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# SO(10) GUTs with large tensor representations on noncommutative space-time

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We construct a noncommutative version of a general renormalizable SO(10) GUT with Higgses in the 210, $\overline{126}$, 45, 10 and 120 irreps of SO(10) and a Peccei-Quinn symmetry. Thus, we formulate the noncommutative counterpart of a nonsupersymmetric SO(10) GUT which has recently been shown to be consistent with all the physics below $M_{\mathrm{GUT}}$. The simplicity of our construction—the simplicity of the Yukawa terms, in particular—stems from the fact that the Higgses of our GUT can be viewed as elements of the Clifford algebra $\mathbb{C}\mathrm{l}_{10}(\mathbb{C})$; elements on which the SO(10) gauge transformations act by conjugation. The noncommutative GUT we build contains tree-level interactions among different Higgs species that are absent in their ordinary counterpart as they are forbidden by SO(10) and Lorentz invariance. The existence of these interactions helps to clearly distinguish noncommutative Minkowski space-time from ordinary Minkowski space-time.

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## I. INTRODUCTION

It has recently been shown in Ref. [1] that a nonsupersymmetric SO(10) GUT with Higgses in the 210, $\overline{126}$, 45 and 10 irreps of SO(10)—and with an intermediate breaking to the Patti-Salam group by the 210—is compatible with all the experimental data currently available, if the naturalness paradigm is put aside. A salient feature of this GUT is that the amount of dark matter that has been observed is accounted for by the existence of an axion which results from the spontaneously broken Peccei-Quinn symmetry of the theory. Under this global U(1) symmetry some Higgses are charged, others are not.

It is almost 15 years [2] since it became well established that ordinary Minkowski space-time might have to be replaced with its noncommutative counterpart as one probes shorter distances. Hence, it is interesting to see whether there can be constructed on noncommutative space-time a field theory which can be considered to be a noncommutative version of the phenomenologically relevant SO(10) GUT of Ref. [1]. The purpose of this paper is to show that, indeed, a noncommutative counterpart of the ordinary GUT in Ref. [1] can be formulated. We shall actually enlarge, for the sake of generality, the Higgs content of that GUT with a Higgs in the 120 irrep of SO (10), for the latter naturally occurs in the most general ordinary SO(10) Yukawa term for fermions in the 16. Ordinary SO(10) GUTs with Higgses only in the 210, $\overline{126}$, 45, 10 and 120 are very suitable for their generalization to GUTs on noncommutative space-time, for the Higgses they involve can naturally be understood as elements of the Clifford algebra $\mathbb{C}\mathrm{l}_{10}(\mathbb{C})$ and SO(10) acts on these elements by conjugation. This feature of the Higgses—which is very appealing from the noncommutative geometry standpoint [3]—is lost if one considers Higgses in the 16 or 54 irreps of SO(10), another popular Higgs irreps in SO(10) model building.

The formulation of the noncommutative counterpart of the SO(10) GUT of Ref. [1] will be carried out with the help of the enveloping-algebra formalism. This formalism was put forward in Refs. [4–6]. The enveloping-algebra framework was employed afterwards to build the noncommutative standard model [7], a noncommutative deformation of the ordinary standard model with no new degrees of freedom—see Refs. [8,9] for alternative noncommutative extensions of the ordinary standard model. The formulation of the gauge and fermionic sectors of noncommutative GUTs with SU(5) and SO(10) as gauge groups was tackled, within the enveloping-algebra framework, in Ref. [10]. The nontrivial issue of constructing noncommutative Yukawa terms with the help of the enveloping-algebra formalism was addressed in Ref. [11]. Outside the enveloping-algebra framework, the formulation of noncommutative gauge theories for SO(N) groups was discussed in Ref. [12].

