutation number. Their linear relations are

$$D_{ij,k}=\sum_{l}A_{ij,kl}E_{ij,l}, \tag{34}$$

where $A_{ij,kl}$ are elements of their relative matrices. Our purpose is to find all of the independent operators of $D_{ij,k}$, and $E_{ij,l}$ are the bridges in finding all linear relations.

E. Linear relations

Generally, for the n_f-flavor case, the linear relations depend only on the small sets $D_{ij,k}$ or $E_{ij,l}$, and our search scope is narrowed. All linear conditions can be added one by one

(i) EOM: With the EOM (21), we can remove all terms including $h^{\mu}{}_{\mu}$ in $D_{ij,k}$ or $\nabla^{\mu}u_{\mu}$ in $E_{ij,l}$.

(ii) Partial integration, Bianchi identity, Schouten identity, and tensor relations: The schemes for these conditions are the same. We can directly remove one term in (20), (22), (23), and (25). Although the relations are too many, remembering which relations have been used or not is not easy, and one does not know which $D_{ij,k}$ or $E_{ij,l}$ should be removed. A crude method is to apply these relations to all operators in $E_{ij,l}$. For partial integration, we only need to deal with the far left covariant derivative in each factor, because $E_{ij,l}$ is a complete set. There indeed exist some operators with another covariant derivative order. For the Bianchi identity, to reveal the covariant derivatives, (22) needs to be changed to

$$\begin{aligned}0=&\nabla^{\mu}\nabla^{\nu}\Gamma^{\lambda}-\nabla^{\mu}\nabla^{\lambda}\Gamma^{\nu}-\nabla^{\mu}\Gamma^{\nu}\Gamma^{\lambda}-\Gamma^{\nu}\nabla^{\mu}\Gamma^{\lambda}+\nabla^{\mu}\Gamma^{\lambda}\Gamma^{\nu}+\Gamma^{\lambda}\nabla^{\mu}\Gamma^{\nu}+\nabla^{\lambda}\nabla^{\mu}\Gamma^{\nu}-\nabla^{\lambda}\nabla^{\nu}\Gamma^{\mu}-\nabla^{\lambda}\Gamma^{\mu}\Gamma^{\nu}-\Gamma^{\mu}\nabla^{\lambda}\Gamma^{\nu}+\nabla^{\lambda}\Gamma^{\nu}\Gamma^{\mu}\\&+\Gamma^{\nu}\nabla^{\lambda}\Gamma^{\mu}+\nabla^{\nu}\nabla^{\lambda}\Gamma^{\mu}-\nabla^{\nu}\nabla^{\mu}\Gamma^{\lambda}-\nabla^{\nu}\Gamma^{\lambda}\Gamma^{\mu}-\Gamma^{\lambda}\nabla^{\nu}\Gamma^{\mu}+\nabla^{\nu}\Gamma^{\mu}\Gamma^{\lambda}+\Gamma^{\mu}\nabla^{\nu}\Gamma^{\lambda}.\end{aligned} \tag{35}$$

We also only need to focus on the covariant derivative factor near a Γ. The other covariant derivatives, even if the first in $\nabla\nabla\Gamma$, need not be considered. For the Schouten identity, we must exchange all indices different from that of ϵ. For the tensor relations, tensors involve one t_- and several t_+ terms. Because the number of t_- is at most one, exchanging it for each t_+ can give a linear relations. In summary, we can write down all linear relations.

$$\sum_{l}R_{ij,rl}E_{ij,l}=0, \tag{36}$$

where r is the number of relations, and $R_{ij,rl}$ are the relation matrices with the row index "ij" and column index "rl." The reduced row echelon form of $R_{ij,rl}$,

$$R_{ij,rl}\to S_{ij,rl}=\begin{pmatrix}1 & O & O & O & O & \cdots & \cdots & \cdots\\ & & 1 & O & O & \cdots & \cdots & \cdots\\ & & & \cdots & \cdots & \cdots & \cdots & \cdots\\ & & & & 1 & \cdots & \cdots & \cdots\\ & & & & & O & O & O\end{pmatrix}, \tag{37}$$

[4]A tensor relation changes t_+ and t_- by couples, so the total numbers of types of external sources remain the same. We also consider (32).

[5]The EOM has been used to exclude the $\nabla^{\mu}u_{\mu}$ factor.

where the O are some appropriate dimension-zero-matrices and "$\cdots$" can be nonzero matrices. The rank of $R_{ij,rl}$ or $S_{ij,rl}$ is equal to the number of linear relations. And each nonzero row-vector in $S_{ij,rl}$ gives a linear relation. We consider numbers of the operators that have number "1" as independent. Hence (34) can be reduced to

$$D_{ij,k} = \sum_l A'_{ij,kl} E_{ij,l}, \tag{38}$$

where using (36) $A'_{ij,kl}$ are the matrices $A_{ij,kl}$, with all linear dependent operators removed in the columns of (37) with "1."

(iii) Cayley-Hamilton relations: Because the Cayley-Hamilton relations can change the trace, we collect all D_{ij} and E_{ij}, as

$$\begin{aligned} D_{ij} &\to D_i = [D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}] \\ E_{ij} &\to E_i = [E_{i1}, E_{i2}, E_{i3}, \cdots, E_{in}], \end{aligned} \tag{39}$$

where n is the number corresponding to the maximum trace. Then, (38) is reduced to

$$D_{i,k} = \sum_l A'_{i,kl} E_{i,l}; \tag{40}$$

the Cayley-Hamilton relations only give new linear relations when one trace contains more than n_f operators. A, B, C in (28) and (29) can be one factor or more. Applying all possible combinations, we can get all Cayley-Hamilton relations. Removing all linear dependent terms, as in (38), we obtain

$$D_{n_f,i,k} = \sum_l A''_{n_f,i,kl} E_{n_f,i,l}, \tag{41}$$

where $D_{n_f,i,k}$ is the kth operator in the n_f-flavor case in the classification i ($n_f = 2$ or 3), $E_{n_f,i,l}$ is the lth revealable covariant derivative operator in the n_f-flavor case in the classification i, and $A''_{n_f,i,kl}$ is the matrix of coefficients relating the two.

(iv) Contact terms: Contact terms can be found in a specialized treatment. Because all the building blocks in Table II mix u (or U) fields and external sources, Constructing the contact terms with such building blocks is not so useful. Hence we construct contact terms with the LR-basis, without U fields. Their relations are

$$\begin{aligned} F_L^{\mu\nu} &= \frac{1}{2} u^\dagger (f_+^{\mu\nu} + f_-^{\mu\nu}) u, \\ F_R^{\mu\nu} &= \frac{1}{2} u (f_+^{\mu\nu} - f_-^{\mu\nu}) u^\dagger, \\ \chi &= \frac{1}{2} u (\chi_+ + \chi_-) u, \\ \chi^\dagger &= \frac{1}{2} u^\dagger (\chi_+ - \chi_-) u^\dagger \\ t^{\mu\nu} &= \frac{1}{2} u (t_+^{\mu\nu} + t_-^{\mu\nu}) u, \\ t^{\dagger\mu\nu} &= \frac{1}{2} u^\dagger (t_+^{\mu\nu} - t_-^{\mu\nu}) u^\dagger \\ \nabla^\mu \hat{\theta} &= \nabla^\mu \hat{\theta}. \end{aligned} \tag{42}$$

To construct the contact terms, one only needs the basis and their covariant derivatives in (42). By performing the same steps above, we can obtain all permutations of the non-U operators. There exist though two exceptions. The first is that different LR-bases have different chiral rotations. Hence, not all permutations, but only the chiral invariant operators remain. The second is that for the SU group, $u(U)$ is unitary, so $\det u(\det U) = 1$. With (27), the combination of the determinant of the LR-basis or its covariant derivatives can be chiral invariant as well. At order p^2, there exists no such operator. For order p^4, there exists one term only when $n_f = 2$,

$$\begin{aligned} \det\chi + \det\chi^\dagger = &-\frac{1}{4}\langle\chi_+^2\rangle - \frac{1}{4}\langle\chi_-^2\rangle + \frac{1}{4}\langle\chi_+\rangle\langle\chi_+\rangle \\ &+ \frac{1}{4}\langle\chi_-\rangle\langle\chi_-\rangle. \end{aligned} \tag{43}$$

For order p^6, there exists one term when $n_f = 3$,

$$\begin{aligned} \det\chi + \det\chi^\dagger = &\frac{1}{24}\langle\chi_+\rangle^3 + \frac{1}{8}\langle\chi_+\rangle\langle\chi_-\rangle^2 - \frac{1}{8}\langle\chi_+\rangle\langle\chi_+^2\rangle \\ &- \frac{1}{8}\langle\chi_+\rangle\langle\chi_-^2\rangle - \frac{1}{4}\langle\chi_+\chi_-\rangle\langle\chi_-\rangle \\ &+ \frac{1}{12}\langle\chi_+^3\rangle + \frac{1}{4}\langle\chi_+\chi_-^2\rangle. \end{aligned} \tag{44}$$

When $n_f = 2$, there exist three terms,

$$\begin{aligned} \det D^\mu\chi + \det D^\mu\chi^\dagger &= -\frac{1}{4}\langle\chi_+^\mu\chi_{+\mu}\rangle - \frac{1}{4}\langle\chi_-^\mu\chi_{-\mu}\rangle + \frac{1}{4}\langle\chi_+^\mu\rangle\langle\chi_{+\mu}\rangle + \frac{1}{4}\langle\chi_-^\mu\rangle\langle\chi_{-\mu}\rangle, \\ (\det\chi + \det\chi^\dagger)\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta} &= \left(-\frac{1}{4}\langle\chi_+^2\rangle - \frac{1}{4}\langle\chi_-^2\rangle + \frac{1}{4}\langle\chi_+\rangle\langle\chi_+\rangle + \frac{1}{4}\langle\chi_-\rangle\langle\chi_-\rangle\right)\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}, \\ (\det\chi + \det\chi^\dagger)\nabla^\mu\nabla_\mu\hat{\theta} &= \left(-\frac{1}{4}\langle\chi_+^2\rangle - \frac{1}{4}\langle\chi_-^2\rangle + \frac{1}{4}\langle\chi_+\rangle\langle\chi_+\rangle + \frac{1}{4}\langle\chi_-\rangle\langle\chi_-\rangle\right)\nabla^\mu\nabla_\mu\hat{\theta}. \end{aligned} \tag{45}$$

The first term is similar to $\langle D_\mu\chi D^\mu\tilde{\chi}\rangle + \text{H.c.}$ given in [8].Following the steps above, we obtain a similar relation as in (38) or (40).

$$D_{n_f,k} = \sum_l A'_{n_f,kl} E_{n_f,l}, n_f = 3 \text{ or } 2. \tag{46}$$

$D_{n_f,k}$ and $E_{n_f,k}$ are all D_{ij} and E_{ij} operators in (39) with respective contact terms.

(v) Eliminating rule: For a given n_f, collecting all the linear relations above, we can obtain

$$\sum_l R_{rl} D_l = 0. \tag{47}$$

The linear independent rows of R_{rl} correspond to independent terms. The question arises: Which terms are considered linear dependent and need to be removed? We follow the rule: The contact terms and terms with the least number of covariant derivatives and traces are retained as far as possible. Furthermore, we retain the smallest-possible numerical representation of the operator (see Sec. IV C).

(vi) Real: The chiral Lagrangian is real. Finally, with the last column in Table I and II, some terms need factoring by "i" to make them real.

(vii) U and SU groups: The above discussions are about the U group. Most of the time, one does not care about the η' particle, and hence the discussion is limited in the SU group. Of course, one can integrate η' to match the SU group results [39], but to do so gives nothing of interest. It is convenient when identifying the terms involving the SU group in the U group. However under the SU group, all of the operators O_m in (30) are even parity. Usually, we focus on traceless sources ($\langle v^\mu\rangle = \langle a^\mu\rangle = 0$ or $\langle u^\mu\rangle = \langle f_+^{\mu\nu}\rangle = 0$) in the SU group, although sometimes we need study to the singlet (axial-)vector external sources, so $\langle u^\mu\rangle \neq 0$ or $\langle f_+^{\mu\nu}\rangle \neq 0$ need to be considered. Fortunately, these situations cannot change the calculations above; we only need to set traceless terms to zero.

F. Discussions

(i) Linear relations of the dependent operators: At this point, the chiral Lagrangian can be obtained, but when we solve other problems, we may meet some dependent operators. Most of their expansions in terms of independent operators are simple, but several are complex; see Appendix B in [5] and [6] for examples. We will introduce a scheme to generate these relations. When we derive the chiral Lagrangian, all the operators are O_n, and all reveal-able covariant derivative standard forms are P_m. We can get their linear relations from (38), and full relations from (17), (18), and (19);

$$O_n = \sum_m B_{nm} P_m, \tag{48}$$

where B_{nm} is the matrix of coefficients, which have been used in all linear relations. If O' is another operator, then

$$O' = \sum_m T_m P_m. \tag{49}$$

Using all linear relations, this can be reduced to

$$O' = \sum_m T'_m P_m. \tag{50}$$

Supposing the relation between O' and the set of O_n is

$$O' = \sum_n C_n O_n. \tag{51}$$

The coefficients C_n are the solution of the linear equation,

$$\sum_n B_{nm} C_n = T'_m. \tag{52}$$

Hence all linear relations between linear dependent terms and linear independent terms can be obtained such as in [6] and Appendix B in [5].

(ii) The steps above can be done directly, and systematically, without any artifices, and hence can be performed on a computer. Furthermore, the accuracy is reliable, and we can obtain the results relatively quickly. This method is not very sensitive with regard to the form of the linear relations; we can add or remove some linear relation in (36), without affecting other steps. Therefore it can be used for other problems where Lagrangian need to be constructed, for instance, the electroweak chiral Lagrangian [40,41]. If we solve some special problems, changing the counting rules can be more effective [42].

V. RESULTS

We can now obtain the chiral Lagrangian to order p^6. We list the results in the following.

A. p^0 and p^2 order

The results for order p^0 and p^2 are the same as in [11],

$$\begin{aligned}\mathcal{L}_{0+2} = &-W_0(X) + W_1(X)\langle u^2\rangle + W_2(X)\langle\chi_+\rangle \\ &+ iW_3(X)\langle\chi_-\rangle - W_4(X)\langle u^\mu\rangle\langle u_\mu\rangle \\ &- iW_5(X)\langle u^\mu\rangle\nabla^\mu\hat{\theta} + W_6(X)\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}.\end{aligned} \tag{53}$$

Minus signs of some of the coefficients are to confirm that in [11].

TABLE V. Numbers of terms in the chiral Lagrangian at order p^6.

Number	$P=+/-$	U(N)			$SU(N)_I$			$SU(N)_{II}$			$SU(N)_{III}$		
Classification	P	n	3	2	n	3	2	n	3	2	n	3	2
$\theta=\bar{t}^{\mu\nu}=0$	287/206	493	451	281	155	133	74	263	235	142	138	116	60
$\theta\neq 0,\bar{t}^{\mu\nu}=0$	261/206	467	455	371	124	118	83	251	243	194	116	110	75
$\theta=0,\bar{t}^{\mu\nu}\neq 0$	159/149	308	297	209	110	104	67	137	131	88	97	91	56
$\theta\neq 0,\bar{t}^{\mu\nu}\neq 0$	64/59	123	123	108	51	51	43	52	52	44	42	42	34
Total	771/620	1391	1326	969	440	406	267	703	661	468	393	359	225

B. p^4 order

For the U groups, results have been given in [11]. For the SU groups, results can be found in [2,3], or [43,44] in terms of the building blocks, or [9] with tensor sources. We have repeated the calculations to check our method. We find one more linear relations than [11] for the U group.

$$i\epsilon_{\mu\nu\lambda\rho}\langle(F_L^{\mu\nu}+U^\dagger F_R^{\mu\nu}U)C^\lambda C^\rho\rangle=\epsilon_{\mu\nu\lambda\rho}\langle F_L^{\mu\nu}U^\dagger F_R^{\lambda\rho}U\rangle,\quad \text{or } O_{29}=O_{28}. \tag{54}$$

The meanings of the notations in (54) can be found in [11]. It can be proved by substituting (8) and $C^\mu=U^\dagger D^\mu U$ into (54) directly. By transforming (54) to the building-block basis, the calculation is much simpler. We list our results in Table VI in Appendix A. Besides (54), there are some differences between Table VI and [11]: Aside from the U(N) and U(3) group, we also include the U(2) groups results and the tensor sources as well. For the SU groups, we considered the following three types of external sources,

$$\begin{aligned}&SU(3)_I:\ \langle f_+^{\mu\nu}\rangle\neq 0,\langle u^\mu\rangle=0,\\ &SU(3)_{II}:\ \langle f_+^{\mu\nu}\rangle=0,\langle u^\mu\rangle\neq 0,\\ &SU(3)_{III}:\ \langle f_+^{\mu\nu}\rangle=0,\langle u^\mu\rangle=0.\end{aligned} \tag{55}$$

The results with $\langle f_+^{\mu\nu}\rangle\neq 0$ and $\langle u^\mu\rangle\neq 0$ can be found within the U group results, so they are not listed here. The $SU(3)_{III}$ case is the conventional SU group case.

C. p^6 order

At order p^6, as much a check of our method as that of existing results, we also compare our results with that of others. We recover the results given in [5,6,8] without tensor sources. With tensor sources, apart from the more relations in [10] than [9], we find one other linear relation in the n_f-flavor case,

$$Y_{102}=\frac{1}{2}Y_{78}-Y_{87}-2Y_{101}-Y_{117}, \tag{56}$$

and another nine linear relations in the two-flavor case,

$$\begin{aligned}Y_{48}&=-Y_{47},\\ Y_{49}&=-Y_{47},\\ Y_{50}&=2Y_{47},\\ Y_{71}&=-Y_{70},\\ Y_{72}&=-Y_{70},\\ Y_{73}&=-2Y_{70},\\ Y_{92}&=0,\\ Y_{106}&=\frac{1}{2}Y_9+\frac{1}{2}Y_{10}-2Y_{11}-Y_{13}+Y_{51}-4Y_{52}+Y_{82}\\ &\quad+2Y_{90}-2Y_{105}+8Y_{118},\\ Y_{108}&=-\frac{1}{8}Y_9-\frac{1}{8}Y_{10}+\frac{1}{2}Y_{11}+\frac{1}{4}Y_{13}+\frac{1}{8}Y_{32}-\frac{1}{16}Y_{35}\\ &\quad-\frac{1}{4}Y_{51}+Y_{52}-\frac{1}{2}Y_{82}-\frac{1}{2}Y_{90}+Y_{105}-2Y_{118}.\end{aligned} \tag{57}$$

At order p^6, the number of independent terms is too large, so we separate the chiral Lagrangian into four types, depending on whether they include $t^{\mu\nu}$ or $\hat{\theta}$. We list these four types in the first column of Table V, and summarize the number of terms in each type. Explicit terms can be found in Section B. Table VII and VIII give the even- and odd-parity results when $\theta=\bar{t}^{\mu\nu}=0$, respectively; similarly for Table IX and X when $\theta\neq 0,\bar{t}^{\mu\nu}=0$, Table XI and XII when $\theta=0,\bar{t}^{\mu\nu}\neq 0$, respectively, and Table XIII and XIV when $\theta\neq 0,\bar{t}^{\mu\nu}\neq 0$. These are some differences in sequence numbers from [5,8,9], but this creates no ambiguity.

VI. SUMMARY

In this paper, we constructed the full p^6-order U group chiral Lagrangian, including n_f-, three- and two-flavor cases, all bilinear currents, and the θ parameter, and three kinds of SU groups results. The number of terms in each case can be found in Table VI, and the detailed form can be found in Appendix B. We also list the full p^4-order results in Appendix A. During the calculations, we found one extra linear relation in the p^4-order under the U(3) group, and one more linear relation in the p^6-order under the SU groups with tensor sources in n_f flavors, and further nine more relations in the two-flavor case. We develop a scheme

to derive the relations among the linear dependent and independent terms which enable us to systematically set and examine various possible relations among the LECs.

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APPENDIX A: THE p^4 ORDER RESULTS

TABLE VI. The p^4 order results in U groups and SU groups. The numbers are the sequence numbers in each case. The details can be found in Sec. V B.

		U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
Operators	P	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu u_\mu u^\nu u_\nu\rangle$	+	1	1	1	1	1	1	1	1	1	1	1	1
$\langle u^\mu u^\nu u_\mu u_\nu\rangle$	+	2	2	2	2	2	2	2	2	2	2	2	2
$\langle u^\mu\rangle\langle u_\mu u^\nu u_\nu\rangle$	+	3	3	3				3	3	3			
$\langle u^\mu u_\mu\rangle\langle u^\nu u_\nu\rangle$	+	4	4	4	3	3		4	4	4	3	3	
$\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu\rangle$	+	5	5	5	4			5	5	5	4		
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u_\nu\rangle$	+	6	6					6	6				
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u_\nu\rangle$	+	7	7					7	7				
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu\rangle$	+	8						8					
$\langle f_-^{\mu\nu} f_{-\mu\nu}\rangle$	+	9	8	6	5	4	3	9	8	6	5	4	3
$i\langle f_+^{\mu\nu} u_\mu u_\nu\rangle$	+	10	9	7	6	5	4	10	9	7	6	5	4
$\langle u^\mu u_\mu \chi_+\rangle$	+	11	10	8	7	6	5	11	10	8	7	6	5
$\langle u^\mu\rangle\langle u_\mu\chi_+\rangle$	+	12	11	9				12	11	9			
$\langle u^\mu u_\mu\rangle\langle\chi_+\rangle$	+	13	12	10	8	7		13	12	10	8	7	
$\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_+\rangle$	+	14	13					14	13				
$\langle\chi_+^2\rangle$	+	15	14	11	9	8	6	15	14	11	9	8	6
$\langle\chi_+\rangle\langle\chi_+\rangle$	+	16	15	12	10	9	7	16	15	12	10	9	7
$\langle\chi_-\rangle\langle\chi_-\rangle$	+	17	16	13	11	10		17	16		11	10	
$\langle F_L^{\mu\nu} F_{L\mu\nu}\rangle + \langle F_R^{\mu\nu} F_{R\mu\nu}\rangle$	+	18	17	14	12	11	8	18	17	13	12	11	8
$\langle F_L^{\mu\nu}\rangle\langle F_{L\mu\nu}\rangle + \langle F_R^{\mu\nu}\rangle\langle F_{R\mu\nu}\rangle$	+	19	18	15	13	12	9	19	18	14			
$\langle F_L^{\mu\nu}\rangle\langle F_{R\mu\nu}\rangle$	+	20	19	16									
$\langle\chi\chi^\dagger\rangle$	+	21	20	17	14	13	10	20	19	15	13	12	9
$\det\chi + \det\chi^\dagger$	+						11			16			10
$i\langle u^\mu u_\mu u^\nu\rangle\nabla_\nu\hat{\theta}$	+	22	21	18	15	14		21	20	17	14	13	
$i\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\nabla_\nu\hat{\theta}$	+	23	22	19				22	21	18			
$i\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\nabla_\mu\hat{\theta}$	+	24	23	20				23	22	19			
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\nabla_\nu\hat{\theta}$	+	25	24					24	23				
$i\langle u^\mu\chi_+\rangle\nabla_\mu\hat{\theta}$	+	26	25	21	16	15	12	25	24	20	15	14	11
$i\langle u^\mu\rangle\langle\chi_+\rangle\nabla_\mu\hat{\theta}$	+	27	26	22				26	25	21			

(Table continued)

TABLE VI. (*Continued*)

Operators	P	U(3) n	U(3) 3	U(3) 2	$SU(3)_I$ n	$SU(3)_I$ 3	$SU(3)_I$ 2	$SU(3)_{II}$ n	$SU(3)_{II}$ 3	$SU(3)_{II}$ 2	$SU(3)_{III}$ n	$SU(3)_{III}$ 3	$SU(3)_{III}$ 2
$\langle\chi_-\rangle\nabla^\mu\nabla_\mu\hat{\theta}$	+	28	27	23	17	16	13	27	26	22	16	15	12
$\langle u^\mu u_\mu\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	+	29	28	24	18	17	14	28	27	23	17	16	13
$\langle u^\mu u^\nu\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	+	30	29	25	19	18	15	29	28	24	18	17	14
$\langle u^\mu\rangle\langle u_\mu\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	+	31	30	26				30	29	25			
$\langle u^\mu\rangle\langle u^\nu\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	+	32	31	27				31	30	26			
$\langle\chi_+\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}$	+	33	32	28	20	19	16	32	31	27	19	18	15
$i\langle u^\mu\rangle\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	+	34	33	29				33	32	28			
$\nabla^\mu\nabla_\mu\hat{\theta}\nabla^\nu\nabla_\nu\hat{\theta}$	+	35	34	30	21	20	17	34	33	29	20	19	16
$\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	+	36	35	31	22	21	18	35	34	30	21	20	17
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u_\nu f_{-\lambda\rho}\rangle$	+	37	36	32				36	35	31			
$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}$	+	38	37	33	23	22	19	37	36	32	22	21	18
$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}$	+	39	38	34				38	37	33			
$i\langle u^\mu u^\nu t_{+\mu\nu}\rangle$	+	40	39	35	24	23	20	39	38	34	23	22	19
$\langle f_+^{\mu\nu} t_{+\mu\nu}\rangle$	+	41	40	36	25	24	21	40	39	35	24	23	20
$\langle f_+^{\mu\nu}\rangle\langle t_{+\mu\nu}\rangle$	+	42	41	37	26	25	22						
$\langle t_+^{\mu\nu} t_{+\mu\nu}\rangle$	+	43	42	38	27	26	23	41	40	36	25	24	21
$\langle t_+^{\mu\nu}\rangle\langle t_{+\mu\nu}\rangle$	+	44	43	39	28	27	24	42	41	37	26	25	22
$\langle u^\mu\rangle\langle u^\nu f_{-\mu\nu}\rangle$	−	45	44	40									
$i\langle u^\mu u_\mu\chi_-\rangle$	−	46	45	41									
$i\langle u^\mu\rangle\langle u_\mu\chi_-\rangle$	−	47	46	42									
$i\langle u^\mu u_\mu\rangle\langle\chi_-\rangle$	−	48	47	43									
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_-\rangle$	−	49	48										
$i\langle\chi_+\chi_-\rangle$	−	50	49	44									
$i\langle\chi_+\rangle\langle\chi_-\rangle$	−	51	50	45									
$i\langle u^\mu f_{-\mu}{}^\nu\rangle\nabla_\nu\hat{\theta}$	−	52	51	46									
$i\langle u^\mu h_\mu{}^\nu\rangle\nabla_\nu\hat{\theta}$	−	53	52	47									
$i\langle u^\mu\rangle\langle f_{-\mu}{}^\nu\rangle\nabla_\nu\hat{\theta}$	−	54	53	48									
$i\langle u^\mu\rangle\langle h_\mu{}^\nu\rangle\nabla_\nu\hat{\theta}$	−	55	54	49									
$i\langle\chi_+\rangle\nabla^\mu\nabla_\mu\hat{\theta}$	−	56	55	50									
$\langle u^\mu\chi_-\rangle\nabla_\mu\hat{\theta}$	−	57	56	51									
$\langle u^\mu\rangle\langle\chi_-\rangle\nabla_\mu\hat{\theta}$	−	58	57	52									
$\langle h^{\mu\nu}\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	−	59	58	53									
$i\langle\chi_-\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}$	−	60	59	54									
$i\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}\nabla^\nu\nabla_\nu\hat{\theta}$	−	61	60	55									
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_\lambda u_\rho\rangle$	−	62	61	56									
$\langle u^\mu u^\nu t_{-\mu\nu}\rangle$	−	63	62	57									
$i\langle f_+^{\mu\nu} t_{-\mu\nu}\rangle$	−	64	63	58									
$i\langle f_+^{\mu\nu}\rangle\langle t_{-\mu\nu}\rangle$	−	65	64	59									
$i\langle t_+^{\mu\nu} t_{-\mu\nu}\rangle$	−	66	65	60									
$i\langle t_+^{\mu\nu}\rangle\langle t_{-\mu\nu}\rangle$	−	67	66	61									

APPENDIX B: THE p^6 ORDER RESULTS

TABLE VII. The p^6 order results in U groups and SU groups, with $t = 0$, $\hat{\theta} = 0$ and the even parity. The numbers are the sequence numbers in each case. The details can be found in Sec. V C.

