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Three-dimensional Lorentz-violating action

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We demonstrate the generation of the three-dimensional Chern-Simons–like Lorentz-breaking "mixed" quadratic action via an appropriate Lorentz-breaking coupling of vector and scalar fields to the spinor field and study some features of the scalar QED with such a term. We show that the same term emerges through a nonperturbative method, namely the Julia-Toulouse approach of condensation of charges and defects.

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I. INTRODUCTION

The Lorentz symmetry breaking is intensively studied now (for some observational results see [1]). One of the most interesting lines of its investigation consists in constructing the Lorentz-breaking extensions of the known physical models. The first description of the possibilities for these extensions was carried out in [2]. Further, many examples of the new Lorentz-breaking terms were generated due to appropriate couplings of scalar, spinor, and gravitational fields with the spinor ones. The most important examples of such terms are, first, the four-dimensional Lorentz-breaking Chern-Simons–like term originally introduced by Jackiw and collaborators [3], second, the non-Abelian generalization of this term [4,5], and third, the gravitational Chern-Simons term [5,6]. We can note also other manners of description of the Lorentz symmetry breaking such as noncommutativity [7] and double special relativity [8].

However, all these results are four-dimensional ones. At the same time, the three-dimensional space-time represents itself as a convenient laboratory for the study of many physical effects. The main reasons for it are the simpler form and one-loop finiteness for almost all field theory models. The main results achieved in the study of the Lorentz symmetry breaking in three-dimensional space-time are, first, the generation of many Lorentz-breaking terms as a consequence of the spontaneous Lorentz symmetry breaking in the three-dimensional bumblebee model through a tadpole method with the use of the reducible representation of the Dirac matrices [9], second, generalization of duality [10] for the Lorentz-breaking models implying in the arising of new couplings between scalar, spinor, and gauge fields [11], and third, the obtaining of new terms via dimensional reduction of the electrodynamics with the four-dimensional Lorentz-breaking Chern-Simons–like term [12]. In all these papers, a new mixed quadratic term involving both scalar and electromagnetic fields was shown to arise. Some possible applications of this term within the confinement context were discussed in [13,14]. Therefore, the very natural question consists in the possibility of generating this term through simpler and more traditional mechanisms of the Lorentz-breaking couplings of scalar and gauge fields to the spinor one, which could be similar to [4,5], and through the Julia-Toulouse approach [13,14].

The Julia-Toulouse approach (JTA) consists in a prescription to obtain a low-energy effective field theory describing a system where a condensation of topological currents has occurred. Initially, these topological currents are sparsely distributed through the system constituting the diluted phase. Then, there is a proliferation of topological currents due to a condensation mechanism that is beyond the scope of JTA since in the Julia-Toulouse method the condensation process is taken for granted. The original proposal of this technique was done in the realm of condensed matter physics in Ref. [15]; later, this procedure was generalized to relativistic quantum fields in Ref. [16]. The original JTA relies on duality transformations, since to apply the JTA the first step is to get the dual theory on the diluted phase before applying the prescription and then obtain the effective theory on the condensed regime on a dual theory. The final step is to dualize again to finally find the effective field theory of the original condensed phase. However, this original procedure that depends on duality tranformations can sometimes be cumbersome, and indeed it is not necessary as shown in Refs. [13,14]. This new procedure, dubbed the generalized Julia-Toulouse approach (GJTA), is based on the JT rationale, and its cornerstone uses the generalized Poisson identity. This identity makes clear the physical content of the condensation of topological currents, and this avoids the two dual transformations to implement the original JTA. Another advantage of GJTA is that it can be applied to models that do not admit a dual theory [17]. For a comprehensive discussion of GJTA with applications, the reader is referred to [14].

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This manuscript is organized as follows: In Sec. II, two different forms to get the "mixed" term are presented: one via Feynman diagram methods (Sec. II A) and using the proper-time approach (Sec. II B). In Sec. III, the GJTA is briefly presented, and it is used to obtain the same mixed term as before. The conclusions are present in Sec. IV, and the corrections on the physical spectra due to the mixed term are given in the Appendix.