In the enveloping-algebra framework, the noncommutative gauge fields are elements of the universal enveloping algebra of the Lie algebra of the ordinary gauge group and the Seiberg-Witten map defines those noncommutative fields in terms of the corresponding ordinary fields. When the Seiberg-Witten map is defined as a formal power series in the noncommutativity matrix parameter $\omega^{\mu\nu}$, the action of the noncommutative theory is a formal power series in $\omega^{\mu\nu}$ with coefficients that are integrated polynomials in the ordinary fields and their derivatives. Quite a few theoretical properties—e.g., renormalizability [13–19], gauge anomalies [20,21], existence of noncommutative

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deformations of ordinary instantons and monopoles [22–24]—of the noncommutative gauge theories so defined have been analyzed by considering the first few terms of the corresponding $\omega^{\mu\nu}$-expanded actions. Some phenomenological properties of the noncommutative gauge theories at hand have been studied in Refs. [25–32].

The UV/IR mixing effects [33] that are a feature of the $\omega^{\mu\nu}$-unexpanded U(N) noncommutative field theories cannot be exhibited in a noncommutative gauge theory built with the help of the Seiberg-Witten map, when this map is defined as a series expansion in $\omega^{\mu\nu}$. To uncover such UV/IR effects in this $\omega^{\mu\nu}$-expanded theories some kind of resummation of an infinite number of terms that are powers of $\omega^{\mu\nu}$ must be worked out: a daunting task. Fortunately, for the enveloping-algebra formalism to work [6] it is not a must that the Seiberg-Witten be given by a formal series expansion in $\omega^{\mu\nu}$. Indeed, the enveloping-algebra framework works equally well if the Seiberg-Witten map is defined by expanding in the number of ordinary fields, thus leaving its dependence on $\omega^{\mu\nu}$ exact. Hence, to study noncommutative UV/IR effects in theories defined within the enveloping-algebra formalism one should use this $\omega^{\mu\nu}$-exact Seiberg-Witten map. This was done for the first time in Ref. [34] where it was shown, in the U(1) case with fermions in the adjoint, that if the $\omega^{\mu\nu}$ dependence of the Seiberg-Witten is handled exactly, then, there is an UV/IR mixing phenomenon in the noncommutative theory defined within the enveloping-algebra formalism. The analysis of the UV/IR mixing effects was later extended [35] to fermions in the fundamental representation coupled to U(1) gauge fields. Very recently the complete one-loop photon and neutrino propagators have been worked out and its full UV/IR mixing structure unveiled—see Ref. [36]. The UV/IR mixing in the one-loop propagator of adjoint fermions coupled to U(1) fields and its very relevant implications on neutrino physics have been studied in Refs. [37–40]—see Ref. [41] for a review. Finally, let us stress that the cohomological techniques developed in Refs. [42,43]—see also [44]—are extremely useful [45] when computing the ($\omega^{\mu\nu}$-exact) expansion of the Seiberg-Witten map in the number of ordinary fields.

As we said at the beginning of this Introduction, the purpose of this paper is to show that there is indeed a noncommutative counterpart of the SO(10) GUT of Ref. [1]. We shall give the complete action of the noncommutative SO(10) GUT: its Yukawa and Higgs parts, in particular. The action will be expressed in terms of noncommutative fields whose noncommutative gauge transformations are the natural generalization of the corresponding ordinary gauge transformations. That this strategy works for the Yukawa and Higgs terms of our SO(10) GUT is a consequence of the fact that the ordinary Higgses of our theory can be viewed as elements of $\mathbb{C}l_{10}(\mathbb{C})$ and that the gauge transformations act on these objects by conjugation. An important by-product of this Clifford algebra construction is that the noncommutative Higgs action naturally contains terms which cannot occur when space-time is commutative and there is Lorentz invariance. These distinct noncommutative terms give rise to tree-level interactions among different species of Higgses and gauge fields, and if experimentally detected will send a clear signal that space-time is noncommutative at short enough distances.

