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Explaining the $B \to K^* \mu^+ \mu^-$ data with scalar interactions

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Recent LHCb results on the decay $B \to K^* \mu^+ \mu^-$ show significant deviations from the standard model estimates in some of the angular correlations. In this paper we study the possibility of explaining these deviations using new scalar interactions. We show that new dimension-6 four-quark operators of scalar and pseudoscalar type can successfully account for the discrepancy even after being consistent with other experimental measurements. We also briefly discuss possible extensions of the standard model where these operators can be generated.

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I. INTRODUCTION

The standard model (SM) has been extremely successful in explaining all the measurements to date in particle-physics experiments. The Higgs boson, the long awaited last missing piece of the SM, was also discovered recently in the LHC experiment [1,2]. At this moment the main goal of LHC will be to look for signals of new physics (NP) and establish experimentally the existence of physics beyond the SM. While direct search experiments are extremely important in this endeavor, the flavor physics and other low-energy experiments will play complimentary roles to the direct search experiments in particular, if the NP scale is rather high or does not couple significantly to the first two generations of quarks. In fact, deviations from the SM expectations at the level of $\sim 2\sigma$–4σ have already been reported in recent years in a few observables involving decays and mixing of B mesons [3–11]. On the theoretical side also, various NP explanations of these deviations have been suggested [12–38].

The decays involving the $b \to s\mu^+\mu^-$ transition are particularly interesting as they are extremely rare in the SM, and many extensions of the SM are capable of producing measurable effects beyond the SM. In particular, the three-body decay $B \to K^* \mu^+ \mu^-$ offers a large number of observables in the kinematic and angular distributions of the final-state particles, and some of these distributions have also been argued to be less prone to hadronic uncertainties [15–17,20,25,30,39–41].

The LHCb collaboration has recently measured four angular observables (P'_4, P'_5, P'_6, and P'_8 in the notation of Ref. [41]), which are largely free from form-factor uncertainties, in particular, in the large recoil limit (i.e., low invariant mass, $\sqrt{q^2}$, of the dilepton system). For each of the four observables, the data were presented in six q^2 bins, and, quite interestingly, a significant deviation of 3.7σ from the SM expectation was observed only in one of the bins ($4.30 < q^2 < 8.68\ \mathrm{GeV}^2$) for only one observable, the P'_5. It is worth mentioning here that there is still a considerable amount of theoretical uncertainty due to (unknown) power corrections to the factorization framework [42]. Hence, there is a possibility that the observed deviation will be resolved once deeper understanding of these corrections is achieved. In this paper we take the observed deviation at the face value and study its possible explanation from physics beyond the SM.

Note that the observable P'_5 is related to the observable $\mathcal{S}_5$ defined in Refs. [15,35], see Table 1 in Ref. [35] for a precise comparison. We would like to mention here that the observable $\mathcal{S}_5$ is exactly the same (apart from an overall normalization factor of $4/3$) as the longitudinal-transverse asymmetry A_{LT}, which we defined in our earlier work [17] as

$$A_{\mathrm{LT}} = \frac{\int_{-\pi/2}^{\pi/2} d\phi \Big\{ (\int_0^1 - \int_{-1}^0) d\cos\theta_K \frac{d^3\Gamma}{dq^2 d\phi d\cos\theta_K} \Big\}}{\int_{-\pi/2}^{\pi/2} d\phi \Big\{ (\int_0^1 + \int_{-1}^0) d\cos\theta_K \frac{d^3\Gamma}{dq^2 d\phi d\cos\theta_K} \Big\}}, \quad (1)$$

where θ_K and ϕ are two of the total three angles (the other angle θ_μ is integrated) in the full angular distribution of $B \to K^*(\to K\pi)\mu^+\mu^-$ (see Fig. 9 in Ref. [17] for a diagrammatic illustration).

In the SM, the $b \to s$ flavor transition is governed by the effective Hamiltonian,

$$\mathcal{H}_{\mathrm{eff}}^{\mathrm{SM}} = -\frac{4G_F}{\sqrt{2}} (V_{ts}^* V_{tb}) \sum_{i=1}^{10} C_i \mathcal{O}_i, \quad (2)$$

and the decay $B \to K^* \mu^+ \mu^-$ proceeds via the three operators, namely,

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$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\alpha\beta} P_R b) F^{\alpha\beta},$$
$$\mathcal{O}_9 = \frac{\alpha_{\rm em}}{4\pi} (\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \mu) \quad \text{and}$$
$$\mathcal{O}_{10} = \frac{\alpha_{\rm em}}{4\pi} (\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \gamma_5 \mu), \tag{3}$$