II. PERTURBATIVE APPROACH

A. Feynman diagram methods

Let us consider the model of fermions interacting with scalar field $\phi(x)$ and vector one $A_\mu(x)$, where the Lorentz symmetry violation is implemented via a constant vector a^μ. We consider the Lagrangian involving the Lorentz-breaking generalization of the Yukawa coupling [18] (we note that this coupling is renormalizable; see the discussion of the renormalizability of the Lorentz-breaking theories in [19]),

$$\mathcal{L}_f = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\phi(\Box + M^2)\phi + \bar{\psi}(i\partial\!\!\!/ - m - eA\!\!\!/ - g a\!\!\!/\phi)\psi. \tag{1}$$

We note that, unlike the four-dimensional theory (see, for example, [4]) where the Lorentz symmetry breaking has been introduced through an additive term $b\!\!\!/\gamma_5$, with b^μ a Lorentz-breaking pseudovector; in three dimensions this manner of implementing the Lorentz symmetry breaking is the most adequate one since the γ_5 matrix now is simply a unit matrix. Thus, the impact of this additive term can be completely removed through an appropriate redefinition of the A^μ field. Integrating out the spinor fields, we arrive at their following complete one-loop effective action:

$$\Gamma^{(1)} = i\mathrm{Tr}\ln(i\partial\!\!\!/ - m - eA\!\!\!/ - g a\!\!\!/\phi). \tag{2}$$

Within this paper, our aim consists in calculating the one-loop Chern-Simons–like mixed effective action of the form

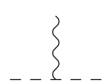
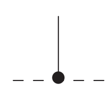
$$\Gamma = \int d^3x \epsilon^{\mu\nu\lambda} a_\mu F_{\nu\lambda}\phi. \tag{3}$$

Some issues related to this effective action were discussed in [9,11–14]. It is natural to suggest that in the momentum space it can be represented as

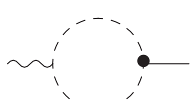
$$\Gamma = \int \frac{d^3q}{(2\pi)^3}\phi(-q)\Pi^\mu(q)A_\mu(q), \tag{4}$$

with $\Pi^\mu(q)$ as the self-energy tensor. We note that in this theory also other quadratic contributions to the action are generated, for example, the Chern-Simons term; however, here we concentrate only on the mixed term (3).

Applying the following Feynman rules:

 $= \frac{i(p\!\!\!/+m)}{p^2-m^2}$  $= -ie\gamma^\mu$  $= -ig a\!\!\!/$

where the dot denotes the Lorentz-breaking insertion in the vertex, we arrive at the following diagram that contributes to the two-point mixed function of the scalar and vector fields:



Here the dashed line is for the propagator of the ψ field, the wavy line is for the external A_μ field, and the single line is for the external ϕ field.

The contribution of this diagram evidently looks like

$$I = -eg\mathrm{Tr}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}A\!\!\!/(-p)(k\!\!\!/+m)\phi(p)a\!\!\!/(k\!\!\!/+p\!\!\!/+m) \times \frac{1}{(k^2-m^2)[(k+p)^2-m^2]}. \tag{5}$$

To obtain the term (3) proportional to the Levi-Cività symbol we must take into account the products of three Dirac matrices only,

$$I = -egm\mathrm{Tr}\int\frac{d^3p}{(2\pi)^3}A^\mu(-p)\phi(p)a^\nu \times \int\frac{d^3k}{(2\pi)^3}\frac{\gamma_\mu\gamma_\alpha\gamma_\nu k^\alpha + \gamma_\mu\gamma_\nu\gamma_\alpha(k^\alpha + p^\alpha)}{(k^2-m^2)[(k+p)^2-m^2]}. \tag{6}$$

We choose the signature diag$(+--)$, the corresponding Dirac matrices are $(\gamma^0)^\alpha{}_\beta = \sigma^2$, $(\gamma^1)^\alpha{}_\beta = i\sigma^1$, $(\gamma^0)^\alpha{}_\beta = i\sigma^3$, and they satisfy relations $\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}$, $\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\lambda) = 2i\epsilon^{\mu\nu\lambda}$. Using these relations, we can simplify the expression for the contribution above,

$$I = -2i\epsilon_{\alpha\mu\nu} egm \int \frac{d^3 p}{(2\pi)^3} p^\alpha A^\mu(-p)\phi(p) a^\nu \times \int \frac{d^3 k}{(2\pi)^3} \frac{1}{(k^2 - m^2)[(k+p)^2 - m^2]}. \tag{7}$$