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu u_\mu u^\nu u_\nu u^\lambda u_\lambda\rangle$	1	1	1	1	1	1	1	1	1	1	1	1
$\langle u^\mu u_\mu u^\nu u^\lambda u_\nu u_\lambda\rangle$	2	2	2	2	2	2	2	2	2	2	2	2
$\langle u^\mu u_\mu u^\nu u^\lambda u_\lambda u_\nu\rangle$	3	3	3	3	3		3	3	3	3	3	
$\langle u^\mu u^\nu u_\mu u^\lambda u_\nu u_\lambda\rangle$	4	4	4	4	4	3	4	4	4	4	4	3
$\langle u^\mu u^\nu u^\lambda u_\mu u_\nu u_\lambda\rangle$	5	5	5	5	5		5	5	5	5	5	
$\langle u^\mu u^\nu u^\lambda\rangle\langle u_\mu u_\lambda u_\nu\rangle$	6	6	6	6	6		6	6	6	6	6	
$\langle u^\mu u^\nu u^\lambda\rangle\langle u_\mu u_\nu u_\lambda\rangle$	7	7		7	7		7	7		7	7	
$\langle u^\mu u_\mu u^\nu\rangle\langle u_\nu u^\lambda u_\lambda\rangle$	8	8	7	8	8		8	8	7	8	8	
$\langle u^\mu u^\nu\rangle\langle u_\mu u^\lambda u_\nu u_\lambda\rangle$	9	9	8	9			9	9	8	9		
$\langle u^\mu u_\mu\rangle\langle u^\nu u^\lambda u_\nu u_\lambda\rangle$	10	10	9	10			10	10	9	10		
$\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu u^\lambda u_\lambda\rangle$	11	11		11			11	11		11		
$\langle u^\mu u_\mu\rangle\langle u^\nu u_\nu u^\lambda u_\lambda\rangle$	12	12		12			12	12		12		
$\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda u_\lambda u_\nu\rangle$	13	13	10				13	13	10			
$\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda u_\nu u_\lambda\rangle$	14	14					14	14				
$\langle u^\mu\rangle\langle u_\mu u^\nu u_\nu u^\lambda u_\lambda\rangle$	15	15					15	15				
$\langle u^\mu u^\nu\rangle\langle u_\mu u^\lambda\rangle\langle u_\nu u_\lambda\rangle$	16	16		13			16	16		13		
$\langle u^\mu u_\mu\rangle\langle u^\nu u^\lambda\rangle\langle u_\nu u_\lambda\rangle$	17	17		14			17	17		14		
$\langle u^\mu u_\mu\rangle\langle u^\nu u_\nu\rangle\langle u^\lambda u_\lambda\rangle$	18	18		15			18	18		15		
$\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\langle u_\mu u_\nu u_\lambda\rangle$	19	19					19	19				
$\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\langle u_\mu u^\lambda u_\lambda\rangle$	20	20					20	20				
$\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\langle u_\nu u^\lambda u_\lambda\rangle$	21	21					21	21				
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u^\lambda u_\nu u_\lambda\rangle$	22	22					22	22				
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u^\lambda u_\nu u_\lambda\rangle$	23	23					23	23				
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u_\nu u^\lambda u_\lambda\rangle$	24						24					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u_\nu u^\lambda u_\lambda\rangle$	25						25					
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u^\lambda\rangle\langle u_\nu u_\lambda\rangle$	26						26					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u^\lambda\rangle\langle u_\nu u_\lambda\rangle$	27						27					

operators	U(3)			$SU(3)_I$			$SU(3)_{III}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle \chi_+ f_-^{\mu\nu} f_{-\mu\nu}\rangle$	146	123	72	73	55	30	135	112	64	68	50	28
$\langle \chi_+ h^{\mu\nu} h_{\mu\nu}\rangle$	147	124	73	74	56	31	136	113	65	69	51	29
$\langle u^\mu \chi_+ \nabla^\nu f_{-\mu\nu}\rangle + \text{H.c.}$	148	125	74	75	57	32	137	114	66	70	52	30
$\langle \chi_+\rangle\langle f_-^{\mu\nu} f_{-\mu\nu}\rangle$	149	126	75	76	58		138	115	67	71	53	
$\langle \chi_+ f_-^{\mu\nu}\rangle\langle f_{-\mu\nu}\rangle$	150	127	76				139	116	68			
$\langle \chi_+\rangle\langle h^{\mu\nu} h_{\mu\nu}\rangle$	151	128	77	77	59		140	117	69	72	54	
$\langle \chi_+ h^{\mu\nu}\rangle\langle h_{\mu\nu}\rangle$	152	129	78				141	118	70			
$\langle u^\mu\rangle\langle \chi_+ \nabla^\nu f_{-\mu\nu}\rangle$	153	130	79				142	119	71			
$\langle u^\mu \chi_+\rangle\langle \nabla^\nu f_{-\mu\nu}\rangle$	154	131	80				143	120	72			
$\langle u^\mu \nabla^\nu f_{-\mu\nu}\rangle\langle \chi_+\rangle$	155	132	81	78	60		144	121	73	73	55	
$\langle \chi_+\rangle\langle f_-^{\mu\nu}\rangle\langle f_{-\mu\nu}\rangle$	156	133					145	122				
$\langle \chi_+\rangle\langle h^{\mu\nu}\rangle\langle h_{\mu\nu}\rangle$	157	134					146	123				
$\langle u^\mu\rangle\langle \chi_+\rangle\langle \nabla^\nu f_{-\mu\nu}\rangle$	158	135					147	124				
$i\langle f_+^{\mu\nu} u_\mu u_\nu \chi_+\rangle + \text{H.c.}$	159	136	82	79	61	33	148	125	74	74	56	31
$i\langle f_+^{\mu\nu} u_\mu \chi_+ u_\nu\rangle$	160	137	83	80	62	34	149	126	75	75	57	
$i\langle f_+^{\mu\nu}\rangle\langle u_\mu u_\nu \chi_+\rangle$	161	138	84	81	63							
$i\langle f_+^{\mu\nu} u_\mu u_\nu\rangle\langle \chi_+\rangle$	162	139		82	64		150	127		76	58	
$i\langle f_+^{\mu\nu} u_\mu \chi_+\rangle\langle u_\nu\rangle + \text{H.c.}$	163	140					151	128				
$\langle f_+^{\mu\nu} f_{+\mu\nu} \chi_+\rangle$	164	141	85	83	65	35	152	129	76	77	59	32
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu\nu} \chi_+\rangle$	165	142	86	84	66	36						
$\langle f_+^{\mu\nu} f_{+\mu\nu}\rangle\langle \chi_+\rangle$	166	143	87	85	67	37	153	130		78	60	
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu\nu}\rangle\langle \chi_+\rangle$	167	144		86	68							
$\langle u^\mu u_\mu \chi_+^2\rangle$	168	145	88	87	69	38	154	131	77	79	61	33
$\langle u^\mu \chi_+ u_\mu \chi_+\rangle$	169	146	89	88	70	39	155	132	78	80	62	34
$\langle u^\mu\rangle\langle u_\mu \chi_+^2\rangle$	170	147	90				156	133	79			
$\langle u^\mu u_\mu\rangle\langle \chi_+^2\rangle$	171	148	91	89	71		157	134	80	81	63	
$\langle u^\mu u_\mu \chi_+\rangle\langle \chi_+\rangle$	172	149	92	90	72	40	158	135	81	82	64	35

(Table continued)

TABLE VII. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u_\nu\rangle\langle u^\lambda u_\lambda\rangle$	28						28					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u_\nu\rangle\langle u^\lambda u_\lambda\rangle$	29						29					
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda\rangle\langle u_\mu u_\nu u_\lambda\rangle$	30						30					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu u^\lambda u_\lambda\rangle$	31						31					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu\rangle\langle u^\lambda u_\lambda\rangle$	32						32					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u^\lambda\rangle\langle u_\nu u_\lambda\rangle$	33						33					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu\rangle\langle u^\lambda\rangle\langle u_\lambda\rangle$	34						34					
$\langle u^\mu u_\mu f_-^{\nu\lambda} f_{-\nu\lambda}\rangle$	35	24	11	16	9	4	35	24	11	16	9	4
$\langle u^\mu u^\nu f_{-\mu}{}^\lambda f_{-\nu\lambda}\rangle$	36	25	12	17	10	5	36	25	12	17	10	5
$\langle u^\mu u^\nu f_{-\nu}{}^\lambda f_{-\mu\lambda}\rangle$	37	26	13	18	11	6	37	26	13	18	11	6
$\langle u^\mu f_{-\mu}{}^\nu u^\lambda f_{-\nu\lambda}\rangle + \text{H.c.}$	38	27	14	19	12	7	38	27	14	19	12	7
$\langle u^\mu f_-^{\nu\lambda} u_\mu f_{-\nu\lambda}\rangle$	39	28	15	20	13	8	39	28	15	20	13	8
$\langle u^\mu u^\nu h_\mu{}^\lambda f_{-\nu\lambda}\rangle + \text{H.c.}$	40	29	16	21	14	9	40	29	16	21	14	9
$\langle u^\mu u^\nu h_\nu{}^\lambda f_{-\mu\lambda}\rangle + \text{H.c.}$	41	30	17	22	15	10	41	30	17	22	15	10
$\langle u^\mu h_\mu{}^\nu u^\lambda f_{-\nu\lambda}\rangle + \text{H.c.}$	42	31	18	23	16	11	42	31	18	23	16	11
$\langle u^\mu u_\mu h^{\nu\lambda} h_{\nu\lambda}\rangle$	43	32	19	24	17	12	43	32	19	24	17	12
$\langle u^\mu u^\nu h_\mu{}^\lambda h_{\nu\lambda}\rangle$	44	33	20	25	18	13	44	33	20	25	18	13
$\langle u^\mu u^\nu h_\nu{}^\lambda\rangle\langle h_{\mu\lambda}\rangle + \text{H.c.}$	45	34	21				45	34	21			
$\langle u^\mu u_\mu h^{\nu\lambda}\rangle\langle h_{\nu\lambda}\rangle$	46	35	22				46	35	22			
$\langle u^\mu u^\nu f_{-\nu}{}^\lambda\rangle\langle h_{\mu\lambda}\rangle + \text{H.c.}$	47	36	23				47	36	23			
$\langle u^\mu u^\nu h_\nu{}^\lambda\rangle\langle f_{-\mu\lambda}\rangle + \text{H.c.}$	48	37	24				48	37	24			
$\langle u^\mu u^\nu f_{-\nu}{}^\lambda\rangle\langle f_{-\mu\lambda}\rangle + \text{H.c.}$	49	38	25				49	38	25			
$\langle u^\mu h^{\nu\lambda}\rangle\langle u_\nu h_{\mu\lambda}\rangle$	50	39	26	26	19		50	39	26	26	19	
$\langle u^\mu h^{\nu\lambda}\rangle\langle u_\mu h_{\nu\lambda}\rangle$	51	40	27	27	20		51	40	27	27	20	
$\langle u^\mu h_\mu{}^\nu\rangle\langle u^\lambda h_{\nu\lambda}\rangle$	52	41	28	28	21		52	41	28	28	21	
$\langle u^\mu u^\nu\rangle\langle u^\lambda\nabla_\lambda h_{\mu\nu}\rangle$	53	42	29	29	22		53	42	29	29	22	
$\langle u^\mu u^\nu\rangle\langle u^\lambda\nabla_\mu h_{\nu\lambda}\rangle$	54	43	30	30			54	43	30	30		
$\langle u^\mu u^\nu\rangle\langle u_\mu\nabla^\lambda h_{\nu\lambda}\rangle$	55	44	31	31	23		55	44	31	31	23	
$\langle u^\mu u_\mu\rangle\langle u^\nu\nabla^\lambda h_{\nu\lambda}\rangle$	56	45	32	32			56	45	32	32		
$\langle u^\mu u^\nu\rangle\langle h_\mu{}^\lambda h_{\nu\lambda}\rangle$	57	46	33	33			57	46	33	33		
$\langle u^\mu\rangle\langle u^\nu u^\lambda\nabla_\lambda h_{\mu\nu}\rangle + \text{H.c.}$	58	47	34				58	47	34			

operators	U(3)			$SU(3)_I$			$SU(3)_{III}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu\chi_+\rangle\langle u_\mu\chi_+\rangle$	173	150	93	91	73		159	136	82	83	65	
$\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_+^2\rangle$	174	151					160	137				
$\langle u^\mu\rangle\langle u_\mu\chi_+\rangle\langle\chi_+\rangle$	175	152					161	138				
$\langle u^\mu u_\mu\rangle\langle\chi_+\rangle\langle\chi_+\rangle$	176	153		92			162	139		84		
$\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_+\rangle\langle\chi_+\rangle$	177						163					
$\langle\chi_+^\mu\chi_{+\mu}\rangle$	178	154	94	93	74	41	164	140	83	85	66	36
$\langle\chi_+^\mu\rangle\langle\chi_{+\mu}\rangle$	179	155	95	94	75	42	165	141	84	86	67	37
$\langle\chi_+^3\rangle$	180	156	96	95	76	43	166	142	85	87	68	38
$\langle\chi_+\rangle\langle\chi_+^2\rangle$	181	157	97	96	77	44	167	143	86	88	69	39
$\langle\chi_+\rangle\langle\chi_+\rangle\langle\chi_+\rangle$	182	158		97	78		168	144		89	70	
$i\langle u^\mu u^\nu f_{-\mu\nu}\chi_-\rangle + \text{H.c.}$	183	159	98	98	79	45	169	145	87	90	71	40
$i\langle u^\mu u^\nu h_{\mu\nu}\chi_-\rangle + \text{H.c.}$	184	160	99	99	80	46	170	146	88	91	72	41
$i\langle u^\mu h_\mu{}^\nu u_\nu\chi_-\rangle$	185	161	100	100	81	47	171	147	89	92	73	42
$i\langle u^\mu\rangle\langle u^\nu f_{-\mu\nu}\chi_-\rangle + \text{H.c.}$	186	162	101				172	148	90			
$i\langle u^\mu f_{-\mu}{}^\nu\rangle\langle u_\nu\chi_-\rangle$	187	163	102	101	82		173	149	91	93	74	
$i\langle u^\mu\rangle\langle u^\nu h_{\mu\nu}\chi_-\rangle + \text{H.c.}$	188	164	103				174	150	92			
$i\langle u^\mu u^\nu\rangle\langle h_{\mu\nu}\chi_-\rangle$	189	165	104	102	83		175	151	93	94	75	
$i\langle u^\mu u^\nu h_{\mu\nu}\rangle\langle\chi_-\rangle$	190	166	105	103	84		176	152	94	95	76	
$i\langle u^\mu h_\mu{}^\nu\rangle\langle u_\nu\chi_-\rangle$	191	167	106	104			177	153	95	96		
$i\langle u^\mu u^\nu\chi_-\rangle\langle h_{\mu\nu}\rangle$	192	168	107				178	154	96			
$i\langle u^\mu\rangle\langle u^\nu f_{-\mu\nu}\rangle\langle\chi_-\rangle$	193	169					179	155				
$i\langle u^\mu\rangle\langle u^\nu\chi_-\rangle\langle f_{-\mu\nu}\rangle$	194	170					180	156				
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle h_{\mu\nu}\chi_-\rangle$	195	171					181	157				
$i\langle u^\mu\rangle\langle u^\nu h_{\mu\nu}\rangle\langle\chi_-\rangle$	196	172					182	158				
$i\langle u^\mu u^\nu\rangle\langle h_{\mu\nu}\rangle\langle\chi_-\rangle$	197	173					183	159				
$i\langle u^\mu\rangle\langle u^\nu\chi_-\rangle\langle h_{\mu\nu}\rangle$	198	174					184	160				
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle h_{\mu\nu}\rangle\langle\chi_-\rangle$	199						185					
$\langle f_+^{\mu\nu} f_{-\mu\nu}\chi_-\rangle + \text{H.c.}$	200	175	108	105	85	48	186	161	97	97	77	43
$\langle\nabla^\mu f_{+\mu}{}^\nu u_\nu\chi_-\rangle + \text{H.c.}$	201	176	109	106	86	49	187	162	98	98	78	44
$i\langle u^\mu\chi_-\chi_{+\mu}\rangle + \text{H.c.}$	202	177	110	107	87	50	188	163	99	99	79	45
$i\langle u^\mu\rangle\langle\chi_-\chi_{+\mu}\rangle$	203	178	111				189	164	100			

(Table continued)

TABLE VII. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu\rangle\langle u^\nu u_\nu\nabla^\lambda h_{\mu\lambda}\rangle$	59	48	35				59	48	35			
$\langle u^\mu\rangle\langle u^\nu u^\lambda\nabla_\mu h_{\nu\lambda}\rangle$	60	49	36				60	49	36			
$\langle u^\mu\rangle\langle u^\nu u_\mu\nabla^\lambda h_{\nu\lambda}\rangle + \text{H.c.}$	61	50	37				61	50	37			
$\langle u^\mu f_-^{\nu\lambda}\rangle\langle u_\nu f_{-\mu\lambda}\rangle$	62	51	38	34	24		62	51	38	34	24	
$\langle u^\mu f_-^{\nu\lambda}\rangle\langle u_\mu f_{-\nu\lambda}\rangle$	63	52	39	35			63	52	39	35		
$\langle u^\mu u^\nu\rangle\langle h_\mu{}^\lambda\rangle\langle h_{\nu\lambda}\rangle$	64	53					64	53				
$\langle u^\mu u_\mu\rangle\langle h^{\nu\lambda}\rangle\langle h_{\nu\lambda}\rangle$	65	54					65	54				
$\langle u^\mu\rangle\langle u^\nu h_\nu{}^\lambda\rangle\langle h_{\mu\lambda}\rangle$	66	55					66	55				
$\langle u^\mu\rangle\langle u^\nu h_\mu{}^\lambda\rangle\langle h_{\nu\lambda}\rangle$	67	56					67	56				
$\langle u^\mu\rangle\langle u_\mu h^{\nu\lambda}\rangle\langle h_{\nu\lambda}\rangle$	68	57					68	57				
$\langle u^\mu\rangle\langle u^\nu f_{-\nu}{}^\lambda\rangle\langle h_{\mu\lambda}\rangle$	69	58					69	58				
$\langle u^\mu\rangle\langle u^\nu f_{-\mu}{}^\lambda\rangle\langle h_{\nu\lambda}\rangle$	70	59					70	59				
$\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\langle\nabla_\nu h_{\mu\lambda}\rangle$	71	60					71	60				
$\langle u^\mu u^\nu\rangle\langle f_{-\mu}{}^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	72	61					72	61				
$\langle u^\mu u_\mu\rangle\langle f_-^{\nu\lambda}\rangle\langle f_{-\nu\lambda}\rangle$	73	62					73	62				
$\langle u^\mu\rangle\langle u^\nu h_\mu{}^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	74	63					74	63				
$\langle u^\mu\rangle\langle u^\nu f_{-\mu}{}^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	75	64					75	64				
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda\nabla_\mu h_{\nu\lambda}\rangle$	76	65					76	65				
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu\nabla^\lambda h_{\nu\lambda}\rangle$	77	66					77	66				
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\nabla^\lambda h_{\nu\lambda}\rangle$	78	67					78	67				
$\langle u^\mu\rangle\langle u_\mu\rangle\langle f_-^{\nu\lambda}\rangle\langle f_{-\nu\lambda}\rangle$	79						79					
$\langle u^\mu\rangle\langle u^\nu\rangle\langle f_{-\mu}{}^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	80						80					
$\langle u^\mu\rangle\langle u^\nu\rangle\langle h_\mu{}^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	81						81					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle h^{\nu\lambda}\rangle\langle h_{\nu\lambda}\rangle$	82						82					
$\langle\nabla^\mu f_{-\mu}{}^\nu\nabla^\lambda f_{-\nu\lambda}\rangle$	83	68	40	36	25	14	83	68	40	36	25	14
$i\langle f_+^{\mu\nu}u_\mu u_\nu u^\lambda u_\lambda\rangle + \text{H.c.}$	84	69	41	37	26	15	84	69	41	37	26	15
$i\langle f_+^{\mu\nu}u_\mu u^\lambda u_\nu u_\lambda\rangle + \text{H.c.}$	85	70	42	38	27	16	85	70	42	38	27	16
$i\langle f_+^{\mu\nu}u_\mu u^\lambda u_\lambda u_\nu\rangle$	86	71	43	39	28		86	71	43	39	28	
$i\langle f_+^{\mu\nu}u^\lambda u_\mu u_\nu u_\lambda\rangle$	87	72	44	40	29		87	72	44	40	29	
$i\langle f_+^{\mu\nu}\rangle\langle u_\mu u_\nu u^\lambda u_\lambda\rangle$	88	73	45	41	30							
$i\langle f_+^{\mu\nu}u_\mu u_\nu\rangle\langle u^\lambda u_\lambda\rangle$	89	74		42	31		88	73		41	30	
$i\langle f_+^{\mu\nu}u_\mu u_\nu u^\lambda\rangle\langle u_\lambda\rangle + \text{H.c.}$	90	75					89	74				
$i\langle u^\mu\chi_-\rangle\langle\chi_{+\mu}\rangle$	204	179	112	108	88	51	190	165	101	100	80	46
$i\langle u^\mu\chi_{+\mu}\rangle\langle\chi_-\rangle$	205	180	113	109	89		191	166	102	101	81	
$i\langle u^\mu\rangle\langle\chi_-\rangle\langle\chi_{+\mu}\rangle$	206	181					192	167				
$\langle u^\mu u_\mu\chi_-^2\rangle$	207	182	114	110	90	52	193	168	103	102	82	47
$\langle u^\mu\chi_- u_\mu\chi_-\rangle$	208	183	115	111	91	53	194	169	104	103	83	48
$\langle u^\mu\rangle\langle u_\mu\chi_-^2\rangle$	209	184	116				195	170	105			
$\langle u^\mu u_\mu\rangle\langle\chi_-^2\rangle$	210	185	117	112	92		196	171	106	104	84	
$\langle u^\mu u_\mu\chi_-\rangle\langle\chi_-\rangle$	211	186	118	113	93	54	197	172	107	105	85	49
$\langle u^\mu\chi_-\rangle\langle u_\mu\chi_-\rangle$	212	187	119	114	94		198	173	108	106	86	
$\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_-^2\rangle$	213	188					199	174				
$\langle u^\mu\rangle\langle u_\mu\chi_-\rangle\langle\chi_-\rangle$	214	189					200	175				
$\langle u^\mu u_\mu\rangle\langle\chi_-\rangle\langle\chi_-\rangle$	215	190		115			201	176		107		
$\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_-\rangle\langle\chi_-\rangle$	216						202					
$\langle\chi_-^\mu\rangle\langle\chi_{-\mu}\rangle$	217	191	120	116	95		203	177		108	87	
$\langle\chi_+\chi_-^2\rangle$	218	192	121	117	96	55	204	178	109	109	88	50
$\langle\chi_+\rangle\langle\chi_-^2\rangle$	219	193	122	118	97	56	205	179	110	110	89	51
$\langle\chi_+\chi_-\rangle\langle\chi_-\rangle$	220	194	123	119	98	57	206	180	111	111	90	52
$\langle\chi_+\rangle\langle\chi_-\rangle\langle\chi_-\rangle$	221	195		120			207			112		
$i\langle F_L^{\mu\nu}F_{L\mu}{}^\lambda F_{L\nu\lambda}\rangle + \text{H.c.}$	222	196	124	121	99	58	208	181	112	113	91	53
$\langle D^\mu F_{L\mu}{}^\nu D^\lambda F_{L\nu\lambda}\rangle + \text{H.c.}$	223	197	125	122	100	59	209	182	113	114	92	54
$\langle D^\mu F_{L\mu}{}^\nu\rangle\langle D^\lambda F_{L\nu\lambda}\rangle + \text{H.c.}$	224	198	126	123	101	60	210	183	114			
$\langle D^\mu F_{L\mu}{}^\nu\rangle\langle D^\lambda F_{R\nu\lambda}\rangle$	225	199	127									
$\langle D^\mu\chi D_\mu\chi^\dagger\rangle$	226	200	128	124	102	61	211	184	115	115	93	55
$\det\chi + \text{H.c.}$					103			185			94	
$\det D^\mu\chi + \text{H.c.}$						62			116			56
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu u_\nu u_\lambda u^\sigma f_{-\rho\sigma}\rangle + \text{H.c.}$	227	201	129	125	104		212	186	117	116	95	
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu u_\nu u^\sigma u_\sigma f_{-\lambda\rho}\rangle + \text{H.c.}$	228	202	130	126	105		213	187	118	117	96	
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu u_\nu u_\lambda u^\sigma h_{\rho\sigma}\rangle + \text{H.c.}$	229	203	131	127	106		214	188	119	118	97	
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u^\sigma u_\nu u_\lambda h_{\rho\sigma}\rangle + \text{H.c.}$	230	204					215	189				
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu u^\sigma u_\sigma\rangle\langle u_\nu f_{-\lambda\rho}\rangle$	231	205	132	128	107		216	190	120	119	98	
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu u^\sigma\rangle\langle u_\sigma u_\nu f_{-\lambda\rho}\rangle + \text{H.c.}$	232	206	133	129			217	191	121	120		
$\epsilon^{\mu\nu\lambda\rho}\langle u^\sigma\rangle\langle u_\sigma u_\mu u_\nu f_{-\lambda\rho}\rangle + \text{H.c.}$	233	207					218	192				

(Table continued)