with the corresponding Wilson coefficients $\{C_7, C_9, C_{10}\} \simeq \{0.3, 4.1, 4.3\}$ at the scale $\mu = 4.8$ GeV. In models beyond the SM, new chirally flipped ($P_{L(R)} \to P_{R(L)}$) operators $\mathcal{O}'_7$, $\mathcal{O}'_9$, and $\mathcal{O}'_{10}$ may also be generated. It was pointed out in Ref. [17] that $A_{\rm LT}$ is particularly sensitive to the operators $\mathcal{O}_9$, $\mathcal{O}'_9$, $\mathcal{O}_{10}$, and $\mathcal{O}'_{10}$. In fact, a global fit to the NP contribution ($\Delta C_{7,9,10}, \Delta C'_{7,9,10}$) to the above six Wilson coefficients taking into account the recent LHCb data along with the existing data on some other rare and radiative $b \to s$ modes was performed in Ref. [34] (see also Ref. [43]) with the conclusion that the deviations seen in the LHCb experiment can be explained by just adding a large negative contribution to the Wilson coefficient C_9[1]:

$$\Delta C_9 \approx -1.5. \tag{4}$$

A similar fit to the Wilson coefficients was also performed in Ref. [35] with a slightly different conclusion. They reported the best-fit solution to be the one with the presence of NP contributions to both C_9 and C'_9:

$$\Delta C_9 \approx -1.0, \qquad \Delta C'_9 \approx 1.0. \tag{5}$$

Note that the solutions above are rather unusual as most NP models would in general produce not only new contributions to C_9 and C'_9 but also to other operators. In fact, the new Z' boson considered in Refs. [36,38] to explain the data indeed had rather nonstandard couplings to the fermions. It is also worth mentioning that the scalar or pseudoscalar operators of the forms $(\bar{s}P_{L(R)}b)(\bar{\mu}\mu)$ and $(\bar{s}P_{L(R)}b)(\bar{\mu}\gamma_5\mu)$ cannot explain the data owing to their very little effect on $A_{\rm LT}$ [17] in particular, once the consistency with the measured branching ratio of $B_s \to \mu^+\mu-$ is taken into account.[2]

In this work we instead consider new four-quark scalar interactions that couple the third-generation quarks. Possible mixing in the quark sector then leads to flavor-changing $b \to s$ transitions. Note that there is no direct contribution to the decay $b \to s\mu^+\mu^-$ in this case, but it can arise at the one-loop level. As we will show explicitly in the next sections, in this way we can generate new contributions to C_9 and C'_9 with a negligible effect on the other operators. In fact, such four-quark scalar operators involving the third generation of quarks are rather motivated after the discovery of the Higgs particle and can arise in many extensions of the SM, e.g., topcolor models [46–48], R-parity violating supersymmetry, and multi-Higgs models [49].

The precise definition of the NP operators will be given in the next section. In Sec. III we will compute the constraints on these operators from $\bar{B}_s - B_s$ mixing. Their effect on the $B \to K^*\mu^+\mu^-$ decays will be discussed in Sec. IV. We will stop in Sec. V after making some concluding remarks.

II. NEW PHYSICS OPERATORS

As we mentioned in the previous section, in this work we consider effective four-quark scalar interactions of the form

$$\mathcal{H}_{\rm eff}^{\rm NP} = -\frac{\mathcal{G}_1}{\Lambda^2}[\bar{s}(1-\gamma^5)b][\bar{b}(1+\gamma^5)b] - \frac{\mathcal{G}_2}{\Lambda^2}[\bar{s}(1+\gamma^5)b][\bar{b}(1-\gamma^5)b] + \text{h.c.}, \tag{6}$$

which are assumed to be generated by unknown short-distance physics beyond the SM. Here Λ is the scale of NP, and $\mathcal{G}_1$ and $\mathcal{G}_2$ are the Wilson coefficients which parameterize our ignorance about the underlying microscopic theory.

To proceed with our calculations, we will not need to work with specific models that can generate these operators, and hence we will take Eq. (6) as the starting point of our phenomenological analysis. However, as an existence proof, we briefly mention here the topcolor model of Ref. [46]. In such models the top quark participates in a new strong interaction, which is assumed to be spontaneously broken at some high energy scale Λ. The strong interaction, though not confining, leads to the formation of a top condensate $\langle \bar{t}_L t_R \rangle$ resulting in scalar bound states in the low-energy spectrum of the theory which couple strongly to the b quark [47,48]. Integrating out these scalar bound states generates, in the weak interaction basis (denoted by b' below), an effective four-fermion operator of the form

$$\bar{b}'(1+\gamma_5)b'\bar{b}'(1-\gamma_5)b', \tag{7}$$

with possibly rather large couplings [47,48]. The above operator then generates the operators in Eq. (6) once the quark mass matrices are diagonalized, making $\mathcal{G}_{1,2}$ dependent also on the mixing matrices of the left and right chiral down-type quarks.

