Jet-induced collective modes in an anisotropic quark-gluon plasma

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We discuss the characteristics of collective modes induced by relativistic jets in a collisionless anisotropic quark-gluon plasma (AQGP) assuming a colorless Tsunami-like momentum distribution of the jet partons. Within the framework of the transport equation, we derive and discuss the dispersion relations for both the stable and unstable modes of the composite system in the Vlasov approximation. We consider the case when the wave vector is parallel to the anisotropy direction as the growth rate of the unstable mode is maximum in this scenario. When the wave vector ($\mathbf{k}$) is perpendicular to the jet velocity ($\mathbf{v}_{\rm jet}$), two stable modes are found (referred to as mode I and mode II hereafter) of which one is independent of the jet velocity. In case of $\mathbf{k}\|\mathbf{v}_{\rm jet}$, we obtain two identical modes (mode I) and one distinct mode (referred to as mode III hereafter). In all of the cases it is found that stable modes shift toward the light cone for nonzero values of the anisotropy parameter (ξ) and the jet strength (η). In case of unstable mode I, the growth rate increases with ξ for fixed η. The growth rate in case of modes II and III increases with ξ and η, and the nature of the increase depends on the jet velocity.

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I. INTRODUCTION

The primary goal of the ultrarelativistic heavy-ion collision experiments at BNL RHIC and at CERN LHC is to study the properties of a deconfined state of the QCD matter, commonly known as quark-gluon plasma (QGP). According to lattice calculation, the novel state of matter is expected to be formed when the temperature of the nuclear matter is raised to the critical value $T_c \sim 170$ MeV or the energy density of the nuclear matter is raised to above 1 GeV$/fm^3$. High-energy partons behave as hard probes which are produced in the early stage of the collision due to hard scattering. In a relativistic heavy-ion collision, jets with high transverse momentum travel through the hot and dense medium losing energy by collisional (interaction with thermal quark and gluon) and radiative processes (bremsstrahlung). As a consequence, high p_T hadrons produced due to parton fragmentation are suppressed. This phenomenon is known as jet quenching [1].

Apart from jet quenching, the jet particles also interact with the plasma leading to modified collective oscillation. Moreover, supersonic jets propagating through the plasma lead to conical flow behind it in the form of shock waves with Mach cone structure in the medium. This produces a color charge density wake and also wake potential [2–5] in the QGP. Such studies have been performed in Refs. [6,7]. The experimental evidence of the azimuthal dihadron correlation at RHIC shows a double-peak structure in the away side [8,9] for the intermediate p_T particles. Such peaks were predicted as a signature of Mach shocks [6,7] and Cherenkov-like radiation [10] created by the partonic jets traveling through the QGP. It should be noted that the interaction of a relativistic stream of charge particles with an electromagnetic plasma also influences the collective modes of the system, which may lead to some observables relevant to the heavy-ion phenomenology. First, Manuel, Mrowczynski, and Mannarelli [11–13] have studied unstable collective modes of a system composed by an equilibrated and isotropic QGP when a relativistic jet of partons passes through the medium. Within the framework of linear response theory, the interaction between the jet and the plasma shows an exponential growth of collective gauge fields, with a colorless tsunamilike initial momentum distribution of the jet. The unstable modes arise at a velocity of the jet larger than the speed of sound in the QGP. At a lower velocity of the jet, the modes are unstable under certain conditions. In another work [14], the energy loss due to stream instabilities induced by two jets has been discussed in great detail. All of the phenomenological treatments are performed in situations where the distributions of the soft partons providing the thermal background are assumed to have isotropic momentum distributions. In a realistic scenario, due to rapid longitudinal expansion at the onset of the QGP phase, anisotropy arises in the $p_T - p_L$ plane with $\langle {p_L}^2\rangle \ll \langle {p_T}^2\rangle$ in the local rest frame [15–17]. Such momentum-space anisotropy leads to the collective modes having a characteristic behavior distinct from what happens in isotropic plasma which has been extensively studied in [18,19] where it is shown that the gluonic collective modes can be unstable.

In the present work we concentrate on the collective modes in an anisotropic quark-gluon plasma (AQGP) induced by relativistic jets. In studying the evolution of such a system, we use the method of the plasma physics within the framework of the quark-gluon transport theory

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[20,21] in weak coupling regime, i.e., $g \ll 1$. The time scale for the evolution of collective modes is assumed to be much shorter than the interparticle collision time. In this approach, we have neglected the hard mode interactions, assuming that the interactions between jet and plasma are only mediated by mean gauge fields. Kinetic instability can occur due to the interaction of the plasma and jet partons which leads to electric or magnetic-type instabilities with the latter being analogous to the Weibel instability [22] in the QGP [20,23–25]. Plasma instabilities could be an explanation for the fast isotropization predicted by the study of elliptic flow at relativistic heavy ion collider (RHIC) [26,27]. In momentum-space anisotropic plasma, the growth rate of the magnetic instability is maximum in the direction of anisotropy [18,19,28]. So we concentrate on this special case in which the momentum of the collective mode is in the direction of the anisotropy. The nonequilibrium jet of particles while traveling through AQGP destabilizes the plasma producing the collective gauge fields. In the present work we give a quantitative estimate of how the passage of the jet affects the dispersion relations of the collective modes and the growth rate of the instabilities in AQGP. For demonstrative purpose, a colorless tsunamilike jet distribution has been considered in which case some portion of the calculation can be done analytically.

The organization of the paper is as follows. In Sec. II we present analytic expression for the hard-loop gluon polarization tensor in case of anisotropic plasma and the polarization tensor of the jet, and then we show how the dispersion relations are modified in the presence of a tsunamilike jet along with the numerical results. Finally, we conclude in Sec. III.

