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Studying light propagation in a locally homogeneous universe through an extended Dyer-Roeder approach

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Light is affected by local inhomogeneities in its propagation, which may alter distances and so cosmological parameter estimation. In the era of precision cosmology, the presence of inhomogeneities may induce systematic errors if not properly accounted. In this vein, a new interpretation of the conventional Dyer-Roeder (DR) approach by allowing light received from distant sources to travel in regions denser than average is proposed. It is argued that the existence of a distribution of small and moderate cosmic voids (or "black regions") implies that its matter content was redistributed to the homogeneous and clustered matter components with the former becoming denser than the cosmic average in the absence of voids. Phenomenologically, this means that the DR smoothness parameter (denoted here by α_E) can be greater than unity, and, therefore, all previous analyses constraining it should be rediscussed with a free upper limit. Accordingly, by performing a statistical analysis involving 557 type Ia supernovae (SNe Ia) from Union2 compilation data in a flat ΛCDM model, we obtain for the extended parameter $\alpha_E = 1.26^{+0.68}_{-0.54}$ (1σ). The effects of α_E are also analyzed for generic ΛCDM models and flat XCDM cosmologies. For both models, we find that a value of α_E greater than unity is able to harmonize SNe Ia and cosmic microwave background observations thereby alleviating the well-known tension between low and high redshift data. Finally, a simple toy model based on the existence of cosmic voids is proposed in order to justify why α_E can be greater than unity as required by supernovae data.

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I. INTRODUCTION

The accelerating cosmic concordance model (flat ΛCDM) is in agreement with all the existing observations both at the background and perturbative levels. However, while more data are being gathered, there is an accumulating evidence that a more realistic description beyond the "precision era" requires a better comprehension of systematic effects in order to have the desirable accuracy.

Local inhomogeneities are not only possible sources of different systematics, but may also signal for an intrinsic incompleteness of the cosmic description. This occurs because the Universe is homogeneous and isotropic only on large scales ($\gtrsim$100 Mpc). However, on smaller scales, a variety of structures involving galaxies, clusters, and superclusters of galaxies are observed. Permeating these structures there are also voids or "black regions" (as dubbed long ago by Zel'dovich [1]) where galaxies are almost or totally absent as recently suggested by the N-body *Millenium* simulations [2]. This means that statistically uniform cosmologies are only coarse-grained representations of what is actually present in the real Universe. As a consequence, the description of light propagation by taking into account such richness of structures is a challenging task to improve the cosmic concordance model, but the correct method still remains far from a consensus [3–8].

Zel'dovich [9], Bertotti [10], Gunn [11], and Kantowski [12] were the first to investigate the influence of small-scale inhomogeneities in the light propagation from distant sources. Later on, Dyer and Roeder (DR) [13] assumed explicitly that only a fraction of the average matter density must affect the light propagation in the intergalactic medium. Phenomenologically, the unknown physical conditions along the path, associated with the clumpiness effects, were described by the smoothness parameter:

$$\alpha = \frac{\rho_h}{\rho_h + \rho_{\rm cl}}, \qquad (1)$$

where ρ_h and $\rho_{\rm cl}$ are the fractions of homogeneous and clumped densities, respectively. This parameter quantifies the fraction of homogeneously distributed matter within a given light cone. For $\alpha = 0$ (empty beam), all matter is clumped while for $\alpha = 1$ the fully homogeneous case is recovered, and for a partial clumpiness the smoothness parameter is restricted over the interval [0,1]. The reader should keep in mind that such a restriction clearly excludes the possibility of light rays traveling in regions denser than

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average. In principle, it should be very interesting to see how the presence of cosmic voids—a key entity nowadays—could be considered in the above prescription.

More recently, many studies concerning the light propagation and its effects on the derived distances have been performed [5,7,14,15]. Current constraints on the smoothness parameter are still weak [16–19]; however, it is intriguing that the quoted analyses had their best fits for α equal to unity which corresponds to a perfectly ΛCDM homogeneous model at all scales [16,17]. More recently, some authors have also argued for a crucial deficiency of the DR approach, and, as such, it should be replaced by a more detailed description, probably, based on the weak lensing approach [5,15].

