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CP asymmetries in three-body final states in charged *D* decays and *CPT* invariance

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The study of regional *CP* asymmetry in Dalitz plots of charm (& beauty) decays gives us more information about the underlying dynamics than the ratio between total rates. In this paper we explore the consequences of the constraint from *CPT* symmetry with emphasis on three-body *D* decays. We show simulations of $D^\pm \to \pi^\pm K^+ K^-$ and discuss correlations with measured $D^\pm \to \pi^\pm \pi^+ \pi^-$. There are important comments about analysis of recent LHCb data in *CP* asymmetries for $B^\pm$ decays to three-body final states.

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I. PROBING *CP* ASYMMETRIES WITH *CPT* INVARIANCE

The study of *CP* violation (CPV) is a portal to new dynamics (ND). Although no obvious signal of ND has been shown in hadronic data, there are still good reasons for its existence: neutrinos oscillations, the existence of dark matter and dark energy, and our own existence are the most obvious ones.

CPV is a well-established phenomenon in decays of *K* and *B* mesons, but no *CP* asymmetry has been found in *D* decays . . . yet. The Standard Model (SM) predicts very small CPV in singly Cabibbo suppressed (SCS) *D* decays and close to zero CPV in doubly Cabibbo suppressed (DCS) ones. The observation of *CP* asymmetries at $\mathcal{O}(10^{-2})$ level in charm decays would be a clear manifestation of ND. The experimental sensitivity, however, is rapidly reaching $\mathcal{O}(10^{-3})$ with no signals of CPV. Although the observation of CPV in charm would be a great achievement in itself, one would still have the difficult problem of disentangling ND effects from SM ones.

The vast majority of experimental searches and theoretical works refer to two-body final states (FS) of the type $D \to P_1 P_2$ (*P* denotes light pseudoscalar mesons). From the theory side there are large uncertainties related to the hadronic degrees of freedom that could easily hide the impact of ND. From the experimental side, the usual *CP* asymmetries in two-body decays give only a single number and this is not enough information to understand the nature of an eventual CPV signal. One needs to go beyond two-body decays and look at new observables. Three- and four-body decays are the natural way. First, local effects may be larger than phase space integrated ones. The asymmetry is modulated by the strong phase variation characteristic of resonant decays [1]. The *CP* asymmetry may change sign across the phase space, and the comparison between integrated rates would dilute an eventual signal. Furthermore, the pattern of the *CP* asymmetry across the phase space could give insights about the underlying operators. CPV searches with charged *D* mesons with three-body FS are therefore a very important program.

In this paper we investigate the possible patterns of CPV in three-body, SCS nonleptonic decays of *D* mesons. ND could produce sizable asymmetries in DCS decays, but DCS rates are small. We focus on direct CPV with $\Delta C = 1$ forces and explore the correlations introduced by strong final state interactions (FSI) and *CPT* conservation, which is assumed to be exact.

Some comments are in order:

(i) Theoretical predictions are more complicated in $D \to PV$ with narrow vector meson resonances *V*, since one deals with three-body FS and interference with other intermediate states must be taken into account. It becomes much more complicated in $D \to PS$ with broad scalar resonances *S*, in particular with σ and κ. From the experimental side, a full Dalitz plot with very large data sets is quite challenging; for example, effects such as FSI among the three hadrons must be included in the decay model. The modeling of the S-waves is another instance of limitations of the existing tools.

(ii) One alternative are model independent searches, comparing directly the D^+ and the D^- Dalitz plots, as in [2,3], which is a convenient first step.

(iii) Additional constraints should be used, such as *CPT* invariance. It comes into 'play' by imposing equalities of total decay rates of particle and antiparticle. Its invariance imply also the equalities of partial rates of classes of FS where their members can rescatter to each other. Given that three-body decays are mostly a sequence of two-body

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transitions, processes such as 2π, $K\bar{K}\leftrightarrow 2\pi$, $K\bar{K}$ and $\pi K\leftrightarrow\pi K$ become crucial. In other words, CPT invariance would relate asymmetries in $D\to\pi\pi\pi$ with $D\to\pi K\bar{K}$.

CPT invariance is a clear instance where the low energy hadron dynamics play an important role: it is an unavoidable ingredient to the decay amplitude models. However, going from quarks to hadrons and understanding the dynamics of three-body FSI are real challenges in a quantitative way. Dispersion relations and chiral perturbation theory are some of the theoretical tools needed for a more realistic description of three-body FS. This is indeed a field plenty of opportunities.

A. CP asymmetries and CPT constraints

Let us consider a decay into a FS f that can proceed through two different amplitudes:

$$T(D^+\to f) = e^{i\phi_1^{\rm weak}}e^{i\delta_1^{\rm FSI}}\mathcal{A}_1 + e^{i\phi_2^{\rm weak}}e^{i\delta_2^{\rm FSI}}|\mathcal{A}_2|, \quad (1)$$

$$T(D^-\to \bar{f}) = e^{-i\phi_1^{\rm weak}}e^{i\delta_1^{\rm FSI}}\mathcal{A}_1 + e^{-i\phi_2^{\rm weak}}e^{i\delta_2^{\rm FSI}}\mathcal{A}_2. \quad (2)$$