III. $\bar{B}_s - B_s$ MIXING

The four-quark operators in Eq. (6) will clearly contribute to the $\bar{B}_s - B_s$ mixing at the one-loop level (see Fig. 1). Taking one operator at a time, the diagram in Fig. 1 will generate the operators

[1]See, however, Ref. [44] for a possible subtlety.

[2]In this context, it is also quite interesting to investigate the effect of tensor operators, which definitely deserves a separate dedicated study and will be presented in a future publication [45].

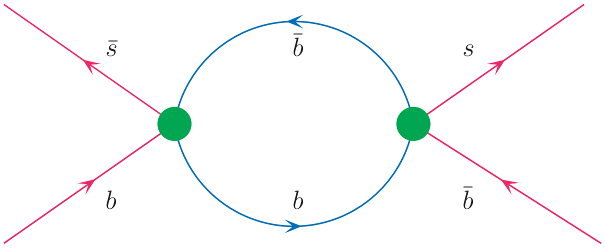


FIG. 1 (color online). Feynman diagram showing $\bar{B}_s - B_s$ mixing generated from the operators in Eq. (6).

$$\mathcal{O}_1 = \mathcal{K}_1[\bar{s}(1-\gamma^5)b][\bar{s}(1-\gamma^5)b] \quad \text{and}$$
$$\mathcal{O}_2 = \mathcal{K}_2[\bar{s}(1+\gamma^5)b][\bar{s}(1+\gamma^5)b], \tag{8}$$

where the effective couplings $\mathcal{K}_1$ and $\mathcal{K}_2$ are given by

$$\mathcal{K}_{1(2)} = -\frac{3\mathcal{G}^2_{1(2)}}{2\pi^2\Lambda^2}\frac{m_b^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{m_b^2}\right). \tag{9}$$

The magnitude of the NP contribution to the mass difference in the $\bar{B}_s - B_s$ system can now be written as

$$|\Delta M^{\rm NP}_{B_s}| = |\mathcal{K}_{1(2)}|\frac{|\langle \bar{B}^0_s|[\bar{s}(1\mp\gamma^5)b][\bar{s}(1\mp\gamma^5)b]|B^0_s\rangle|}{2m_{B_s}}, \tag{10}$$

where m_{B_s} is the mass of the B_s meson. With the following definition of the matrix element [50],

$$\langle \bar{B}^0_s|[\bar{s}(1-\gamma^5)b][\bar{s}(1-\gamma^5)]|B^0_s\rangle$$
$$= -\frac{5}{3}\left(\frac{m_{B_s}}{m_b+m_s}\right)^2 m_{B_s}{}^2 f_{B_s}{}^2\mathcal{B}_{B_s}, \tag{11}$$

where f_{B_s} and $\mathcal{B}_{B_s}$ are the decay constant and relevant bag parameter, respectively, one can now write

$$|\Delta M^{\rm NP}_{B_s}| = \frac{5}{6}\left(\frac{m_{B_s}}{m_b+m_s}\right)^2 m_{B_s} f_{B_s}{}^2\mathcal{B}_{B_s}|\mathcal{K}_{1(2)}|. \tag{12}$$

In Fig. 2 we show the contours of $|\Delta M^{\rm NP}_{B_s}|$ in the $\alpha_{\mathcal{G}_{1(2)}} - \Lambda$ plane ($\alpha_{\mathcal{G}_{1(2)}} \equiv \mathcal{G}^2_{1(2)}/4\pi$), taking the values of the other parameters to be $m_b = 4.8$ GeV, $m_{B_s} = 5.37$ GeV, $f_{B_s} = 225$ MeV, and $\mathcal{B}_{B_s}(m_b) = 0.80$.