In this paper we advocate a slightly different but complementary point of view. It will be assumed that the DR approach is a useful tool in the sense that it provides the simplest one-parametric description of the effects caused by local inhomogeneities, but its initial conception needs to be somewhat extended. This is done in two steps: (i) by allowing α (here denoted by α_E) to be greater than unity in the statistical data analyses, and (ii) by interpreting the obtained results in terms of the existence of a distribution of cosmic voids or "black regions" in the Universe (see Sec. V). As we shall see, by performing a statistical analysis involving 557 SNe Ia from the Union2 compilation data [20], we obtain $\alpha_E = 1.26^{+0.68}_{-0.54}$ (1σ) for a flat ΛCDM model. This 1σ confidence region shows that $\alpha_E > 1$ has a very significant probability. We also show that α_E greater than unity is also able to harmonize the low redshift (SNe Ia) and baryon acoustic oscillations (BAO) data with the observations from cosmic microwave background (CMB).

II. THE DYER-ROEDER DISTANCE

The differential equation driving the light propagation in curved spacetimes is the Sachs optical equation

$$\sqrt{A}'' + \frac{1}{2}R_{\mu\nu}k^{\mu}k^{\nu}\sqrt{A} = 0, \tag{2}$$

where a prime denotes differentiation with respect to the affine parameter λ, A is the cross-sectional area of the light beam, $R_{\mu\nu}$ the Ricci tensor, k^{μ} the photon four-momentum ($k^{\mu}k_{\mu} = 0$), and the shear was neglected [21].

Five steps are needed to achieve the luminosity distance in the Dyer-Roeder approach:

(i) the assumption that the angular diameter distance $d_A \propto \sqrt{A}$;

(ii) the relation between the Ricci tensor and the energy-momentum tensor $T_{\mu\nu}$ through Eintein's field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{3}$$

where in our units $c = 1$, R is the scalar curvature, $g_{\mu\nu}$ is the metric describing a Friedmann-Robertson-Walker geometry, G is Newton's constant, and $R_{\mu\nu}k^{\mu}k^{\nu} = 8\pi G T_{\mu\nu}k^{\mu}k^{\nu}$;

(iii) the relation between the affine parameter λ and the redshift z

$$\frac{dz}{d\lambda} = (1+z)^2\frac{H(z)}{H_0}, \tag{4}$$

where $H(z)$ is the Hubble parameter whose present-day value, H_0, is the Hubble's constant;

(iv) the ansatz ρ_m goes to $\alpha\rho_m$, since light experiences the local gravitational field, the matter density felt by a light ray will be $\alpha\rho_m$ instead of ρ_m, which is assumed to be valid in the line of sight of a typical light ray; and, finally,

(v) the validity of the duality relation between the angular diameter and luminosity distances [22–24]

$$d_L(z) = (1+z)^2 d_A(z). \tag{5}$$

For a general XCDM model, where the Universe is composed by cold dark matter (CDM) and a fluid X with equation of state $p_X = w\rho_X$ (w constant) representing dark energy, the Dyer-Roeder distance ($d_L = H_0^{-1}D_L$) can be written as

$$\frac{3}{2}[\alpha_E(z)\Omega_m(1+z)^3 + \Omega_X(1+w)(1+z)^{3(1+w)}]D_L(z) + (1+z)^2E(z)\frac{d}{dz}\left[(1+z)^2E(z)\frac{d}{dz}\frac{D_L(z)}{(1+z)^2}\right] = 0, \tag{6}$$

where $\alpha_E(z)$ denotes the extended Dyer-Roeder parameter, Ω_X, w, are the density and equation of state parameters of dark energy while the dimensionless Hubble parameter, $E(z) = H/H_0$, reads

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_X(1+z)^{3(1+w)} + \Omega_k(1+z)^2}, \tag{7}$$

where $\Omega_k = (1 - \Omega_m - \Omega_X)$ and the limiting case ($w = -1$, $\Omega_X = \Omega_\Lambda$) of all the above expressions describe an arbitrary ΛCDM model. The above Eq. (6) must be solved with two initial conditions, namely, $D_L(z=0) = 0$ and $\frac{dD_L}{dz}|_{z=0} = 1$. As in the original DR approach, from now on it will be assumed that α_E is a constant parameter (see, however, [18,25]).