In charged D mesons only direct CP violation is possible, which means $\Gamma(D^+\to f)\neq\Gamma(D^-\to\bar{f})$. Computing the CP asymmetry in the partial width one has

$$\frac{\Gamma(D^+\to f)-\Gamma(D^-\to\bar{f})}{\Gamma(D^+\to f)+\Gamma(D^-\to\bar{f})} = -\frac{2\sin(\Delta\phi_W)\sin(\Delta\delta^{\rm FSI})|\mathcal{A}_2/\mathcal{A}_1|}{1+|\mathcal{A}_2/\mathcal{A}_1|^2+2|\mathcal{A}_2/\mathcal{A}_1|\cos(\Delta\phi_W)\cos(\Delta\delta^{\rm FSI})|}, \quad (3)$$

with $\Delta\phi_W=\phi_1^{\rm weak}-\phi_2^{\rm weak}$ and $\Delta\delta^{\rm FSI}=\delta_1^{\rm FSI}-\delta_2^{\rm FSI}$. We see clearly how CP asymmetries arise when there are differences in both weak and strong phases.

However, the constraints from CPT invariance are not apparent. Suppose the decay mode f belongs to a family of n final states f_n connected to each other via rescattering. The consequences of CPT invariance (general comments on CPT invariance are given in Refs. [4–9]) become visible if we rewrite the decay amplitude in the form

$$T(D^+\to f_j) = e^{i\delta_{f_j}}\left[T_{f_j}+\sum_{f_j\neq f_k}T_{f_k}iT^{\rm resc}_{f_jf_k}\right] \quad (4)$$

$$T(D^-\to \bar{f}_j) = e^{i\delta_{f_j}}\left[T^*_{\bar{f}_j}+\sum_{f_k\neq f_j}T^*_{\bar{f}_j}iT^{\rm resc}_{f_jf_k}\right], \quad (5)$$

where amplitudes $T^{\rm resc}_{f_jf_k}$ describe FSI connecting f_j and f_k. One gets, in addition to the direct term, a contribution to the CP asymmetry of the form

$$\Delta\gamma(a) = 4\sum_{f_k\neq f_j}T^{\rm resc}_{f_jf_k}{\rm Im}T^*_{f_j}T_{f_k}. \quad (6)$$

The following are subtle but very important statements about using these equations:

(i) Final states f_n should also include modes with neutrals. In practice, decays like $D+\to\pi^+\pi^0\pi^0$ are really hard to obtain.

(ii) CPT invariance can be satisfied in two ways: one can find that none of the decays, $D^\pm\to\pi^\pm\pi^+\pi^-/\pi^\pm\pi^0\pi^0/\pi^\pm K^+K^-/\pi^\pm K^0\bar{K}^0$, shows evidence for CP asymmetry, or at least two of the decays find CP violation with opposite signs.

So far $D^\pm\to\pi^\pm\pi^+\pi^-$ shows no evidence about CP asymmetry [10], but the two roads are still open.

B. Scattering in $\pi\pi\leftrightarrow KK$, $\pi\pi\leftrightarrow\pi\pi$, $KK\leftrightarrow KK$, $\pi K\leftrightarrow\pi K$

Early experimental results from $\pi\pi$ scattering presented a significant deviation from the elastic regime of the S-wave in the region between 1.0–1.5 GeV [11,12]. The inelasticity parameter decreases, starting at 1.0 GeV, get a minimum at 1.2 GeV and come back again to the unitary circle at around 1.5 GeV, going counterclockwise in the Argand circle. A similar inspection was performed for the P- and D-waves, but no significant deviation from the elastic regime was found in this energy interval.

The deviation of the inelasticity in the S-wave $\pi\pi\to\pi\pi$ scattering is associated to a corresponding increase of the cross section of $\pi\pi\to KK$ [13], in the same energy region. Notice that due to G-parity conservation a pair of pions can only scatter into an even number of pions. In other words, an initial state of two pions can produce either two pions or two kaons.

The same study performed to $K\pi$ elastic scattering by the LASS experiment [14] showed that both S- and P-waves have an inelasticity parameter close to unity in the Argand circle, up to 1.4 GeV in the P-wave and 1.5 GeV in the S-wave. The D-wave is dominated by the resonance $K_2(1430)$ that decays to $K\pi$ with a branching fraction of about 50% [15]. Therefore, rescattering of the $K\pi$ system is not relevant to this discussion.

The energy range of the K^+K^- pair is $2m_K\leq m(KK)\leq m_D-m_\pi$, which coincides with the range where the inelasticity of the $\pi\pi$ scattering deviates from unity. CPT invariance, therefore, connects the $D^+\to\pi^+\pi^+\pi^-$ and the $D^+\to\pi^+K^+K^-$ decays, through the S-wave $\pi\pi\leftrightarrow KK$ scattering. A comprehensive argument should include $\pi^0\pi^0$ and $K^0\bar{K}^0$ as well, but this will not be addressed in this paper.

C. Some intriguing results: Charmless three-body $B^\pm$ decays

Recent LHCb results on charmless three-body $B^\pm$ decays show sizable averaged CP asymmetries over the FS with correlations [16]:

$$A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}) = +0.032 \pm 0.008_{\rm stat} \pm 0.004_{\rm syst} \pm 0.007_{\psi K^{\pm}} \quad (7)$$

$$A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-}) = -0.043 \pm 0.009_{\rm stat} \pm 0.003_{\rm syst} \pm 0.007_{\psi K^{\pm}}. \quad (8)$$

It is important to note that these CP asymmetries come with opposite signs.