The mass difference ΔM_{B_s} has been very precisely measured with its value given by [51]

$$\Delta M^{\rm Exp}_{B_s} = 17.69 \pm 0.08\ {\rm ps}^{-1}, \tag{13}$$

which is consistent with the SM expectation [52],

$$\Delta M^{\rm SM}_{B_s} = 17.3 \pm 2.6\ {\rm ps}^{-1}. \tag{14}$$

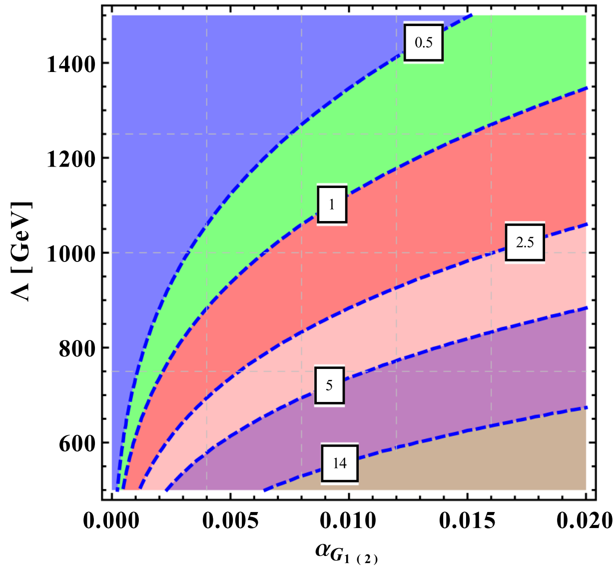


FIG. 2 (color online). Contours of $|\Delta M^{\rm NP}_{B_s}|$ (ps^{-1}) in the $\alpha_{\mathcal{G}_{1(2)}}$-$\Lambda$ plane.

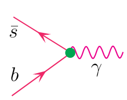
We will conservatively demand that the coupling $\mathcal{G}_{1(2)}$ and the NP scale Λ satisfy the constraint

$$|\Delta M^{\rm NP}_{B_s}| \lesssim 2.5\ {\rm ps}^{-1}. \tag{15}$$

IV. CONTRIBUTION TO $b \to s\mu^+\mu^-$

The effective Hamiltonian $\mathcal{H}^{\rm NP}_{\rm eff}$ of Eq. (6) generates the effective vertices $\bar{s}b\gamma$, $\bar{s}bg$, and $\bar{s}bZ$ at the one-loop level, as shown in Fig. 3. The vertices with a γ or a Z can now contribute to $b \to sl^+l^-$ decay once a lepton pair is attached to them. Note that the operators $\mathcal{O}_7$ or $\mathcal{O}'_7$ are not generated in this way (we will see this explicitly below), and hence there is no new contribution to the decay $b \to s\gamma$.

A computation of the diagram in Fig. 3 (without the lepton pair attached) gives the effective vertex for $\bar{s}b\gamma$ to be



$$= -\sqrt{4\pi\alpha_{em}}\frac{e_b}{\Lambda^2}\bar{s}\,[\mathcal{G}_1\mathcal{R}^\mu_1 + \mathcal{G}_2\mathcal{R}^\mu_2]\,b\,A_\mu \tag{16}$$

where

$$\mathcal{R}^\mu_{1(2)} = \frac{1}{2\pi^2}\int_0^1 dx x(1-x)$$
$$\times\left\{{\rm Ln}\left(\frac{\Lambda^2}{m_b^2}\right) - {\rm Ln}\left(1 - \frac{q^2}{m_b^2}x(1-x)\right)\right\}$$
$$\times[\gamma^\mu q^2 - q^\mu \not{q}]\frac{(1\pm\gamma_5)}{2}. \tag{17}$$

Here $e_b = -\frac{1}{3}$, the electric charge of the b quark in units of electron charge, and q^μ is the 4-momenta of the photon. It is clear from the above expression that the amplitude for

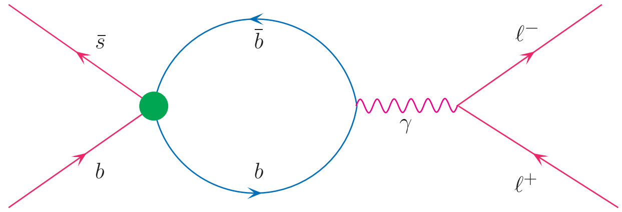
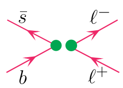


FIG. 3 (color online). Feynman diagram showing how the operator in Eq. (6) contributes to the decay $b \to s l^+ l^-$.

on-shell photon production is identically zero, as claimed in the previous paragraph.

It is now straightforward to calculate the effective vertex for the decay of our interest $b \to s l^+ l^-$ by attaching a lepton pair to the virtual photon. This gives



$$= -(4\pi\alpha_{em})\frac{e_b}{\Lambda^2}\frac{1}{2\pi^2} \times \int_0^1 dx\, x(1-x)\left\{\mathrm{Ln}\left(\frac{\Lambda^2}{m_b^2}\right) - \mathrm{Ln}\left(1 - \frac{q^2}{m_b^2}x(1-x)\right)\right\} [\mathcal{G}_2\mathcal{O}_9 + \mathcal{G}_1\mathcal{O}'_9] \tag{18}$$

Note that the q^μ term in Eq. (16) does not contribute due to electromagnetic gauge invariance. As the contribution coming from a Z exchange is suppressed with respect to the γ exchange by a factor of q^2/M_Z^2, we do not include the Z contribution. This also means the new contributions to C_{10} and C'_{10} are extremely tiny.