III. DETERMINING α_E FROM SUPERNOVA DATA

In order to show the physical interest of the approach proposed here we have performed a statistical analysis involving 557 SNe Ia from the Union2 compilation data [20]. Following standard lines, we have applied the

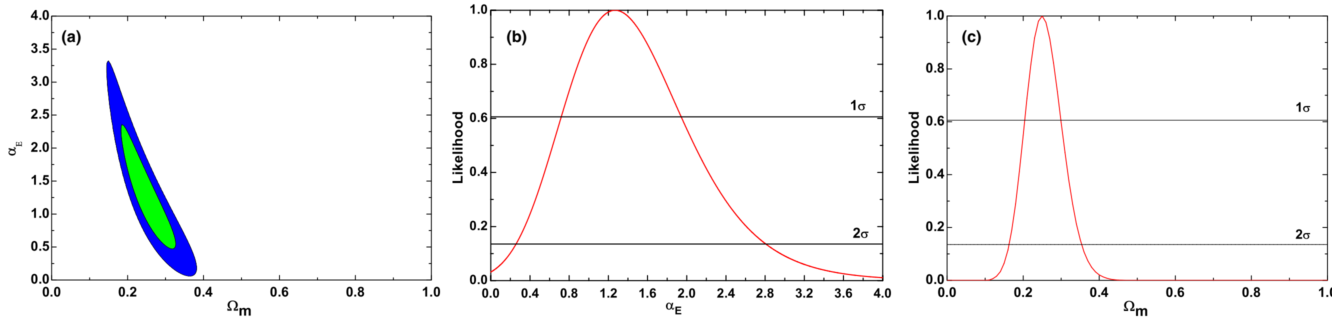

FIG. 1 (color online). (a) The (Ω_m, α_E) plane for a flat ΛCDM model. The contours represent the 68.3% and 95.4% confidence levels. The best fit is $\Omega_m = 0.25$ and $\alpha_E = 1.26$. Note that a flat model with only matter and inhomogeneities ($\Omega_m = 1$) is ruled out with great statistical confidence. (b) Likelihood of α_E. The smoothness parameter is restricted on the interval $0.72 \leq \alpha_E \leq 1.94$ (1σ). (c) Likelihood of Ω_m. We see that the density parameter Ω_m is restricted to the interval $0.21 \leq \Omega_m \leq 0.29$ (1σ).

maximum likelihood estimator (we refer the reader to Ref. [16,20] for details on statistical analysis involving Supernovae data).

In Fig. 1(a) we display the results obtained by assuming a flat ΛCDM model. The contours correspond to 68.3% (1σ) and 95.4% (2σ) confidence levels. The best fits are $\Omega_m = 0.25$ and $\alpha_E = 1.26$. As we can see from Figs. 1(b) and 1(c) the matter density parameter is well constrained, being restricted over the interval $0.21 \leq \Omega_m \leq 0.29$ (1σ), while the smoothness parameter is in the interval $0.72 \leq \alpha_E \leq 1.94$ (1σ). Although α_E is poorly constrained, we see that the probability peaks in $\alpha_E > 1$, and, therefore, denser than average regions in the line of sight are fully compatible with the data. It is interesting to compare the bounds over Ω_m with our previous analysis with the restriction $\alpha_E \leq 1.0$ [16]. The interval $0.24 \leq \Omega_m \leq 0.35$ (2σ) was obtained. As should be expected, by dropping the restriction $\alpha_E \leq 1.0$ lesser values of Ω_m are allowed by data.

IV. SNE IA-CMB TENSION AND α_E

The tension between low and high redshift data has been reported by many authors (see, for instance, [26]). A numerical weak lensing approach to solve this problem was recently discussed by Amendola *et al.* [4] based on a meatball model. Can such a tension be alleviated by our extended DR approach?