The CP asymmetry was measured across the Dalitz plot and this is the most interesting result. "Local" CP asymmetries come also with opposite signs, but are much larger:

$$A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-})_{\text{'local'}} = +0.678 \pm 0.078_{\rm stat} \pm 0.032_{\rm syst} \pm 0.007_{\psi K^{\pm}} \quad (9)$$

$$A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-})_{\text{'local'}} = -0.226 \pm 0.020_{\rm stat} \pm 0.004_{\rm syst} \pm 0.007_{\psi K^{\pm}}. \quad (10)$$

"Local" CP asymmetries here mean the following:

(i) positive asymmetry at low $m_{\pi^+\pi^-}$ just below m_{ρ^0}

(ii) negative asymmetry both at low and high $m_{K^+K^-}$ values.

There is another important aspect: asymmetries are observed in regions of the phase space not associated to any particular resonance.

A very similar effect was observed in even more CKM suppressed three-body FS, namely $B^+ \to \pi^+\pi^-\pi^+$ and $B^+ \to \pi^+K^-K^+$. The LHCb experiment has measured these averaged and local CP asymmetries [17]:

$$A_{CP}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) = +0.120 \pm 0.020_{\rm stat} \pm 0.019_{\rm syst} \pm 0.007_{J/\psi K^{\pm}} \quad (11)$$

$$A_{CP}(B^{\pm} \to \pi^{\pm}K^{+}K^{-}) = -0.153 \pm 0.046_{\rm stat} \pm 0.019_{\rm syst} \pm 0.007_{J/\psi K^{\pm}} \quad (12)$$

$$A_{CP}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-})|_{\text{'local'}} = +0.584 \pm 0.082_{\rm stat} \pm 0.027_{\rm syst} \pm 0.007_{\psi K^{\pm}} \quad (13)$$

$$A_{CP}(B^{\pm} \to \pi^{\pm}K^{+}K^{-})|_{\text{'local'}} = -0.648 \pm 0.070_{\rm stat} \pm 0.013_{\rm syst} \pm 0.007_{\psi K^{\pm}}. \quad (14)$$

Again it is very interesting that LHCb data in Eqs. (11)–(14) show CP asymmetries with opposite signs as "natural" by CPT invariance, no matter what forces produce them. Again, a CPT symmetry argument has to include neutral bosons as well.

In summary, the results from charmless three-body $B^{\pm}$ decays are very intriguing. Large regional effects, diluted when phase space integration is performed, appear in regions not associated to resonances, and with opposite signs in FS that are related by rescattering. Do they show impact of ND? We refer to [8,9,18,19] for additional discussions on this issue.

II. SIMULATIONS OF $D^{\pm} \to \pi^{\pm}K^{-}K^{+}$ AND $D^{\pm} \to \pi^{\pm}\pi^{-}\pi^{+}$

A. Correlations between $D^{\pm} \to \pi^{\pm}K^{+}K^{-}$ and $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$

In this section we perform simulations of the $D^{\pm} \to \pi^{\pm}K^{-}K^{+}$ Dalitz plot to illustrate the rescattering effects discussed above. The simulations are performed in the framework of the isobar model. It is well known that a sum of Breit-Wigners plus a nonresonant term is not a correct representation of the S-wave [20], but the goal here is not to extract quantitative information on the resonant structure of the decay. Rather, we are interested in the differences between D^+ and D^- Dalitz plots that reflect CPV effects with and without the constraints of CPT constraint.

For the decay amplitude we use the resonant substructure is based on Dalitz plot analysis performed by CLEO-c Collaboration [21]. For simplicity, we neglect contributions from amplitudes that result in decay fractions smaller than 1%. The resonant amplitudes are written as a product of form factors, relativistic Breit-Wigners and spin amplitudes. We use the following amplitudes: $A_{\phi\pi} = A(D^+ \to \phi\pi^+)$, $A_{K*K} = A(D^+ \to K^*(892)^0K^+)$, $A_{K_0^*K} = A(D^+ \to K_0^*(1430)^0K^+)$, $A_{a_0\pi} = A(D^+ \to a_0(1450)^0\pi^+)$, and $A_{\kappa K} = A(D^+ \to \kappa(800)K^+)$.

The decay amplitudes are written as

$$\mathcal{A} = \sum c_j A_j \quad (15)$$

$$\bar{\mathcal{A}} = \sum \bar{c}_j A_j \quad (16)$$

with $c_j \equiv a_j e^{i\delta_j}$, $j = \phi\pi,\ K*K,\ K_0^*K,\ a_0\pi,\ \kappa K$. The amplitudes A_j involve only CP-even, strong phases from the Breit-Wigner functions. Weak phases are included in the phase of the c_j coefficients. CP conservation imply $c_j = \bar{c}_j$ for all j.

The couplings c_j between the jth resonant mode and the initial state are complex for two reasons:

(i) Weak forces between quarks may produce phases that are opposite for antiquarks.

(ii) the decay amplitude is affected by hadronic FSI. Strong phases due to the resonance-bachelor rescattering are included in δ_j and they are the same for hadrons and antihadrons.