Comparing Eq. (18) with Eq. (6), we can now calculate the NP contribution to the Wilson coefficient C_9 and C'_9. This reads

$$\Delta C_9 = \frac{2\sqrt{2}e_b\mathcal{G}_2}{G_F\Lambda^2(V_{ts}^*V_{tb})}\int_0^1 dx x(1-x) \times \left\{\mathrm{Ln}\left(\frac{\Lambda^2}{m_b^2}\right) - \mathrm{Ln}\left(1 - \frac{q^2}{m_b^2}x(1-x)\right)\right\}$$
$$= \frac{2\sqrt{2}e_b\mathcal{G}_2}{G_F\Lambda^2(V_{ts}^*V_{tb})}\left\{\frac{1}{6}\mathrm{Ln}\left(\frac{\Lambda^2}{m_b^2}\right) - \int_0^1 dx x(1-x)\mathrm{Ln} \times \left(1 - \frac{q^2}{m_b^2}x(1-x)\right)\right\}. \tag{19}$$

The expression for $\Delta C'_9$ can be obtained from Eq. (19) after replacing $\mathcal{G}_2$ by $\mathcal{G}_1$. Although ΔC_9 is a function of the dilepton invariant mass q^2, the variation in ΔC_9 in the whole q^2 range is less than 1%, and thus we will neglect this variation below.

In Fig. 4 we show the contours of ΔC_9 in the $\alpha_{\mathcal{G}_{1(2)}}$-$\Lambda$ plane. The green shaded region above the red (dotted) contour satisfies the constraint $|\Delta M_{B_s}^{\mathrm{NP}}| < 2.5\ \mathrm{ps}^{-1}$. Thus, Fig. 4 clearly reveals that the value $\Delta C_9 \approx -1.5$ can indeed be achieved keeping the $\bar{B}_s - B_s$ mixing completely under control and for reasonable choices of $\mathcal{G}_2$ and Λ. In fact, turning on both the couplings $\mathcal{G}_1$ and $\mathcal{G}_2$ with opposite sign can even reproduce the solution in Eq. (5).

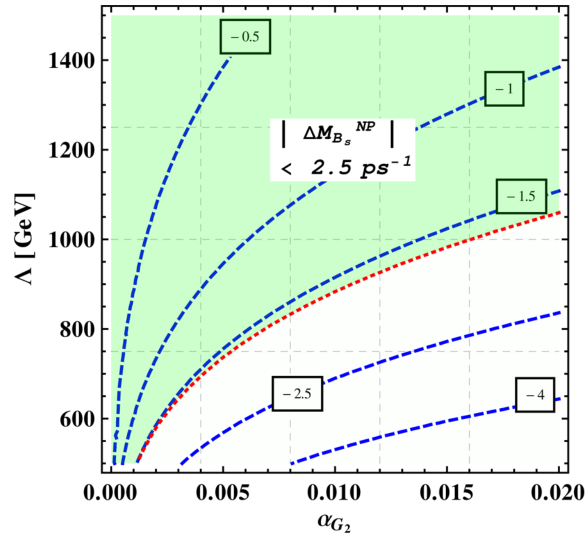


FIG. 4 (color online). Contours (blue, dashed) of ΔC_9 in the $\alpha_{\mathcal{G}_2}$-Λ plane. The green (shaded) region above the red (dotted) curve has $|\Delta M_{B_s}^{\mathrm{NP}}| < 2.5\ \mathrm{ps}^{-1}$.

V. CONCLUSION

In this paper we have studied the possibility of explaining certain deviations from the SM expectations in the angular distribution of the decay $B \to K^*\mu^+\mu^-$ observed recently by the LHCb collaboration. We have shown that new dimension-6 four-quark operators of scalar and pseudoscalar type can naturally account for these deviations without conflicting with other experimental measurements. This is in contrast to generic scalar 4-fermion operators that would in general give rise to new contributions to other decays like $b \to s\gamma$, $B_s \to \mu^+\mu^-$, etc., and hence would be very tightly constrained. We have also briefly mentioned how well-known extensions of the SM can generate these dimension-6 operators. Detailed phenomenological analysis of these models in particular, in view of the large amount of available experimental data, should be carried out and will be presented elsewhere.

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