II. POLARIZATION TENSORS AND COLLECTIVE MODES IN AQGP INDUCED BY A JET

In this section we briefly mention how the polarization tensors can be obtained in a purely anisotropic QGP and in an AQGP with a jet propagating through it. Then we write down the dispersion relations in AQGP (without the jet) as well for the composite system comprising of the jet. To do this it may be recalled that with the linear approximation of transport equations(Vlasov approximation) one can solve the polarization tensor for particles species λ [12,13,18,28,29]:

$$\Pi_\lambda^{\mu\nu}(K) = g^2 \int_p p^\mu \frac{\partial f_\lambda(\mathbf{p})}{\partial p_\beta}\left(g^{\beta\nu} - \frac{p^\nu k^\beta}{P.K + i\epsilon}\right), \quad (1)$$

where λ specify the quarks, antiquarks, gluons, or partials of jet. $f(\mathbf{p})$ is an arbitrary distribution. This tensor is symmetric, $\Pi^{\mu\nu}(K) = \Pi^{\nu\mu}(K)$, and transverse, $K^\mu \Pi^{\mu\nu} = 0$.

We first discuss the structure of the polarization function in a purely AQGP, i.e., in absence of the jet. To include the local momentum-space anisotropy in the plasma, one has to calculate the gluon polarization tensor incorporating anisotropic distribution function of the particles. This subsequently can be used to construct the hard thermal loop (HTL)-corrected gluon propagator which, in general, assumes a very complicated form. Such an HTL propagator was first derived in [18] in the temporal-axial gauge.

In an anisotropic plasma (without the jet), the spacelike component of the self-energy tensor can be written as [18]

$$\Pi_p^{ij}(K) = -g^2 \int \frac{d^3p}{(2\pi)^3} v^i \partial^l f(\mathbf{p})\left(\delta^{jl} + \frac{v^j k^l}{K.V + i\epsilon}\right). \quad (2)$$

The phase-space distribution is assumed to be given by the following ansatz [18]:

$$f(\mathbf{p}) = f_\xi(\mathbf{p}) = f_{\rm iso}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}.\hat{\mathbf{n}})^2}\right). \quad (3)$$

Here $f_{\rm iso}$ is an arbitrary isotropic distribution function, and $\hat{\mathbf{n}}$ is the direction of anisotropy. The parameter ξ is the degree of anisotropy $(-1 < \xi < \infty)$ and is given by $\xi = \frac{1}{2}\frac{\langle p_T^2\rangle}{\langle p_z^2\rangle} - 1$. By making a change of variable $(\tilde{p}^2 = p^2(1 + \xi(\mathbf{p}.\hat{\mathbf{n}})^2))$ the spatial components can be written as

$$\Pi_p^{ij}(K) = m_D^2 \int \frac{d\Omega}{(4\pi)} v^i \frac{v^l + \xi(\mathbf{v}.\hat{\mathbf{n}})n^l}{(1+\xi(\mathbf{v}.\hat{\mathbf{n}})^2)^2}\left(\delta^{jl} + \frac{v^j k^l}{(K.V + i\epsilon)}\right), \quad (4)$$

where

$$m_D^2 = -\frac{g^2}{2\pi^2}\int_0^\infty dp\, p^2 \frac{df_{\rm iso}(p^2)}{dp} \quad (5)$$

is the isotropic Debye mass which depends on $f_{\rm iso}$. The self-energy, apart from four-momentum (K^μ), also depends on the anisotropic vector $(n^\mu = (1, \mathbf{n}))$. Using the proper tensor basis [18,30], one can decompose the self-energy into the four-structure functions as

$$\Pi_p^{ij}(K) = \alpha A^{ij} + \beta B^{ij} + \gamma C^{ij} + \delta D^{ij}, \quad (6)$$

where

$$A^{ij} = \delta^{ij} - k^i k^j/\mathbf{k^2}, \quad B^{ij} = k^i k^j/\mathbf{k^2},$$
$$C^{ij} = \tilde{n}^i \tilde{n}^j/\tilde{n}^2, \quad D^{ij} = k^i \tilde{n}^j + k^j \tilde{n}^i, \quad (7)$$

with $\tilde{n}^i = A^{ij} n^j$, which obeys $\tilde{\mathbf{n}}.\mathbf{k} = 0$ and $n^2 = 1$. Now α, β, γ and δ are determined by the following contractions:

$$k^i \Pi^{ij} k^j = \mathbf{k}^2 \beta, \quad \tilde{n}^i \Pi^{ij} k^j = \tilde{n}^2 \mathbf{k}^2 \delta,$$
$$\tilde{n}^i \Pi^{ij} \tilde{n}^j = \tilde{n}^2(\alpha + \gamma), \quad {\rm Tr}\Pi^{ij} = 2\alpha + \beta + \gamma. \quad (8)$$

The expressions for structure functions have been given in Ref. [18]. In the isotropic limit, $\xi \to 0$, the structure functions γ and δ vanish, and α and β are directly related to the transverse and longitudinal components of the polarization tensor of the plasma, respectively.

The dispersion law for the collective modes of anisotropic plasma in temporal axial gauge can be determined by finding the poles of the propagator $\tilde{\mathbf{\Delta}}^{ij}$

$$[\tilde{\mathbf{\Delta}}^{-1}(K)]^{ij} = (\mathbf{k}^2 - \omega^2)\delta^{ij} - k^i k^j + \Pi_p^{ij}(K). \tag{9}$$

By substituting Eq. (7) in the above equation and performing the inverse formula [18] one finds

$$\tilde{\mathbf{\Delta}}(K) = \tilde{\mathbf{\Delta}}_A[\mathbf{A} - \mathbf{C}] + \tilde{\mathbf{\Delta}}_G[(\mathbf{k}^2 - \omega^2 + \alpha + \gamma)\mathbf{B} + (\beta - \omega^2)\mathbf{C} - \delta\mathbf{D}]. \tag{10}$$

The dispersion relation for the gluonic modes in anisotropic plasma (without jet) is given by the zeroes of

$$\tilde{\mathbf{\Delta}}_A^{-1}(K) = k^2 - \omega^2 + \alpha = 0, \tag{11}$$

$$\tilde{\mathbf{\Delta}}_G^{-1}(K) = (k^2 - \omega^2 + \alpha + \gamma)(\beta - \omega^2) - k^2\tilde{n}^2\delta^2 = 0. \tag{12}$$

If we examine the propagators (11) and (12) in the static limit($\omega \to 0$), we find that there are three mass scales [18]: m_α and $m_\pm$. In isotropic limit, $\xi \to 0$, $m_\alpha^2 = m_-^2 = 0$ and $m_+^2 = m_D^2$.