After Wick rotation and integration over momenta we arrive at

$$I = \epsilon_{\alpha\mu\nu} eg \frac{m}{4\pi|m|} \int \frac{d^3 p}{(2\pi)^3} p^\alpha A^\mu(-p)\phi(p) a^\nu. \tag{8}$$

Carrying out the inverse Wick rotation and inverse Fourier transform, we find after some simple transformations

$$I = -eg \frac{m}{8\pi|m|} \int d^3 x \epsilon^{\alpha\mu\nu} F_{\alpha\mu} a_\nu \phi. \tag{9}$$

This is a desired mixed term (3). It possesses a restricted gauge invariance (cf. [11]); i.e., it is invariant under the gauge transformations $\delta A_\mu = \partial_\mu \xi$, where the scalar ϕ stays untouched. We note that the dependence of this result on the sign of the mass m originates from the ambiguity of choice of the direction of the Lorentz-breaking vector a^μ [20].

B. The Schwinger proper-time method

Alternatively, we can also calculate the same term via the proper-time method. To do it, we study the expression (2). First, we can rewrite this expression in the form $\mathrm{Tr}\ln(\Box + \mathcal{M})$, adding to the right-hand side of (2) a constant $i\mathrm{Tr}\ln(i\partial\!\!\!/ + m)$, similar to [5]. As a result, the one-loop effective action (2) takes the form

$$\Gamma^{(1)} = i\mathrm{Tr}\ln(-\Box - m^2 - eA\!\!\!/(i\partial\!\!\!/ + m) - g\phi a\!\!\!/(i\partial\!\!\!/ + m)), \tag{10}$$

We can expand this expression up to the first order in a^μ, which looks like

$$\Gamma_1^{(1)} = ig\mathrm{Tr}[[\Box + m^2 + eA\!\!\!/(i\partial\!\!\!/ + m)]^{-1}\phi a\!\!\!/(i\partial\!\!\!/ + m)]. \tag{11}$$

Now, we can use the Schwinger proper-time representation $A^{-1} = i\int_0^\infty e^{isA} ds$,

$$\Gamma_1^{(1)} = -g\mathrm{Tr}\left[\int_0^\infty ds e^{is(\Box + m^2 + eA\!\!\!/(i\partial\!\!\!/ + m))}\phi a\!\!\!/(i\partial\!\!\!/ + m)\right]. \tag{12}$$

To evaluate the exponential, we use the Hausdorf formula whose sufficient form in our case is $e^{A+B} = e^A e^B e^{-\frac{[A,B]}{2}}$. Thus, taking into account only the first derivatives of A_μ and using the cyclic property of the trace, we find

$$\Gamma_1^{(1)} = -g\mathrm{Tr}\left[\int_0^\infty ds e^{ism^2} e^{iseA\!\!\!/(i\partial\!\!\!/ + m)} e^{-es^2(\partial^\mu A\!\!\!/)(i\partial\!\!\!/ + m)\partial_\mu}\phi a\!\!\!/(i\partial\!\!\!/ + m) e^{is\Box}\right]. \tag{13}$$

The derivatives act on all on the right. Now, we can keep in this expression only the first order in A_μ,

$$\Gamma_1^{(1)} = -eg\mathrm{Tr}\left[\int_0^\infty ds e^{ism^2}(isA\!\!\!/(i\partial\!\!\!/ + m) - s^2(\partial^\mu A\!\!\!/)(i\partial\!\!\!/ + m)\partial_\mu)\phi a\!\!\!/(i\partial\!\!\!/ + m) e^{is\Box}\right]. \tag{14}$$

It remains to calculate a trace. To obtain a desired term, we must take into account only contributions involving exactly three Dirac matrices and involving an even number of the derivatives acting on $e^{is\Box}$. As $\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\lambda) = 2i\epsilon^{\mu\nu\lambda}$, we arrive at

$$\Gamma_1^{(1)} = -2egm \int d^3 x \int_0^\infty ds s e^{ism^2} \epsilon^{\mu\nu\lambda} A_\mu (\partial_\nu \phi) a_\lambda e^{is\Box} \delta^3(x - x')|_{x=x'}. \tag{15}$$

After the Fourier transform and Wick rotation we arrive at

$$\Gamma_1^{(1)} = -2egm \int d^3 x \int \frac{d^3 k}{(2\pi)^3} \int ds s e^{-sm^2} \epsilon^{\mu\nu\lambda} A_\mu (\partial_\nu \phi) a_\lambda e^{-sk^2}. \tag{16}$$

The calculation of the integrals over momenta and, then, over s is straightforward, and we again arrive at

$$I = -eg \frac{m}{8\pi|m|} \int d^3 x \epsilon^{\alpha\mu\nu} F_{\alpha\mu} a_\nu \phi. \tag{17}$$

This result is identical to the one obtained using the Feynman diagram approach. It is very natural since this contribution is superficially finite and hence does not involve any ambiguities.