In order to answer that, let us consider an arbitrary ΛCDM model and plot the bounds on the $(\Omega_m, \Omega_\Lambda)$ plane by fixing three different values of α_E. By selecting $\alpha_E = 0.7$, 1.0, and 1.3 we may study what happens with the $(\Omega_m, \Omega_\Lambda)$ contours when higher values are considered. In Fig. 2(a) we show the contours obtained for the chosen values of α_E. Note that when α_E grows from 0.7 to 1.3 the best fit moves of around 1σ towards lower values of the pair $(\Omega_m, \Omega_\Lambda)$ thereby becoming more compatible with the cosmic concordance flat ΛCDM model. This is a remarkable result since it improves the agreement with independent constraints coming from BAO and the angular power spectrum of the CMB, and, more importantly, maintaining the same reduced $\chi^2_{\rm red}$.

In Table I, the basic results are summarized. Note that the greatest value of α_E yields the minimum reduced $\chi^2_{\rm red} = \chi^2/\nu$ (ν is number of d.o.f.).

In Fig. 2(b), we display the statistical results for a flat XCDM model and the same values for α_E adopted in the

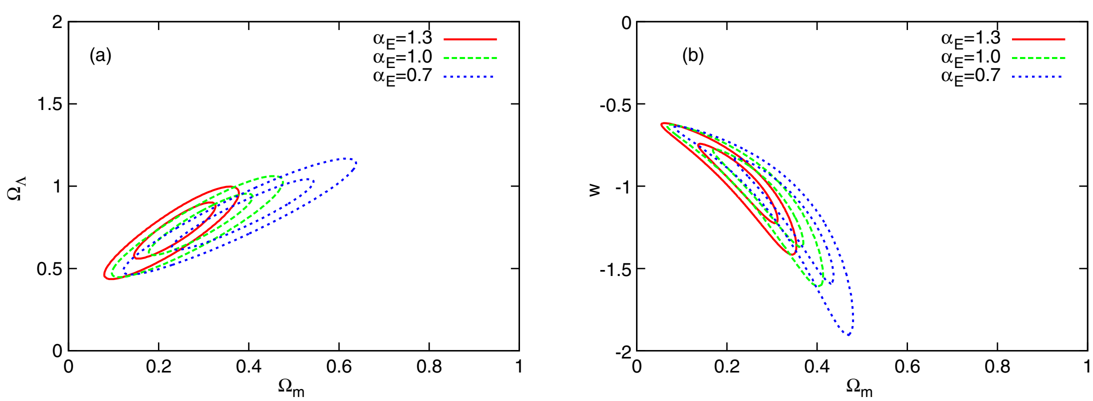

FIG. 2 (color online). (a) The influence of the smoothness parameter on the $(\Omega_m, \Omega_\Lambda)$ plane. The contours for three values of the smoothness parameter α_E using 557 SNe Ia from the Union2 Compilation Data [20] correspond to 1, 2, and 3σ. Greater values of α_E provide results more compatible with a flat model. (b) Contours for the (Ω_m, w) plane in a flat XCDM model. The same trend is observed, greater values of α_E imply greater values of w thereby alleviating the tension among the low and high redshift data.

TABLE I. Best fits for Ω_m and Ω_Λ.

α_E	Ω_m	Ω_Λ	$\chi^2_{\rm red}$
0.7	0.39	0.83	0.978
1.0	0.30	0.78	0.977
1.3	0.24	0.74	0.977

TABLE II. Best fits for Ω_m and w.

α_E	Ω_m	w	$\chi^2_{\rm red}$
0.7	0.35	−1.18	0.978
1.0	0.29	−1.06	0.978
1.3	0.23	−0.96	0.977

previous ΛCDM analysis. Again, we see that for higher values of α_E, the contours are displaced towards regions with higher values for w and smaller values for Ω_m, again contributing to cancel the tension between the low and high redshift data.

In Table II, we summarize the best fits for Ω_m and w along with their respective minimum reduced $\chi^2_{\rm red}$.