The Dalitz plot of the $D^{\pm} \to \pi^{\pm}K^{-}K^{+}$ decay is shown in Fig. 1. The prominent contributions from the $\phi\pi$ and $K^*(892)^0K^+$ are clearly visible. The contribution from the broad S-wave $K^-\pi^+$ resonances can be seen at the edges of the $s_{K^-\pi^+}$ axis.

The set of coefficients c_j ($\bar{c}_j$) defines, thus, the decay amplitude $\mathcal{A}$ ($\bar{\mathcal{A}}$). In our simulations we assume no

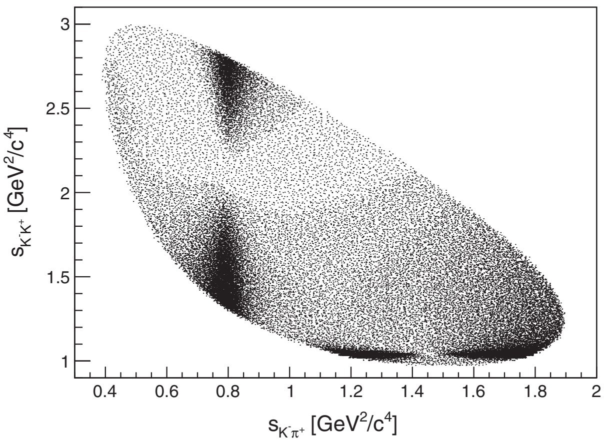

FIG. 1. A simulation of the Dalitz plot of the decay $D \to K^- K^+ \pi^+$. The decay model is taken from CLEO-c (see text for details) and is used as the starting point of our studies.

production asymmetries and identical detection efficiencies, so the number of D^+ and D^- decays is proportional to the integral of the decay amplitudes over the Dalitz plot,

$$N_{D^+} \propto \int |\mathcal{A}|^2 ds_{KK} ds_{K\pi} \qquad N_{D^-} \propto \int |\bar{\mathcal{A}}|^2 ds_{KK} ds_{K\pi}. \tag{17}$$

In the case of CP conservation we have exactly the same number of D^+ and D^-. But if there is CPV the values of the two integrals will differ, in general. We simulate the D^+ and the D^- Dalitz plot separately, seeding CPV in the latter. We always simulate 3×10^6 $D^+ \to K^- K^+ \pi^+$ decays. The number of generated D^- decays is defined according to the ratio of the above integrals, which depends on how CPV is seeded.

The averaged CP asymmetry is computed as

$$A_{CP} = \frac{\int |\mathcal{A}|^2 ds_{KK} ds_{K\pi} - \int |\bar{\mathcal{A}}|^2 ds_{KK} ds_{K\pi}}{\int |A|^2 ds_{KK} ds_{K\pi} + \int |\bar{\mathcal{A}}|^2 ds_{KK} ds_{K\pi}}. \tag{18}$$

The D^+ and D^- Dalitz plots, simulated as described above, are compared using the Miranda method [2,3]. In this method the $D^\pm$ Dalitz plot is divided into bins; a comparison between the D^+ and D^- Dalitz plot is performed directly in a bin-by-bin basis, computing, for each bin, the anisotropy variable

$$\mathcal{S}^i_{CP} = \frac{N^+_i - N^-_i}{\sqrt{N^+_i + N^-_i}}, \tag{19}$$

with N^+_i and N^-_i being the ith bin content of the D^+ and D^- Dalitz plots, respectively.

The value of $\mathcal{S}^i_{CP}$ is a measure of the significance of the excess of one charge over the other in the ith bin. Notice that $\mathcal{S}^i_{CP}$ may be positive or negative. If CP is conserved, N^+_i and N^-_i will differ only by statistical fluctuations. The values of $\mathcal{S}^i_{CP}$, in this case, are distributed according to a unit Gaussian centered at zero. As an example, we show in Fig. 2 a simulation in which CP is conserved—the same number of D^+ and D^- decays are simulated with $c_j = \bar{c}_j$. The plot on the left has the distribution of $\mathcal{S}^i_{CP}$ across the Dalitz plot. No region show any excess of on charge over the other, as expected. The distribution of $\mathcal{S}^i_{CP}$ is shown on the plot on the left, with a unit Gaussian centered at zero superimposed.

There are a number of models for CPV beyond the SM. In this exercise we assume a simple scenario, consistent with the SM, in which CPV manifests as a difference in