III. LORENTZ-BREAKING MIXED TERM AND THE JULIA-TOULOUSE APPROACH

In the previous sections we have showed that the mixed quadratic term can be successfully generated within the traditional perturbative approach. In this section, we show

how the same term can be generated within an alternative, nonperturbative technique, that is, the Julia-Toulouse method.

To proceed with the Julia-Toulouse approach [14], we start with the Lagrangian (1), at the zero mass, and introduce the corresponding generating functional in the diluted phase,

$$Z_d[j^\mu] = \int DA^\mu D\phi \exp\left[-i\int d^3x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\phi\Box\phi + (-eA_\mu + g\phi a_\mu)j^\mu\right)\right], \tag{18}$$

where we absorbed the fermionic coupling into the current j^μ. We choose the current to be of the form $j^\mu = \epsilon^{\mu\nu\alpha}\partial_\nu\chi_\alpha$, to be topologically conserved. The vector χ_α is called the Chern kernel [14]. Then, following [14], we add a so-called activation term $\int d^3x \frac{j^\mu j_\mu}{2\Lambda}$ to the classical action (that is, the argument of the exponential) to introduce a defect condensation. The parameter Λ is related to the density of the condensate. It is a free parameter of the procedure, and it can be fixed after comparing the effective field theory obtained by the JTA to the same theory computed by other methods [13,14,16,21,22]. In particular, for the three-dimensional quantum electrodynamics (QED) with magnetic monopoles, this parameter is fixed to maintain the consistency of this theory [13,14]. Thus, the generating functional is modified, and we arrive at the new generation functional Z_c describing the condensed phase:

$$Z_c[j^\mu] = \sum_{\{\chi_\alpha\}}\int DA^\mu D\phi \exp\left[-i\int d^3x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\phi\Box\phi + (-eA_\mu + g\phi a_\mu)\epsilon^{\mu\nu\alpha}\partial_\nu\chi_\alpha + \frac{\epsilon^{\mu\nu\alpha}\partial_\nu\chi_\alpha\epsilon_{\mu\lambda\beta}\partial^\lambda\chi^\beta}{2\Lambda}\right)\right]. \tag{19}$$

Here we suggest the formal sum over the branes χ_α. Let us promote their condensation. During this process, they convert to a vector field B_α, which is formally described by introducing the integral over B_α and the functional delta function $\delta(\chi_\alpha - B_\alpha)$, so we have

$$Z_c[j^\mu] = \sum_{\{\chi_\alpha\}}\int DA^\mu D\phi DB^\alpha\delta(\chi_\alpha - B_\alpha) \times\exp\left[-i\int d^3x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\phi\Box\phi + (-eA_\mu + g\phi a_\mu)\epsilon^{\mu\nu\alpha}\partial_\nu\chi_\alpha + \frac{\epsilon^{\mu\nu\alpha}\partial_\nu\chi_\alpha\epsilon_{\mu\lambda\beta}\partial^\lambda\chi^\beta}{2\Lambda}\right)\right], \tag{20}$$

which is equivalent to

$$Z_c[j^\mu] = \sum_{\{\chi_\alpha\}}\int DA^\mu D\phi DB^\alpha\delta(\chi_\alpha - B_\alpha) \times\exp\left[-i\int d^3x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\phi\Box\phi + (-eA_\mu + g\phi a_\mu)\epsilon^{\mu\nu\alpha}\partial_\nu B_\alpha + \frac{\epsilon^{\mu\nu\alpha}\partial_\nu B_\alpha\epsilon_{\mu\lambda\beta}\partial^\lambda B^\beta}{2\Lambda}\right)\right], \tag{21}$$

where the sum is taken over the branes. Then we use a generalized Poisson identity [14]