TABLE VII. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$			operators	U(3)			$SU(3)_I$			$SU(3)_{III}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2		n	3	2	n	3	2	n	3	2	n	3	2
$i\langle f_+^{\mu\nu}u_\mu u^\lambda u_\lambda\rangle\langle u_\nu\rangle + \text{H.c.}$	91	76					90	75					$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u^\sigma u_\sigma u_\nu f_{-\lambda\rho}\rangle + \text{H.c.}$	234	208	134				219	193	122			
$i\langle f_+^{\mu\nu}u^\lambda\rangle\langle u_\mu u_\nu u_\lambda\rangle$	92	77		43			91	76		42			$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u^\sigma u_\nu u_\sigma f_{-\lambda\rho}\rangle$	235	209					220	194				
$i\langle f_+^{\mu\nu}u_\mu u^\lambda\rangle\langle u_\nu u_\lambda\rangle + \text{H.c.}$	93	78		44			92	77		43			$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u_\nu u^\sigma u_\sigma\rangle\langle f_{-\lambda\rho}\rangle$	236	210					221	195				
$i\langle f_+^{\mu\nu}u_\mu u^\lambda u_\nu\rangle\langle u_\lambda\rangle$	94	79					93	78					$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u_\nu u^\sigma\rangle\langle u_\lambda h_{\rho\sigma}\rangle$	237	211					222	196				
$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\mu u_\nu u_\lambda\rangle$	95	80											$\epsilon^{\mu\nu\lambda\rho}\langle u^\sigma\rangle\langle u_\mu u_\sigma\rangle\langle u_\nu f_{-\lambda\rho}\rangle$	238	212					223	197				
$i\langle f_+^{\mu\nu}u_\mu u_\nu\rangle\langle u^\lambda\rangle\langle u_\lambda\rangle$	96						94						$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u^\sigma u_\sigma\rangle\langle u_\nu f_{-\lambda\rho}\rangle$	239	213					224	198				
$i\langle f_+^{\mu\nu}u_\mu u^\lambda\rangle\langle u_\nu\rangle\langle u_\lambda\rangle + \text{H.c.}$	97						95						$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u_\nu u^\sigma\rangle\langle u_\sigma f_{-\lambda\rho}\rangle$	240	214					225	199				
$i\langle f_+^{\mu\nu}f_{-\mu}{}^\lambda f_{-\nu\lambda}\rangle$	98	81	46	45	32	17	96	79	45	44	31	17	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u^\sigma\rangle\langle u_\sigma u_\nu f_{-\lambda\rho}\rangle + \text{H.c.}$	241	215					226	200				
$i\langle f_+^{\mu\nu}h_\mu{}^\lambda f_{-\nu\lambda}\rangle + \text{H.c.}$	99	82	47	46	33	18	97	80	46	45	32	18	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u^\sigma\rangle\langle u_\nu u_\sigma\rangle\langle f_{-\lambda\rho}\rangle$	242						227					
$i\langle f_+^{\mu\nu}h_\mu{}^\lambda h_{\nu\lambda}\rangle$	100	83	48	47	34	19	98	81	47	46	33	19	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u^\sigma\rangle\langle u_\sigma\rangle\langle u_\nu f_{-\lambda\rho}\rangle$	243						228					
$i\langle\nabla^\mu f_{+\mu}{}^\nu u^\lambda f_{-\nu\lambda}\rangle + \text{H.c.}$	101	84	49	48	35	20	99	82	48	47	34	20	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\nabla_\nu f_{-\lambda}{}^\sigma f_{-\rho\sigma}\rangle + \text{H.c.}$	244	216	135	130	108		229	201	123	121	99	
$\langle f_+^{\mu\nu}f_{+\mu\nu}u^\lambda u_\lambda\rangle$	102	85	50	49	36	21	100	83	49	48	35	21	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\nabla_\nu h_\lambda{}^\sigma\rangle\langle h_{\rho\sigma}\rangle$	245	217	136				230	202	124			
$\langle f_+^{\mu\nu}f_{+\mu}{}^\lambda u_\nu u_\lambda\rangle$	103	86	51	50	37	22	101	84	50	49	36	22	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu h_\nu{}^\sigma\rangle\langle\nabla_\lambda h_{\rho\sigma}\rangle$	246	218	137				231	203	125			
$\langle f_+^{\mu\nu}f_{+\mu}{}^\lambda u_\lambda u_\nu\rangle$	104	87	52	51	38	23	102	85	51	50	37	23	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle h_\nu{}^\sigma\nabla_\lambda h_{\rho\sigma}\rangle$	247	219	138				232	204	126			
$\langle f_+^{\mu\nu}u_\mu f_{+\nu}{}^\lambda u_\lambda\rangle + \text{H.c.}$	105	88	53	52	39	24	103	86	52	51	38	24	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle f_{-\nu}{}^\sigma\rangle\langle\nabla_\lambda f_{-\rho\sigma}\rangle$	248	220					233	205				
$\langle f_+^{\mu\nu}u^\lambda f_{+\mu\nu}u_\lambda\rangle$	106	89	54	53	40	25	104	87	53	52	39	25	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}f_{-\lambda\rho}u^\sigma u_\sigma\rangle + \text{H.c.}$	249	221	139	131	109		234	206	127	122	100	
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu\nu}u^\lambda u_\lambda\rangle$	107	90	55	54	41	26							$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}f_{-\lambda}{}^\sigma u_\rho u_\sigma\rangle + \text{H.c.}$	250	222	140	132	110	63	235	207	128	123	101	
$\langle f_+^{\mu\nu}f_{+\mu\nu}\rangle\langle u^\lambda u_\lambda\rangle$	108	91	56	55	42		105	88		53	40		$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}f_{-\lambda}{}^\sigma u_\sigma u_\rho\rangle + \text{H.c.}$	251	223	141	133	111		236	208	129	124	102	
$\langle f_+^{\mu\nu}f_{+\mu\nu}u^\lambda\rangle\langle u_\lambda\rangle$	109	92	57				106	89	54				$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_\lambda f_{-\rho}{}^\sigma u_\sigma\rangle + \text{H.c.}$	252	224	142	134	112		237	209	130	125	103	
$\langle f_+^{\mu\nu}u^\lambda\rangle\langle f_{+\mu\nu}u_\lambda\rangle$	110	93	58	56	43		107	90		54			$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}h_\lambda{}^\sigma u_\rho u_\sigma\rangle + \text{H.c.}$	253	225	143	135	113	64	238	210	131	126	104	
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu}{}^\lambda u_\nu u_\lambda\rangle + \text{H.c.}$	111	94	59	57	44	27							$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}h_\lambda{}^\sigma u_\sigma u_\rho\rangle + \text{H.c.}$	254	226	144	136	114		239	211	132	127	105	
$\langle f_+^{\mu\nu}f_{+\mu}{}^\lambda\rangle\langle u_\nu u_\lambda\rangle$	112	95	60	58	45		108	91		55	41		$i\epsilon^{\mu\nu\lambda\rho}\langle\nabla^\sigma f_{+\mu\sigma}u_\nu u_\lambda\rangle\langle u_\rho\rangle$	255	227	145				240	212				
$\langle f_+^{\mu\nu}f_{+\mu}{}^\lambda u_\nu\rangle\langle u_\lambda\rangle + \text{H.c.}$	113	96	61				109	92	55				$i\epsilon^{\mu\nu\lambda\rho}\langle\nabla^\sigma f_{+\mu\nu}u_\sigma u_\lambda\rangle\langle u_\rho\rangle + \text{H.c.}$	256	228					241	213				
$\langle f_+^{\mu\nu}u^\lambda\rangle\langle f_{+\mu\lambda}u_\nu\rangle$	114	97	62	59	46		110	93		56	42		$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma h_{\nu\sigma}u_\lambda\rangle\langle u_\rho\rangle + \text{H.c.}$	257	229	146				242	214				
$\langle f_+^{\mu\nu}u_\mu\rangle\langle f_{+\nu}{}^\lambda u_\lambda\rangle$	115	98	63	60	47		111	94		57			$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}h_\lambda{}^\sigma u_\sigma\rangle\langle u_\rho\rangle + \text{H.c.}$	258	230					243	215				
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu\nu}\rangle\langle u^\lambda u_\lambda\rangle$	116	99		61									$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma f_{-\nu\sigma}u_\lambda\rangle\langle u_\rho\rangle + \text{H.c.}$	259	231					244	216				
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu\nu}u^\lambda\rangle\langle u_\lambda\rangle$	117	100											$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma f_{-\nu\lambda}u_\sigma\rangle\langle u_\rho\rangle + \text{H.c.}$	260	232					245	217				
$\langle f_+^{\mu\nu}f_{+\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\lambda\rangle$	118	101					112						$i\epsilon^{\mu\nu\lambda\rho}\langle\nabla^\sigma f_{+\mu\sigma}\rangle\langle u_\nu u_\lambda u_\rho\rangle$	261	233		137	115							
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu}{}^\lambda\rangle\langle u_\nu u_\lambda\rangle$	119	102		62									$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma\rangle\langle u_\nu u_\lambda h_{\rho\sigma}\rangle$	262	234		138	116							
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu}{}^\lambda u_\nu\rangle\langle u_\lambda\rangle$	120	103											$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_\lambda\nabla^\sigma f_{+\rho\sigma}\rangle + \text{H.c.}$	263	235	147	139	117	65	246	218	133	128	106	
$\langle f_+^{\mu\nu}f_{+\mu}{}^\lambda\rangle\langle u_\nu\rangle\langle u_\lambda\rangle$	121	104					113						$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda}{}^\sigma f_{-\rho\sigma}\rangle$	264	236	148	140	118	66						

(Table continued)

TABLE VII. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu}{}^{\lambda}u_\lambda\rangle\langle u_\nu\rangle$	122	105										
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\lambda\rangle$	123											
$\langle f_+^{\mu\nu}\rangle\langle f_{+\mu}{}^{\lambda}\rangle\langle u_\nu\rangle\langle u_\lambda\rangle$	124											
$\langle u^\mu u_\mu u^\nu u_\nu\chi_+\rangle$	125	106	64	63	48	28	114	95	56	58	43	26
$\langle u^\mu u^\nu u_\mu u_\nu\chi_+\rangle$	126	107	65	64	49	29	115	96	57	59	44	27
$\langle u^\mu u^\nu u_\mu u_\nu\chi_+\rangle$	127	108	66	65	50		116	97	58	60	45	
$\langle u^\mu\rangle\langle u_\mu u^\nu u_\nu\chi_+\rangle + \text{H.c.}$	128	109	67				117	98	59			
$\langle u^\mu u_\mu\rangle\langle u^\nu u_\nu\chi_+\rangle$	129	110	68	66	51		118	99	60	61	46	
$\langle u^\mu u_\mu u^\nu\rangle\langle u_\nu\chi_+\rangle$	130	111	69	67	52		119	100	61	62	47	
$\langle u^\mu u_\mu u^\nu u_\nu\rangle\langle\chi_+\rangle$	131	112	70	68	53		120	101	62	63	48	
$\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu\chi_+\rangle$	132	113	71	69			121	102	63	64		
$\langle u^\mu\rangle\langle u^\nu u_\mu u_\nu\chi_+\rangle$	133	114					122	103				
$\langle u^\mu u^\nu u_\mu u_\nu\rangle\langle\chi_+\rangle$	134	115		70	54		123	104		65	49	
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u_\nu\chi_+\rangle$	135	116					124	105				
$\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\langle u_\nu\chi_+\rangle$	136	117					125	106				
$\langle u^\mu\rangle\langle u_\mu u^\nu u_\nu\rangle\langle\chi_+\rangle$	137	118					126	107				
$\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\langle u_\mu\chi_+\rangle$	138	119					127	108				
$\langle u^\mu u_\mu\rangle\langle u^\nu u_\nu\rangle\langle\chi_+\rangle$	139	120		71			128	109		66		
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u_\nu\chi_+\rangle$	140	121					129	110				
$\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu\rangle\langle\chi_+\rangle$	141	122		72			130	111		67		
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu\chi_+\rangle$	142						131					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u_\nu\rangle\langle\chi_+\rangle$	143						132					
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u_\nu\rangle\langle\chi_+\rangle$	144						133					
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu\rangle\langle\chi_+\rangle$	145						134					

operators	U(3)			$SU(3)_I$			$SU(3)_{III}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda}{}^{\sigma}h_{\rho\sigma}\rangle$	265	237	149	141	119	67						
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle\nabla^\sigma f_{+\lambda\sigma}u_\rho\rangle$	266	238	150	142	120							
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\nabla^\sigma f_{+\lambda\sigma}\rangle\langle u_\rho\rangle$	267	239	151				247	219				
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle\nabla^\sigma f_{+\lambda\sigma}\rangle\langle u_\rho\rangle$	268	240										
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu u_\nu f_{-\lambda\rho}\chi_+\rangle + \text{H.c.}$	269	241	152	143	121	68	248	220	134	129	107	57
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u_\nu f_{-\lambda\rho}\chi_+\rangle + \text{H.c.}$	270	242	153				249	221	135			
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\chi_+\rangle\langle u_\nu f_{-\lambda\rho}\rangle$	271	243	154	144	122		250	222	136	130	108	
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u_\nu f_{-\lambda\rho}\rangle\langle\chi_+\rangle$	272	244					251	223				
$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u_\nu\chi_+\rangle\langle f_{-\lambda\rho}\rangle$	273	245					252	224				
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\chi_+ f_{-\lambda\rho}\rangle + \text{H.c.}$	274	246	155	145	123	69	253	225	137	131	109	58
$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu u_\nu u_\lambda u_\rho\chi_-\rangle$	275	247		146	124		254	226		132	110	
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{-\mu\nu}f_{-\lambda\rho}\chi_-\rangle$	276	248	156	147	125	70	255	227	138	133	111	59
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{-\mu\nu}\rangle\langle f_{-\lambda\rho}\chi_-\rangle$	277	249	157				256	228	139			
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{-\mu\nu}f_{-\lambda\rho}\rangle\langle\chi_-\rangle$	278	250	158	148	126		257	229	140	134	112	
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{-\mu\nu}\rangle\langle f_{-\lambda\rho}\rangle\langle\chi_-\rangle$	279	251					258	230				
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\chi_- u_\lambda u_\rho\rangle + \text{H.c.}$	280	252	159	149	127	71	259	231	141	135	113	60
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_\lambda\chi_- u_\rho\rangle$	281	253	160	150	128	72	260	232	142	136	114	
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle u_\lambda u_\rho\chi_-\rangle$	282	254	161	151	129							
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\chi_- u_\lambda\rangle\langle u_\rho\rangle + \text{H.c.}$	283	255					261	233				
$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_\lambda u_\rho\rangle\langle\chi_-\rangle$	284	256		152	130		262	234		137	115	
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}f_{+\lambda\rho}\chi_-\rangle$	285	257	162	153	131	73	263	235		138	116	
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda\rho}\chi_-\rangle$	286	258	163	154	132	74						
$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda\rho}\rangle\langle\chi_-\rangle$	287	259		155	133							

TABLE VIII. The p^6 order results in U groups and SU groups, with $t = 0$, $\hat{\theta} = 0$ and the odd parity. The numbers are the sequence numbers in each case. The details can be found in Sec. V C.

Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2
$\langle u^\mu u_\mu u^\nu u^\lambda f_{-\nu\lambda}\rangle + \text{H.c.}$	288	260	164	$\langle u^\mu \chi_+ u^\nu h_{\mu\nu}\rangle$	340	308	192	$\langle f_+^{\mu\nu} u_\mu u_\nu \chi_-\rangle + \text{H.c.}$	392	355	221	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u_\lambda u_\rho u^\sigma u_\sigma\rangle + \text{H.c.}$	444	405	254
$\langle u^\mu u^\nu u_\mu u^\lambda f_{-\nu\lambda}\rangle + \text{H.c.}$	289	261	165	$\langle u^\mu\rangle\langle u^\nu \chi_+ f_{-\mu\nu}\rangle + \text{H.c.}$	341	309	193	$\langle f_+^{\mu\nu} u_\mu \chi_- u_\nu\rangle$	393	356	222	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u_\lambda u^\sigma u_\rho u_\sigma\rangle + \text{H.c.}$	445	406	255
$\langle u^\mu u_\mu u^\nu u^\lambda h_{\nu\lambda}\rangle + \text{H.c.}$	290	262	166	$\langle u^\mu \chi_+\rangle\langle u^\nu f_{-\mu\nu}\rangle$	342	310	194	$\langle f_+^{\mu\nu}\rangle\langle u_\mu u_\nu \chi_-\rangle$	394	357	223	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u_\lambda u^\sigma u_\sigma u_\rho\rangle$	446	407	256
$\langle u^\mu u^\nu u_\mu u^\lambda\rangle\langle h_{\nu\lambda}\rangle$	291	263	167	$\langle u^\mu\rangle\langle u^\nu \chi_+ h_{\mu\nu}\rangle + \text{H.c.}$	343	311	195	$\langle f_+^{\mu\nu} u_\mu u_\nu\rangle\langle \chi_-\rangle$	395	358		$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u^\sigma u_\lambda u_\rho u_\sigma\rangle$	447	408	257
$\langle u^\mu u_\mu u^\nu u^\lambda\rangle\langle h_{\nu\lambda}\rangle$	292	264		$\langle u^\mu u^\nu\rangle\langle \chi_+ h_{\mu\nu}\rangle$	344	312	196	$\langle f_+^{\mu\nu} u_\mu \chi_-\rangle\langle u_\nu\rangle + \text{H.c.}$	396	359		$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma u_\sigma u_\nu u_\lambda\rangle\langle u_\rho\rangle + \text{H.c.}$	448	409	258
$\langle u^\mu u^\nu u^\lambda\rangle\langle u_\mu h_{\nu\lambda}\rangle$	293	265	168	$\langle u^\mu u^\nu \chi_+\rangle\langle h_{\mu\nu}\rangle$	345	313	197	$i\langle f_+^{\mu\nu} f_{+\mu\nu}\chi_-\rangle$	397	360	224	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma u_\nu u_\sigma u_\lambda\rangle\langle u_\rho\rangle$	449	410	
$\langle u^\mu u_\mu u^\nu\rangle\langle u^\lambda h_{\nu\lambda}\rangle$	294	266	169	$\langle u^\mu \chi_+\rangle\langle u^\nu h_{\mu\nu}\rangle$	346	314	198	$i\langle f_+^{\mu\nu}\rangle\langle f_{+\mu\nu}\chi_-\rangle$	398	361	225	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u^\sigma u_\sigma u_\lambda\rangle\langle u_\rho\rangle + \text{H.c.}$	450	411	
$\langle u^\mu u^\nu\rangle\langle u^\lambda u_\lambda h_{\mu\nu}\rangle$	295	267	170	$\langle u^\mu u^\nu h_{\mu\nu}\rangle\langle \chi_+\rangle$	347	315	199	$i\langle f_+^{\mu\nu} f_{+\mu\nu}\rangle\langle \chi_-\rangle$	399	362	226	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma u_\nu u_\lambda\rangle\langle u_\rho u_\sigma\rangle$	451	412	
$\langle u^\mu u^\nu\rangle\langle u^\lambda u_\mu h_{\nu\lambda}\rangle + \text{H.c.}$	296	268	171	$\langle u^\mu\rangle\langle u^\nu \chi_+\rangle\langle f_{-\mu\nu}\rangle$	348	316		$i\langle f_+^{\mu\nu}\rangle\langle f_{+\mu\nu}\rangle\langle \chi_-\rangle$	400	363		$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u^\sigma u_\lambda\rangle\langle u_\rho u_\sigma\rangle + \text{H.c.}$	452	413	
$\langle u^\mu\rangle\langle u^\nu u^\lambda u_\lambda h_{\mu\nu}\rangle + \text{H.c.}$	297	269	172	$\langle u^\mu\rangle\langle u^\nu f_{-\mu\nu}\rangle\langle \chi_+\rangle$	349	317		$i\langle u^\mu u_\mu \chi_+ \chi_-\rangle + \text{H.c.}$	401	364	227	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma u_\sigma\rangle\langle u_\nu u_\lambda u_\rho\rangle$	453	414	
$\langle u^\mu\rangle\langle u^\nu u^\lambda u_\nu h_{\mu\lambda}\rangle$	298	270		$\langle u^\mu\rangle\langle u^\nu\rangle\langle \chi_+ h_{\mu\nu}\rangle$	350	318		$i\langle u^\mu \chi_+ u_\mu \chi_-\rangle$	402	365	228	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma\rangle\langle u_\nu u_\lambda u_\rho u_\sigma\rangle$	454	415	
$\langle u^\mu\rangle\langle u^\nu u^\lambda u_\mu h_{\nu\lambda}\rangle + \text{H.c.}$	299	271		$\langle u^\mu\rangle\langle u^\nu \chi_+\rangle\langle h_{\mu\nu}\rangle$	351	319		$i\langle u^\mu\rangle\langle u_\mu \chi_+ \chi_-\rangle + \text{H.c.}$	403	366	229	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle u^\sigma\rangle\langle u_\lambda u_\rho u_\sigma\rangle$	455	416	
$\langle u^\mu u^\nu\rangle\langle u_\mu u^\lambda\rangle\langle h_{\nu\lambda}\rangle$	300	272		$\langle u^\mu u^\nu\rangle\langle \chi_+\rangle\langle h_{\mu\nu}\rangle$	352	320		$i\langle u^\mu u_\mu\rangle\langle \chi_+ \chi_-\rangle$	404	367	230	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u_\lambda u_\rho\rangle\langle u^\sigma\rangle\langle u_\sigma\rangle$	456		
$\langle u^\mu u_\mu\rangle\langle u^\nu u^\lambda\rangle\langle h_{\nu\lambda}\rangle$	301	273		$\langle u^\mu\rangle\langle u^\nu h_{\mu\nu}\rangle\langle \chi_+\rangle$	353	321		$i\langle u^\mu u_\mu \chi_+\rangle\langle \chi_-\rangle$	405	368	231	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u_\lambda u^\sigma\rangle\langle u_\rho\rangle\langle u_\sigma\rangle + \text{H.c.}$	457		
$\langle u^\mu\rangle\langle u^\nu u_\nu u^\lambda\rangle\langle h_{\mu\lambda}\rangle$	302	274		$\langle u^\mu\rangle\langle u^\nu\rangle\langle \chi_+\rangle\langle h_{\mu\nu}\rangle$	354			$i\langle u^\mu \chi_+\rangle\langle u_\mu \chi_-\rangle$	406	369	232	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} f_{-\lambda}{}^\sigma f_{-\rho\sigma}\rangle$	458	417	259
$\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda\rangle\langle h_{\nu\lambda}\rangle$	303	275		$i\langle f_+^{\mu\nu}\chi_+ f_{-\mu\nu}\rangle + \text{H.c.}$	355	322	200	$i\langle u^\mu u_\mu \chi_-\rangle\langle \chi_+\rangle$	407	370	233	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} f_{-\lambda}{}^\sigma h_{\rho\sigma}\rangle + \text{H.c.}$	459	418	260
$\langle u^\mu\rangle\langle u^\nu u_\nu u^\lambda\rangle\langle f_{-\mu\lambda}\rangle$	304	276		$i\langle \nabla^\mu f_{+\mu}{}^\nu u_\nu \chi_+\rangle + \text{H.c.}$	356	323	201	$i\langle u^\mu\rangle\langle u_\mu\rangle\langle \chi_+ \chi_-\rangle$	408	371		$i\epsilon^{\mu\nu\lambda\rho}\langle \nabla_\mu f_{+\nu}{}^\sigma f_{-\lambda\sigma} u_\rho\rangle + \text{H.c.}$	460	419	261
$\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\langle u_\nu h_{\mu\lambda}\rangle$	305	277		$\langle u^\mu \chi_+\rangle\langle \chi_{+\mu}\rangle$	357	324	202	$i\langle u^\mu\rangle\langle u_\mu \chi_+\rangle\langle \chi_-\rangle$	409	372		$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u^\sigma f_{+\lambda\rho} u_\sigma\rangle$	461	420	262
$\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\langle u^\lambda h_{\mu\lambda}\rangle$	306	278		$i\langle u^\mu u_\mu u^\nu u_\nu \chi_-\rangle$	358	325	203	$i\langle u^\mu u_\mu\rangle\langle \chi_+\rangle\langle \chi_-\rangle$	410	373		$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} f_{+\lambda\rho} u^\sigma u_\sigma\rangle$	462	421	263
$\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\langle u_\mu h_{\nu\lambda}\rangle$	307	279		$i\langle u^\mu u^\nu u_\mu u_\nu \chi_-\rangle$	359	326	204	$i\langle u^\mu\rangle\langle u_\mu \chi_-\rangle\langle \chi_+\rangle$	411	374		$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} f_{+\lambda}{}^\sigma u_\rho u_\sigma\rangle + \text{H.c.}$	463	422	264
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda u_\lambda h_{\mu\nu}\rangle$	308	280		$i\langle u^\mu u^\nu u_\nu u_\mu \chi_-\rangle$	360	327	205	$i\langle u^\mu\rangle\langle u_\mu\rangle\langle \chi_+\rangle\langle \chi_-\rangle$	412			$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda\rho} u^\sigma u_\sigma\rangle$	464	423	265
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u^\lambda f_{-\nu\lambda}\rangle$	309			$i\langle u^\mu\rangle\langle u_\mu u^\nu u_\nu \chi_-\rangle + \text{H.c.}$	361	328	206	$i\langle \chi_+^\mu \chi_{+\mu}\rangle$	413	375	234	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u^\sigma\rangle\langle f_{+\lambda\rho} u_\sigma\rangle$	465	424	266
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	310			$i\langle u^\mu u_\mu\rangle\langle u^\nu u_\nu \chi_-\rangle$	362	329	207	$i\langle \chi_-^\mu\rangle\langle \chi_{+\mu}\rangle$	414	376	235	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} f_{+\lambda\rho} u^\sigma\rangle\langle u_\sigma\rangle$	466	425	
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u^\lambda h_{\nu\lambda}\rangle$	311			$i\langle u^\mu u_\mu u^\nu\rangle\langle u_\nu \chi_-\rangle$	363	330	208	$i\langle \chi_+^2 \chi_-\rangle$	415	377	236	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda}{}^\sigma u_\rho u_\sigma\rangle + \text{H.c.}$	467	426	267
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u^\lambda\rangle\langle h_{\nu\lambda}\rangle$	312			$i\langle u^\mu u_\mu u^\nu u_\nu\rangle\langle \chi_-\rangle$	364	331	209	$i\langle \chi_+\rangle\langle \chi_+ \chi_-\rangle$	416	378	237	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu} u^\sigma\rangle\langle f_{+\lambda\sigma} u_\rho\rangle$	468	427	268
$\langle h^{\mu\nu} f_{-\mu}{}^\lambda f_{-\nu\lambda}\rangle$	313	281	173	$i\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu \chi_-\rangle$	365	332	210	$i\langle \chi_+^2\rangle\langle \chi_-\rangle$	417	379	238	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda\rho} u^\sigma\rangle\langle u_\sigma\rangle$	469	428	
$\langle h^{\mu\nu}\rangle\langle f_{-\mu}{}^\lambda f_{-\nu\lambda}\rangle$	314	282	174	$i\langle u^\mu\rangle\langle u^\nu u_\mu u_\nu \chi_-\rangle$	366	333		$i\langle \chi_+\rangle\langle \chi_+\rangle\langle \chi_-\rangle$	418	380		$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda}{}^\sigma u_\rho\rangle\langle u_\sigma\rangle$	470	429	
$\langle h^{\mu\nu} f_{-\mu}{}^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	315	283	175	$i\langle u^\mu u^\nu u_\mu u_\nu\rangle\langle \chi_-\rangle$	367	334		$\langle u^\mu \chi_-\rangle\langle \chi_{-\mu}\rangle$	419	381	239	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda}{}^\sigma u_\sigma\rangle\langle u_\rho\rangle$	471	430	
$\langle h^{\mu\nu}\rangle\langle h_\mu{}^\lambda f_{-\nu\lambda}\rangle$	316	284	176	$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u_\nu \chi_-\rangle$	368	335		$i\langle \chi_-^3\rangle$	420	382	240	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda\rho}\rangle\langle u^\sigma u_\sigma\rangle$	472	431	
$\langle u^\mu f_{-\mu}{}^\nu\rangle\langle \nabla^\lambda f_{-\nu\lambda}\rangle$	317	285	177	$i\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\langle u_\nu \chi_-\rangle$	369	336		$i\langle \chi_-\rangle\langle \chi_-^2\rangle$	421	383	241	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda\rho}\rangle\langle u^\sigma\rangle\langle u_\sigma\rangle$	473		