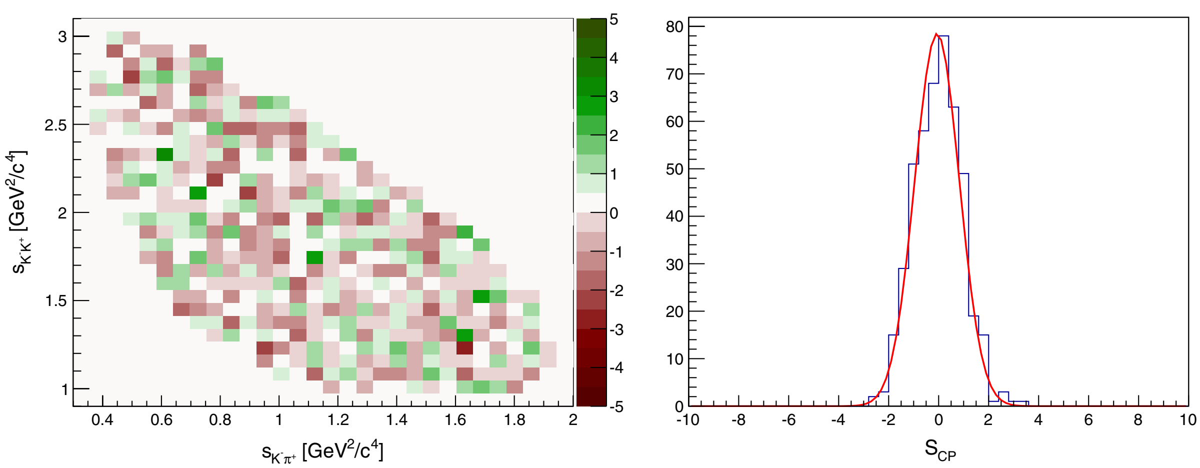

FIG. 2 (color online). A simulation of the Dalitz plot of the decays $D^+ \to K^- K^+ \pi^+$ and $D^- \to K^+ K^- \pi^-$. No CPV is seeded: the same set of coefficients c_j is used for the simulation of the D^+ and the D^- samples. Values of $\mathcal{S}^i_{CP}$ are, in this case, distributed according to a unit Gaussian centered at zero. No excess of one charge over the other is observed in any region of the Dalitz plot, apart from statistical fluctuations.

relative phase of the $K^-\pi^+$ and K^-K^+ resonances in the D^+ and the D^- Dalitz plots. We refer to this as the SM scenario. We first simulate CPV in this SM scenario (SM CPV, for short). Then we simulate the contribution to $D^+ \to K^-K^+\pi^+$ from the $D^+ \to \pi^-\pi^+\pi^+$ decay via $\pi^+\pi^- \to K^+K^-$ rescattering In this simulation we "turn off" the SM CPV and introduce a small CPV effect in $D^+ \to \pi^-\pi^+\pi^+$. Finally the full simulation including both effects is performed.

Our SM CPV consists in introducing a 3° difference in the relative phases of the K^-K^+ and $K^-\pi^+$ resonances when the D^- sample is generated. This 3° difference causes a minor excess of D^- over D^+ resulting in an averaged asymmetry of 0.08%, beyond the current experimental sensitivity. The one- and two-dimensional distributions of $\mathcal{S}^i_{CP}$ for the SM CPV simulation are shown in Fig. 3. Large local asymmetries are observed, mostly in regions where the K^-K^+ and $K^-\pi^+$ amplitudes overlap. The asymmetry is modulated by the strong phase variation of the resonances, leading to negative values of $\mathcal{S}^i_{CP}$ in some regions of the Dalitz plot and positive in another ones. We see how large local effects can result in a very small averaged asymmetry. The distribution of S_{CP} values is no longer centered at zero ($\mu = -0.395 \pm 0.076$) and is significantly wider than a unit Gaussian ($\sigma = 1.56 \pm 0.06$).

We now illustrate the effect of the *CPT* constraint. In the $D^+ \to K^-K^+\pi^+$ decay amplitude, we now introduce the contribution from the $D \to \pi^-\pi^+\pi^+$ decay through through the $\pi\pi \leftrightarrow KK$ scattering, but keeping $c_j = \bar{c}_j$. Weak phases are in general obscured by the strong ones, but here is an instance where the existence of the strong phase favors the observation of small differences in weak phases.

The rescattering term is inspired by Eqs. (4), (5). For simplicity, the weak amplitude for the $D^+ \to \pi^-\pi^+\pi^+$ decay is represented by a complex constant, $T_{D\to 3\pi}$, with an unknown modulus and *CP* odd phase.

The $\pi\pi \to KK$ scattering amplitude is written as $T_{\pi\pi\to KK} = A_{\pi\pi\to KK}e^{i\delta_{\pi\pi\to KK}}$. The real functions $A_{\pi\pi\to KK}$ and $\delta_{\pi\pi\to KK}$ are taken from [13]. $T_{\pi\pi\to KK}$ is *CP* invariant. The decay amplitudes become

$$\mathcal{A} = c_{\phi\pi}A_{\phi\pi} + c_{a_0\pi}A_{a_0\pi} + c_{\kappa K}A_{\kappa K} + c_{K^*K}A_{K^*K} + c_{K_0^*K}A_{K_0^*K} + T_{D\to 3\pi}T_{\pi\pi\to KK}, \tag{20}$$

$$\bar{\mathcal{A}} = \bar{c}_{\phi\pi}A_{\phi\pi} + \bar{c}_{a_0\pi}A_{a_0\pi} + c_{\kappa K}A_{\kappa K} + c_{K^*K}A_{K^*K} + \bar{c}_{K_0^*K}A_{K_0^*K} + \bar{T}_{D\to 3\pi}T_{\pi\pi\to KK}. \tag{21}$$

Before performing the full simulation, we investigate the effect of the rescattering term *alone*, which means $c_j = \bar{c}_j$. In Eqs. (20), (21) we set $|\bar{T}_{D\to 3\pi}| = 0.9|T_{D\to 3\pi}|$ and $\arg(\bar{T}_{D\to 3\pi}) = \arg(T_{D\to 3\pi}) + 5°$. The values of $|T_{D\to 3\pi}|$ and $\arg(T_{D\to 3\pi})$ are unknown. We chose arbitrary values that yield a small decay fraction of approximately 2% for the rescattering contribution. This small amount of rescattering and the small difference introduced between $T_{D\to 3\pi}$ and $\bar{T}_{D\to 3\pi}$ are sufficient to yield a *CP* asymmetry of approximately 0.7%, well within the current experimental sensitivity.