The solutions of the above two equations depend on $m_D, \omega, \mathbf{k}, \xi$ and $\hat{\mathbf{k}}.\hat{\mathbf{n}} = \cos\theta_n$. For $\xi > 0$ one finds that there are at most three stable and two unstable modes which depend on θ_n, and for $\xi < 0$, there are three stable modes; however, only one is unstable [18,19]. In general, the expressions for the structure functions are very complicated [18,19] and cannot be evaluated analytically for the general case, i.e., for the arbitrary orientation of the wave vector with respect to the anisotropy direction. However, if we concentrate on the case where the momentum $\mathbf{k}$ is in the direction of anisotropy $\hat{\mathbf{n}}(\mathbf{k}||\hat{\mathbf{n}})$, analytical evaluations of the structure functions are possible. Moreover, in such a case, the structure function γ and $\tilde{n}^2 = 1 - (\hat{\mathbf{k}}.\hat{\mathbf{n}})^2$ are identically equal to zero. The other structure functions are obtained using the contractions given in Eq. (8), and the final results simplify to [19]

$$\begin{aligned}
\alpha(\omega,k,\xi) &= \frac{m_D^2}{4\sqrt{\xi}(1+\xi z^2)^2}\bigg[(1+z^2+\xi[(\xi-1)z^4+(\xi+6)z^2-1])\arctan\Big[\sqrt{\xi}\Big] \\
&\quad + \sqrt{\xi}(z^2-1)\left(1+\xi z^2 - z(1+\xi)\ln\frac{z+1+i\varepsilon}{z-1+i\varepsilon}\right)\bigg], \\
\beta(\omega,k,\xi) &= -\frac{m_D^2 z^2}{2\sqrt{\xi}(1+\xi z^2)^2}\left[(1+\xi)(1-\xi z^2)\arctan\sqrt{\xi} + \sqrt{\xi}\left(1+\xi z^2 - z(1+\xi)\ln\frac{z+1+i\varepsilon}{z-1+i\varepsilon}\right)\right], \\
\delta(\omega,k,\xi) &= \frac{m_D^2 z}{4\sqrt{\xi}(1+\xi z^2)^3}\bigg[(\xi[\xi z^4(\xi+3) - 2z^2(\xi^2+6\xi+3)+6\xi+3]-1)z\arctan\sqrt{\xi} \\
&\quad + \sqrt{\xi}\left(z(1+\xi z^2)(1+4\xi-3\xi z^2) + \xi(z^2-1)(4z^2+3\xi z^2-1)\ln\frac{z+1+i\varepsilon}{z-1+i\varepsilon}\right)\bigg],
\end{aligned} \tag{13}$$

where $z = \omega/k$.

In order to study the collective modes due to the propagation of an energetic jet, we have to find also the structure of the polarization tensor induced by the jet. For simplicity, this can be done by assuming a colorless tsunamilike momentum distribution of the jet [13,31]:

$$f_{\mathrm{jet}}(\mathbf{p}) = \bar{n}\bar{u}^0\delta^{(3)}(\mathbf{p} - \Lambda\bar{\mathbf{u}}). \tag{14}$$

Here $\bar{n}$ is a parameter proportional to the density, and $\bar{u}^\mu = \gamma(1, \mathbf{v}_{\mathrm{jet}})$ is the four-velocity, where γ is the Lorentz factor and $\mathbf{v}_{\mathrm{jet}}$ is the velocity of the jet parton. The parameter Λ fixes the scale of energy of particles. By substituting Eq. (14) in Eq. (2), one deduces the following expression of the polarization tensor for the jet partons:

$$\Pi_{\mathrm{jet}}^{ij}(K) = \omega_{\mathrm{jet}}^2\left(\delta^{ij} + \frac{k^i v_{\mathrm{jet}}^j + k^j v_{\mathrm{jet}}^i}{\omega - \mathbf{k}.\mathbf{v}_{\mathrm{jet}}} - \frac{(\omega^2 - \mathbf{k}^2)v_{\mathrm{jet}}^i v_{\mathrm{jet}}^j}{(\omega - \mathbf{k}.\mathbf{v}_{\mathrm{jet}})^2}\right), \tag{15}$$

where v_{jet} is the velocity of jet and $\omega_{\mathrm{jet}}^2 = \frac{g^2\bar{n}}{2\Lambda}$ is the plasma frequency of the jet. The dispersion relations of the collective modes of the system due to jet are determined by searching the poles of the propagator of Eq. (9) by replacing Π_p^{ij} with Π_{jet}^{ij}, i.e., by finding the solution $\omega(k)$.

In this work, our main aim is to study the collective modes of the anisotropic plasma due to the propagation of an energetic jet. To do this, we note that in a very short time regime, where the Vlasov approximation is valid, the total polarization of the system is given by the sum of the two polarization tensors:

$$\Pi_t^{\mu\nu}(K) = \Pi_p^{\mu\nu}(K) + \Pi_{\mathrm{jet}}^{\mu\nu}(K). \tag{16}$$

The dispersion relation for the collective modes of the total system can be found by solving the equation

$$\det[(k^2-\omega^2)\delta^{ij}-k^ik^j+\Pi_t^{ij}(K)]=0. \tag{17}$$

In order to simplify the dispersion relation [Eq. (17)], we define a dimensionless quantity $\eta=\omega_{\rm jet}^2/\omega_t^2$, where $\omega_t^2=\omega_{\rm jet}^2+m_D^2/3$ which is related to Debye mass and the plasma frequency of the jet. The solution of the dispersion relation depends on $|\mathbf{k}|$, jet velocity($|\mathbf{v}_{\rm jet}|$), η and the angle between momentum $\mathbf{k}$ and the jet velocity $\mathbf{v}_{\rm jet}$. It is clearly seen that the jet does not interact with the plasma when $\eta=0$. In the following subsection we analyze jet-induced collective modes in AQGP for the two special cases, when the jet velocity is (a) perpendicular to the momentum, i.e., $\mathbf{k}\perp\mathbf{v}_{\rm jet}$, and (b) parallel to the momentum, i.e., $\mathbf{k}||\mathbf{v}_{\rm jet}$.