(Table continued)

TABLE VIII. (*Continued*)

Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2
$\langle h^{\mu\nu}\rangle\langle f_{-\mu}{}^{\lambda}\rangle\langle f_{-\nu\lambda}\rangle$	318	286		$i\langle u^{\mu}\rangle\langle u_{\mu}u^{\nu}u_{\nu}\rangle\langle\chi_{-}\rangle$	370	337		$i\langle\chi_{-}\rangle\langle\chi_{-}\rangle\langle\chi_{-}\rangle$	422	384		$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}u_{\nu}u_{\lambda}u_{\rho}\chi_{+}\rangle$	474	432	
$i\langle f_{+}^{\mu\nu}u_{\mu}u^{\lambda}f_{-\nu\lambda}\rangle + \text{H.c.}$	319	287	178	$i\langle u^{\mu}\rangle\langle u^{\nu}u_{\nu}\rangle\langle u_{\mu}\chi_{-}\rangle$	371	338		$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}u_{\nu}u^{\sigma}u_{\lambda}u_{\rho}u_{\sigma}\rangle$	423	385		$\epsilon^{\mu\nu\lambda\rho}\langle\chi_{+}f_{-\mu\nu}f_{-\lambda\rho}\rangle$	475	433	269
$i\langle f_{+}^{\mu\nu}u^{\lambda}u_{\mu}f_{-\nu\lambda}\rangle + \text{H.c.}$	320	288	179	$i\langle u^{\mu}u_{\mu}\rangle\langle u^{\nu}u_{\nu}\rangle\langle\chi_{-}\rangle$	372	339		$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}f_{-\nu}{}^{\sigma}u_{\sigma}f_{-\lambda\rho}\rangle + \text{H.c.}$	424	386	242	$\epsilon^{\mu\nu\lambda\rho}\langle\chi_{+}\rangle\langle f_{-\mu\nu}f_{-\lambda\rho}\rangle$	476	434	270
$i\langle f_{+}^{\mu\nu}u^{\lambda}u_{\lambda}f_{-\mu\nu}\rangle + \text{H.c.}$	321	289	180	$i\langle u^{\mu}\rangle\langle u^{\nu}\rangle\langle u_{\mu}u_{\nu}\chi_{-}\rangle$	373	340		$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}u_{\nu}f_{-\lambda}{}^{\sigma}f_{-\rho\sigma}\rangle$	425	387	243	$\epsilon^{\mu\nu\lambda\rho}\langle\chi_{+}f_{-\mu\nu}\rangle\langle f_{-\lambda\rho}\rangle$	477	435	271
$i\langle f_{+}^{\mu\nu}u_{\mu}f_{-\nu}{}^{\lambda}u_{\lambda}\rangle + \text{H.c.}$	322	290	181	$i\langle u^{\mu}u^{\nu}\rangle\langle u_{\mu}u_{\nu}\rangle\langle\chi_{-}\rangle$	374	341		$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}u^{\sigma}f_{-\nu\lambda}f_{-\rho\sigma}\rangle + \text{H.c.}$	426	388	244	$\epsilon^{\mu\nu\lambda\rho}\langle\chi_{+}\rangle\langle f_{-\mu\nu}\rangle\langle f_{-\lambda\rho}\rangle$	478	436	
$i\langle f_{+}^{\mu\nu}u_{\mu}u^{\lambda}h_{\nu\lambda}\rangle + \text{H.c.}$	323	291	182	$i\langle u^{\mu}\rangle\langle u_{\mu}\rangle\langle u^{\nu}\rangle\langle u_{\nu}\chi_{-}\rangle$	375			$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}u_{\nu}f_{-\lambda}{}^{\sigma}h_{\rho\sigma}\rangle + \text{H.c.}$	427	389	245	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\chi_{+}u_{\lambda}u_{\rho}\rangle + \text{H.c.}$	479	437	272
$i\langle f_{+}^{\mu\nu}u^{\lambda}u_{\mu}h_{\nu\lambda}\rangle + \text{H.c.}$	324	292	183	$i\langle u^{\mu}\rangle\langle u_{\mu}\rangle\langle u^{\nu}u_{\nu}\rangle\langle\chi_{-}\rangle$	376			$\epsilon^{\mu\nu\lambda\rho}\langle u^{\sigma}u_{\mu}f_{-\nu\lambda}\rangle\langle h_{\rho\sigma}\rangle + \text{H.c.}$	428	390	246	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_{\lambda}\chi_{+}u_{\rho}\rangle$	480	438	273
$i\langle\nabla^{\mu}f_{+}^{\nu\lambda}u_{\nu}u_{\lambda}\rangle\langle u_{\mu}\rangle$	325	293		$i\langle u^{\mu}\rangle\langle u^{\nu}\rangle\langle u_{\mu}u_{\nu}\rangle\langle\chi_{-}\rangle$	377			$\epsilon^{\mu\nu\lambda\rho}\langle u^{\sigma}u_{\mu}h_{\nu\sigma}\rangle\langle f_{-\lambda\rho}\rangle + \text{H.c.}$	429	391	247	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle u_{\lambda}u_{\rho}\chi_{+}\rangle$	481	439	274
$i\langle\nabla^{\mu}f_{+\mu}{}^{\nu}u^{\lambda}u_{\nu}\rangle\langle u_{\lambda}\rangle + \text{H.c.}$	326	294		$i\langle u^{\mu}\rangle\langle u_{\mu}\rangle\langle u^{\nu}\rangle\langle u_{\nu}\rangle\langle\chi_{-}\rangle$	378			$\epsilon^{\mu\nu\lambda\rho}\langle u^{\sigma}u_{\sigma}f_{-\mu\nu}\rangle\langle f_{-\lambda\rho}\rangle$	430	392	248	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\chi_{+}u_{\lambda}\rangle\langle u_{\rho}\rangle + \text{H.c.}$	482	440	
$i\langle f_{+}^{\mu\nu}h_{\mu}{}^{\lambda}u_{\lambda}\rangle\langle u_{\nu}\rangle + \text{H.c.}$	327	295	184	$i\langle f_{-}^{\mu\nu}f_{-\mu\nu}\chi_{-}\rangle$	379	342	211	$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}h_{\nu}{}^{\sigma}\rangle\langle u_{\lambda}h_{\rho\sigma}\rangle$	431	393	249	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_{\lambda}u_{\rho}\rangle\langle\chi_{+}\rangle$	483	441	
$i\langle f_{+}^{\mu\nu}h_{\mu}{}^{\lambda}u_{\nu}\rangle\langle u_{\lambda}\rangle + \text{H.c.}$	328	296		$i\langle h^{\mu\nu}h_{\mu\nu}\chi_{-}\rangle$	380	343	212	$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}u^{\sigma}\rangle\langle u_{\nu}\nabla_{\lambda}h_{\rho\sigma}\rangle$	432	394	250	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}f_{+\lambda\rho}\chi_{+}\rangle$	484	442	275
$i\langle f_{+}^{\mu\nu}f_{-\mu}{}^{\lambda}u_{\lambda}\rangle\langle u_{\nu}\rangle + \text{H.c.}$	329	297		$i\langle u^{\mu}\nabla^{\nu}f_{-\mu\nu}\chi_{-}\rangle + \text{H.c.}$	381	344	213	$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}\rangle\langle u^{\sigma}u_{\nu}\nabla_{\lambda}h_{\rho\sigma}\rangle + \text{H.c.}$	433	395	251	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda\rho}\chi_{+}\rangle$	485	443	276
$i\langle f_{+}^{\mu\nu}f_{-\mu\nu}u^{\lambda}\rangle\langle u_{\lambda}\rangle + \text{H.c.}$	330	298		$i\langle f_{-}^{\mu\nu}\rangle\langle f_{-\mu\nu}\chi_{-}\rangle$	382	345	214	$\epsilon^{\mu\nu\lambda\rho}\langle u^{\sigma}h_{\mu\sigma}\rangle\langle u_{\nu}f_{-\lambda\rho}\rangle$	434	396	252	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda\rho}\rangle\langle\chi_{+}\rangle$	486	444	
$i\langle f_{+}^{\mu\nu}\rangle\langle u^{\lambda}u_{\mu}h_{\nu\lambda}\rangle + \text{H.c.}$	331	299		$i\langle f_{-}^{\mu\nu}f_{-\mu\nu}\rangle\langle\chi_{-}\rangle$	383	346	215	$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}h_{\nu}{}^{\sigma}\rangle\langle u_{\sigma}f_{-\lambda\rho}\rangle$	435	397	253	$i\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}u_{\nu}\chi_{-}f_{-\lambda\rho}\rangle + \text{H.c.}$	487	445	277
$\langle f_{+}^{\mu\nu}f_{+\mu}{}^{\lambda}h_{\nu\lambda}\rangle$	332	300	185	$i\langle h^{\mu\nu}\rangle\langle h_{\mu\nu}\chi_{-}\rangle$	384	347	216	$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}\rangle\langle u_{\nu}h_{\lambda}{}^{\sigma}\rangle\langle h_{\rho\sigma}\rangle$	436	398		$i\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}\rangle\langle u_{\nu}\chi_{-}f_{-\lambda\rho}\rangle + \text{H.c.}$	488	446	278
$\langle f_{+}^{\mu\nu}\rangle\langle f_{+\mu}{}^{\lambda}f_{-\nu\lambda}\rangle$	333	301	186	$i\langle h^{\mu\nu}h_{\mu\nu}\rangle\langle\chi_{-}\rangle$	385	348	217	$\epsilon^{\mu\nu\lambda\rho}\langle u^{\sigma}\rangle\langle u_{\mu}f_{-\nu\lambda}\rangle\langle h_{\rho\sigma}\rangle$	437	399		$i\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}f_{-\nu\lambda}\rangle\langle u_{\rho}\chi_{-}\rangle$	489	447	279
$\langle f_{+}^{\mu\nu}\rangle\langle f_{+\mu}{}^{\lambda}h_{\nu\lambda}\rangle$	334	302	187	$i\langle u^{\mu}\rangle\langle\nabla^{\nu}f_{-\mu\nu}\chi_{-}\rangle$	386	349	218	$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}\rangle\langle u^{\sigma}f_{-\nu\lambda}\rangle\langle h_{\rho\sigma}\rangle$	438	400		$i\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}\rangle\langle u_{\nu}\chi_{-}\rangle\langle f_{-\lambda\rho}\rangle$	490	448	
$\langle f_{+}^{\mu\nu}f_{+\mu}{}^{\lambda}\rangle\langle h_{\nu\lambda}\rangle$	335	303	188	$i\langle u^{\mu}\nabla^{\nu}f_{-\mu\nu}\rangle\langle\chi_{-}\rangle$	387	350	219	$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}\rangle\langle u_{\nu}u^{\sigma}\rangle\langle\nabla_{\lambda}h_{\rho\sigma}\rangle$	439	401		$i\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}\rangle\langle u_{\nu}f_{-\lambda\rho}\rangle\langle\chi_{-}\rangle$	491	449	
$\langle f_{+}^{\mu\nu}\rangle\langle\nabla_{\mu}f_{+\nu}{}^{\lambda}u_{\lambda}\rangle$	336	304	189	$i\langle u^{\mu}\chi_{-}\rangle\langle\nabla^{\nu}f_{-\mu\nu}\rangle$	388	351	220	$\epsilon^{\mu\nu\lambda\rho}\langle u^{\sigma}u_{\sigma}\rangle\langle f_{-\mu\nu}\rangle\langle f_{-\lambda\rho}\rangle$	440	402		$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}f_{-\lambda\rho}\chi_{-}\rangle + \text{H.c.}$	492	450	280
$\langle f_{+}^{\mu\nu}\rangle\langle f_{+\mu}{}^{\lambda}\rangle\langle h_{\nu\lambda}\rangle$	337	305		$i\langle f_{-}^{\mu\nu}\rangle\langle f_{-\mu\nu}\rangle\langle\chi_{-}\rangle$	389	352		$\epsilon^{\mu\nu\lambda\rho}\langle u^{\sigma}\rangle\langle u_{\mu}h_{\nu\sigma}\rangle\langle f_{-\lambda\rho}\rangle$	441	403		$i\epsilon^{\mu\nu\lambda\rho}\langle F_{L\mu\nu}F_{L\lambda}{}^{\sigma}F_{L\rho\sigma}\rangle + \text{H.c.}$	493	451	281
$\langle u^{\mu}u^{\nu}\chi_{+}f_{-\mu\nu}\rangle + \text{H.c.}$	338	306	190	$i\langle h^{\mu\nu}\rangle\langle h_{\mu\nu}\rangle\langle\chi_{-}\rangle$	390	353		$\epsilon^{\mu\nu\lambda\rho}\langle u^{\sigma}\rangle\langle u_{\sigma}f_{-\mu\nu}\rangle\langle f_{-\lambda\rho}\rangle$	442	404					
$\langle u^{\mu}u^{\nu}\chi_{+}h_{\mu\nu}\rangle + \text{H.c.}$	339	307	191	$i\langle u^{\mu}\rangle\langle\nabla^{\nu}f_{-\mu\nu}\rangle\langle\chi_{-}\rangle$	391	354		$\epsilon^{\mu\nu\lambda\rho}\langle u_{\mu}\rangle\langle u^{\sigma}\rangle\langle f_{-\nu\lambda}\rangle\langle f_{-\rho\sigma}\rangle$	443						

TABLE IX. The p^6 order results in U groups and SU groups, with $t = 0, \hat{\theta} \neq 0$ and the even parity. The numbers are the sequence numbers in each case. The details can be found in Sec. V C.

	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$				U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
Operators	n	3	2	n	3	2	n	3	2	n	3	2	Operators	n	3	2	n	3	2	n	3	2	n	3	2
$i\langle u^\mu u_\mu u^\nu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	1	1	1	1	1		1	1	1	1	1		$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	133	128					127	122				
$i\langle u^\mu u_\mu u^\nu u^\lambda u_\nu\rangle\nabla_\lambda\hat{\theta}$	2	2	2	2	2		2	2	2	2	2		$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	134	129					128	123				
$i\langle u^\mu u^\nu u_\mu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	3	3	3	3	3		3	3	3	3	3		$\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda u_\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	135	130					129	124				
$i\langle u^\mu\rangle\langle u_\mu u^\nu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta} + \text{H.c.}$	4	4	4				4	4	4				$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	136						130					
$i\langle u^\mu u_\mu\rangle\langle u^\nu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	5	5	5	4	4		5	5	5	4	4		$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	137						131					
$i\langle u^\mu u^\nu\rangle\langle u_\mu u^\lambda u_\lambda\rangle\nabla_\nu\hat{\theta}$	6	6	6	5			6	6	6	5			$\langle f_-^{\mu\nu} f_{-\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	138	131	95	57	52	25	132	125	88	53	48	21
$i\langle u^\mu\rangle\langle u^\nu u_\nu u^\lambda u_\lambda\rangle\nabla_\mu\hat{\theta}$	7	7	7				7	7	7				$\langle f_-^{\mu\nu} f_{-\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	139	132	96	58	53	26	133	126	89	54	49	22
$i\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	8	8	8	6			8	8	8	6			$\langle h^{\mu\nu} f_{-\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	140	133	97	59	54	27	134	127	90	55	50	23
$i\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda u_\nu\rangle\nabla_\lambda\hat{\theta}$	9	9					9	9					$\langle h^{\mu\nu} h_{\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	141	134	98	60	55	28	135	128	91	56	51	24
$i\langle u^\mu\rangle\langle u^\nu u^\lambda u_\nu u_\lambda\rangle\nabla_\mu\hat{\theta}$	10	10					10	10					$\langle h^{\mu\nu} h_\mu{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	142	135	99	61	56	29	136	129	92	57	52	25
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	11	11					11	11					$\langle u^\mu\nabla^\nu f_{-\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	143	136	100	62	57	30	137	130	93	58	53	26
$i\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\langle u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	12	12					12	12					$\langle u^\mu\nabla^\nu f_{-\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	144	137	101	63	58	31	138	131	94	59	54	27
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u^\lambda u_\lambda\rangle\nabla_\nu\hat{\theta}$	13	13					13	13					$\langle u^\mu\nabla^\nu f_{-\nu}{}^\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\lambda\hat{\theta}$	145	138	102	64	59	32	139	132	95	60	55	28
$i\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\langle u^\lambda u_\lambda\rangle\nabla_\nu\hat{\theta}$	14	14					14	14					$\langle u^\mu h_\mu{}^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	146	139	103	65	60	33	140	133	96	61	56	29
$i\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\langle u^\lambda u_\lambda\rangle\nabla_\mu\hat{\theta}$	15	15					15	15					$\langle u^\mu u_\mu\rangle\nabla^\nu\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	147	140	104	66	61	34	141	134	97	62	57	30
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	16	16					16	16					$\langle u^\mu u^\nu\rangle\nabla_\mu\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	148	141	105	67	62	35	142	135	98	63	58	31
$i\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\langle u_\nu u_\lambda\rangle\nabla_\mu\hat{\theta}$	17	17					17	17					$\langle h^{\mu\nu}\rangle\langle f_{-\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	149	142	106				143	136	99			
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	18						18						$\langle h^{\mu\nu}\rangle\langle h_{\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	150	143	107				144	137	100			
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u^\lambda u_\lambda\rangle\nabla_\nu\hat{\theta}$	19						19						$\langle h^{\mu\nu}\rangle\langle h_\mu{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	151	144	108				145	138	101			
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda\rangle\langle u_\mu u_\nu\rangle\nabla_\lambda\hat{\theta}$	20						20						$\langle u^\mu\rangle\langle\nabla^\nu f_{-\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	152	145	109				146	139	102			
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle u_\nu\rangle\langle u^\lambda\rangle\nabla_\lambda\hat{\theta}$	21						21						$\langle u^\mu\rangle\langle\nabla^\nu f_{-\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	153	146	110				147	140	103			
$i\langle u^\mu f_{-\mu}{}^\nu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta} + \text{H.c.}$	22	18	9	7	5		22	18	9	7	5		$\langle u^\mu\rangle\langle\nabla^\nu f_{-\nu}{}^\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\lambda\hat{\theta}$	154	147	111				148	141	104			
$i\langle u^\mu f_-^{\nu\lambda} f_{-\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	23	19	10	8	6		23	19	10	8	6		$\langle u^\mu\rangle\langle h_\mu{}^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	155	148	112				149	142	105			
$i\langle u^\mu h_\mu{}^\nu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta} + \text{H.c.}$	24	20	11	9	7		24	20	11	9	7		$\langle u^\mu\rangle\langle u_\mu\rangle\nabla^\nu\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	156	149	113				150	143	106			
$i\langle u^\mu h^{\nu\lambda} f_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta} + \text{H.c.}$	25	21	12	10	8		25	21	12	10	8		$\langle u^\mu\rangle\langle u^\nu\rangle\nabla_\mu\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	157	150	114				151	144	107			
$i\langle u^\mu h_\mu{}^\nu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta} + \text{H.c.}$	26	22	13	11	9		26	22	13	11	9		$i\langle f_+^{\mu\nu} u_\mu u_\nu\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	158	151	115	68	63	36	152	145	108	64	59	32
$i\langle u^\mu h^{\nu\lambda} h_{\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	27	23	14	12	10		27	23	14	12	10		$i\langle f_+^{\mu\nu} u_\mu u^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta} + \text{H.c.}$	159	152	116	69	64	37	153	146	109	65	60	33
$i\langle u^\mu u^\nu\nabla_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta} + \text{H.c.}$	28	24	15	13	11		28	24	15	13	11		$\langle u^\mu u_\mu\chi_+\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	160	153	117	70	65	38	154	147	110	66	61	34
$i\langle u^\mu h^{\nu\lambda}\rangle\langle h_{\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	29	25	16				29	25	16				$\langle u^\mu u^\nu\chi_+\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	161	154	118	71	66	39	155	148	111	67	62	35
$i\langle u^\mu h^{\nu\lambda}\rangle\langle h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	30	26	17				30	26	17				$\langle u^\mu\rangle\langle u_\mu\chi_+\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	162	155	119				156	149	112			

(Table continued)

TABLE IX. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$i\langle u^\mu h_\mu{}^\nu\rangle\langle h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	31	27	18				31	27	18			
$i\langle u^\mu f_-^{\nu\lambda}\rangle\langle h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	32	28	19				32	28	19			
$i\langle u^\mu f_{-\mu}{}^\nu\rangle\langle h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	33	29	20				33	29	20			
$i\langle u^\mu u^\nu\rangle\langle\nabla^\lambda h_{\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	34	30	21				34	30	21			
$i\langle u^\mu u^\nu\rangle\langle\nabla^\lambda h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	35	31	22				35	31	22			
$i\langle u^\mu u_\mu\rangle\langle\nabla^\nu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	36	32	23				36	32	23			
$i\langle u^\mu h^{\nu\lambda}\rangle\langle f_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	37	33	24				37	33	24			
$i\langle u^\mu h_\mu{}^\nu\rangle\langle f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	38	34	25				38	34	25			
$i\langle u^\mu f_-^{\nu\lambda}\rangle\langle f_{-\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	39	35	26				39	35	26			
$i\langle u^\mu\rangle\langle u^\nu\nabla^\lambda h_{\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	40	36	27				40	36	27			
$i\langle u^\mu\rangle\langle u^\nu\nabla^\lambda h_{\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	41	37	28				41	37	28			
$i\langle u^\mu\rangle\langle u^\nu\nabla^\lambda h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	42	38	29				42	38	29			
$i\langle u^\mu\rangle\langle u^\nu\nabla_\nu h_\mu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	43	39	30				43	39	30			
$i\langle u^\mu\rangle\langle u^\nu\nabla_\mu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	44	40	31				44	40	31			
$i\langle u^\mu\rangle\langle u_\mu\nabla^\nu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	45	41	32				45	41	32			
$i\langle u^\mu\rangle\langle h^{\nu\lambda}h_{\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	46	42	33				46	42	33			
$i\langle u^\mu\rangle\langle f_{-\mu}{}^\nu\rangle\langle f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	47	43					47	43				
$i\langle u^\mu\rangle\langle f_-^{\nu\lambda}\rangle\langle f_{-\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	48	44					48	44				
$i\langle u^\mu\rangle\langle h_\mu{}^\nu\rangle\langle f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	49	45					49	45				
$i\langle u^\mu\rangle\langle h^{\nu\lambda}\rangle\langle f_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	50	46					50	46				
$i\langle u^\mu\rangle\langle h_\mu{}^\nu\rangle\langle h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	51	47					51	47				
$i\langle u^\mu\rangle\langle h^{\nu\lambda}\rangle\langle h_{\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	52	48					52	48				
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle\nabla_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	53	49					53	49				
$\langle f_+^{\mu\nu}u_\mu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	54	50	34	14	12	1	54	50	34	14	12	
$\langle f_+^{\mu\nu}u_\mu u^\lambda u_\nu\rangle\nabla_\lambda\hat{\theta}$	55	51	35	15	13		55	51	35	15	13	
$\langle f_+^{\mu\nu}u_\mu u^\lambda u_\lambda\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	56	52	36	16	14		56	52	36	16	14	
$\langle f_+^{\mu\nu}\rangle\langle u_\mu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}$	57	53	37	17	15							
$\langle f_+^{\mu\nu}u_\mu u_\nu\rangle\langle u^\lambda\rangle\nabla_\lambda\hat{\theta}$	58	54					57	53				
$\langle f_+^{\mu\nu}u_\mu u^\lambda\rangle\langle u_\nu\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	59	55					58	54				
$\langle f_+^{\mu\nu}u_\mu u^\lambda\rangle\langle u_\lambda\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	60	56					59	55				
$i\langle f_+^{\mu\nu}f_{+\mu\nu}u^\lambda\rangle\nabla_\lambda\hat{\theta}$	61	57	38	18	16	2	60	56	37	17	15	

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu u_\mu\rangle\langle\chi_+\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	163	156	120	72	67		157	150	113	68	63	
$\langle u^\mu\rangle\langle u^\nu\chi_+\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	164	157	121				158	151	114			
$\langle u^\mu u^\nu\rangle\langle\chi_+\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	165	158	122	73	68		159	152	115	69	64	
$\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_+\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	166	159					160	153				
$\langle u^\mu\rangle\langle u^\nu\rangle\langle\chi_+\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	167	160					161	154				
$\langle\chi_+^\mu\rangle\nabla_\mu\hat{\theta}\nabla^\nu\nabla_\nu\hat{\theta}$	168	161	123	74	69	40	162	155	116	70	65	36
$\langle\chi_+^\mu\rangle\nabla^\nu\hat{\theta}\nabla_\mu\nabla_\nu\hat{\theta}$	169	162	124	75	70	41	163	156	117	71	66	37
$\langle\chi_+\rangle\nabla^\mu\nabla_\mu\hat{\theta}\nabla^\nu\nabla_\nu\hat{\theta}$	170	163	125	76	71	42	164	157	118	72	67	38
$\langle\chi_+^2\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}$	171	164	126	77	72	43	165	158	119	73	68	39
$\langle\chi_+\rangle\langle\chi_+\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}$	172	165	127	78	73	44	166	159	120	74	69	40
$i\langle h^{\mu\nu}\chi_-\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	173	166	128	79	74	45	167	160	121	75	70	41
$i\langle u^\mu\chi_{-\mu}\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	174	167	129	80	75	46	168	161	122	76	71	42
$i\langle u^\mu\chi_-^\nu\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	175	168	130	81	76	47	169	162	123	77	72	43
$i\langle h^{\mu\nu}\rangle\langle\chi_-\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	176	169	131				170	163	124			
$i\langle u^\mu\rangle\langle\chi_{-\mu}\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	177	170	132				171	164	125			
$i\langle u^\mu\rangle\langle\chi_-^\nu\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	178	171	133				172	165	126			
$\langle\chi_-\rangle\langle\chi_-\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}$	179	172	134	82	77		173	166		78	73	
$i\langle u^\mu u_\mu u^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	180	173	135	83	78		174	167	127	79	74	
$i\langle u^\mu u^\nu u^\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	181	174	136	84	79		175	168	128	80	75	
$i\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	182	175	137				176	169	129			
$i\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\nabla_\mu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	183	176	138				177	170	130			
$i\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	184	177	139				178	171	131			
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	185	178					179	172				
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	186	179					180	173				
$i\langle h^{\mu\nu}\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	187	180	140				181	174	132			
$i\langle h^{\mu\nu}\rangle\nabla_\mu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\nu\nabla_\lambda\hat{\theta}$	188	181	141				182	175	133			
$i\langle u^\mu\rangle\nabla_\mu\hat{\theta}\nabla^\nu\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	189	182	142				183	176	134			
$i\langle u^\mu\rangle\nabla^\nu\hat{\theta}\nabla_\mu\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	190	183	143				184	177	135			
$i\langle u^\mu\chi_+\rangle\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	191	184	144	85	80	48	185	178	136	81	76	44
$i\langle u^\mu\rangle\langle\chi_+\rangle\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	192	185	145				186	179	137			
$\langle\chi_-^\mu\rangle\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	193	186	146	86	81	49	187	180	138	82	77	45