The one- and two-dimensional distributions of $\mathcal{S}^i_{CP}$ for this simulation are shown in Fig. 4. The effect of the global asymmetry is a displacement of the mean of the $\mathcal{S}^i_{CP}$ distribution (right plot). The width of the Gaussian, $\sigma = 1.747 \pm 0.067$, deviates significantly from unity. The Dalitz plot exhibits a clear excess of D^- over D^+ events towards lower values of $m^2_{K^+K^-}$, as expected since $|T_{\pi\pi\to KK}|$ has a maximum near 1.2 GeV$/c^2$.

We are now ready for the full simulation. The D^- sample is generated with the set of $\bar{c}_j$ coefficients used in the SM CPV example, whereas the rescattering term is as described above. The $\mathcal{S}^i_{CP}$ distributions are shown in Fig. 5.

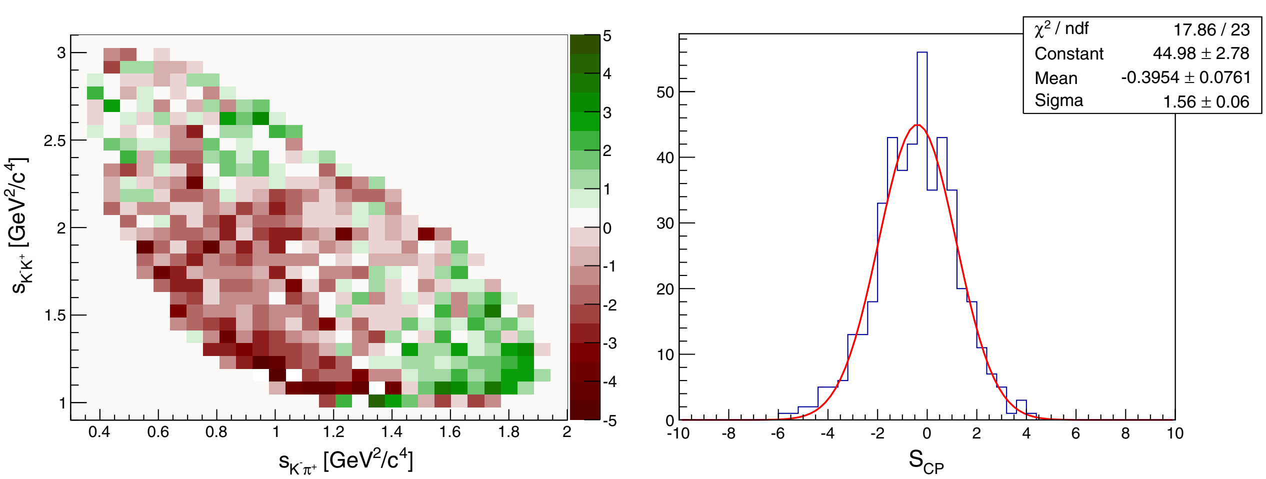

FIG. 3 (color online). A simulation of *CP* violation in the decay $D \to K^-K^+\pi^+$. A 3° difference in the K^*K^+ and $\phi\pi^+$ relative phase between D^+ and D^- is introduced. The difference in relative phase cause the *CP* asymmetry to change sign across the Dalitz plot, according to the phase variation of the interfering Breit-Wigners functions.

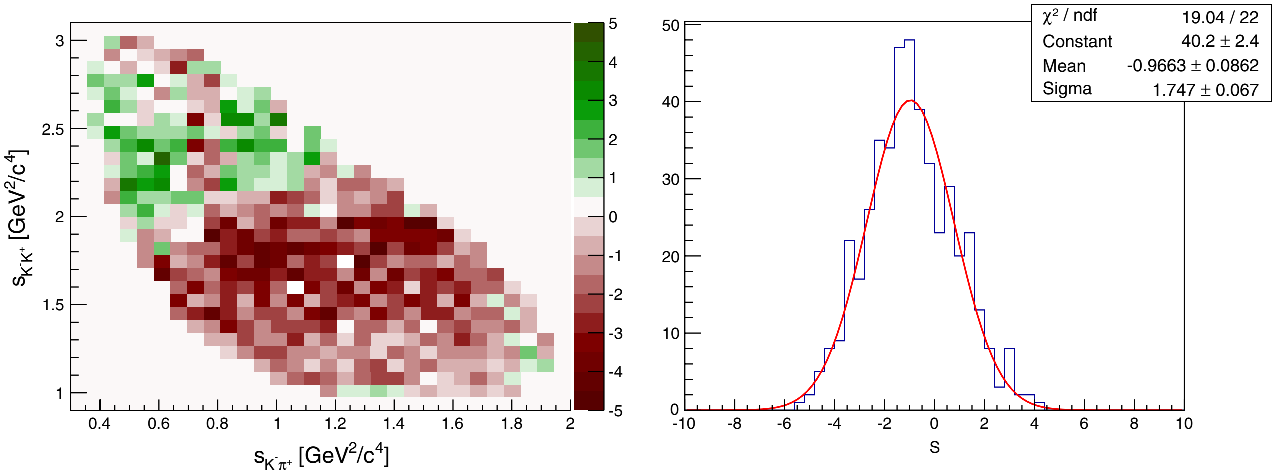

FIG. 4 (color online). A simulation of the Dalitz plot of the decay $D^+ \to K^-K^+\pi^+$. The same set of coefficients c_j are used for both D^+ and D^-. A rescattering term in the decay amplitude is introduced (see text for details), being different for D^+ and D^-. The distribution in the left panel is fitted to a Gaussian with free mean and width.