A. k orthogonal to $\mathbf{v}_{\rm jet}$

Here we choose the coordinates $\mathbf{k}=(0,0,k)$ and $\mathbf{v}_{\rm jet}=(v_{\rm jet},0,0)$. In this case the condition (17) reads as

$$d_{22}(d_{11}d_{33}-d_{13}^2)=0, \tag{18}$$

where

$$\begin{aligned}
d_{11}&=(k^2-\omega^2)+\alpha(\omega,k,\xi)+\eta\omega_t^2\left(1-\frac{(\omega^2-k^2)v_{\rm jet}^2}{\omega^2}\right),\\
d_{13}&=\eta\omega_t^2\frac{k}{\omega}v_{\rm jet},\\
d_{22}&=(k^2-\omega^2)+\alpha(\omega,k,\xi)+\eta\omega_t^2,\\
d_{33}&=\beta(\omega,k,\xi)-\omega^2+\eta\omega_t^2.
\end{aligned} \tag{19}$$

1. Stable modes

First, we find the stable collective modes which have poles at real-valued $\omega>|\mathbf{k}|$. The dispersion relation for all of the collective modes of the composite system can be determined by finding the solution to the equations

$$d_{22}=(k^2-\omega^2)+\alpha(\omega,k,\xi)+\eta\omega_t^2=0, \tag{20}$$

and

$$\begin{aligned}
d_{11}d_{33}-d_{13}^2=\Bigg[&(k^2-\omega^2)+\alpha(\omega,k,\xi)\\
&+\eta\omega_t^2\left(1-\frac{(\omega^2-k^2)v_{\rm jet}^2}{\omega^2}\right)\Bigg][\beta(\omega,k,\xi)\\
&-\omega^2+\eta\omega_t^2]-\left(\eta\omega_t^2\frac{k}{\omega}v_{\rm jet}\right)^2=0,
\end{aligned} \tag{21}$$

which is referred to as mode I and mode II, respectively. It may be noted that when $\eta=0$ one recovers the usual modes as in AQGP given by Eqs. (11) and (12) with $\gamma=0$ and $\tilde{n}^2=0$. It is clearly seen that the collective mode I is independent of jet velocity. However, it depends on the plasma frequency of the jet. In Fig. 1 we present the dispersion relation for the mode I for different values of the anisotropy parameter $\xi(0,1,5)$, $m_D=\sqrt{3}$ and $\eta=0.1$. It is clearly noticed that the collective modes are very sensitive to the anisotropy parameter, and with the increase of ξ, it is diminished. The results for stable mode II are shown in Fig. 2 for two different $\eta(0.05,0.1)$ at two different jet velocities (0.55, 0.99). In all of the cases the collective modes shift toward the light cone with the increase of ξ. At the low-momentum region, the collective modes are enhanced with the increase of the strength of the jet(η) for both the jet velocities considered here. Moreover, at fixed values of η and ξ, the modes are increased with the increase of the jet velocity.

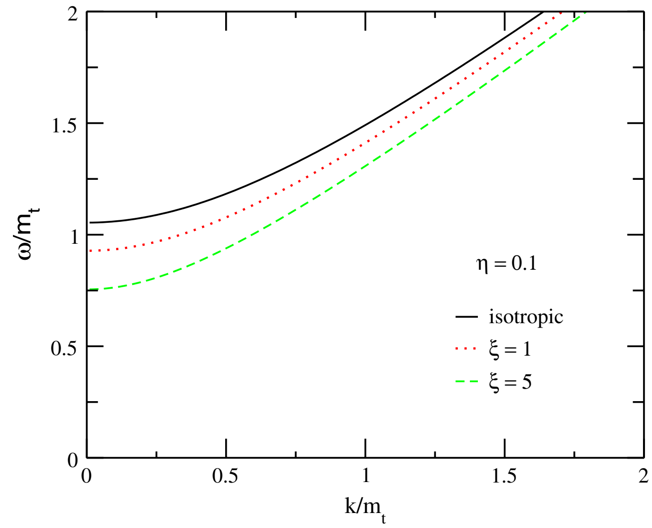

FIG. 1 (color online). Dispersion relation for the stable mode I for an anisotropy parameter $\xi=\{0,1,5\}$ and fixed $\eta=0.1$.

2. Unstable modes

In an AQGP, when the momentum is parallel to the anisotropy direction, i.e., $\mathbf{k}||\hat{\mathbf{n}}$, the scale m_α^2 and m_-^2 are negative at $\omega\to0$ [18]. It indicates that for $\xi>0$ the system possesses an magnetic instability [32,33]. This can be identified as the so-called filamentation or Weibel instability [22]. The instability is driven by the energy transferred from the particles to the field, which leads to a more rapid thermalization and equilibration of QGP.

We now investigate how the colorless tsunamilike jet distribution affects the growth rates of these instabilities. The dispersion relation Eqs. (20) and (21) also have poles along the imaginary ω axis. The dispersion relation for these modes can be calculated by taking $\omega=\omega_0+i\Gamma$ with ω_0 and Γ are real valued. Numerically we find that the collective modes of Eqs. (20) and (21) are nonpropagating ($\omega_0=0$). The solution $\omega(k)=i\Gamma(k)$ gives the growth rate