(Table continued)

TABLE IX. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$			Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2		n	3	2	n	3	2	n	3	2	n	3	2
$i\langle f_+^{\mu\nu} f_{+\mu}{}^{\lambda} u_\nu\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	62	58	39	19	17	3	61	57	38	18	16		$\langle \chi_-\rangle \nabla^\mu \hat{\theta} \nabla_\mu \hat{\theta} \nabla^\nu \nabla_\nu \hat{\theta}$	194	187	147	87	82	50	188	181	139	83	78	46
$i\langle f_+^{\mu\nu}\rangle \langle f_{+\mu\nu} u^\lambda\rangle \nabla_\lambda \hat{\theta}$	63	59	40	20	18								$\langle u^\mu u_\mu\rangle \nabla^\nu \hat{\theta} \nabla_\nu \hat{\theta} \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	195	188	148	88	83	51	189	182	140	84	79	47
$i\langle f_+^{\mu\nu} f_{+\mu\nu}\rangle \langle u^\lambda\rangle \nabla_\lambda \hat{\theta}$	64	60	41				62	58					$\langle u^\mu u^\nu\rangle \nabla_\mu \hat{\theta} \nabla_\nu \hat{\theta} \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	196	189	149	89	84	52	190	183	141	85	80	48
$i\langle f_+^{\mu\nu}\rangle \langle f_{+\mu}{}^{\lambda} u_\nu\rangle \nabla_\lambda \hat{\theta}$	65	61	42	21	19	4							$\langle u^\mu\rangle \langle u_\mu\rangle \nabla^\nu \hat{\theta} \nabla_\nu \hat{\theta} \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	197	190	150				191	184	142			
$i\langle f_+^{\mu\nu} f_{+\mu}{}^{\lambda}\rangle \langle u_\nu\rangle \nabla_\lambda \hat{\theta}$	66	62	43				63	59					$\langle u^\mu\rangle \langle u^\nu\rangle \nabla_\mu \hat{\theta} \nabla_\nu \hat{\theta} \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	198	191	151				192	185	143			
$i\langle f_+^{\mu\nu}\rangle \langle f_{+\mu}{}^{\lambda} u_\lambda\rangle \nabla_\nu \hat{\theta}$	67	63	44	22	20								$\langle \chi_+\rangle \nabla^\mu \hat{\theta} \nabla_\mu \hat{\theta} \nabla^\nu \hat{\theta} \nabla_\nu \hat{\theta}$	199	192	152	90	85	53	193	186	144	86	81	49
$i\langle f_+^{\mu\nu}\rangle \langle f_{+\mu\nu}\rangle \langle u^\lambda\rangle \nabla_\lambda \hat{\theta}$	68	64											$i\langle u^\mu\rangle \nabla_\mu \hat{\theta} \nabla^\nu \hat{\theta} \nabla_\nu \hat{\theta} \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	200	193	153				194	187	145			
$i\langle f_+^{\mu\nu}\rangle \langle f_{+\mu}{}^{\lambda}\rangle \langle u_\nu\rangle \nabla_\lambda \hat{\theta}$	69	65											$\nabla^\mu \hat{\theta} \nabla^\nu \hat{\theta} \nabla^\lambda \nabla_\mu \hat{\theta} \nabla_\lambda \nabla_\nu \hat{\theta}$	201	194	154	91	86	54	195	188	146	87	82	50
$i\langle u^\mu u_\mu u^\nu \chi_+\rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	70	66	45	23	21	5	64	60	39	19	17	1	$\nabla^\mu \hat{\theta} \nabla_\mu \hat{\theta} \nabla^\nu \nabla^\lambda \hat{\theta} \nabla_\lambda \nabla_\nu \hat{\theta}$	202	195	155	92	87	55	196	189	147	88	83	51
$i\langle u^\mu u^\nu u_\mu \chi_+\rangle \nabla_\nu \hat{\theta}$	71	67	46	24	22	6	65	61	40	20	18	2	$\nabla^\mu \hat{\theta} \nabla^\nu \hat{\theta} \nabla_\mu \nabla_\nu \hat{\theta} \nabla^\lambda \nabla_\lambda \hat{\theta}$	203	196	156	93	88	56	197	190	148	89	84	52
$i\langle u^\mu\rangle \langle u_\mu u^\nu \chi_+\rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	72	68	47				66	62	41				$\langle F_L^{\mu\nu} F_{L\mu}{}^{\lambda}\rangle \nabla_\nu \hat{\theta} \nabla_\lambda \hat{\theta} + \text{H.c.}$	204	197	157	94	89	57	198	191	149	90	85	53
$i\langle u^\mu u_\mu\rangle \langle u^\nu \chi_+\rangle \nabla_\nu \hat{\theta}$	73	69	48	25	23		67	63	42	21	19		$\langle F_L^{\mu\nu} F_{L\mu\nu}\rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta} + \text{H.c.}$	205	198	158	95	90	58	199	192	150	91	86	54
$i\langle u^\mu u_\mu u^\nu\rangle \langle \chi_+\rangle \nabla_\nu \hat{\theta}$	74	70	49	26	24		68	64	43	22	20		$\langle \chi \chi^\dagger\rangle \nabla^\mu \hat{\theta} \nabla_\mu \hat{\theta}$	206	199	159	96	91	59	200	193	151	92	87	55
$i\langle u^\mu u^\nu\rangle \langle u_\mu \chi_+\rangle \nabla_\nu \hat{\theta}$	75	71	50	27			69	65	44	23			$i\langle D^\mu F_{L\mu}{}^{\nu}\rangle \nabla_\nu \hat{\theta} \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta} + \text{H.c.}$	207	200	160				201	194	152			
$i\langle u^\mu\rangle \langle u^\nu u_\nu \chi_+\rangle \nabla_\mu \hat{\theta}$	76	72	51				70	66	45				$\nabla^\mu \hat{\theta} \nabla_\mu \hat{\theta} \nabla^\nu \hat{\theta} \nabla_\nu \hat{\theta} \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	208	201	161	97	92	60	202	195	153	93	88	56
$i\langle u^\mu\rangle \langle u_\mu\rangle \langle u^\nu \chi_+\rangle \nabla_\nu \hat{\theta}$	77	73					71	67					$\nabla^\mu \nabla^\nu \nabla^\lambda \hat{\theta} \nabla_\mu \nabla_\lambda \nabla_\nu \hat{\theta}$	209	202	162	98	93	61	203	196	154	94	89	57
$i\langle u^\mu\rangle \langle u_\mu u^\nu\rangle \langle \chi_+\rangle \nabla_\nu \hat{\theta}$	78	74					72	68					$\langle F_L^{\mu\nu}\rangle \langle F_{L\mu}{}^{\lambda}\rangle \nabla_\nu \hat{\theta} \nabla_\lambda \hat{\theta} + \text{H.c.}$	210	203	163	99	94	62	204	197	155			
$i\langle u^\mu\rangle \langle u^\nu u_\nu\rangle \langle \chi_+\rangle \nabla_\mu \hat{\theta}$	79	75					73	69					$\langle F_L^{\mu\nu}\rangle \langle F_{L\mu\nu}\rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta} + \text{H.c.}$	211	204	164	100	95	63	205	198	156			
$i\langle u^\mu\rangle \langle u^\nu\rangle \langle u_\mu \chi_+\rangle \nabla_\nu \hat{\theta}$	80	76					74	70					$\langle F_L^{\mu\nu}\rangle \langle F_{R\mu}{}^{\lambda}\rangle \nabla_\nu \hat{\theta} \nabla_\lambda \hat{\theta}$	212	205	165									
$i\langle u^\mu\rangle \langle u_\mu\rangle \langle u^\nu\rangle \langle \chi_+\rangle \nabla_\nu \hat{\theta}$	81						75						$\langle F_L^{\mu\nu}\rangle \langle F_{R\mu\nu}\rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	213	206	166									
$i\langle \chi_+ \nabla^\mu f_{-\mu}{}^{\nu}\rangle \nabla_\nu \hat{\theta}$	82	77	52	28	25	7	76	71	46	24	21	3	$\det \chi \nabla^\mu \hat{\theta} \nabla_\mu \hat{\theta} + \text{H.c.}$						64			157			58
$i\langle h^{\mu\nu} \chi_{+\mu}\rangle \nabla_\nu \hat{\theta}$	83	78	53	29	26	8	77	72	47	25	22	4	$\det \chi \nabla^\mu \nabla_\mu \hat{\theta} + \text{H.c.}$						65			158			59
$i\langle u^\mu \chi_{+\mu}\rangle \nabla^\nu \nabla_\nu \hat{\theta}$	84	79	54	30	27	9	78	73	48	26	23	5	$i\epsilon^{\mu\nu\lambda\rho} \langle u^\sigma u_\mu u_\nu f_{-\lambda\rho}\rangle \nabla_\sigma \hat{\theta} + \text{H.c.}$	214	207	167	101	96	66	206	199	159	95	90	60
$i\langle \chi_+\rangle \langle \nabla^\mu f_{-\mu}{}^{\nu}\rangle \nabla_\nu \hat{\theta}$	85	80	55				79	74	49				$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu u_\nu u^\sigma f_{-\lambda\sigma}\rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	215	208	168	102	97	67	207	200	160	96	91	61
$i\langle h^{\mu\nu}\rangle \langle \chi_{+\mu}\rangle \nabla_\nu \hat{\theta}$	86	81	56				80	75	50				$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu u^\sigma u_\sigma f_{-\nu\lambda}\rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	216	209	169	103	98	68	208	201	161	97	92	62
$i\langle u^\mu\rangle \langle \chi_{+\mu}\rangle \nabla^\nu \nabla_\nu \hat{\theta}$	87	82	57				81	76	51				$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu u_\nu u^\sigma h_{\lambda\sigma}\rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	217	210	170	104	99	69	209	202	162	98	93	63
$\langle f_+^{\mu\nu} u_\mu \chi_+\rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	88	83	58	31	28	10	82	77	52	27	24	6	$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu u^\sigma u_\sigma\rangle \langle f_{-\nu\lambda}\rangle \nabla_\rho \hat{\theta}$	218	211	171				210	203	163			
$i\langle u^\mu \chi_+^2\rangle \nabla_\mu \hat{\theta}$	89	84	59	32	29	11	83	78	53	28	25	7	$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu u^\sigma\rangle \langle u_\nu h_{\lambda\sigma}\rangle \nabla_\rho \hat{\theta}$	219	212	172	105	100		211	204	164	99	94	
$i\langle u^\mu\rangle \langle \chi_+^2\rangle \nabla_\mu \hat{\theta}$	90	85	60				84	79	54				$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu\rangle \langle u^\sigma u_\nu h_{\lambda\sigma}\rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	220	213	173				212	205	165			
$i\langle u^\mu \chi_+\rangle \langle \chi_+\rangle \nabla_\mu \hat{\theta}$	91	86	61	33	30		85	80	55	29	26		$i\epsilon^{\mu\nu\lambda\rho} \langle u^\sigma u_\sigma\rangle \langle u_\mu f_{-\nu\lambda}\rangle \nabla_\rho \hat{\theta}$	221	214	174	106	101		213	206	166	100	95	
$i\langle u^\mu\rangle \langle \chi_+\rangle \langle \chi_+\rangle \nabla_\mu \hat{\theta}$	92	87					86	81					$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu u^\sigma\rangle \langle u_\sigma f_{-\nu\lambda}\rangle \nabla_\rho \hat{\theta}$	222	215	175	107			214	207	167	101		

(Table continued)

TABLE IX. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu f_{-\mu}{}^\nu \chi_- \rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	93	88	62	34	31	12	87	82	56	30	27	8
$\langle u^\mu h_\mu{}^\nu \chi_- \rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	94	89	63	35	32	13	88	83	57	31	28	9
$\langle u^\mu u_\mu \chi_-^\nu \rangle \nabla_\nu \hat{\theta}$	95	90	64	36	33	14	89	84	58	32	29	10
$\langle u^\mu u^\nu \chi_{-\mu} \rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	96	91	65	37	34	15	90	85	59	33	30	11
$\langle u^\mu \rangle \langle f_{-\mu}{}^\nu \chi_- \rangle \nabla_\nu \hat{\theta}$	97	92	66				91	86	60			
$\langle u^\mu f_{-\mu}{}^\nu \rangle \langle \chi_- \rangle \nabla_\nu \hat{\theta}$	98	93	67	38	35		92	87	61	34	31	
$\langle u^\mu \chi_- \rangle \langle f_{-\mu}{}^\nu \rangle \nabla_\nu \hat{\theta}$	99	94	68				93	88	62			
$\langle u^\mu \rangle \langle h_\mu{}^\nu \chi_- \rangle \nabla_\nu \hat{\theta}$	100	95	69				94	89	63			
$\langle u^\mu h_\mu{}^\nu \rangle \langle \chi_- \rangle \nabla_\nu \hat{\theta}$	101	96	70	39	36		95	90	64	35	32	
$\langle u^\mu \chi_- \rangle \langle h_\mu{}^\nu \rangle \nabla_\nu \hat{\theta}$	102	97	71				96	91	65			
$\langle u^\mu \rangle \langle u_\mu \chi_-^\nu \rangle \nabla_\nu \hat{\theta}$	103	98	72				97	92	66			
$\langle u^\mu u_\mu \rangle \langle \chi_-^\nu \rangle \nabla_\nu \hat{\theta}$	104	99	73	40	37		98	93	67	36	33	
$\langle u^\mu \rangle \langle u^\nu \chi_{-\mu} \rangle \nabla_\nu \hat{\theta}$	105	100	74				99	94	68			
$\langle u^\mu u^\nu \rangle \langle \chi_{-\mu} \rangle \nabla_\nu \hat{\theta}$	106	101	75	41	38		100	95	69	37	34	
$\langle u^\mu \rangle \langle f_{-\mu}{}^\nu \rangle \langle \chi_- \rangle \nabla_\nu \hat{\theta}$	107	102					101	96				
$\langle u^\mu \rangle \langle h_\mu{}^\nu \rangle \langle \chi_- \rangle \nabla_\nu \hat{\theta}$	108	103					102	97				
$\langle u^\mu \rangle \langle u_\mu \rangle \langle \chi_-^\nu \rangle \nabla_\nu \hat{\theta}$	109	104					103	98				
$\langle u^\mu \rangle \langle u^\nu \rangle \langle \chi_{-\mu} \rangle \nabla_\nu \hat{\theta}$	110	105					104	99				
$\langle \chi_-^\mu \rangle \nabla_\mu \nabla^\nu \nabla_\nu \hat{\theta}$	111	106	76	42	39	16	105	100	70	38	35	12
$\langle \chi_- \chi_+^\mu \rangle \nabla_\mu \hat{\theta}$	112	107	77	43	40	17	106	101	71	39	36	13
$\langle \chi_+ \chi_-^\mu \rangle \nabla_\mu \hat{\theta}$	113	108	78	44	41	18	107	102	72	40	37	14
$\langle \chi_- \rangle \langle \chi_+^\mu \rangle \nabla_\mu \hat{\theta}$	114	109	79	45	42	19	108	103	73	41	38	15
$\langle \chi_+ \rangle \langle \chi_-^\mu \rangle \nabla_\mu \hat{\theta}$	115	110	80	46	43		109	104		42	39	
$i \langle u^\mu \chi_-^2 \rangle \nabla_\mu \hat{\theta}$	116	111	81	47	44	20	110	105	74	43	40	16
$i \langle u^\mu \rangle \langle \chi_-^2 \rangle \nabla_\mu \hat{\theta}$	117	112	82				111	106	75			
$i \langle u^\mu \chi_- \rangle \langle \chi_- \rangle \nabla_\mu \hat{\theta}$	118	113	83	48	45		112	107	76	44	41	
$i \langle u^\mu \rangle \langle \chi_- \rangle \langle \chi_- \rangle \nabla_\mu \hat{\theta}$	119	114					113	108				
$\langle u^\mu u_\mu u^\nu u_\nu \rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	120	115	84	49	46	21	114	109	77	45	42	17
$\langle u^\mu u_\mu u^\nu u^\lambda \rangle \nabla_\nu \hat{\theta} \nabla_\lambda \hat{\theta}$	121	116	85	50	47	22	115	110	78	46	43	18
$\langle u^\mu u^\nu u_\mu u_\nu \rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	122	117	86	51	48	23	116	111	79	47	44	19
$\langle u^\mu u^\nu u_\mu u^\lambda \rangle \nabla_\nu \hat{\theta} \nabla_\lambda \hat{\theta}$	123	118	87	52	49	24	117	112	80	48	45	20

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu u^\sigma \rangle \langle u_\nu f_{-\lambda\sigma} \rangle \nabla_\rho \hat{\theta}$	223	216	176	108	102		215	208	168	102	96	
$i\epsilon^{\mu\nu\lambda\rho} \langle u^\sigma \rangle \langle u_\sigma u_\mu f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	224	217	177				216	209	169			
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u^\sigma u_\sigma f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta}$	225	218	178				217	210	170			
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u^\sigma \rangle \langle u_\nu f_{-\lambda\rho} \rangle \nabla_\sigma \hat{\theta}$	226	219					218	211				
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u_\nu u^\sigma \rangle \langle f_{-\lambda\rho} \rangle \nabla_\sigma \hat{\theta}$	227	220					219	212				
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u_\nu u^\sigma \rangle \langle f_{-\lambda\sigma} \rangle \nabla_\rho \hat{\theta}$	228	221					220	213				
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u^\sigma \rangle \langle u_\nu f_{-\lambda\sigma} \rangle \nabla_\rho \hat{\theta}$	229	222					221	214				
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u^\sigma \rangle \langle u_\sigma f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta}$	230	223					222	215				
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u^\sigma u_\sigma \rangle \langle f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta}$	231	224					223	216				
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u_\nu u^\sigma \rangle \langle h_{\lambda\sigma} \rangle \nabla_\rho \hat{\theta}$	232	225					224	217				
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u^\sigma \rangle \langle u_\sigma \rangle \langle f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta}$	233						225					
$i\epsilon^{\mu\nu\lambda\rho} \langle f_{-\mu\nu} \nabla^\sigma f_{-\lambda\sigma} \rangle \nabla_\rho \hat{\theta}$	234	226	179	109	103	70	226	218	171	103	97	64
$\epsilon^{\mu\nu\lambda\rho} \langle f_{+\mu}{}^\sigma f_{-\nu\lambda} u_\rho \rangle \nabla_\sigma \hat{\theta} + \text{H.c.}$	235	227	180	110	104	71	227	219	172	104	98	65
$\epsilon^{\mu\nu\lambda\rho} \langle f_{+\mu\nu} f_{-\lambda}{}^\sigma u_\rho \rangle \nabla_\sigma \hat{\theta} + \text{H.c.}$	236	228	181	111	105	72	228	220	173	105	99	66
$\epsilon^{\mu\nu\lambda\rho} \langle f_{+\mu\nu} f_{-\lambda}{}^\sigma u_\sigma \rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	237	229	182	112	106	73	229	221	174	106	100	67
$\epsilon^{\mu\nu\lambda\rho} \langle f_{+\mu\nu} h_\lambda{}^\sigma u_\rho \rangle \nabla_\sigma \hat{\theta} + \text{H.c.}$	238	230	183	113	107	74	230	222	175	107	101	68
$\epsilon^{\mu\nu\lambda\rho} \langle f_{+\mu\nu} h_\lambda{}^\sigma u_\sigma \rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	239	231	184	114	108	75	231	223	176	108	102	69
$\epsilon^{\mu\nu\lambda\rho} \langle \nabla^\sigma f_{+\mu\nu} u_\lambda u_\rho \rangle \nabla_\sigma \hat{\theta}$	240	232	185	115	109	76	232	224	177	109	103	70
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \chi_+ f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	241	233	186	116	110	77	233	225	178	110	104	71
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle \chi_+ f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta}$	242	234	187				234	226	179			
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \chi_+ \rangle \langle f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta}$	243	235	188				235	227	180			
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu f_{-\nu\lambda} \rangle \langle \chi_+ \rangle \nabla_\rho \hat{\theta}$	244	236	189	117	111		236	228	181	111	105	
$i\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle \chi_+ \rangle \langle f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta}$	245	237					237	229				
$i\epsilon^{\mu\nu\lambda\rho} \langle f_{+\mu\nu} u_\lambda \chi_- \rangle \nabla_\rho \hat{\theta} + \text{H.c.}$	246	238	190	118	112	78	238	230	182	112	106	72
$\epsilon^{\mu\nu\lambda\rho} \langle u^\sigma u_\mu f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta} \nabla_\sigma \hat{\theta} + \text{H.c.}$	247	239	191	119	113		239	231	183	113	107	
$\epsilon^{\mu\nu\lambda\rho} \langle u^\sigma \rangle \langle u_\mu f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta} \nabla_\sigma \hat{\theta}$	248	240	192				240	232	184			
$\epsilon^{\mu\nu\lambda\rho} \langle u_\mu u^\sigma \rangle \langle f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta} \nabla_\sigma \hat{\theta}$	249	241	193				241	233	185			
$\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u^\sigma f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta} \nabla_\sigma \hat{\theta}$	250	242	194				242	234	186			
$\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u_\nu f_{-\lambda}{}^\sigma \rangle \nabla_\rho \hat{\theta} \nabla_\sigma \hat{\theta}$	251	243	195				243	235	187			
$\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u_\nu h_\lambda{}^\sigma \rangle \nabla_\rho \hat{\theta} \nabla_\sigma \hat{\theta}$	252	244	196				244	236	188			
$\epsilon^{\mu\nu\lambda\rho} \langle u_\mu \rangle \langle u^\sigma \rangle \langle f_{-\nu\lambda} \rangle \nabla_\rho \hat{\theta} \nabla_\sigma \hat{\theta}$	253	245					245	237				

(Table continued)

TABLE IX. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu\rangle\langle u_\mu u^\nu u_\nu\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	124	119	88				118	113	81			
$\langle u^\mu u_\mu\rangle\langle u^\nu u_\nu\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	125	120	89	53	50		119	114	82	49	46	
$\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	126	121	90	54			120	115	83	50		
$\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	127	122	91				121	116	84			
$\langle u^\mu u_\mu\rangle\langle u^\nu u^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	128	123	92	55	51		122	117	85	51	47	
$\langle u^\mu\rangle\langle u^\nu u_\nu u^\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\lambda\hat{\theta}$	129	124	93				123	118	86			
$\langle u^\mu u^\nu\rangle\langle u_\mu u^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	130	125	94	56			124	119	87	52		
$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u_\nu\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	131	126					125	120				
$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u_\nu\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	132	127					126	121				

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}\nabla^\sigma\hat{\theta}\nabla_\sigma\hat{\theta}$	254	246	197	120	114	79	246	238	189	114	108	73
$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}\nabla^\sigma\hat{\theta}\nabla_\sigma\hat{\theta}$	255	247	198				247	239	190			
$i\epsilon^{\mu\nu\lambda\rho}\langle {F_{L\mu}}^\sigma D_\sigma F_{L\nu\lambda}\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	256	248	199	121	115	80	248	240	191	115	109	74
$i\epsilon^{\mu\nu\lambda\rho}\langle F_{L\mu\nu}D^\sigma F_{L\lambda\sigma}\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	257	249	200	122	116	81	249	241	192	116	110	75
$i\epsilon^{\mu\nu\lambda\rho}\langle {F_{L\mu}}^\sigma\rangle\langle D_\sigma F_{L\nu\lambda}\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	258	250	201	123	117	82	250	242	193			
$i\epsilon^{\mu\nu\lambda\rho}\langle F_{L\mu\nu}\rangle\langle D^\sigma F_{L\lambda\sigma}\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	259	251	202	124	118	83	251	243	194			
$i\epsilon^{\mu\nu\lambda\rho}\langle D^\sigma F_{L\mu\sigma}\rangle\langle F_{R\nu\lambda}\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	260	252	203									
$i\epsilon^{\mu\nu\lambda\rho}\langle D^\sigma F_{L\mu\nu}\rangle\langle F_{R\lambda\sigma}\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	261	253	204									

TABLE X. The p^6 order results in U groups and SU groups, with $t=0$, $\hat{\theta}\neq 0$ and the odd parity. The numbers are the sequence numbers in each case. The details can be found in Sec. V C.