V. WHY IS α_E BIGGER THAN UNITY?

Here we propose a simple toy model based on the existence of cosmic voids in order to explain why α_E can be bigger than unity. Recent studies have pointed out that cosmic voids not only represent a key constituent of the cosmic mass distribution, but, potentially, may become one of the cleanest probes to constrain cosmological parameters [27]. The idea is to consider that very large voids are relatively rare entities, i.e. their formation suffers from the same kind of size (mass) segregation “felt” by the largest galaxies and clusters. By assuming that the three basic entities filling the observed Universe are (i) matter homogeneously distributed (ρ_h), (ii) the clustered component ($\rho_{\rm cl}$), and (iii) voids (ρ_{vd}) of small and moderate sizes, we define the extended DR parameter [see Eq. (1)]:

$$\alpha_E = \frac{\rho_h}{\rho_h + \rho_{\rm cl} + \rho_{vd}}. \tag{8}$$

The important task now is to quantify the contribution of voids representing the local underdensities in the Universe. The presence of a void means that its matter was somehow redistributed to the clustered and the homogeneous components. The gravitational effect of a void in an initially homogeneous distribution is equivalent to superimposing a negative density (for small densities the nonrelativistic superposition principle is approximately valid). For simplicity, it will be assumed here that the overall contribution of the void component can be approximated by the linear expression, $\rho_{vd} = -\delta(\rho_h + \rho_{\rm cl})$, where δ is a positive number smaller than unity. Therefore, α_E given Eq. (8) can be rewritten as

$$\alpha_E = \frac{\rho_h}{(\rho_h + \rho_{\rm cl})(1-\delta)} \equiv \frac{\alpha}{1-\delta}, \tag{9}$$

which clearly satisfies the inequality $\alpha_E \geq \alpha$, where α is the standard DR parameter. In particular, when the clustered component does not contribute we find $\alpha_E = \frac{1}{1-\delta} \geq 1$.

Note that negative density never happens in our approach. What really happens is that the voids behave as effectively negative because they give mass to the other components. The negative sign comes only to provide a means to the homogeneous part to acquire a higher density. For example, consider we have only two components: a homogeneous part (ρ_h) and voids (ρ_{vd}). Obviously, as voids have low density, they should give part of their mass to the homogeneous part. As a consequence, the homogeneous part will have a higher density than average. Note that we do not have negative densities anywhere, but the voids can be treated with negative sign since they are donating their mass.

Despite being only a toy model, there is a limitation of our model which is important to stress. We consider light propagates in the homogeneous part, not crossing voids. While this can happen, it is very unlikely since voids dominate the Universe by volume. Therefore, an important step forward is to estimate in a statistical way how often light propagates in voids and how α_E is affected, truly incorporating the presence of voids in our approach.

The previous analyses using supernovae data implies that we have effectively constrained the extended parameter, α_E. How does one roughly estimate the void contribution from this crude model? By applying the standard DR approach to the Union2 sample, the best fit is $\alpha = 1$, and combining with the result for a flat ΛCDM model (Sec. III), one may check that the void contribution has a best fit of $\delta \sim 0.2$. It should be important to search for a possible connection between the present approach and more sophisticated methods from weak lensing.

VI. CONCLUSIONS

In this paper we have discussed the role played by local inhomogeneities on the light propagation based on an extended Dyer-Roeder approach. In the new interpretation light can travel in regions denser than average, a possibility phenomenologically described by a smoothness parameter $\alpha_E > 1$.

In order to test such a hypothesis we have performed a statistical analysis in a flat ΛCDM model and the best fit achieved was $\alpha_E = 1.26$ and $\Omega_m = 0.25$, the parameters being restricted to the intervals $0.72 \leq \alpha_E \leq 1.94$ and $0.21 \leq \Omega_m \leq 0.29$ within the 68.3% confidence level. Although α_E is poorly constrained, the results are fully compatible with the hypothesis of light traveling in denser than average regions. We have also analyzed how different values for the smoothness parameter affect the bounds over $(\Omega_m, \Omega_\Lambda)$ in an arbitrary ΛCDM model. Interestingly,

$\alpha_E > 1$ improves the cosmic concordance model since it provides a better agreement between low and high redshift data (Supernovae, CMB, and BAO). The same happened when a flat XCDM model was considered with the assumption that $\alpha_E > 1$.

The obtained results reinforce the interest on the influence of local inhomogeneities and may pave the way for a more fundamental description of light propagation in the real Universe.

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