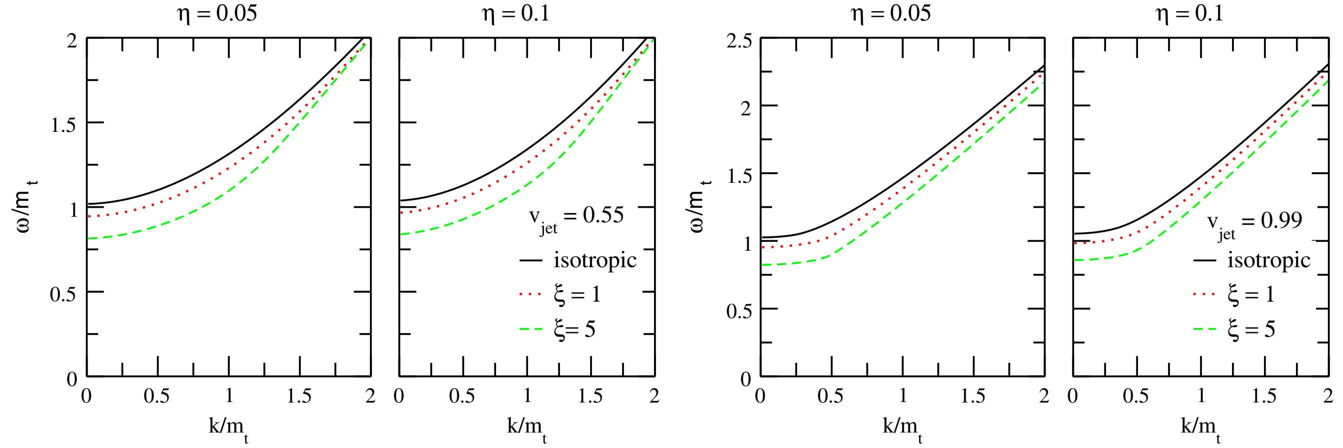

FIG. 2 (color online). Dispersion relation for the stable mode II for an anisotropy parameter $\xi = \{0, 1, 5\}$ and different $\eta = \{0.05, 0.1\}$ at two jet velocities $v_{\mathrm{jet}} = \{0.55, 0.99\}$.

$\Gamma(k)$. At $\xi = 0$ we do not find any unstable collective mode for mode I. In Fig. 3 we present the unstable collective mode I at two different strength of the anisotropy ($\xi = \{1, 5\}$) for three different values of η. In both cases we find that with the increasing values of η the maximum values of Γ decrease and the corresponding value of the momentum remains approximately the same at fixed ξ. However, at fixed η, the maximum value of the growth rate Γ increases with ξ and shifted toward the higher momentum. Figure 4 describes the imaginary part of the dispersion relation for the unstable collective mode II for two different velocities of jet. When the velocity of the jet is less than the average speed of the plasmon, the growth rate increases with the increase of ξ at fixed η, and the maximum point switches toward the higher value of the momentum, as shown in Fig. 4(a). It is also seen that the maximum value of Γ is increased with increasing value of η at same values of ξ, and the corresponding value of the momentum remains approximately the same [13]. For $v_{\mathrm{jet}} = 0.99$, i.e., for the jet velocity greater than the phase velocity of plasmon, the unstable collective mode II of the composite system is shown in Fig. 4(b). It is clearly noticed that the value of Γ increases with the increasing values of ξ and η. But the values of the momenta at which the maxima of Γ occur are independent of ξ and η. In order to find the wave number $k_{\mathrm{max}}(\xi, v_{\mathrm{jet}}, \eta)$ at which the unstable modes of the spectrum terminates, we take the limit $\Gamma \to 0$ to obtain

$$k_{\mathrm{max}} = \frac{\omega_t}{2\sqrt{1-v_{\mathrm{jet}}^2}}\left[3 - 2v_{\mathrm{jet}}^2((\eta-3)-7\eta) - 3(\eta-1) \right.$$
$$\left. \times(-1+\xi+2v_{\mathrm{jet}}(1+\xi))\frac{\arctan[\sqrt{\xi}]}{\sqrt{\xi}}\right]^{1/2}. \qquad (22)$$

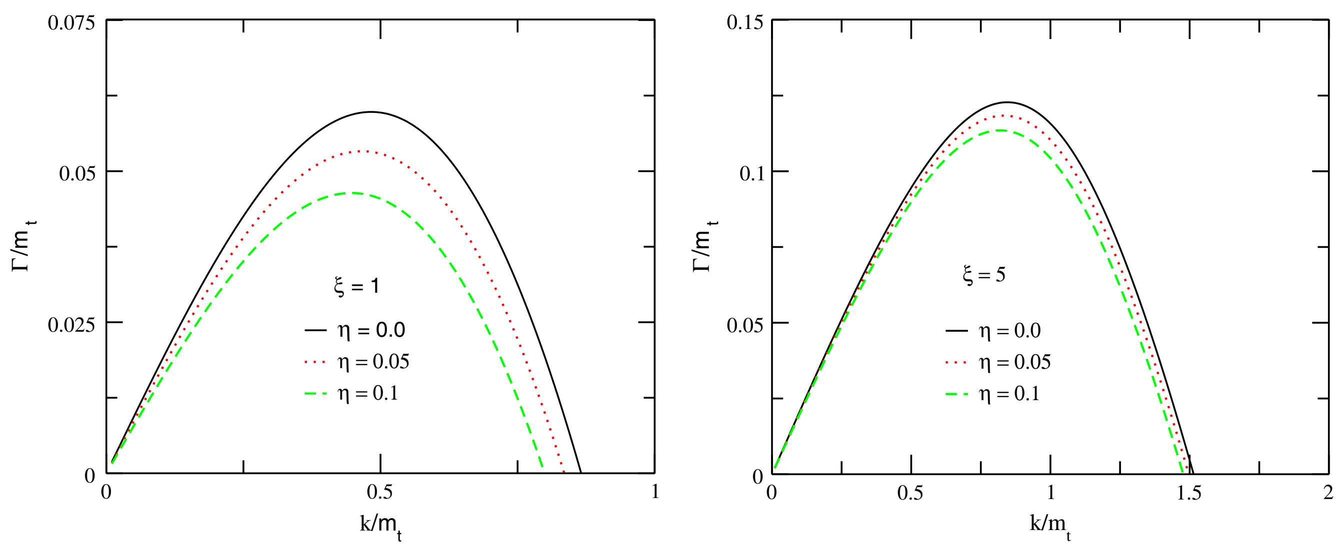

FIG. 3 (color online). Imaginary part of the dispersion relation of the unstable mode I at $\xi = 1$ (left) and at $\xi = 5$ (right) for different values of $\eta = \{0, 0.05, 0.1\}$.