Operators	U(3)			Operators	U(3)			Operators	U(3)			Operators	U(3)		
	n	3	2		n	3	2		n	3	2		n	3	2
$i\langle u^\mu u_\mu u^\nu {f_{-\nu}}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	262	254	205	$\langle\nabla^\mu {f_{+\mu}}^\nu u_\nu u^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	314	303	241	$\langle u^\mu\rangle\langle u^\nu {f_{-\mu}}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	366	354	283	$i\langle F_L^{\mu\nu}D^\lambda F_{L\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	418	406	329
$i\langle u^\mu u^\nu u_\mu {f_{-\nu}}^\lambda\rangle\nabla_\lambda\hat{\theta}$	263	255	206	$i\langle u^\mu\chi_+ {f_{-\mu}}^\nu\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	315	304	242	$\langle u^\mu u^\nu\rangle\langle {f_{-\mu}}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	367	355	284	$i\langle D^\mu\chi\chi^\dagger\rangle\nabla_\mu\hat{\theta}+\text{H.c.}$	419	407	330
$i\langle u^\mu u^\nu u^\lambda f_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	264	256	207	$i\langle u^\mu\chi_+ {h_\mu}^\nu\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	316	305	243	$\langle u^\mu\rangle\langle u^\nu {f_{-\nu}}^\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\lambda\hat{\theta}$	368	356	285	$i\nabla^\mu\nabla^\nu\hat{\theta}\nabla_\nu\nabla^\lambda\hat{\theta}\nabla_\lambda\nabla_\mu\hat{\theta}$	420	408	331
$i\langle u^\mu u_\mu u^\nu {h_\nu}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	265	257	208	$i\langle u^\mu u_\mu\chi_+^\nu\rangle\nabla_\nu\hat{\theta}$	317	306	244	$\langle u^\mu\rangle\langle u_\mu h^{\nu\lambda}\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	369	357	286	$i\nabla^\mu\nabla_\mu\hat{\theta}\nabla^\nu\nabla^\lambda\hat{\theta}\nabla_\lambda\nabla_\nu\hat{\theta}$	421	409	332
$i\langle u^\mu u^\nu u_\mu {h_\nu}^\lambda\rangle\nabla_\lambda\hat{\theta}$	266	258	209	$i\langle u^\mu u^\nu\chi_{+\mu}\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	318	307	245	$\langle u^\mu u_\mu\rangle\langle h^{\nu\lambda}\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	370	358	287	$\langle D^\mu F_L^{\nu\lambda}\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\nabla_\mu\hat{\theta}+\text{H.c.}$	422	410	333
$i\langle u^\mu u^\nu u^\lambda h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	267	259	210	$i\langle u^\mu\rangle\langle\chi_+ {f_{-\mu}}^\nu\rangle\nabla_\nu\hat{\theta}$	319	308	246	$\langle u^\mu\rangle\langle u^\nu h_{\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	371	359	288	$i\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\nu\nabla_\lambda\hat{\theta}$	423	411	334
$i\langle u^\mu u^\nu u^\lambda h_{\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	268	260	211	$i\langle u^\mu\chi_+\rangle\langle {f_{-\mu}}^\nu\rangle\nabla_\nu\hat{\theta}$	320	309	247	$\langle u^\mu u^\nu\rangle\langle h_{\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	372	360	289	$i\langle F_L^{\mu\nu}\rangle\langle D^\lambda F_{L\mu\lambda}\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	424	412	335
$i\langle u^\mu u^\nu u^\lambda\rangle\langle h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	269	261	212	$i\langle u^\mu {f_{-\mu}}^\nu\rangle\langle\chi_+\rangle\nabla_\nu\hat{\theta}$	321	310	248	$\langle u^\mu\rangle\langle u^\nu {h_\mu}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	373	361	290	$i\langle F_L^{\mu\nu}\rangle\langle D^\lambda F_{L\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	425	413	336
$i\langle u^\mu u_\mu u^\nu\rangle\langle {h_\nu}^\lambda\rangle\nabla_\lambda\hat{\theta}$	270	262	213	$i\langle u^\mu\rangle\langle\chi_+ {h_\mu}^\nu\rangle\nabla_\nu\hat{\theta}$	322	311	249	$\langle u^\mu u^\nu\rangle\langle {h_\mu}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	374	362	291	$i\langle D^\mu F_L^{\nu\lambda}\rangle\langle F_{R\nu\lambda}\rangle\nabla_\mu\hat{\theta}+\text{H.c.}$	426	414	337
$i\langle u^\mu u_\mu u^\nu\rangle\langle {f_{-\nu}}^\lambda\rangle\nabla_\lambda\hat{\theta}$	271	263	214	$i\langle u^\mu\chi_+\rangle\langle {h_\mu}^\nu\rangle\nabla_\nu\hat{\theta}$	323	312	250	$\langle u^\mu\rangle\langle u^\nu {h_\nu}^\lambda\rangle\nabla_\mu\hat{\theta}\nabla_\lambda\hat{\theta}$	375	363	292	$i\langle D^\mu {F_{L\mu}}^\nu\rangle\langle {F_{R\nu}}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	427	415	338
$i\langle u^\mu u^\nu\rangle\langle u^\lambda h_{\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	272	264	215	$i\langle u^\mu h_\mu{}^\nu\rangle\langle\chi_+\rangle\nabla_\nu\hat{\theta}$	324	313	251	$\langle u^\mu\rangle\langle u^\nu\rangle\langle {f_{-\mu}}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	376	364		$i\epsilon^{\mu\nu\lambda\rho}\langle u^\sigma f_{-\mu\nu}f_{-\lambda\rho}\rangle\nabla_\sigma\hat{\theta}$	428	416	339

(Table continued)

TABLE X. (*Continued*)

Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2
$i\langle u^\mu u^\nu\rangle\langle u^\lambda h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	273	265	216	$i\langle u^\mu\rangle\langle u_\mu\chi_+^\nu\rangle\nabla_\nu\hat{\theta}$	325	314	252	$\langle u^\mu\rangle\langle u_\mu\rangle\langle h^{\nu\lambda}\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	377	365		$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu f_{-\nu\lambda}h_\rho{}^\sigma\rangle\nabla_\sigma\hat{\theta}+\text{H.c.}$	429	417	340
$i\langle u^\mu u^\nu\rangle\langle u_\mu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	274	266	217	$i\langle u^\mu u_\mu\rangle\langle\chi_+^\nu\rangle\nabla_\nu\hat{\theta}$	326	315	253	$\langle u^\mu\rangle\langle u^\nu\rangle\langle h_{\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	378	366		$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu h_\nu{}^\sigma\rangle\langle h_{\lambda\sigma}\rangle\nabla_\rho\hat{\theta}$	430	418	341
$i\langle u^\mu u_\mu\rangle\langle u^\nu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	275	267	218	$i\langle u^\mu\rangle\langle u^\nu\chi_{+\mu}\rangle\nabla_\nu\hat{\theta}$	327	316	254	$\langle u^\mu\rangle\langle u^\nu\rangle\langle h_\mu{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	379	367		$i\epsilon^{\mu\nu\lambda\rho}\langle u^\sigma f_{-\mu\nu}\rangle\langle h_{\lambda\sigma}\rangle\nabla_\rho\hat{\theta}$	431	419	342
$i\langle u^\mu\rangle\langle u^\nu u^\lambda h_{\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	276	268	219	$i\langle u^\mu u^\nu\rangle\langle\chi_{+\mu}\rangle\nabla_\nu\hat{\theta}$	328	317	255	$\langle h^{\mu\nu}\rangle\nabla_\mu\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	380	368	293	$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu f_{-\nu}{}^\sigma\rangle\langle h_{\lambda\sigma}\rangle\nabla_\rho\hat{\theta}$	432	420	343
$i\langle u^\mu\rangle\langle u^\nu u^\lambda h_{\mu\lambda}\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	277	269	220	$i\langle u^\mu\rangle\langle\chi_+\rangle\langle f_{-\mu}{}^\nu\rangle\nabla_\nu\hat{\theta}$	329	318		$\langle\chi_+h^{\mu\nu}\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	381	369	294	$i\epsilon^{\mu\nu\lambda\rho}\langle u^\sigma h_{\mu\sigma}\rangle\langle f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}$	433	421	344
$i\langle u^\mu\rangle\langle u^\nu u_\nu h_\mu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	278	270	221	$i\langle u^\mu\rangle\langle\chi_+\rangle\langle h_\mu{}^\nu\rangle\nabla_\nu\hat{\theta}$	330	319		$\langle u^\mu\chi_{+\mu}\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	382	370	295	$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu h_\nu{}^\sigma\rangle\langle f_{-\lambda\sigma}\rangle\nabla_\rho\hat{\theta}$	434	422	345
$i\langle u^\mu\rangle\langle u^\nu u_\mu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	279	271	222	$i\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_+^\nu\rangle\nabla_\nu\hat{\theta}$	331	320		$\langle u^\mu\chi_+^\nu\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	383	371	296	$i\epsilon^{\mu\nu\lambda\rho}\langle u^\sigma f_{-\mu\nu}\rangle\langle f_{-\lambda\sigma}\rangle\nabla_\rho\hat{\theta}$	435	423	346
$i\langle u^\mu u^\nu\rangle\langle u^\lambda f_{-\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	280	272	223	$i\langle u^\mu\rangle\langle u^\nu\rangle\langle\chi_{+\mu}\rangle\nabla_\nu\hat{\theta}$	332	321		$\langle\chi_+\rangle\langle h^{\mu\nu}\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	384	372	297	$i\epsilon^{\mu\nu\lambda\rho}\langle u^\sigma\rangle\langle u_\mu\nabla_\nu h_{\lambda\sigma}\rangle\nabla_\rho\hat{\theta}$	436	424	347
$i\langle u^\mu u^\nu\rangle\langle u_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	281	273	224	$i\langle\chi_+^\mu\rangle\nabla_\mu\nabla^\nu\nabla_\nu\hat{\theta}$	333	322	256	$\langle u^\mu\rangle\langle\chi_{+\mu}\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	385	373	298	$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u^\sigma\nabla_\nu h_{\lambda\sigma}\rangle\nabla_\rho\hat{\theta}$	437	425	348
$i\langle u^\mu u_\mu\rangle\langle u^\nu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	282	274	225	$i\langle\chi_+\chi_+^\mu\rangle\nabla_\mu\hat{\theta}$	334	323	257	$\langle u^\mu\rangle\langle\chi_+^\nu\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	386	374	299	$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle u_\nu\nabla^\sigma h_{\lambda\sigma}\rangle\nabla_\rho\hat{\theta}$	438	426	349
$i\langle u^\mu\rangle\langle u^\nu u^\lambda f_{-\mu\lambda}\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	283	275	226	$i\langle\chi_+\rangle\langle\chi_+^\mu\rangle\nabla_\mu\hat{\theta}$	335	324	258	$i\langle u^\mu u_\mu\chi_-\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	387	375	300	$i\epsilon^{\mu\nu\lambda\rho}\langle u^\sigma\rangle\langle f_{-\mu\nu}\rangle\langle f_{-\lambda\rho}\rangle\nabla_\sigma\hat{\theta}$	439	427	
$i\langle u^\mu\rangle\langle u^\nu u_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	284	276	227	$\langle u^\mu u_\mu u^\nu\chi_-\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	336	325	259	$i\langle u^\mu u^\nu\chi_-\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	388	376	301	$i\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle h_\nu{}^\sigma\rangle\langle f_{-\lambda\rho}\rangle\nabla_\sigma\hat{\theta}$	440	428	
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	285	277		$\langle u^\mu u^\nu u_\mu\chi_-\rangle\nabla_\nu\hat{\theta}$	337	326	260	$i\langle u^\mu\rangle\langle u_\mu\chi_-\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	389	377	302	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma u_\nu u_\lambda u_\rho\rangle\nabla_\sigma\hat{\theta}$	441	429	350
$i\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\langle f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	286	278		$\langle u^\mu\rangle\langle u_\mu u^\nu\chi_-\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	338	327	261	$i\langle u^\mu u_\mu\rangle\langle\chi_-\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	390	378	303	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u^\sigma u_\lambda u_\rho\rangle\nabla_\sigma\hat{\theta}+\text{H.c.}$	442	430	351
$i\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\langle f_{-\mu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	287	279		$\langle u^\mu u_\mu\rangle\langle u^\nu\chi_-\rangle\nabla_\nu\hat{\theta}$	339	328	262	$i\langle u^\mu\rangle\langle u^\nu\chi_-\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	391	379	304	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_\lambda u^\sigma u_\sigma\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	443	431	352
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	288	280		$\langle u^\mu u_\mu u^\nu\rangle\langle\chi_-\rangle\nabla_\nu\hat{\theta}$	340	329	263	$i\langle u^\mu u^\nu\rangle\langle\chi_-\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	392	380	305	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma\rangle\langle u_\nu u_\lambda u_\rho\rangle\nabla_\sigma\hat{\theta}$	444	432	353
$i\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\langle f_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	289	281		$\langle u^\mu u^\nu\rangle\langle u_\mu\chi_-\rangle\nabla_\nu\hat{\theta}$	341	330	264	$i\langle u^\mu\rangle\langle u_\mu\rangle\langle\chi_-\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	393	381		$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma u_\nu u_\lambda\rangle\langle u_\rho\rangle\nabla_\sigma\hat{\theta}$	445	433	
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda f_{-\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	290	282		$\langle u^\mu\rangle\langle u^\nu u_\nu\chi_-\rangle\nabla_\mu\hat{\theta}$	342	331	265	$i\langle u^\mu\rangle\langle u^\nu\rangle\langle\chi_-\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}$	394	382		$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u^\sigma u_\lambda\rangle\langle u_\rho\rangle\nabla_\sigma\hat{\theta}+\text{H.c.}$	446	434	
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	291	283		$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\chi_-\rangle\nabla_\nu\hat{\theta}$	343	332		$i\langle\chi_-^\mu\rangle\nabla_\mu\hat{\theta}\nabla^\nu\nabla_\nu\hat{\theta}$	395	383	306	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_\lambda u^\sigma\rangle\langle u_\sigma\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	447	435	
$i\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\langle h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	292	284		$\langle u^\mu\rangle\langle u_\mu u^\nu\rangle\langle\chi_-\rangle\nabla_\nu\hat{\theta}$	344	333		$i\langle\chi_-^\mu\rangle\nabla^\nu\hat{\theta}\nabla_\mu\nabla_\nu\hat{\theta}$	396	384	307	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma f_{+\nu\lambda}u_\rho\rangle\nabla_\sigma\hat{\theta}+\text{H.c.}$	448	436	354
$i\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\langle h_\mu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	293	285		$\langle u^\mu\rangle\langle u^\nu u_\nu\rangle\langle\chi_-\rangle\nabla_\mu\hat{\theta}$	345	334		$i\langle\chi_-\rangle\nabla^\mu\nabla_\mu\hat{\theta}\nabla^\nu\nabla_\nu\hat{\theta}$	397	385	308	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma\rangle\langle f_{+\nu\lambda}u_\rho\rangle\nabla_\sigma\hat{\theta}$	449	437	355
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	294	286		$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu\chi_-\rangle\nabla_\nu\hat{\theta}$	346	335		$i\langle\chi_+\chi_-\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}$	398	386	309	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda}{}^\sigma u_\rho\rangle\nabla_\sigma\hat{\theta}$	450	438	356
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	295	287		$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle\chi_-\rangle\nabla_\nu\hat{\theta}$	347			$i\langle\chi_+\rangle\langle\chi_-\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}$	399	387	310	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda}{}^\sigma u_\sigma\rangle\nabla_\rho\hat{\theta}$	451	439	357
$i\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\langle h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	296	288		$\langle\nabla^\mu f_{-\mu}{}^\nu\chi_-\rangle\nabla_\nu\hat{\theta}$	348	336	266	$i\langle u^\mu f_{-\mu}{}^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	400	388	311	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}\rangle\langle f_{+\lambda}{}^\sigma\rangle\langle u_\rho\rangle\nabla_\sigma\hat{\theta}$	452	440	
$i\langle u^\mu\rangle\langle u^\nu u^\lambda\rangle\langle h_{\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	297	289		$\langle h^{\mu\nu}\chi_{-\mu}\rangle\nabla_\nu\hat{\theta}$	349	337	267	$i\langle u^\mu h_\mu{}^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	401	389	312	$\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u_\lambda\chi_+\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	453	441	358
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	298			$\langle u^\mu\chi_{-\mu}\rangle\nabla^\nu\nabla_\nu\hat{\theta}$	350	338	268	$i\langle u^\mu h^{\nu\lambda}\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	402	390	313	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu f_{-\nu\lambda}\chi_-\rangle\nabla_\rho\hat{\theta}+\text{H.c.}$	454	442	359
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu\rangle\langle h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	299			$\langle\nabla^\mu f_{-\mu}{}^\nu\rangle\langle\chi_-\rangle\nabla_\nu\hat{\theta}$	351	339	269	$i\langle u^\mu u_\mu\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	403	391	314	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle f_{-\nu\lambda}\chi_-\rangle\nabla_\rho\hat{\theta}$	455	443	360
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u^\lambda\rangle\langle h_{\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	300			$\langle h^{\mu\nu}\rangle\langle\chi_{-\mu}\rangle\nabla_\nu\hat{\theta}$	352	340	270	$i\langle u^\mu u^\nu\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	404	392	315	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu f_{-\nu\lambda}\rangle\langle\chi_-\rangle\nabla_\rho\hat{\theta}$	456	444	361
$i\langle f_-^{\mu\nu}\nabla_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	301	290	228	$\langle u^\mu\rangle\langle\chi_{-\mu}\rangle\nabla^\nu\nabla_\nu\hat{\theta}$	353	341	271	$i\langle u^\mu\rangle\langle f_{-\mu}{}^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	405	393	316	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\chi_-\rangle\langle f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}$	457	445	362
$i\langle f_-^{\mu\nu}\nabla^\lambda f_{-\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	302	291	229	$i\langle f_+^{\mu\nu}u_\mu\chi_-\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	354	342	272	$i\langle u^\mu\rangle\langle h_\mu{}^\nu\rangle\nabla_\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	406	394	317	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle f_{-\nu\lambda}\rangle\langle\chi_-\rangle\nabla_\rho\hat{\theta}$	458	446	
$i\langle h^{\mu\nu}\nabla_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	303	292	230	$\langle u^\mu\chi_+\chi_-\rangle\nabla_\mu\hat{\theta}+\text{H.c.}$	355	343	273	$i\langle u^\mu\rangle\langle h^{\nu\lambda}\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	407	395	318	$\epsilon^{\mu\nu\lambda\rho}\langle h_\mu{}^\sigma f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}$	459	447	363

(Table continued)

TABLE X. (*Continued*)

Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2
$i\langle h^{\mu\nu}h_{\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	304	293	231	$\langle u^\mu\rangle\langle\chi_+\chi_-\rangle\nabla_\mu\hat{\theta}$	356	344	274	$i\langle u^\mu\rangle\langle u_\mu\rangle\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	408	396	319	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\nabla^\sigma f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}$	460	448	364
$i\langle u^\mu\nabla^\nu f_{-\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	305	294	232	$\langle u^\mu\chi_+\rangle\langle\chi_-\rangle\nabla_\mu\hat{\theta}$	357	345	275	$i\langle u^\mu\rangle\langle u^\nu\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	409	397	320	$\epsilon^{\mu\nu\lambda\rho}\langle h_\mu{}^\sigma\rangle\langle f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}$	461	449	365
$i\langle h^{\mu\nu}\rangle\langle\nabla_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	306	295	233	$\langle u^\mu\chi_-\rangle\langle\chi_+\rangle\nabla_\mu\hat{\theta}$	358	346	276	$i\langle\chi_+^\mu\rangle\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	410	398	321	$\epsilon^{\mu\nu\lambda\rho}\langle u_\mu\rangle\langle\nabla^\sigma f_{-\nu\lambda}\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}$	462	450	366
$i\langle h^{\mu\nu}\rangle\langle h_{\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	307	296	234	$\langle u^\mu\rangle\langle\chi_+\rangle\langle\chi_-\rangle\nabla_\mu\hat{\theta}$	359	347		$i\langle\chi_+\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}\nabla^\nu\nabla_\nu\hat{\theta}$	411	399	322	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu}{}^\sigma u_\nu u_\lambda\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}$	463	451	367
$i\langle u^\mu\rangle\langle\nabla^\nu f_{-\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	308	297	235	$i\langle\chi_-\rangle\langle\chi_-^\mu\rangle\nabla_\mu\hat{\theta}$	360	348	277	$\langle u^\mu\chi_-\rangle\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	412	400	323	$i\epsilon^{\mu\nu\lambda\rho}\langle f_{+\mu\nu}u^\sigma u_\lambda\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}+\text{H.c.}$	464	452	368
$\langle f_+^{\mu\nu}u_\mu f_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	309	298	236	$\langle u^\mu u^\nu f_{-\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}+\text{H.c.}$	361	349	278	$\langle u^\mu\rangle\langle\chi_-\rangle\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	413	401	324	$\epsilon^{\mu\nu\lambda\rho}\langle F_{L\mu\nu}F_{L\lambda}{}^\sigma\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}+\text{H.c.}$	465	453	369
$\langle f_+^{\mu\nu}u^\lambda f_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	310	299	237	$\langle u^\mu u_\mu h^{\nu\lambda}\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	362	350	279	$\langle h^{\mu\nu}\rangle\nabla_\mu\hat{\theta}\nabla_\nu\hat{\theta}\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	414	402	325	$\epsilon^{\mu\nu\lambda\rho}\langle F_{L\mu\nu}\rangle\langle F_{L\lambda}{}^\sigma\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}+\text{H.c.}$	466	454	370
$\langle f_+^{\mu\nu}u^\lambda f_{-\mu\lambda}\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	311	300	238	$\langle u^\mu u^\nu h_{\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	363	351	280	$\langle u^\mu\rangle\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}\nabla^\lambda\nabla_\lambda\hat{\theta}$	415	403	326	$\epsilon^{\mu\nu\lambda\rho}\langle F_{L\mu}{}^\sigma\rangle\langle F_{R\nu\lambda}\rangle\nabla_\rho\hat{\theta}\nabla_\sigma\hat{\theta}+\text{H.c.}$	467	455	371
$\langle f_+^{\mu\nu}u_\mu h_\nu{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	312	301	239	$\langle u^\mu u^\nu h_\mu{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}+\text{H.c.}$	364	352	281	$i\langle\chi_-\rangle\nabla^\mu\hat{\theta}\nabla_\mu\hat{\theta}\nabla^\nu\hat{\theta}\nabla_\nu\hat{\theta}$	416	404	327							
$\langle f_+^{\mu\nu}u^\lambda h_{\mu\lambda}\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	313	302	240	$\langle u^\mu\rangle\langle u^\nu f_{-\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	365	353	282	$i\langle F_L^{\mu\nu}D^\lambda F_{L\mu\lambda}\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	417	405	328							

TABLE XI. The p^6 order results in U groups and SU groups, with $t\neq 0$, $\hat{\theta}=0$ and the even parity. The numbers are the sequence numbers in each case. The details can be found in Sec. V C.

Operators	U(3) n	3	2	SU(3)_I n	3	2	SU(3)_II n	3	2	SU(3)_III n	3	2	Operators	U(3) n	3	2	SU(3)_I n	3	2	SU(3)_II n	3	2	SU(3)_III n	3	2
$i\langle u^\mu u^\nu u_\mu u^\lambda t_{+\nu\lambda}\rangle+\text{H.c.}$	2	2	2	2	2	2	2	2	2	2	2	2	$\langle u^\mu\rangle\langle u^\nu t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	82	78					70	66						
$i\langle u^\mu u^\nu u_\nu u^\lambda t_{+\mu\lambda}\rangle$	3	3	3	3	3		3	3	3	3	3		$\langle u^\mu u^\nu\rangle\langle t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	83	79		57			71	67		50		
$i\langle u^\mu u^\nu u^\lambda u_\mu t_{+\nu\lambda}\rangle$	4	4	4	4	4		4	4	4	4	4		$\langle u^\mu\rangle\langle u^\nu t_{+\nu}{}^\lambda\rangle\langle t_{+\mu\lambda}\rangle$	84	80					72	68				
$i\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda t_{+\nu\lambda}\rangle+\text{H.c.}$	5	5	5				5	5	5				$\langle u^\mu\rangle\langle u_\mu\rangle\langle t_+^{\nu\lambda}\rangle\langle t_{+\nu\lambda}\rangle$	85						73					
$i\langle u^\mu u_\mu\rangle\langle u^\nu u^\lambda t_{+\nu\lambda}\rangle$	6	6		5	5		6	6		5	5		$\langle u^\mu\rangle\langle u^\nu\rangle\langle t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	86						74					
$i\langle u^\mu u_\mu u^\nu u^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	7	7		6	6		7	7		6	6		$\langle\nabla^\mu t_{+\mu}{}^\nu\nabla^\lambda t_{+\nu\lambda}\rangle$	87	81	54	58	52	33	75	69	44	51	45	28
$i\langle u^\mu u^\nu\rangle\langle u_\mu u^\lambda t_{+\nu\lambda}\rangle+\text{H.c.}$	8	8		7			8	8		7			$\langle\nabla^\mu t_{+\mu}{}^\nu\rangle\langle\nabla^\lambda t_{+\nu\lambda}\rangle$	88	82	55	59	53	34	76	70	45	52	46	29
$i\langle u^\mu u^\nu u^\lambda\rangle\langle u_\mu t_{+\nu\lambda}\rangle$	9	9		8			9	9		8			$\langle\nabla^\mu t_+^{\nu\lambda}\rangle\langle\nabla_\mu t_{+\nu\lambda}\rangle$	89	83	56	60	54	35	77	71	46	53	47	30
$i\langle u^\mu\rangle\langle u^\nu u_\nu u^\lambda t_{+\mu\lambda}\rangle+\text{H.c.}$	10	10					10	10					$i\langle f_+^{\mu\nu}t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle$	90	84	57	61	55	36	78	72	47	54	48	31
$i\langle u^\mu\rangle\langle u^\nu u_\mu u^\lambda t_{+\nu\lambda}\rangle$	11	11					11	11					$\langle\chi_+ t_+^{\mu\nu}t_{+\mu\nu}\rangle$	91	85	58	62	56	37	79	73	48	55	49	32
$i\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u^\lambda t_{+\nu\lambda}\rangle$	12	12					12	12					$\langle\chi_+\rangle\langle t_+^{\mu\nu}t_{+\mu\nu}\rangle$	92	86	59	63	57	38	80	74	49	56	50	33
$i\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	13						13						$\langle\chi_+ t_+^{\mu\nu}\rangle\langle t_{+\mu\nu}\rangle$	93	87	60	64	58	39	81	75	50	57	51	34

(Table continued)

TABLE XI. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$i\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	14						14					
$i\langle f_-^{\mu\nu} f_{-\mu}{}^\lambda t_{+\nu\lambda}\rangle$	15	13	6	9	7	3	15	13	6	9	7	3
$i\langle h^{\mu\nu} f_{-\mu}{}^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	16	14	7	10	8	4	16	14	7	10	8	4
$i\langle h^{\mu\nu} h_\mu{}^\lambda t_{+\nu\lambda}\rangle$	17	15	8	11	9	5	17	15	8	11	9	5
$i\langle u^\mu \nabla_\mu f_-^{\nu\lambda} t_{+\nu\lambda}\rangle + \text{H.c.}$	18	16	9	12	10	6	18	16	9	12	10	6
$i\langle u^\mu \nabla^\nu f_{-\nu}{}^\lambda t_{+\mu\lambda}\rangle + \text{H.c.}$	19	17	10	13	11	7	19	17	10	13	11	7
$\langle f_+^{\mu\nu} u_\mu u^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	20	18	11	14	12	8	20	18	11	14	12	8
$\langle f_+^{\mu\nu} u^\lambda u_\mu t_{+\nu\lambda}\rangle + \text{H.c.}$	21	19	12	15	13	9	21	19	12	15	13	9
$\langle f_+^{\mu\nu} u^\lambda u_\lambda t_{+\mu\nu}\rangle + \text{H.c.}$	22	20	13	16	14	10	22	20	13	16	14	10
$\langle f_+^{\mu\nu} u_\mu t_{+\nu}{}^\lambda u_\lambda\rangle + \text{H.c.}$	23	21	14	17	15	11	23	21	14	17	15	11
$\langle f_+^{\mu\nu} u^\lambda t_{+\mu\nu} u_\lambda\rangle$	24	22	15	18	16	12	24	22	15	18	16	12
$\langle f_+^{\mu\nu}\rangle\langle u_\mu u^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	25	23	16	19	17	13						
$\langle f_+^{\mu\nu} u_\mu\rangle\langle u^\lambda t_{+\nu\lambda}\rangle$	26	24	17	20	18		25	23	16	19	17	
$\langle f_+^{\mu\nu} u_\mu u^\lambda\rangle\langle t_{+\nu\lambda}\rangle + \text{H.c.}$	27	25	18	21	19		26	24	17	20	18	
$\langle f_+^{\mu\nu} u^\lambda t_{+\mu\lambda}\rangle\langle u_\nu\rangle + \text{H.c.}$	28	26	19				27	25				
$\langle f_+^{\mu\nu} t_{+\mu}{}^\lambda\rangle\langle u_\nu u_\lambda\rangle$	29	27	20	22	20		28	26		21	19	
$\langle f_+^{\mu\nu} u_\mu t_{+\nu}{}^\lambda\rangle\langle u_\lambda\rangle + \text{H.c.}$	30	28	21				29	27	18			
$\langle f_+^{\mu\nu} u^\lambda\rangle\langle u_\mu t_{+\nu\lambda}\rangle$	31	29	22	23	21		30	28		22		
$\langle f_+^{\mu\nu}\rangle\langle u^\lambda u_\lambda t_{+\mu\nu}\rangle$	32	30	23	24	22	14						
$\langle f_+^{\mu\nu} u^\lambda\rangle\langle u_\lambda t_{+\mu\nu}\rangle$	33	31	24	25	23		31	29	19	23	20	
$\langle f_+^{\mu\nu} u^\lambda u_\lambda\rangle\langle t_{+\mu\nu}\rangle$	34	32	25	26	24		32	30	20	24	21	
$\langle f_+^{\mu\nu} u^\lambda t_{+\mu\nu}\rangle\langle u_\lambda\rangle + \text{H.c.}$	35	33	26				33	31				
$\langle f_+^{\mu\nu} t_{+\mu\nu}\rangle\langle u^\lambda u_\lambda\rangle$	36	34	27	27	25		34	32		25		
$\langle f_+^{\mu\nu}\rangle\langle u_\mu\rangle\langle u^\lambda t_{+\nu\lambda}\rangle$	37	35										
$\langle f_+^{\mu\nu}\rangle\langle u_\mu u^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	38	36		28								
$\langle f_+^{\mu\nu} u_\mu\rangle\langle u^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	39	37					35	33				
$\langle f_+^{\mu\nu} t_{+\mu}{}^\lambda\rangle\langle u_\nu\rangle\langle u_\lambda\rangle$	40	38					36	34				
$\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\mu t_{+\nu\lambda}\rangle$	41	39										
$\langle f_+^{\mu\nu} u^\lambda\rangle\langle u_\mu\rangle\langle t_{+\nu\lambda}\rangle$	42	40					37					
$\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\lambda t_{+\mu\nu}\rangle$	43	41										
$\langle f_+^{\mu\nu}\rangle\langle u^\lambda u_\lambda\rangle\langle t_{+\mu\nu}\rangle$	44	42		29								