We do not know how large the strong rescattering term should be, or what value the weak phase of $T_{D\to 3\pi}$ should take. The effect of the rescattering in the CP asymmetry depends, naturally, on the assumed difference between $T_{D\to 3\pi}$ and $\bar{T}_{D\to 3\pi}$. One should keep in mind that decays with neutrals must be considered in a comprehensive treatment. But with this simple simulation we show how the addition of a rescattering contribution may change not only the pattern of the SM-CPV asymmetry of Fig. 3, but also give rise to a global CP asymmetry. Different combinations of $|T_{D\to 3\pi}|$ and $\arg(T_{D\to 3\pi})$ yielding decay fractions up to a few percent were tested, always with similar results. With this investigation we want to call attention to the importance of exploring the constraints of CPT symmetry, showing how the rescattering contribution may increase both local and phase space integrated effects.

B. ND in $D^\pm \to \pi^\pm K^+K^-$ with $D^\pm \to \pi^\pm\pi^+\pi^-$

We discuss now one last example: how one can use the Dalitz plot to access the impact of ND. There are a number of extensions of the SM. We explore a scenario in which ND manifests as an enhancement of em CP violation effects associated with the broad scalars (like charged Higgs exchanges). These resonances populates the whole Dalitz plot, interfering with all other components. The resulting asymmetries would be spread all over the phase space.

As discussed before, for the sake of simulations the use of Breit-Wigners parameterization in the context of the isobar model is good enough to highlight the impact of CPV. Better tools—like refined dispersion relations [12] based on the data of low-energy strong scattering—have to be developed when it comes to analyse the large data sets from LHCb.

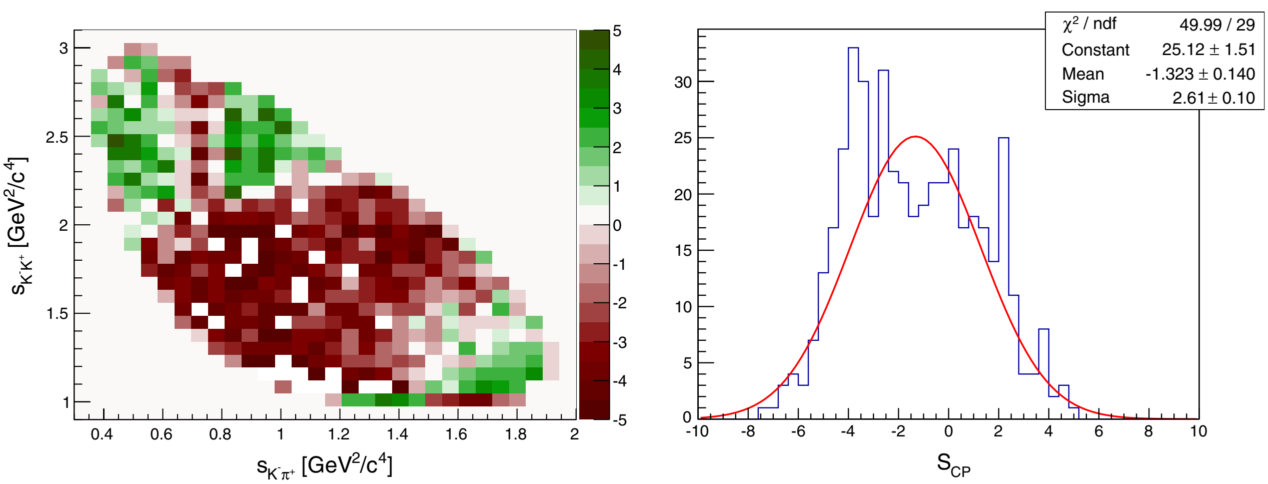

FIG. 5 (color online). A simulation of CP violation in the decay $D^+ \to K^-K^+\pi^+$. A 3° difference in the K^*K^+ and $\phi\pi^+$ relative phase between D^+ and D^- is introduced, in addition to the difference in the rescattering term.