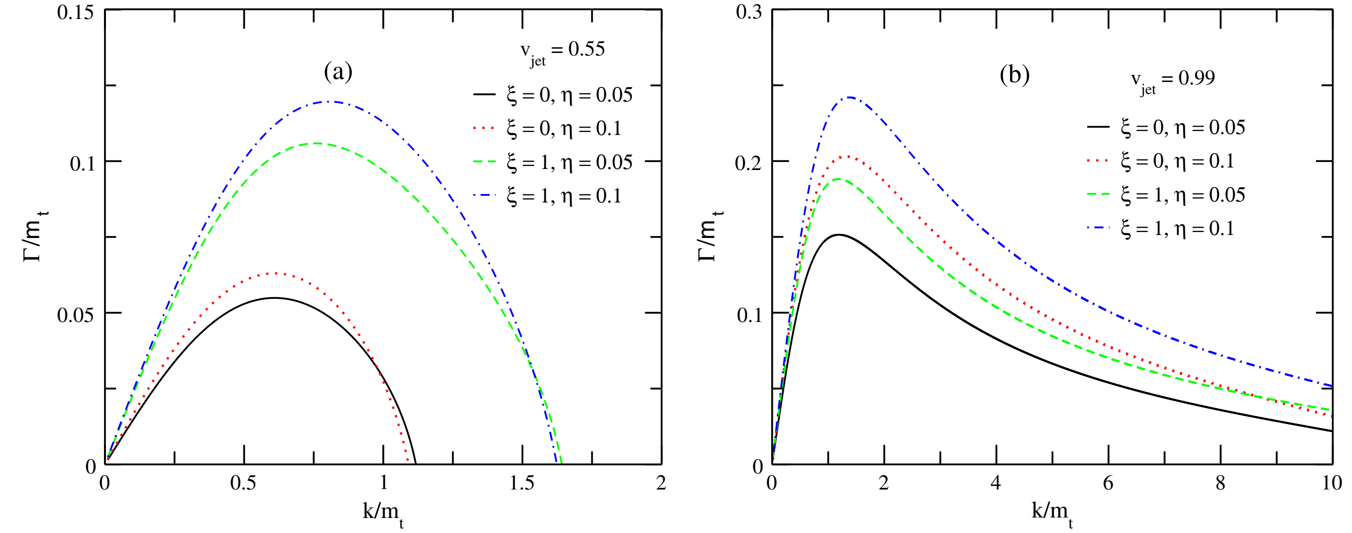

FIG. 4 (color online). Imaginary part of the dispersion relation of the unstable mode II as a function of k for two different values of the velocity of the jet with the variation of ξ and η. The left (right) panel corresponds to $v = 0.55(0.99)$.

The minimum value for the jet velocity where the unstable mode is generated is given by

$$v_{\min} = \frac{1}{\sqrt{2}} \frac{\sqrt{3 - 7\eta - 3(\eta - 1)(\xi - 1)\frac{\arctan[\sqrt{\xi}]}{\sqrt{\xi}}}}{\sqrt{-3 + \eta + 3(\eta - 1)(1 + \xi)\frac{\arctan[\sqrt{\xi}]}{\sqrt{\xi}}}}. \tag{23}$$

Therefore, the threshold value of the jet velocity depends both on η and the anisotropy parameter ξ.

B. k parallel to $\mathbf{v}_{\mathrm{jet}}$

We now discuss the unstable collective mode for the system composed by the AQGP and the jet in the case where the jet velocity is parallel to the momentum of the collective modes. The jet velocity and the wave vector are chosen as $\mathbf{k} = (0, 0, k)$ and $\mathbf{v}_{\mathrm{jet}} = (0, 0, v_{\mathrm{jet}})$, and the corresponding solution of Eq. (17) is

$$d'_{11} d'_{22} d'_{33} = 0, \tag{24}$$

where

$$d'_{11} = (k^2 - \omega^2) + \alpha(\omega, k, \xi) + \eta\omega_t^2 = d'_{22}, \tag{25}$$

$$d'_{33} = \beta(\omega, k, \xi) - \omega^2 + \eta\omega_t^2 \frac{\omega^2(1 - v_{\mathrm{jet}}^2)}{(\omega - k v_{\mathrm{jet}})^2}. \tag{26}$$

The solution of the dispersion relation shows that the collective modes of the composite system have three solutions. But the expressions d'_{11} and d'_{22} are the same and identical to Eq. (20). Consequently, we found similar collective modes as collective mode I in the case when $\mathbf{k}||\mathbf{v}_{\mathrm{jet}}$. Hence, the dispersion relation for the other collective mode can be determined by solving

$$(\beta - \omega^2)(\omega - k v_{\mathrm{jet}})^2 + \eta\omega_t^2\omega^2(1 - v_{\mathrm{jet}}^2) = 0, \tag{27}$$

which is referred to as mode III.

1. Stable modes

In Fig. 5 we present the dispersion relation for the collective mode III for two different values of $\eta(0.05, 0.1)$ at two different values of velocity (0.55, 0.99) of the jet with the variation of anisotropy parameter ξ. In all of the cases we find that the collective modes depend on the anisotropy parameter, and with the increase of ξ, the modes are shifted toward the light cone. At $v_{\mathrm{jet}} = 0.55$, due to the effect of η, a marginally enhancement of collective modes is seen in the lower momentum region at fixed ξ, but at a higher value of the jet velocity, it is unaffected.

2. Unstable modes

Numerically we find that the solutions of Eq. (27) are an unstable propagating mode, i.e., $\omega_0 \neq 0$. At $v_{\mathrm{jet}} = 0.55$, the maximum value of the growth rate Γ increases with the increase of the anisotropy parameter ξ and the strength of the jet(η) is shown in Fig. 6(a). The value of the momentum corresponding to the maximum of Γ is the same for all the cases considered here. The behavior of the growth rate is completely different when the value of the jet velocity is greater than the plasmon phase velocity, as shown in Fig. 6(b). At fixed value of ξ, the maximum value of Γ increases with η, and the corresponding value of the momentum is approximately independent of η. Due to the effect of the anisotropy, the maximum value of the growth rate Γ does not change at fixed η, but the value of the momentum is shifted toward the origin with the increase of ξ.