$$\sum_{\{\chi_\alpha\}}\int DB^\alpha\delta(\chi_\alpha - B_\alpha) = \sum_{\{\Omega_{\mu\nu}\}}\exp\left(2\pi i\int d^3x\epsilon^{\mu\nu\rho}\Omega_{\mu\nu}B_\rho\right), \tag{22}$$

where $\Omega_{\mu\nu}$ is a magnetic vortex over the condensate [14], and arrive at

$$Z_c[j^\mu] = \sum_{\{\Omega_{\mu\nu}\}}\int DA^m D\phi DB^\alpha \exp\left[-i\int d^3x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\phi\Box\phi + (-eA_\mu + g\phi a_\mu)\epsilon^{\mu\nu\alpha}\partial_\nu B_\alpha + \frac{\epsilon^{\mu\nu\alpha}\partial_\nu B_\alpha\epsilon_{\mu\lambda\beta}\partial^\lambda B^\beta}{2\Lambda} + 2\pi\epsilon^{\mu\nu\rho}\Omega_{\mu\nu}B_\rho\right)\right]. \tag{23}$$

It remains only to integrate over the field A_μ. Since it is gauge invariant, we add to the argument of the exponential the Feynman gauge fixing term $-\frac{1}{2}(\partial_\mu A^\mu)^2$, after which the integral over A_μ turns out to be straightforward, by the rule

$$\int DA^\mu \exp\left(i\left(-\frac{1}{2}A_\mu\Box A^\mu + A_\nu j^\nu\right)\right) = \exp\left(i\left(\frac{1}{2}j_\mu\Box^{-1}j^\mu\right)\right). \tag{24}$$

Then, we redefine $B_\mu \to \sqrt{\Lambda}B_\mu$ and arrive at

$$Z_c[j^\mu] = \sum_{\{\Omega_{\mu\nu}\}}\int D\phi DB^\alpha \times\exp\left[-i\int d^3x\left(-\frac{1}{4}F_{\mu\nu}[B]\left(\frac{e^2\Lambda}{\Box} - 1\right)F^{\mu\nu}[B] - \frac{1}{2}\phi\Box\phi + g\phi\sqrt{\Lambda}a_\mu\epsilon^{\mu\nu\alpha}F_{\nu\alpha}[B] + 2\pi\epsilon^{\mu\nu\rho}\Omega_{\mu\nu}B_\rho\right)\right], \tag{25}$$

where $F_{\mu\nu}[B] = \partial_\mu B_\nu - \partial_\nu B_\mu$. The last term vanishes since one considers the phase where the magnetic vortices are absent, which represents a complete condensed phase. Notice also that the term $-\frac{1}{4}F_{\mu\nu}[B]\frac{e^2\Lambda}{\Box}F^{\mu\nu}[B]$ represents a gauge invariant mass term for B^μ that can be seen straightforwardly performing integration by parts. This mass generating mechanism is a signature of the JTA. Hence, we succeed in generating the mixed term $g\phi\sqrt{\Lambda}a_\mu\epsilon^{\mu\nu\alpha}F_{\nu\alpha}[B]$ via GJTA. Interestingly, for the four-dimensional QED with Lorentz breaking, a very similar term, the Carrol-Field-Jackiw term, is induced by GJTA [14].

IV. CONCLUSIONS

In this manuscript, we generated the mixed quadratic term involving both scalar and vector fields in a traditional way, similar to [4], based on the explicitly Lorentz-breaking coupling of the scalar, vector, and spinor fields. This term is naturally finite. Then, it turns out to possess a "restricted" gauge invariance, that is, it is invariant if only the vector field suffers gauge transformations. However, this situation is common in many theories obtained via the dual embedding procedure (see, e.g., [10,11]). Also, we succeeded in generating this term through the proper application of the Julia-Toulouse methodology. Finally, we studied the dispersion relations in the electrodynamics involving this term as an additive one.

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APPENDIX: THE PHYSICAL SPECTRA OF THE MIXED MODEL

As an application of the perturbative methods discussed in Sec. II, the one-loop corrected effective Lagrangian of A^μ and ϕ being the sum of the classical Lagrangian of these fields [see (1) with the one-loop correction given by (17)] looks like

$$L_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\phi(\Box + M^2)\phi + \epsilon^{\alpha\mu\nu}F_{\alpha\mu}v_\nu\phi, \quad \text{(A1)}$$

where $v_\nu = -eg\frac{m}{8\pi|m|}a_\nu$.