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle\chi_+\rangle\langle t_+^{\mu\nu}\rangle\langle t_{+\mu\nu}\rangle$	94	88		65	59		82	76		58	52	
$i\langle t_+^{\mu\nu} t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle$	95	89	61	66	60	40	83	77	51	59	53	35
$\langle u^\mu u_\mu f_-^{\nu\lambda} t_{-\nu\lambda}\rangle + \text{H.c.}$	96	90	62	67	61		84	78	52	60	54	
$\langle u^\mu u^\nu f_{-\mu}{}^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	97	91	63	68	62	41	85	79	53	61	55	36
$\langle u^\mu u^\nu f_{-\nu}{}^\lambda t_{-\mu\lambda}\rangle + \text{H.c.}$	98	92	64	69	63		86	80	54	62	56	
$\langle u^\mu f_{-\mu}{}^\nu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	99	93	65	70	64		87	81	55	63	57	
$\langle u^\mu u^\nu h_\mu{}^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	100	94	66	71	65	42	88	82	56	64	58	37
$\langle u^\mu u^\nu h_\nu{}^\lambda t_{-\mu\lambda}\rangle + \text{H.c.}$	101	95	67	72	66		89	83	57	65	59	
$\langle u^\mu h_\mu{}^\nu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	102	96	68	73	67		90	84	58	66	60	
$\langle u^\mu u^\nu t_{-\nu}{}^\lambda\rangle\langle h_{\mu\lambda}\rangle + \text{H.c.}$	103	97	69				91	85	59			
$\langle u^\mu u^\nu t_{-\nu}{}^\lambda\rangle\langle f_{-\mu\lambda}\rangle + \text{H.c.}$	104	98	70				92	86	60			
$\langle u^\mu u^\nu u^\lambda\rangle\langle \nabla_\mu t_{-\nu\lambda}\rangle$	105	99		74	68		93	87		67	61	
$\langle u^\mu\rangle\langle u^\nu u^\lambda \nabla_\lambda t_{-\mu\nu}\rangle + \text{H.c.}$	106	100					94	88				
$\langle u^\mu\rangle\langle u^\nu u^\lambda \nabla_\mu t_{-\nu\lambda}\rangle$	107	101					95	89				
$\langle u^\mu\rangle\langle u^\nu u_\mu \nabla^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	108	102					96	90				
$\langle u^\mu\rangle\langle u^\nu t_{-\nu}{}^\lambda h_{\mu\lambda}\rangle + \text{H.c.}$	109	103					97	91				
$\langle u^\mu\rangle\langle u^\nu t_{-\mu}{}^\lambda h_{\nu\lambda}\rangle + \text{H.c.}$	110	104					98	92				
$\langle u^\mu u^\nu h_\nu{}^\lambda\rangle\langle t_{-\mu\lambda}\rangle + \text{H.c.}$	111	105		75	69		99	93		68	62	
$i\langle f_+^{\mu\nu} f_{-\mu}{}^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	112	106	71	76	70	43	100	94	61	69	63	38
$i\langle f_+^{\mu\nu} h_\mu{}^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	113	107	72	77	71	44	101	95	62	70	64	39
$i\langle \nabla^\mu f_{+\mu}{}^\nu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	114	108	73	78	72	45	102	96	63	71	65	40
$i\langle \nabla^\mu f_+^{\nu\lambda} u_\mu t_{-\nu\lambda}\rangle + \text{H.c.}$	115	109	74	79	73	46	103	97	64	72	66	41
$i\langle f_+^{\mu\nu}\rangle\langle f_{-\mu}{}^\lambda t_{-\nu\lambda}\rangle$	116	110	75	80	74	47						
$i\langle f_+^{\mu\nu} f_{-\mu}{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	117	111	76	81	75		104	98	65	73	67	
$i\langle f_+^{\mu\nu} t_{-\mu}{}^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	118	112	77				105	99				
$i\langle f_+^{\mu\nu}\rangle\langle h_\mu{}^\lambda t_{-\nu\lambda}\rangle$	119	113	78	82	76	48						
$i\langle f_+^{\mu\nu} h_\mu{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	120	114	79	83	77		106	100	66	74	68	
$i\langle f_+^{\mu\nu} t_{-\mu}{}^\lambda\rangle\langle h_{\nu\lambda}\rangle$	121	115	80				107	101				
$i\langle \nabla^\mu f_{+\mu}{}^\nu\rangle\langle u^\lambda t_{-\nu\lambda}\rangle$	122	116	81	84	78	49						
$i\langle \nabla^\mu f_{+\mu}{}^\nu u^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	123	117	82	85	79		108	102	67	75	69	
$i\langle \nabla^\mu f_{+\mu}{}^\nu t_{-\nu}{}^\lambda\rangle\langle u_\lambda\rangle$	124	118	83				109	103				

(Table continued)

TABLE XI. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$			Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2		n	3	2	n	3	2	n	3	2	n	3	2
$\langle f_+^{\mu\nu}u^\lambda\rangle\langle u_\lambda\rangle\langle t_{+\mu\nu}\rangle$	45	43					38	35					$i\langle\nabla^\mu f_+^{\nu\lambda}\rangle\langle u_\mu t_{-\nu\lambda}\rangle$	125	119	84	86	80	50						
$\langle f_+^{\mu\nu}t_{+\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\lambda\rangle$	46	44					39						$i\langle\nabla^\mu f_+^{\nu\lambda}u_\mu\rangle\langle t_{-\nu\lambda}\rangle$	126	120	85	87	81		110	104	68	76	70	
$\langle f_+^{\mu\nu}\rangle\langle u_\mu\rangle\langle u^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	47												$i\langle\nabla^\mu f_+^{\nu\lambda}t_{-\nu\lambda}\rangle\langle u_\mu\rangle$	127	121	86				111	105				
$\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\lambda\rangle\langle t_{+\mu\nu}\rangle$	48												$i\langle f_+^{\mu\nu}\rangle\langle f_{-\mu}{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	128	122										
$\langle\nabla^\mu f_{+\mu}{}^\nu\nabla^\lambda t_{+\nu\lambda}\rangle$	49	45	28	30	26	15	40	36	21	26	22	13	$i\langle f_+^{\mu\nu}\rangle\langle h_\mu{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	129	123										
$\langle\nabla^\mu f_{+\mu}{}^\nu\rangle\langle\nabla^\lambda t_{+\nu\lambda}\rangle$	50	46	29	31	27	16							$i\langle\nabla^\mu f_{+\mu}{}^\nu\rangle\langle u^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	130	124										
$i\langle f_+^{\mu\nu}f_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle$	51	47	30	32	28	17	41	37	22	27	23	14	$i\langle\nabla^\mu f_+^{\nu\lambda}\rangle\langle u_\mu\rangle\langle t_{-\nu\lambda}\rangle$	131	125										
$i\langle u^\mu u^\nu\chi_+t_{+\mu\nu}\rangle+\text{H.c.}$	52	48	31	33	29	18	42	38	23	28	24	15	$\langle\chi_+f_-^{\mu\nu}t_{-\mu\nu}\rangle+\text{H.c.}$	132	126	87	88	82	51	112	106	69	77	71	42
$i\langle u^\mu\chi_+u^\nu t_{+\mu\nu}\rangle$	53	49	32	34	30	19	43	39	24	29	25	16	$\langle u^\mu\chi_+^\nu t_{-\mu\nu}\rangle+\text{H.c.}$	133	127	88	89	83	52	113	107	70	78	72	43
$i\langle u^\mu\rangle\langle u^\nu\chi_+t_{+\mu\nu}\rangle+\text{H.c.}$	54	50	33				44	40	25				$i\langle u^\mu u^\nu\chi_-t_{-\mu\nu}\rangle+\text{H.c.}$	134	128	89	90	84	53	114	108	71	79	73	44
$i\langle u^\mu u^\nu\chi_+\rangle\langle t_{+\mu\nu}\rangle$	55	51		35	31		45	41		30	26		$i\langle u^\mu\chi_-u^\nu t_{-\mu\nu}\rangle$	135	129	90	91	85	54	115	109	72	80	74	45
$i\langle u^\mu u^\nu t_{+\mu\nu}\rangle\langle\chi_+\rangle$	56	52		36	32		46	42		31	27		$i\langle u^\mu\rangle\langle u^\nu\chi_-t_{-\mu\nu}\rangle+\text{H.c.}$	136	130	91				116	110	73			
$\langle f_+^{\mu\nu}\chi_+t_{+\mu\nu}\rangle+\text{H.c.}$	57	53	34	37	33	20	47	43	26	32	28	17	$i\langle u^\mu u^\nu\chi_-\rangle\langle t_{-\mu\nu}\rangle$	137	131		92	86		117	111		81	75	
$\langle f_+^{\mu\nu}\rangle\langle\chi_+t_{+\mu\nu}\rangle$	58	54	35	38	34	21							$i\langle u^\mu u^\nu t_{-\mu\nu}\rangle\langle\chi_-\rangle$	138	132		93	87		118	112		82	76	
$\langle f_+^{\mu\nu}\chi_+\rangle\langle t_{+\mu\nu}\rangle$	59	55	36	39	35	22	48	44	27	33	29	18	$\langle f_+^{\mu\nu}\chi_-\rangle\langle t_{-\mu\nu}\rangle$	139	133	92	94	88	55	119	113	74	83	77	46
$\langle f_+^{\mu\nu}t_{+\mu\nu}\rangle\langle\chi_+\rangle$	60	56	37	40	36	23	49	45		34	30		$\langle f_+^{\mu\nu}t_{-\mu\nu}\rangle\langle\chi_-\rangle$	140	134	93	95	89	56	120	114		84	78	
$\langle f_+^{\mu\nu}\rangle\langle\chi_+\rangle\langle t_{+\mu\nu}\rangle$	61	57		41	37								$\langle f_+^{\mu\nu}\rangle\langle\chi_-\rangle\langle t_{-\mu\nu}\rangle$	141	135		96	90							
$\langle f_-^{\mu\nu}\chi_-t_{+\mu\nu}\rangle+\text{H.c.}$	62	58	38	42	38	24	50	46	28	35	31	19	$i\langle u^\mu\nabla^\nu t_{+\mu}{}^\lambda t_{-\nu\lambda}\rangle+\text{H.c.}$	142	136	94	97	91	57	121	115	75	85	79	47
$\langle u^\mu\chi_-^\nu t_{+\mu\nu}\rangle+\text{H.c.}$	63	59	39	43	39	25	51	47	29	36	32	20	$i\langle u^\mu\nabla^\nu t_{+\nu}{}^\lambda t_{-\mu\lambda}\rangle+\text{H.c.}$	143	137	95	98	92	58	122	116	76	86	80	48
$\langle u^\mu u_\mu t_+^{\nu\lambda}t_{+\nu\lambda}\rangle$	64	60	40	44	40	26	52	48	30	37	33	21	$i\langle f_-^{\mu\nu}t_{+\mu}{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	144	138	96	99	93	59	123	117	77	87	81	49
$\langle u^\mu u^\nu t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle$	65	61	41	45	41	27	53	49	31	38	34	22	$i\langle h^{\mu\nu}t_{+\mu}{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	145	139	97	100	94	60	124	118	78	88	82	50
$\langle u^\mu u^\nu t_{+\nu}{}^\lambda t_{+\mu\lambda}\rangle$	66	62	42	46	42	28	54	50	32	39	35	23	$i\langle u^\mu\nabla_\mu t_+^{\nu\lambda}\rangle\langle t_{-\nu\lambda}\rangle$	146	140	98	101	95	61	125	119	79	89	83	51
$\langle u^\mu t_{+\mu}{}^\nu u^\lambda t_{+\nu\lambda}\rangle+\text{H.c.}$	67	63	43	47	43	29	55	51	33	40	36	24	$i\langle u^\mu\rangle\langle\nabla^\nu t_{+\mu}{}^\lambda t_{-\nu\lambda}\rangle$	147	141	99				126	120	80			
$\langle u^\mu t_+^{\nu\lambda}u_\mu t_{+\nu\lambda}\rangle$	68	64	44	48	44	30	56	52	34	41	37	25	$i\langle u^\mu\nabla^\nu t_{+\mu}{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	148	142	100	102	96		127	121	81	90	84	
$\langle u^\mu\rangle\langle u_\mu t_+^{\nu\lambda}t_{+\nu\lambda}\rangle$	69	65	45				57	53	35				$i\langle u^\mu\rangle\langle\nabla^\nu t_{+\nu}{}^\lambda t_{-\mu\lambda}\rangle$	149	143	101				128	122	82			
$\langle u^\mu u_\mu\rangle\langle t_+^{\nu\lambda}t_{+\nu\lambda}\rangle$	70	66	46	49	45		58	54	36	42	38		$i\langle u^\mu\nabla^\nu t_{+\nu}{}^\lambda\rangle\langle t_{-\mu\lambda}\rangle$	150	144	102	103	97		129	123	83	91	85	
$\langle u^\mu u_\mu t_+^{\nu\lambda}\rangle\langle t_{+\nu\lambda}\rangle$	71	67	47	50	46	31	59	55	37	43	39	26	$i\langle u^\mu\rangle\langle\nabla^\nu t_{+\mu}{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	151	145					130	124				
$\langle u^\mu t_+^{\nu\lambda}\rangle\langle u_\mu t_{+\nu\lambda}\rangle$	72	68	48	51	47		60	56	38	44	40		$i\langle u^\mu\rangle\langle\nabla^\nu t_{+\nu}{}^\lambda\rangle\langle t_{-\mu\lambda}\rangle$	152	146					131	125				
$\langle u^\mu\rangle\langle u^\nu t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle+\text{H.c.}$	73	69	49				61	57	39				$\langle\chi_-t_+^{\mu\nu}t_{-\mu\nu}\rangle+\text{H.c.}$	153	147	103	104	98	62	132	126	84	92	86	52
$\langle u^\mu u^\nu\rangle\langle t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle$	74	70	50	52	48		62	58	40	45	41		$\langle\chi_-\rangle\langle t_+^{\mu\nu}t_{-\mu\nu}\rangle$	154	148	104	105	99	63	133	127	85	93	87	53
$\langle u^\mu u^\nu t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle+\text{H.c.}$	75	71	51	53	49	32	63	59	41	46	42	27	$\langle\chi_-t_+^{\mu\nu}\rangle\langle t_{-\mu\nu}\rangle$	155	149	105	106	100	64	134	128	86	94	88	54

(Table continued)

TABLE XI. (*Continued*)

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu t_+^{\nu\lambda}\rangle\langle u_\nu t_{+\mu\lambda}\rangle$	76	72	52	54	50		64	60	42	47	43	
$\langle u^\mu t_{+\mu}{}^\nu\rangle\langle u^\lambda t_{+\nu\lambda}\rangle$	77	73	53	55	51		65	61	43	48	44	
$\langle u^\mu\rangle\langle u_\mu\rangle\langle t_+^{\nu\lambda} t_{+\nu\lambda}\rangle$	78	74					66	62				
$\langle u^\mu\rangle\langle u_\mu t_+^{\nu\lambda}\rangle\langle t_{+\nu\lambda}\rangle$	79	75					67	63				
$\langle u^\mu u_\mu\rangle\langle t_+^{\nu\lambda}\rangle\langle t_{+\nu\lambda}\rangle$	80	76		56			68	64		49		

Operators	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
	n	3	2	n	3	2	n	3	2	n	3	2
$\langle\chi_-\rangle\langle t_+^{\mu\nu}\rangle\langle t_{-\mu\nu}\rangle$	156	150		107	101		135	129		95	89	
$\langle D^\mu t^{\nu\lambda} D_\nu t_{\mu\lambda}^\dagger\rangle$	157	151	106	108	102	65	136	130	87	96	90	55
$\langle F_L^{\mu\nu} t_{\mu\nu}^\dagger\chi\rangle + \text{H.c.} + \text{c.c.}$	158	152	107	109	103	66	137	131	88	97	91	56
$\langle F_L^{\mu\nu}\rangle\langle\chi^\dagger t_{\mu\nu}\rangle + \text{H.c.} + \text{c.c.}$	159	153	108	110	104	67						

TABLE XII. The p^6 order results in U groups and SU groups, with $t \neq 0, \hat{\theta} = 0$ and the odd parity. The numbers are the sequence numbers in each case. The details can be found in Sec. V C.

Operators	U(3)			Operators	U(3)			Operators	U(3)			Operators	U(3)		
	n	3	2		n	3	2		n	3	2		n	3	2
$i\langle u^\mu u_\mu f_-^{\nu\lambda} t_{+\nu\lambda}\rangle + \text{H.c.}$	160	154	109	$\langle u^\mu u^\nu\chi_- t_{+\mu\nu}\rangle + \text{H.c.}$	198	192	136	$\langle u^\mu\rangle\langle u^\nu\rangle\langle u_\mu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	236			$\langle u^\mu u^\nu\chi_+ t_{-\mu\nu}\rangle + \text{H.c.}$	274	264	185
$i\langle u^\mu u^\nu f_{-\mu}{}^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	161	155	110	$\langle u^\mu\chi_- u^\nu t_{+\mu\nu}\rangle$	199	193	137	$\langle f_-^{\mu\nu} f_{-\mu}{}^\lambda t_{-\nu\lambda}\rangle$	237	229	160	$\langle u^\mu\chi_+ u^\nu t_{-\mu\nu}\rangle$	275	265	186
$i\langle u^\mu u^\nu f_{-\nu}{}^\lambda t_{+\mu\lambda}\rangle + \text{H.c.}$	162	156	111	$\langle u^\mu\rangle\langle u^\nu\chi_- t_{+\mu\nu}\rangle + \text{H.c.}$	200	194	138	$\langle h^{\mu\nu} f_{-\mu}{}^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	238	230	161	$\langle u^\mu\rangle\langle u^\nu\chi_+ t_{-\mu\nu}\rangle + \text{H.c.}$	276	266	187
$i\langle u^\mu f_{-\mu}{}^\nu u^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	163	157	112	$\langle u^\mu u^\nu\chi_-\rangle\langle t_{+\mu\nu}\rangle$	201	195		$\langle h^{\mu\nu} h_\mu{}^\lambda t_{-\nu\lambda}\rangle$	239	231	162	$\langle u^\mu u^\nu\chi_+\rangle\langle t_{-\mu\nu}\rangle$	277	267	
$i\langle u^\mu u^\nu h_\mu{}^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	164	158	113	$\langle u^\mu u^\nu t_{+\mu\nu}\rangle\langle\chi_-\rangle$	202	196		$\langle u^\mu\nabla_\mu f_-^{\nu\lambda} t_{-\nu\lambda}\rangle + \text{H.c.}$	240	232	163	$\langle u^\mu u^\nu t_{-\mu\nu}\rangle\langle\chi_+\rangle$	278	268	
$i\langle u^\mu u^\nu h_\nu{}^\lambda t_{+\mu\lambda}\rangle + \text{H.c.}$	165	159	114	$i\langle f_+^{\mu\nu}\chi_- t_{+\mu\nu}\rangle + \text{H.c.}$	203	197	139	$\langle u^\mu\nabla^\nu f_{-\nu}{}^\lambda t_{-\mu\lambda}\rangle + \text{H.c.}$	241	233	164	$i\langle f_+^{\mu\nu}\chi_+ t_{-\mu\nu}\rangle + \text{H.c.}$	279	269	188
$i\langle u^\mu h_\mu{}^\nu u^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	166	160	115	$i\langle f_+^{\mu\nu}\rangle\langle\chi_- t_{+\mu\nu}\rangle$	204	198	140	$i\langle f_+^{\mu\nu} u_\mu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	242	234	165	$i\langle f_+^{\mu\nu}\rangle\langle\chi_+ t_{-\mu\nu}\rangle$	280	270	189
$i\langle u^\mu u^\nu t_{+\nu}{}^\lambda\rangle\langle h_{\mu\lambda}\rangle + \text{H.c.}$	167	161	116	$i\langle f_+^{\mu\nu}\chi_-\rangle\langle t_{+\mu\nu}\rangle$	205	199	141	$i\langle f_+^{\mu\nu} u^\lambda u_\mu t_{-\nu\lambda}\rangle + \text{H.c.}$	243	235	166	$i\langle f_+^{\mu\nu}\chi_+\rangle\langle t_{-\mu\nu}\rangle$	281	271	190
$i\langle u^\mu u^\nu t_{+\nu}{}^\lambda\rangle\langle f_{-\mu\lambda}\rangle + \text{H.c.}$	168	162	117	$i\langle f_+^{\mu\nu} t_{+\mu\nu}\rangle\langle\chi_-\rangle$	206	200	142	$i\langle f_+^{\mu\nu} u^\lambda u_\lambda t_{-\mu\nu}\rangle + \text{H.c.}$	244	236	167	$i\langle f_+^{\mu\nu} t_{-\mu\nu}\rangle\langle\chi_+\rangle$	282	272	191
$i\langle u^\mu u^\nu u^\lambda\rangle\langle\nabla_\mu t_{+\nu\lambda}\rangle$	169	163		$i\langle f_+^{\mu\nu}\rangle\langle\chi_-\rangle\langle t_{+\mu\nu}\rangle$	207	201		$i\langle f_+^{\mu\nu} u_\mu t_{-\nu}{}^\lambda u_\lambda\rangle + \text{H.c.}$	245	237	168	$i\langle f_+^{\mu\nu}\rangle\langle\chi_+\rangle\langle t_{-\mu\nu}\rangle$	283	273	
$i\langle u^\mu\rangle\langle u^\nu u^\lambda\nabla_\lambda t_{+\mu\nu}\rangle + \text{H.c.}$	170	164		$\langle h^{\mu\nu} t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle$	208	202	143	$i\langle f_+^{\mu\nu} u^\lambda t_{-\mu\nu} u_\lambda\rangle$	246	238	169	$i\langle f_-^{\mu\nu}\chi_- t_{-\mu\nu}\rangle + \text{H.c.}$	284	274	192
$i\langle u^\mu\rangle\langle u^\nu u^\lambda\nabla_\mu t_{+\nu\lambda}\rangle$	171	165		$\langle u^\mu\nabla^\nu t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	209	203	144	$i\langle f_+^{\mu\nu}\rangle\langle u_\mu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	247	239	170	$i\langle u^\mu\chi_-^\nu t_{-\mu\nu}\rangle + \text{H.c.}$	285	275	193

(Table continued)

TABLE XII. (*Continued*)