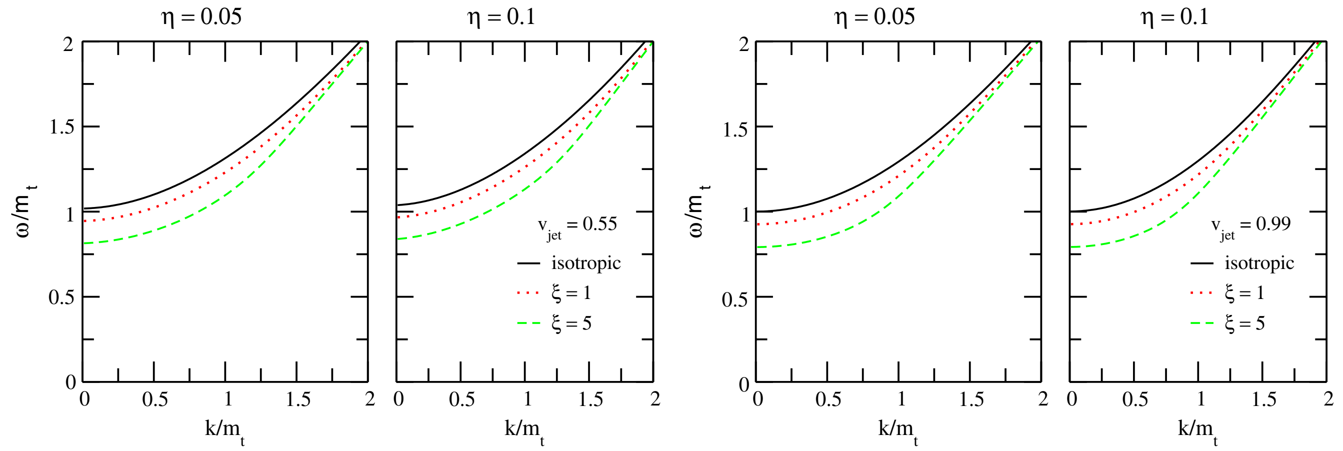

FIG. 5 (color online). Dispersion relation for the stable mode III for an anisotropy parameter $\xi = \{0, 1, 5\}$ and different $\eta = \{0.05, 0.1\}$ at two jet velocities $v_{\text{jet}} = \{0.55, 0.99\}$.

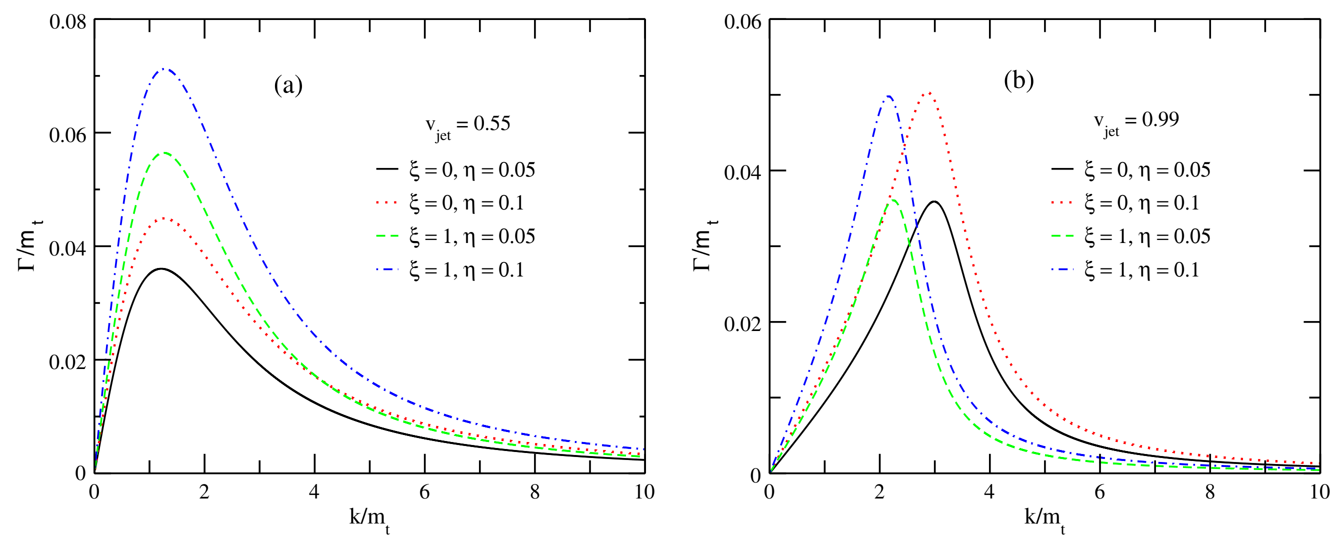

FIG. 6 (color online). Imaginary part of the dispersion relation of the unstable mode III as a function of k for two different values of the velocity of the jet with the variation of ξ and η. The left (right) panel corresponds to $v = 0.55(0.99)$.

III. SUMMARY

In this paper we have studied the effects of the jet of particles on the collective modes of a QCD plasma which is anisotropic in momentum-space using transport theory. To simplify the analysis, we considered the initial distribution of the jet of particles to be a colorless tsunamilike distribution. As we know, the filamentation instability of AQGP is a maximum when the direction of the wave vector is parallel to the anisotropy($\mathbf{k}||\hat{\mathbf{n}}$); we concentrate our calculation on this particular case of such a composite system. For the case when $\mathbf{k}\perp\mathbf{v}_{\text{jet}}$, we find two stable modes (I and II), of which one is independent of the jet velocity; however, both the modes strongly depend on the value of ξ, and for nonzero ξ, the modes shift toward the light cone. In case of $\mathbf{k}||\mathbf{v}_{\text{jet}}$ we obtain two identical modes as mode I and one distinct mode (III) which also shifts toward the light cone for nonzero ξ.