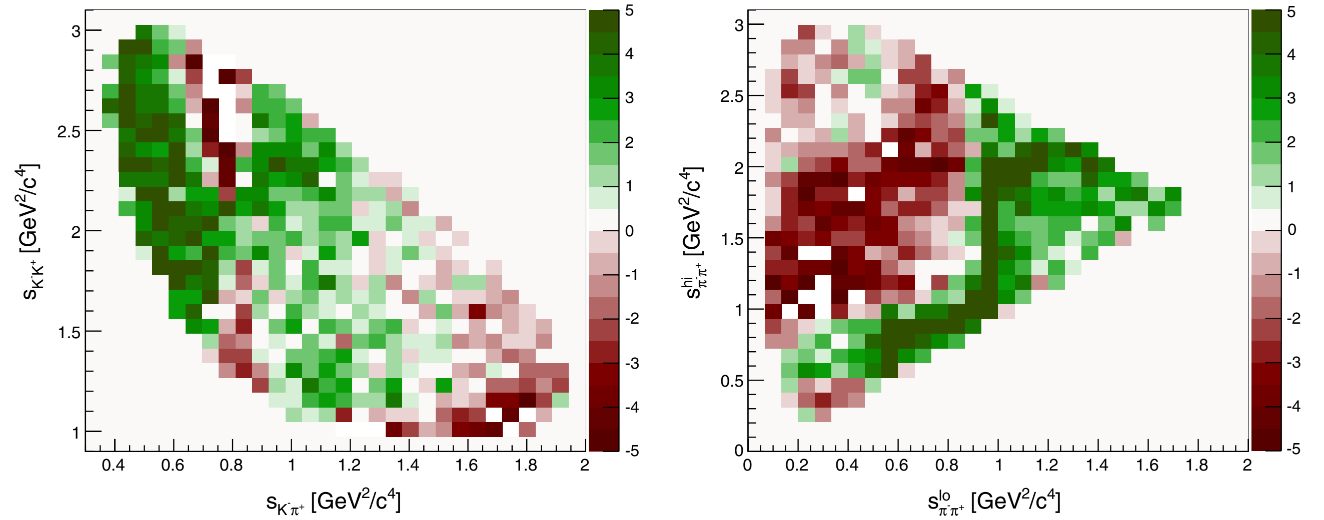

FIG. 6 (color online). Simulation the decays $D^+ \to K^-K^+\pi^+$ (left) and $D^+ \to \pi^-\pi^+\pi^+$ (right). *CP* violation is seeded inspired in a ND scenario in which there is an asymmetry between the coupling between the D meson and the light scalars.

As in our previous simulations, we use the decay amplitude from CLEO-c [21], for the $D^\pm \to \pi^\pm K^+K^-$. For the $D^\pm \to \pi^\pm\pi^+\pi^-$ we use the results from E791 [22]. *CP* violation is seeded as a 1% difference in the strength of the coupling of the D^+ and D^- mesons to the light scalars κ and σ, plus a 1° phase difference.

The distributions of the values from the Miranda procedure across the Dalitz plot with CPV seeded as described above are shown in Fig. 6. The broadness of the scalars cause the *CP* violation effects to be spread over a large portions of the Dalitz plots, being more intense as one approaches the resonance nominal mass. The asymmetry pattern in this example is significantly different from that of the SM CPV of Fig. 3.

III. DISCUSSION

In B transitions one has to find a nonleading source of *CP* violation. We had emphasized the need to go beyond the phase space integrated *CP* asymmetries and probe regional effects on Dalitz plots of three-body B decays [2,3,8,9,18]. It is crucial to understand the impact of $\pi\pi \leftrightarrow \pi\pi$, $K\bar{K} \leftrightarrow K\bar{K}$, $K\pi \leftrightarrow K\pi$ and more.

The landscape is very different for charm decays, where no *CP* violation has been found yet. So far theoretical and experimental efforts have focused mostly on two-body FS of charm mesons. This is no surprise since two-body decays are much simpler to treat than three-body ones. However, in order to understand the possible impact of ND in an eventual observations of *CP* violation in charm decays, one definetely needs to go beyond the ratio of integrated rates and study the pattern of regional CPV. This is the main message of this paper. One has to do it in steps to understand the information that the data will give us.

The SM produces only small *CP* asymmetries in SCS decays and very close to zero in DCS one. In this respect, the mere observation of CPV in DCS decays would be a strong indication of ND. DCS rates, however, are very small and very large data sets would be required.

Singly Cabibbo suppressed decays are much more promising. Very large data sets already exist. In this paper we have produced simulations of three-body singly Cabibbo suppressed $D^\pm$ decays. We focused on the $D^\pm \to \pi^\pm\pi^+\pi^-/\pi^\pm K^+K^-$ and explored the consequences of the *CPT* invariance. It is crucial to understand the impact of $\pi\pi \leftrightarrow \pi\pi$, $\pi\pi \leftrightarrow K\bar{K}$, $K\pi \leftrightarrow K\pi$ etc.

This is obviously very challenging. CPV in decays of heavy flavor involves an interplay between the degrees of freedom at the quark level and long distance effects of low energy hadron physics. One needs to think beyond the simple valence quark diagrams. The U-spin symmetry was invented by Lipkin [23]. Later it was applied to B decays many times, as one can see in these Refs. [24]; in [25] it was suggested that one might to deal with U-spin violation of the order of 10%–20%.

As discussed in Ref. [1], the data indicate much larger violations in exclusive decays $D^0 \to K^+K^-$ vs $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-\pi^+\pi^-$ vs $D^0 \to \pi^+\pi^-\pi^+\pi^-$ but much smaller in the sum of D these decays.

The simulations we performed illustrate the impact of the correlations due to *CPT* invariance, which establishes useful connections between different FS related to each other via strong rescattering. FSI interactions are indeed a crucial ingredient for any accurate Dalitz plot analysis with the contemporary data sets. Much more theoretical work is necessary in order to produce better decay models.

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