Notice that the above effective Lagrangian is not the complete model as there are other terms that can potentially contribute to Eq. (A1). In this manuscript these terms are neglected since we are only interested in the influence of this mixed term on the physical spectra.

Let us briefly discuss the physical spectra of this mixed model. This theory is a partial case of the theory considered in [11,12] arising through a dimensional reduction of the electrodynamics with the Carroll-Field-Jackiw term. Therefore the propagator and, consequently, dispersion relations in our case are similar to the propagator and dispersion relations found in [11,12] (however, unlike [12], we have here $M^2 \neq 0$; i.e. the scalar field is massive, but, unlike [11], we have $m = 0$; i.e. there is no Chern-Simons term). So, we can merely quote the results from [11], which allows us to write the propagators in the form

$$\begin{aligned}
\langle A^\mu A^\nu\rangle &= (\Delta_{11})^{\mu\nu} = [(\Box - M^2)M_{\mu\nu} - T_\mu T_\nu]^{-1}(\Box - M^2),\\
\langle\phi\phi\rangle &= \Delta_{22} = [(\Box - M^2)M_{\mu\nu} - T_\mu T_\nu]^{-1}M_{\mu\nu},\\
\langle A^\mu\phi\rangle &= -\langle\phi A^\mu\rangle = \Delta_{12}^\mu = -\Delta_{21}^\mu\\
&= -T_\nu[(\Box - M^2)M_{\mu\nu} - T_\mu T_\nu]^{-1}. \quad \text{(A2)}
\end{aligned}$$

Therefore, the problem is reduced to finding the operator $\Delta^{\mu\nu} = [(\Box - M^2)M_{\mu\nu} - T_\mu T_\nu]^{-1}$ (with $M_{\mu\nu} = \Box\theta_{\mu\nu} + \frac{\Box}{\xi}\omega_{\mu\nu}$), which we do with the use of a special ansatz [11,23]

$$\begin{aligned}
\Delta^{\nu\alpha} = {} & a_1\theta^{\nu\alpha} + a_2\omega^{\nu\alpha} + a_3 S^{\nu\alpha} + a_4\Lambda^{\nu\alpha} + a_5 T^\nu T^\alpha\\
& + a_6 Q^{\nu\alpha} + a_7 Q^{\alpha\nu} + a_8\Sigma^{\nu\alpha} + a_9\Sigma^{\alpha\nu} + a_{10}\Phi^{\nu\alpha} + a_{11}\Phi^{\alpha\nu}, \quad \text{(A3)}
\end{aligned}$$

where $S_{\mu\nu} = \epsilon_{\mu\lambda\nu}\partial^\lambda$, $T_\nu = S_{\mu\nu}v^\mu$, $\omega_{\mu\nu} = \frac{\partial_\mu\partial_\nu}{\Box}$ is a longitudinal projector, $\theta_{\mu\nu} = \eta_{\mu\nu} - \omega_{\mu\nu}$ is a transverse projector, $Q_{\mu\nu} = v_\mu T_\nu$, $\Lambda_{\mu\nu} = v_\mu v_\nu$, $\Sigma_{\mu\nu} = v_\mu\partial_\nu$, $\Phi_{\mu\nu} = T_\mu\partial_\nu$, and $\lambda = v^\mu\partial_\mu$. These coefficients were found in [11] for $m \neq 0$ and reduce in our case to

$$\begin{aligned}
& a_1 = a_2 = \frac{1}{\Box(\Box - M^2)};\\
& a_3 = a_4 = 0; \quad a_5 = \frac{1}{\Box(\Box - M^2)\mathcal{R}}; \quad a_6 = a_7 = 0;\\
& a_8 = a_9 = a_{10} = a_{11} = 0. \quad \text{(A4)}
\end{aligned}$$

Here we denoted $\mathcal{R} = \Box(\Box - M^2) - T^2$. Proceeding in a manner similar to [11,12], besides the usual dispersion relations $E^2 = \vec{p}^2$ and $E^2 = \vec{p}^2 + M^2$ we also find $(E^2 - \vec{p}^2)(E^2 - \vec{p}^2 - M^2 + v^2) + (\vec{v}\cdot\vec{p} - v_0 E)^2 = 0$. The last relation can be physical only if v^μ is spacelike.

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