We note that the velocity dependence of modes II and III strongly affects the growth rate (Γ) of the unstable modes. In case of mode I, the maximum value of the growth rate increases with the strength of the anisotropy. In case of mode II, Γ_{max} increases with the velocity of the jet. With the increase of η or ξ, Γ_{max} also increases for both the jet velocities considered here. It is also seen that for jet velocity less than the plasmon speed the values of the momenta for which Γ is maximum are approximately independent of the jet strength for fixed ξ. For nonzero ξ, the value of the momentum for which Γ is maximum shifts toward the higher value. It is also important to note that the nature of the growth rate strongly depends on the jet velocity. We also notice that if the jet velocity is greater than the plasmon speed, the maximum growth rate is independent of ξ and η. When the momentum of the collective modes is collinear with the velocity of the jet, the maximum growth rate of the instability increases with ξ and η at the jet velocity $v_{\rm jet} = 0.55$, and the corresponding momentum is $k \approx 1.75\,\omega_t$ for mode III. However, $\Gamma_{\rm max}$ is independent of ξ when the jet velocity is greater than the phase velocity of plasmon at fixed η. Due to the increase of the strength of the jet, the growth rate is also increased. Moreover, the stable collective mode III is almost unaffected by the jet velocity. In our numerical analysis, we find that the maximum value of the growth rate of such a system varies from $0.05\omega_t$ to $0.25\omega_t$. Therefore, the plasma instability fully develops on the time scale of the order of $t \sim (4-20)/\omega_t$, and we can estimate the upper bound of this time scale evaluating the plasma frequency in a weak coupling scenario at $T \sim 350$ MeV, finding $t \sim 1-3$ fm/c.

[1] M. Gyulassy, P. Levai, and I. Vitev, Nucl. Phys. **B571**, 197 (2000); M. Djordjevic and U. Heinz, Phys. Rev. Lett. **101**, 022302 (2008); R. Baier, Y. L. Dokshitzer, A. H. Mueller, and D. Schiff, J. High Energy Phys. 09 (2001), 033; A. K. Dutt-Mazumder, J. Alam, P. Roy, and B. Sinha, Phys. Rev. D **71**, 094016 (2005); P. Roy and A. K. Dutt-Mazumder, Phys. Rev. C **83**, 044904 (2011).

[2] J. Ruppert and B. Muller, Phys. Lett. B **618**, 123 (2005); J. Ruppert and B. Muller, Nucl. Phys. **A774**, 397 (2006).

[3] P. Chakraborty, M. G. Mustafa, and M. H. Thoma, Phys. Rev. D **74**, 094002 (2006); P. Chakraborty, M. G. Mustafa, R. Ray, and M. H. Thoma, J. Phys. G **34**, 2141 (2007).

[4] B.-f. Jiang and J.-r. Li, Nucl. Phys. **A856**, 121 (2011); J. Phys. G **39**, 025007 (2012).

[5] M. Mandal and P. Roy, Phys. Rev. D **86**, 114002 (2012); Phys. Rev. D **88**, 074013 (2013).

[6] J. Casalderrey-Solana, E. V. Shuryak, and D. Teaney, J. Phys. Conf. Ser. **27**, 22 (2005); J. Casalderrey-Solana, J. Phys. G **34**, S345 (2007).

[7] H. Stoecker, Nucl. Phys. **A750**, 121 (2005).

[8] J. Adams *et al.* (STAR Collaboration), Phys. Rev. Lett. **95**, 152301 (2005).

[9] S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **97**, 052301 (2006).

[10] V. Koch, A. Majumder, and X.-N. Wang, Phys. Rev. Lett. **96**, 172302 (2006); Phys. Rev. C **73**, 051901 (2006).

[11] C. Manuel and S. Mrowczynski, Phys. Rev. D **74**, 105003 (2006).

[12] M. Mannarelli and C. Manuel, Phys. Rev. D **76**, 094007 (2007).

[13] M. Mannarelli and C. Manuel, Phys. Rev. D **77**, 054018 (2008).

[14] M. Mannarelli, C. Manuel, S. Gonzalez-Solis, and M. Strickland, Phys. Rev. D **81**, 074036 (2010).

[15] M. Martiez and M. Strickland, Phys. Rev. Lett. **100**, 102301 (2008).

[16] L. Bhattacharya and P. Roy, Phys. Rev. C **78**, 064904 (2008).

[17] M. Mandal and P. Roy, Adv. High Energy Phys. **2013**, 371908 (2013).

[18] P. Romatschke and M. Strickland, Phys. Rev. D **68**, 036004 (2003).

[19] P. Romatschke and M. Strickland, Phys. Rev. D **70**, 116006 (2004).

[20] S. Mrowczynski, Phys. Rev. C **49**, 2191 (1994).

[21] H. T. Elze and U. Heinz, Phys. Rep. **183**, 81 (1989).

[22] E. Weibel, Phys. Rev. Lett. **2**, 83 (1959).

[23] S. Mrowczynski, Phys. Lett. B **314**, 118 (1993).

[24] S. Mrowczynski, Phys. Lett. B **393**, 26 (1997).

[25] S. Mrowczynski, Acta Phys. Pol. B **37**, 427 (2006).

[26] P. Arnold, J. Lenaghan, G. D. Moore, and L. G. Yaffe, Phys. Rev. Lett. **94**, 072302 (2005).

[27] A. Rebhan, P. Romatschke, and M. Strickland, Phys. Rev. Lett. **94** 102303 (2005).

[28] B. Schenke, M. Strickland, C. Greiner, and M. H. Thoma, Phys. Rev. D **73**, 125004 (2006).

[29] P. Arnold, D. T. Son, and L. G. Yaffe, Phys. Rev. D **59**, 105020 (1999).

[30] A. Dumitru, Y. Guo, and M. Strickland, Phys. Lett. B **662**, 37 (2008).

[31] R. D. Pisarski, arXiv:hep-ph/9710370.

[32] M. Strickland, Braz. J. Phys. **37**, 762 (2007).

[33] P. Arnold, J. Lenaghan, and G. D. Moor, J. High Energy Phys. 08 (2003) 002; P. Arnold, G. D. Moor, and L. G. Yaffe, J. High Energy Phys. 01 (2003) 030.