Operators	n	U(3) 3	U(3) 2	Operators	n	U(3) 3	U(3) 2	Operators	n	U(3) 3	U(3) 2	Operators	n	U(3) 3	U(3) 2
$i\langle u^\mu\rangle\langle u^\nu u_\mu\nabla^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	172	166		$\langle f_-^{\mu\nu} t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	210	204	145	$i\langle f_+^{\mu\nu} u_\mu\rangle\langle u^\lambda t_{-\nu\lambda}\rangle$	248	240	171	$i\langle u^\mu u_\mu t_+^{\nu\lambda} t_{-\nu\lambda}\rangle + \text{H.c.}$	286	276	194
$i\langle u^\mu\rangle\langle u^\nu t_{+\nu}{}^\lambda h_{\mu\lambda}\rangle + \text{H.c.}$	173	167		$\langle h^{\mu\nu}\rangle\langle t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle$	211	205	146	$i\langle f_+^{\mu\nu} u_\mu u^\lambda\rangle\langle t_{-\nu\lambda}\rangle + \text{H.c.}$	249	241	172	$i\langle u^\mu u^\nu t_{+\mu}{}^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	287	277	195
$i\langle u^\mu\rangle\langle u^\nu t_{+\mu}{}^\lambda h_{\nu\lambda}\rangle + \text{H.c.}$	174	168		$\langle h^{\mu\nu} t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	212	206	147	$i\langle f_+^{\mu\nu} u^\lambda t_{-\mu\lambda}\rangle\langle u_\nu\rangle + \text{H.c.}$	250	242	173	$i\langle u^\mu t_{+\mu}{}^\nu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	288	278	196
$i\langle u^\mu u^\nu h_\nu{}^\lambda\rangle\langle t_{+\mu\lambda}\rangle + \text{H.c.}$	175	169		$\langle u^\mu\nabla_\mu t_+^{\nu\lambda}\rangle\langle t_{+\nu\lambda}\rangle$	213	207	148	$i\langle f_+^{\mu\nu} t_{-\mu}{}^\lambda\rangle\langle u_\nu u_\lambda\rangle$	251	243	174	$i\langle u^\mu\rangle\langle u_\mu t_+^{\nu\lambda} t_{-\nu\lambda}\rangle + \text{H.c.}$	289	279	197
$\langle f_+^{\mu\nu} f_{-\mu}{}^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	176	170	118	$\langle u^\mu\rangle\langle\nabla^\nu t_{+\mu}{}^\lambda t_{+\nu\lambda}\rangle$	214	208	149	$i\langle f_+^{\mu\nu} u_\mu t_{-\nu}{}^\lambda\rangle\langle u_\lambda\rangle + \text{H.c.}$	252	244	175	$i\langle u^\mu u_\mu\rangle\langle t_+^{\nu\lambda} t_{-\nu\lambda}\rangle$	290	280	198
$\langle f_+^{\mu\nu} h_\mu{}^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	177	171	119	$\langle u^\mu\nabla^\nu t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	215	209	150	$i\langle f_+^{\mu\nu} u^\lambda\rangle\langle u_\mu t_{-\nu\lambda}\rangle$	253	245	176	$i\langle u^\mu u_\mu t_+^{\nu\lambda}\rangle\langle t_{-\nu\lambda}\rangle$	291	281	199
$\langle\nabla^\mu f_{+\mu}{}^\nu u^\lambda t_{+\nu\lambda}\rangle + \text{H.c.}$	178	172	120	$\langle u^\mu t_+^{\nu\lambda}\rangle\langle\nabla_\nu t_{+\mu\lambda}\rangle$	216	210	151	$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda u_\lambda t_{-\mu\nu}\rangle$	254	246	177	$i\langle u^\mu t_+^{\nu\lambda}\rangle\langle u_\mu t_{-\nu\lambda}\rangle$	292	282	200
$\langle\nabla^\mu f_+^{\nu\lambda} u_\mu t_{+\nu\lambda}\rangle + \text{H.c.}$	179	173	121	$\langle h^{\mu\nu}\rangle\langle t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	217	211		$i\langle f_+^{\mu\nu} u^\lambda\rangle\langle u_\lambda t_{-\mu\nu}\rangle$	255	247	178	$i\langle u^\mu u^\nu t_{+\mu}{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle + \text{H.c.}$	293	283	201
$\langle f_+^{\mu\nu}\rangle\langle f_{-\mu}{}^\lambda t_{+\nu\lambda}\rangle$	180	174	122	$\langle u^\mu\rangle\langle\nabla^\nu t_{+\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	218	212		$i\langle f_+^{\mu\nu} u^\lambda u_\lambda\rangle\langle t_{-\mu\nu}\rangle$	256	248	179	$i\langle u^\mu t_+^{\nu\lambda}\rangle\langle u_\nu t_{-\mu\lambda}\rangle$	294	284	202
$\langle f_+^{\mu\nu} f_{-\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	181	175	123	$i\langle\chi_- t_+^{\mu\nu} t_{+\mu\nu}\rangle$	219	213	152	$i\langle f_+^{\mu\nu} u^\lambda t_{-\mu\nu}\rangle\langle u_\lambda\rangle + \text{H.c.}$	257	249	180	$i\langle u^\mu\rangle\langle u_\mu\rangle\langle t_+^{\nu\lambda} t_{-\nu\lambda}\rangle$	295	285	
$\langle f_+^{\mu\nu} t_{+\mu}{}^\lambda\rangle\langle f_{-\nu\lambda}\rangle$	182	176	124	$i\langle\chi_-\rangle\langle t_+^{\mu\nu} t_{+\mu\nu}\rangle$	220	214	153	$i\langle f_+^{\mu\nu} t_{-\mu\nu}\rangle\langle u^\lambda u_\lambda\rangle$	258	250	181	$i\langle u^\mu\rangle\langle u_\mu t_+^{\nu\lambda}\rangle\langle t_{-\nu\lambda}\rangle$	296	286	
$\langle f_+^{\mu\nu}\rangle\langle h_\mu{}^\lambda t_{+\nu\lambda}\rangle$	183	177	125	$i\langle\chi_- t_+^{\mu\nu}\rangle\langle t_{+\mu\nu}\rangle$	221	215	154	$i\langle f_+^{\mu\nu}\rangle\langle u_\mu\rangle\langle u^\lambda t_{-\nu\lambda}\rangle$	259	251		$i\langle u^\mu u_\mu\rangle\langle t_+^{\nu\lambda}\rangle\langle t_{-\nu\lambda}\rangle$	297	287	
$\langle f_+^{\mu\nu} h_\mu{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	184	178	126	$i\langle\chi_-\rangle\langle t_+^{\mu\nu}\rangle\langle t_{+\mu\nu}\rangle$	222	216		$i\langle f_+^{\mu\nu}\rangle\langle u_\mu u^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	260	252		$i\langle u^\mu\rangle\langle u^\nu t_{+\mu}{}^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	298	288	
$\langle f_+^{\mu\nu} t_{+\mu}{}^\lambda\rangle\langle h_{\nu\lambda}\rangle$	185	179	127	$\langle u^\mu u_\mu u^\nu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	223	217	155	$i\langle f_+^{\mu\nu} u_\mu\rangle\langle u^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	261	253		$i\langle u^\mu\rangle\langle u^\nu t_{-\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	299	289	
$\langle\nabla^\mu f_{+\mu}{}^\nu\rangle\langle u^\lambda t_{+\nu\lambda}\rangle$	186	180	128	$\langle u^\mu u^\nu u_\mu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	224	218	156	$i\langle f_+^{\mu\nu} t_{-\mu}{}^\lambda\rangle\langle u_\nu\rangle\langle u_\lambda\rangle$	262	254		$i\langle u^\mu\rangle\langle u_\mu\rangle\langle t_+^{\nu\lambda}\rangle\langle t_{-\nu\lambda}\rangle$	300		
$\langle\nabla^\mu f_{+\mu}{}^\nu u^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	187	181	129	$\langle u^\mu u^\nu u_\nu u^\lambda t_{-\mu\lambda}\rangle$	225	219	157	$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\mu t_{-\nu\lambda}\rangle$	263	255		$i\langle\nabla^\mu t_{+\mu}{}^\nu\nabla^\lambda t_{-\nu\lambda}\rangle$	301	290	203
$\langle\nabla^\mu f_{+\mu}{}^\nu t_{+\nu}{}^\lambda\rangle\langle u_\lambda\rangle$	188	182	130	$\langle u^\mu u^\nu u^\lambda u_\mu t_{-\nu\lambda}\rangle$	226	220	158	$i\langle f_+^{\mu\nu} u^\lambda\rangle\langle u_\mu\rangle\langle t_{-\nu\lambda}\rangle$	264	256		$i\langle\nabla^\mu t_{+\mu}{}^\nu\rangle\langle\nabla^\lambda t_{-\nu\lambda}\rangle$	302	291	204
$\langle\nabla^\mu f_+^{\nu\lambda}\rangle\langle u_\mu t_{+\nu\lambda}\rangle$	189	183	131	$\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	227	221	159	$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\lambda t_{-\mu\nu}\rangle$	265	257		$\langle f_+^{\mu\nu} t_{+\mu}{}^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	303	292	205
$\langle\nabla^\mu f_+^{\nu\lambda} u_\mu\rangle\langle t_{+\nu\lambda}\rangle$	190	184	132	$\langle u^\mu u_\mu\rangle\langle u^\nu u^\lambda t_{-\nu\lambda}\rangle$	228	222		$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda u_\lambda\rangle\langle t_{-\mu\nu}\rangle$	266	258		$i\langle\chi_+ t_+^{\mu\nu} t_{-\mu\nu}\rangle + \text{H.c.}$	304	293	206
$\langle\nabla^\mu f_+^{\nu\lambda} t_{+\nu\lambda}\rangle\langle u_\mu\rangle$	191	185	133	$\langle u^\mu u_\mu u^\nu u^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	229	223		$i\langle f_+^{\mu\nu} u^\lambda\rangle\langle u_\lambda\rangle\langle t_{-\mu\nu}\rangle$	267	259		$i\langle\chi_+\rangle\langle t_+^{\mu\nu} t_{-\mu\nu}\rangle$	305	294	207
$\langle f_+^{\mu\nu}\rangle\langle f_{-\mu}{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	192	186		$\langle u^\mu u^\nu\rangle\langle u_\mu u^\lambda t_{-\nu\lambda}\rangle + \text{H.c.}$	230	224		$i\langle f_+^{\mu\nu} t_{-\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\lambda\rangle$	268	260		$i\langle\chi_+ t_+^{\mu\nu}\rangle\langle t_{-\mu\nu}\rangle$	306	295	208
$\langle f_+^{\mu\nu}\rangle\langle h_\mu{}^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	193	187		$\langle u^\mu u^\nu u^\lambda\rangle\langle u_\mu t_{-\nu\lambda}\rangle$	231	225		$i\langle f_+^{\mu\nu}\rangle\langle u_\mu\rangle\langle u^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	269			$i\langle\chi_+\rangle\langle t_+^{\mu\nu}\rangle\langle t_{-\mu\nu}\rangle$	307	296	
$\langle\nabla^\mu f_{+\mu}{}^\nu\rangle\langle u^\lambda\rangle\langle t_{+\nu\lambda}\rangle$	194	188		$\langle u^\mu\rangle\langle u^\nu u_\nu u^\lambda t_{-\mu\lambda}\rangle + \text{H.c.}$	232	226		$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle u_\lambda\rangle\langle t_{-\mu\nu}\rangle$	270			$\langle t_+^{\mu\nu} t_{+\mu}{}^\lambda t_{-\nu\lambda}\rangle$	308	297	209
$\langle\nabla^\mu f_+^{\nu\lambda}\rangle\langle u_\mu\rangle\langle t_{+\nu\lambda}\rangle$	195	189		$\langle u^\mu\rangle\langle u^\nu u_\mu u^\lambda t_{-\nu\lambda}\rangle$	233	227		$i\langle\nabla^\mu f_{+\mu}{}^\nu\nabla^\lambda t_{-\nu\lambda}\rangle$	271	261	182				
$i\langle\chi_+ f_-^{\mu\nu} t_{+\mu\nu}\rangle + \text{H.c.}$	196	190	134	$\langle u^\mu\rangle\langle u_\mu\rangle\langle u^\nu u^\lambda t_{-\nu\lambda}\rangle$	234	228		$i\langle\nabla^\mu f_{+\mu}{}^\nu\rangle\langle\nabla^\lambda t_{-\nu\lambda}\rangle$	272	262	183				
$i\langle u^\mu\chi_+^\nu t_{+\mu\nu}\rangle + \text{H.c.}$	197	191	135	$\langle u^\mu\rangle\langle u_\mu u^\nu u^\lambda\rangle\langle t_{-\nu\lambda}\rangle$	235			$\langle f_+^{\mu\nu} f_{+\mu}{}^\lambda t_{-\nu\lambda}\rangle$	273	263	184				

TABLE XIII. The p^6 order results in U groups and SU groups, with $t \neq 0$, $\hat{\theta} \neq 0$ and the even parity. The numbers are the sequence numbers in each case. The details can be found in Sec. V C.

	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
m	n	3	2	n	3	2	n	3	2	n	3	2
$\langle u^\mu u_\mu u^\nu t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	1	1	1	1	1		1	1	1	1	1	
$\langle u^\mu u^\nu u^\lambda t_{+\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	2	2	2	2	2	1	2	2	2	2	2	1
$\langle u^\mu u^\nu u^\lambda t_{+\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	3	3	3	3	3		3	3	3	3	3	
$\langle u^\mu\rangle\langle u_\mu u^\nu t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	4	4					4	4				
$\langle u^\mu\rangle\langle u^\nu u^\lambda t_{+\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	5	5	4				5	5	4			
$\langle u^\mu u^\nu u^\lambda\rangle\langle t_{+\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	6	6		4	4		6	6		4	4	
$\langle u^\mu\rangle\langle u^\nu u^\lambda t_{+\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	7	7					7	7				
$i\langle f_+^{\mu\nu} u_\mu t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	8	8	5	5	5	2	8	8	5	5	5	2
$i\langle f_+^{\mu\nu} u^\lambda t_{+\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	9	9	6	6	6	3	9	9	6	6	6	3
$i\langle f_+^{\mu\nu} u^\lambda t_{+\mu\lambda}\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	10	10	7	7	7	4	10	10	7	7	7	4
$i\langle f_+^{\mu\nu}\rangle\langle u_\mu t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	11	11	8	8	8	5						
$i\langle f_+^{\mu\nu} u_\mu\rangle\langle t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	12	12	9	9	9		11	11	8	8	8	
$i\langle f_+^{\mu\nu} t_{+\mu}{}^\lambda\rangle\langle u_\nu\rangle\nabla_\lambda\hat{\theta}$	13	13	10				12	12				
$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda t_{+\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	14	14	11	10	10	6						
$i\langle f_+^{\mu\nu} u^\lambda\rangle\langle t_{+\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	15	15	12	11	11		13	13	9	9	9	
$i\langle f_+^{\mu\nu} t_{+\mu\nu}\rangle\langle u^\lambda\rangle\nabla_\lambda\hat{\theta}$	16	16	13				14	14				
$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda t_{+\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	17	17	14	12	12	7						
$i\langle f_+^{\mu\nu} u^\lambda\rangle\langle t_{+\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	18	18	15	13	13		15	15	10	10	10	
$i\langle f_+^{\mu\nu} t_{+\mu}{}^\lambda\rangle\langle u_\lambda\rangle\nabla_\nu\hat{\theta}$	19	19	16				16	16				
$i\langle f_+^{\mu\nu}\rangle\langle u_\mu\rangle\langle t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	20	20										
$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle t_{+\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	21	21										
$i\langle f_+^{\mu\nu}\rangle\langle u^\lambda\rangle\langle t_{+\mu\lambda}\rangle\nabla_\nu\hat{\theta}$	22	22										
$\langle u^\mu\chi_+ t_{+\mu}{}^\nu\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	23	23	17	14	14	8	17	17	11	11	11	5
$i\langle u^\mu u^\nu t_{+\mu\nu}\rangle\nabla\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	24	24	18	15	15	9	18	18	12	12	12	6
$i\langle u^\mu u^\nu t_{+\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}+\text{H.c.}$	25	25	19	16	16	10	19	19	13	13	13	7
$\langle f_+^{\mu\nu} t_{+\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	26	26	20	17	17	11	20	20	14	14	14	8
$\langle f_+^{\mu\nu} t_{+\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	27	27	21	18	18	12	21	21	15	15	15	9
$\langle f_+^{\mu\nu}\rangle\langle t_{+\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	28	28	22	19	19	13						
$\langle f_+^{\mu\nu}\rangle\langle t_{+\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	29	29	23	20	20	14						
$i\langle u^\mu t_{+\mu}{}^\nu t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	30	30	24	21	21	15	22	22	16	16	16	10
$i\langle u^\mu t_+^{\nu\lambda} t_{+\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	31	31	25	22	22	16	23	23	17	17	17	11
$i\langle u^\mu\rangle\langle t_{+\mu}{}^\nu t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	32	32	26				24	24	18			

	U(3)			$SU(3)_I$			$SU(3)_{II}$			$SU(3)_{III}$		
m	n	3	2	n	3	2	n	3	2	n	3	2
$i\langle u^\mu t_{+\mu}{}^\nu\rangle\langle t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	33	33	27	23	23	17	25	25	19	18	18	12
$i\langle u^\mu t_+^{\nu\lambda}\rangle\langle t_{+\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	34	34	28	24	24		26	26	20	19	19	
$i\langle u^\mu\rangle\langle t_+^{\nu\lambda} t_{+\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	35	35	29				27	27	21			
$i\langle u^\mu t_+^{\nu\lambda}\rangle\langle t_{+\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	36	36	30	25	25		28	28	22	20	20	
$i\langle u^\mu\rangle\langle t_{+\mu}{}^\nu\rangle\langle t_{+\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	37	37					29	29				
$i\langle u^\mu\rangle\langle t_+^{\nu\lambda}\rangle\langle t_{+\nu\lambda}\rangle\nabla_\mu\hat{\theta}$	38	38					30	30				
$\langle t_+^{\mu\nu} t_{+\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	39	39	31	26	26	18	31	31	23	21	21	13
$\langle t_+^{\mu\nu}\rangle\langle t_{+\mu\nu}\rangle\nabla^\lambda\hat{\theta}\nabla_\lambda\hat{\theta}$	40	40	32	27	27	19	32	32	24	22	22	14
$\langle t_+^{\mu\nu}\rangle\langle t_{+\mu}{}^\lambda\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	41	41	33	28	28	20	33	33	25	23	23	15
$i\langle u^\mu f_{-\mu}{}^\nu t_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	42	42	34	29	29	21	34	34	26	24	24	16
$i\langle u^\mu f_-^{\nu\lambda} t_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	43	43	35	30	30	22	35	35	27	25	25	17
$i\langle u^\mu f_-^{\nu\lambda} t_{-\nu\lambda}\rangle\nabla_\mu\hat{\theta}+\text{H.c.}$	44	44	36	31	31	23	36	36	28	26	26	18
$i\langle u^\mu h_\mu{}^\nu t_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	45	45	37	32	32	24	37	37	29	27	27	19
$i\langle u^\mu h^{\nu\lambda} t_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	46	46	38	33	33	25	38	38	30	28	28	20
$i\langle u^\mu u^\nu t_{-\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	47	47	39	34	34	26	39	39	31	29	29	21
$i\langle u^\mu u^\nu t_{-\mu}{}^\lambda\rangle\nabla_\nu\nabla_\lambda\hat{\theta}+\text{H.c.}$	48	48	40	35	35	27	40	40	32	30	30	22
$\langle\nabla^\mu f_{+\mu}{}^\nu t_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	49	49	41	36	36	28	41	41	33	31	31	23
$\langle\nabla^\mu f_+^{\nu\lambda} t_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	50	50	42	37	37	29	42	42	34	32	32	24
$\langle f_+^{\mu\nu} t_{-\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	51	51	43	38	38	30	43	43	35	33	33	25
$\langle f_+^{\mu\nu} t_{-\mu}{}^\lambda\rangle\nabla_\nu\nabla_\lambda\hat{\theta}$	52	52	44	39	39	31	44	44	36	34	34	26
$\langle\nabla^\mu f_{+\mu}{}^\nu\rangle\langle t_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	53	53	45	40	40	32						
$\langle\nabla^\mu f_+^{\nu\lambda}\rangle\langle t_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	54	54	46	41	41	33						
$\langle f_+^{\mu\nu}\rangle\langle t_{-\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	55	55	47	42	42	34						
$\langle f_+^{\mu\nu}\rangle\langle t_{-\mu}{}^\lambda\rangle\nabla_\nu\nabla_\lambda\hat{\theta}$	56	56	48	43	43	35						
$\langle u^\mu\chi_- t_{-\mu}{}^\nu\rangle\nabla_\nu\hat{\theta}+\text{H.c.}$	57	57	49	44	44	36	45	45	37	35	35	27
$\langle t_+^{\mu\nu} t_{-\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	58	58	50	45	45	37	46	46	38	36	36	28
$\langle\nabla^\mu t_{+\mu}{}^\nu t_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	59	59	51	46	46	38	47	47	39	37	37	29
$\langle t_+^{\mu\nu}\rangle\langle t_{-\mu\nu}\rangle\nabla^\lambda\nabla_\lambda\hat{\theta}$	60	60	52	47	47	39	48	48	40	38	38	30
$\langle\nabla^\mu t_{+\mu}{}^\nu\rangle\langle t_{-\nu}{}^\lambda\rangle\nabla_\lambda\hat{\theta}$	61	61	53	48	48	40	49	49	41	39	39	31
$\langle\nabla^\mu t_+^{\nu\lambda}\rangle\langle t_{-\mu\nu}\rangle\nabla_\lambda\hat{\theta}$	62	62	54	49	49	41	50	50	42	40	40	32
$\langle D^\mu t_\mu{}^\nu t_\nu^{\dagger\lambda}\rangle\nabla_\lambda\hat{\theta}+\text{H.c.}$	63	63	55	50	50	42	51	51	43	41	41	33
$\langle t^{\mu\nu} t_\mu^{\dagger\lambda}\rangle\nabla_\nu\hat{\theta}\nabla_\lambda\hat{\theta}$	64	64	56	51	51	43	52	52	44	42	42	34

TABLE XIV. The p^6 order results in U groups and SU groups, with $t \neq 0, \hat{\theta} \neq 0$ and the odd parity. The numbers are the sequence numbers in each case. The details can be found in Sec. V C.

Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2	Operators	U(3) n	3	2
$\langle u^\mu f_{-\mu}{}^{\nu} t_{+\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	65	65	57	$i\langle u^\mu \chi_- t_{+\mu}{}^{\nu}\rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	80	80	72	$\langle f_+^{\mu\nu} u^\lambda t_{-\mu\lambda}\rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	95	95	84	$\langle u^\mu u^\nu t_{-\mu}{}^{\lambda}\rangle \nabla_\nu \hat{\theta} \nabla_\lambda \hat{\theta} + \text{H.c.}$	110	110	96
$\langle u^\mu f_-^{\nu\lambda} t_{+\mu\nu}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	66	66	58	$i\langle t_+^{\mu\nu} t_{+\mu\nu}\rangle \nabla^\lambda \nabla_\lambda \hat{\theta}$	81	81	73	$\langle f_+^{\mu\nu}\rangle \langle u_\mu t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	96	96	85	$i\langle f_+^{\mu\nu} t_{-\mu\nu}\rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	111	111	97
$\langle u^\mu f_-^{\nu\lambda} t_{+\nu\lambda}\rangle \nabla_\mu \hat{\theta} + \text{H.c.}$	67	67	59	$i\langle \nabla^\mu t_{+\mu}{}^{\nu} t_{+\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	82	82	74	$\langle f_+^{\mu\nu} u_\mu\rangle \langle t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	97	97	86	$i\langle f_+^{\mu\nu} t_{-\mu}{}^{\lambda}\rangle \nabla_\nu \hat{\theta} \nabla_\lambda \hat{\theta}$	112	112	98
$\langle u^\mu h_\mu{}^{\nu} t_{+\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	68	68	60	$i\langle t_+^{\mu\nu}\rangle \langle t_{+\mu\nu}\rangle \nabla^\lambda \nabla_\lambda \hat{\theta}$	83	83	75	$\langle f_+^{\mu\nu} t_{-\mu}{}^{\lambda}\rangle \langle u_\nu\rangle \nabla_\lambda \hat{\theta}$	98	98	87	$i\langle f_+^{\mu\nu}\rangle \langle t_{-\mu\nu}\rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	113	113	99
$\langle u^\mu h^{\nu\lambda} t_{+\mu\nu}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	69	69	61	$i\langle t_+^{\mu\nu}\rangle \langle t_{+\mu}{}^{\lambda}\rangle \nabla_\nu \nabla_\lambda \hat{\theta}$	84	84	76	$\langle f_+^{\mu\nu}\rangle \langle u^\lambda t_{-\mu\nu}\rangle \nabla_\lambda \hat{\theta}$	99	99	88	$i\langle f_+^{\mu\nu}\rangle \langle t_{-\mu}{}^{\lambda}\rangle \nabla_\nu \hat{\theta} \nabla_\lambda \hat{\theta}$	114	114	100
$\langle u^\mu u^\nu t_{+\mu\nu}\rangle \nabla^\lambda \nabla_\lambda \hat{\theta}$	70	70	62	$i\langle \nabla^\mu t_{+\mu}{}^{\nu}\rangle \langle t_{+\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	85	85	77	$\langle f_+^{\mu\nu} u^\lambda\rangle \langle t_{-\mu\nu}\rangle \nabla_\lambda \hat{\theta}$	100	100	89	$\langle u^\mu t_{+\mu}{}^{\nu} t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	115	115	101
$\langle u^\mu u^\nu t_{+\mu}{}^{\lambda}\rangle \nabla_\nu \nabla_\lambda \hat{\theta} + \text{H.c.}$	71	71	63	$i\langle u^\mu u_\mu u^\nu t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	86	86	78	$\langle f_+^{\mu\nu} t_{-\mu\nu}\rangle \langle u^\lambda\rangle \nabla_\lambda \hat{\theta}$	101	101	90	$\langle u^\mu\rangle \langle t_{+\mu}{}^{\nu} t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	116	116	102
$i\langle \nabla^\mu f_{+\mu}{}^{\nu} t_{+\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	72	72	64	$i\langle u^\mu u^\nu u^\lambda t_{-\mu\nu}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	87	87	79	$\langle f_+^{\mu\nu}\rangle \langle u^\lambda t_{-\mu\lambda}\rangle \nabla_\nu \hat{\theta}$	102	102	91	$\langle u^\mu t_{+\mu}{}^{\nu}\rangle \langle t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	117	117	103
$i\langle \nabla^\mu f_+^{\nu\lambda} t_{+\mu\nu}\rangle \nabla_\lambda \hat{\theta}$	73	73	65	$i\langle u^\mu u^\nu u^\lambda t_{-\mu\lambda}\rangle \nabla_\nu \hat{\theta}$	88	88	80	$\langle f_+^{\mu\nu} u^\lambda\rangle \langle t_{-\mu\lambda}\rangle \nabla_\nu \hat{\theta}$	103	103	92	$\langle u^\mu t_-^{\nu\lambda}\rangle \langle t_{+\mu\nu}\rangle \nabla_\lambda \hat{\theta}$	118	118	104
$i\langle f_+^{\mu\nu} t_{+\mu\nu}\rangle \nabla^\lambda \nabla_\lambda \hat{\theta}$	74	74	66	$i\langle u^\mu\rangle \langle u_\mu u^\nu t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	89	89		$\langle f_+^{\mu\nu} t_{-\mu}{}^{\lambda}\rangle \langle u_\lambda\rangle \nabla_\nu \hat{\theta}$	104	104	93	$\langle u^\mu t_+^{\nu\lambda}\rangle \langle t_{-\mu\nu}\rangle \nabla_\lambda \hat{\theta}$	119	119	105
$i\langle f_+^{\mu\nu} t_{+\mu}{}^{\lambda}\rangle \nabla_\nu \nabla_\lambda \hat{\theta}$	75	75	67	$i\langle u^\mu\rangle \langle u^\nu u^\lambda t_{-\mu\nu}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	90	90	81	$\langle f_+^{\mu\nu}\rangle \langle u_\mu\rangle \langle t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	105	105		$\langle u^\mu\rangle \langle t_{+\mu}{}^{\nu}\rangle \langle t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	120	120	
$i\langle \nabla^\mu f_{+\mu}{}^{\nu}\rangle \langle t_{+\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta}$	76	76	68	$i\langle u^\mu u^\nu u^\lambda\rangle \langle t_{-\mu\nu}\rangle \nabla_\lambda \hat{\theta}$	91	91		$\langle f_+^{\mu\nu}\rangle \langle u^\lambda\rangle \langle t_{-\mu\nu}\rangle \nabla_\lambda \hat{\theta}$	106	106		$i\langle t_+^{\mu\nu} t_{-\mu\nu}\rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	121	121	106
$i\langle \nabla^\mu f_+^{\nu\lambda}\rangle \langle t_{+\mu\nu}\rangle \nabla_\lambda \hat{\theta}$	77	77	69	$i\langle u^\mu\rangle \langle u^\nu u^\lambda t_{-\nu\lambda}\rangle \nabla_\mu \hat{\theta}$	92	92		$\langle f_+^{\mu\nu}\rangle \langle u^\lambda\rangle \langle t_{-\mu\lambda}\rangle \nabla_\nu \hat{\theta}$	107	107		$i\langle t_+^{\mu\nu}\rangle \langle t_{-\mu\nu}\rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	122	122	107
$i\langle f_+^{\mu\nu}\rangle \langle t_{+\mu\nu}\rangle \nabla^\lambda \nabla_\lambda \hat{\theta}$	78	78	70	$\langle f_+^{\mu\nu} u_\mu t_{-\nu}{}^{\lambda}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	93	93	82	$i\langle u^\mu \chi_+ t_{-\mu}{}^{\nu}\rangle \nabla_\nu \hat{\theta} + \text{H.c.}$	108	108	94	$i\langle D^\mu t^{\nu\lambda} t_{\mu\nu}^\dagger\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	123	123	108
$i\langle f_+^{\mu\nu}\rangle \langle t_{+\mu}{}^{\lambda}\rangle \nabla_\nu \nabla_\lambda \hat{\theta}$	79	79	71	$\langle f_+^{\mu\nu} u^\lambda t_{-\mu\nu}\rangle \nabla_\lambda \hat{\theta} + \text{H.c.}$	94	94	83	$\langle u^\mu u^\nu t_{-\mu\nu}\rangle \nabla^\lambda \hat{\theta} \nabla_\lambda \hat{\theta}$	109	109	95				

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