At this point, the reader may rightly ask why should anyone seriously consider the task of formulating the SO(10) model of Ref. [1] on noncommutative space-time. I shall answer the reader's question as follows. It is quite possible that space-time ceases to be a smooth manifold—a set of points, in particular—as one probes short enough distances. Now, noncommutative manifolds are geometrical objects that generalize the notion of smooth manifold in the sense that the former are not made out of points. The natural, and easiest, noncommutative generalization of our old Minkowski space-time is noncommutative Minkowski space-time, a noncommutative manifold which occurs in string theory. It is therefore very advisable to study what are the effects on the physics of elementary particles of replacing ordinary Minkowski space-time with its noncommutative counterpart. Since no signal of having to deal with a noncommutative manifold has come out of the LHC as yet, it seems likely that the noncommutative character of space-time will reveal itself at energies where grand unification effects are relevant. Hence, the noncommutative counterparts of phenomenologically relevant grand unification models must be formulated and studied. It cannot be denied that the model of Ref. [1] has very appealing features from the point of phenomenology; further, its field content is also very appealing from the standpoint of generalizing the model to a model on noncommutative space-time. Indeed, as we said above, the Higgs fields of the model can be thought as elements of the algebra $\mathbb{C}l_{10}(\mathbb{C})$ and on those elements—and this is a key fact—the gauge transformations act by conjugation. By using this fact, as we shall see below, natural noncommutative generalizations of the Yukawa terms and Higgs potential can be formulated. This is not so for other SO(10) GUTs as they contain Higgses—e.g., Higgses in the 16, the 54…—that cannot be viewed as elements of $\mathbb{C}l_{10}(\mathbb{C})$ and, therefore, would demand the introduction of contrived Higgs potential terms involving noncommutative fields that are the image of composite ordinary operators.

All in all, the reader should bear in mind that, in general, the $\omega^{\mu\nu}$-exact noncommutative models constructed by using the enveloping-algebra formalism are to be understood as effective field theories—they will not describe physics above a certain energy scale—since they fail to be renormalizable and they are not UV complete due to the famous noncommutative UV/IR mixing effects. This comment applies, of course, to the noncommutative model introduced below.

The layout of this paper is as follows. In Sec. II, we display the field content and gauge transformations of the ordinary SO(10) GUT whose noncommutative version we shall construct afterwards. The noncommutative fields of our SO(10) GUT, along with the Seiberg-Witten map equations that define them in terms of their ordinary counterparts, are given in Sec. III. The action of the Peccei-Quinn symmetry on the noncommutative fields is discussed in Sec. IV. Section V is devoted to the construction of the action of our noncommutative SO(10) GUT. Some future research directions are given Sec. VI. We include an Appendix where the $\omega$-exact Seiberg-Witten map for the Higgs fields viewed as elements of $\mathbb{C}l_{10}(\mathbb{C})$ is given up to order 2 in the number of gauge fields.

## II. THE FIELD CONTENT OF THE ORDINARY SO(10) GUT AND ITS GAUGE INVARIANCE

Let us list the matter field content of our ordinary SO(10) GUT, a particular instance of which is the SO(10) GUT of Ref. [1]. First, three—one for each family in the standard model—left-handed fermionic fields $\psi_\alpha^{(16)f}, f = 1, 2, 3$, transforming under the 16 irrep of SO(10) and the (1/2,0) representation of the Lorentz group. Each $\psi_\alpha^{(16)f}$ contains the fermionic fields of a family of the standard model plus the degrees of freedom corresponding a right-handed neutrino. Second, five Higgs fields, namely, $\varphi^{(210)}_{i_1i_2i_3i_4}, \varphi^{(10)}_{i_1}, \varphi^{(45)}_{i_1i_2}, \varphi^{(\overline{126})}_{i_1i_2i_3i_4i_5}$, and $\varphi^{(120)}_{i_1i_2i_3}$, carrying, respectively, the 210, the 10, the 45, the $\overline{126}$ and the 120 irreps of SO(10). The indices $i_1, i_2, \ldots$ run from 1 to 10, and $\varphi^{(210)}_{i_1i_2i_3i_4}$, $\varphi^{(\overline{126})}_{i_1i_2i_3i_4i_5}$ and $\varphi^{(120)}_{i_1i_2i_3}$ are totally antisymmetric, with regard to its $i_1, i_2, \ldots$ indices, SO(10) tensors. Further, $\varphi^{(\overline{126})}_{i_1i_2i_3i_4i_5}$ satisfies the following duality equation:

$$\varphi^{(\overline{126})}_{i_1i_2i_3i_4i_5} = +\frac{i}{5!}\epsilon_{i_1i_2i_3i_4i_5i_6i_7i_8i_9i_{10}}\varphi^{(\overline{126})}_{i_6i_7i_8i_9i_{10}}. \tag{2.1}$$

The symbol $\varphi_I^{(H)}$, $I = 1\ldots dimH$, $H = 210, 10, 45, \overline{126}$ and 120, will stand for the independent components of $\varphi^{(210)}_{i_1i_2i_3i_4}, \varphi^{(10)}_{i_1}, \varphi^{(45)}_{i_1i_2}, \varphi^{(126)}_{i_1i_2i_3i_4i_5}, \varphi^{(\overline{126})}_{i_1i_2i_3i_4i_5}$, and $\varphi^{(120)}_{i_1i_2i_3}$, respectively. $dimH$ is the dimension of the representation $H$.

The gauge field content of our GUT is furnished by the 45 gauge fields $a_\mu^{ij}$, with $a_\mu^{ij} = -a_\mu^{ji}$ and $i, j = 1\ldots10$, which constitute the 45 irrep of SO(10).

Let $\Gamma^i$ denote the Hermitian Dirac matrices in 10 Euclidean dimensions. These matrices generate the Clifford algebra $\mathbb{C}l_{10}(\mathbb{C})$. We shall see later that noncommutative counterparts of the Yukawa terms and some Higgs potential terms of our ordinary SO(10) GUT can be formulated very neatly by using the $\mathbb{C}l_{10}(\mathbb{C})$ Clifford algebra valued Higgs fields:

$$\begin{aligned} &\phi^{(210)} = \Gamma^{i_1}\Gamma^{i_2}\Gamma^{i_3}\Gamma^{i_4}\phi^{(210)}_{i_1i_2i_3i_4}, \qquad \phi^{(10)} = \Gamma^{i_1}\phi^{(10)}_{i_1}, \\ &\phi^{(45)} = i\Gamma^{i_1}\Gamma^{i_2}\varphi^{(45)}_{i_1i_2}, \qquad \phi^{(1\bar{2}6)} = \Gamma^{i_1}\Gamma^{i_2}\Gamma^{i_3}\Gamma^{i_4}\Gamma^{i_5}\phi^{(1\bar{2}6)}_{i_1i_2i_3i_4i_5}, \\ &\phi^{(120)} = i\Gamma^{i_1}\Gamma^{i_2}\Gamma^{i_3}\varphi^{(120)}_{i_1i_2i_3}, \end{aligned} \tag{2.2}$$

rather than the SO(10) tensor fields $\varphi^{(210)}_{i_1i_2i_3i_4}$, $\varphi^{(10)}_{i_1}$, $\varphi^{(45)}_{i_1i_2}$, $\varphi^{(\overline{126})}_{i_1i_2i_3i_4i_5}$ and $\varphi^{(120)}_{i_1i_2i_3i_4}$, which give rise to the former.

From now on, the symbol $a_\mu$ will stand for the following gauge field taking values in the in the Lie algebra of SO(10) in the $16\bigoplus\bar{1}6$ representation:

$$a_\mu = \frac{1}{2}\Sigma^{ij}a_\mu^{ij}, \qquad \Sigma^{ij} = \frac{1}{4i}[\Gamma^i, \Gamma^j], \qquad i, j = 1\ldots10. \tag{2.3}$$

The real fields $a_\mu^{ij}$ carry the 45 irrep of SO(10) and has been introduced above. Notice that $a_\mu$ is an element of the Clifford algebra $\mathbb{C}l_{10}(\mathbb{C})$. Besides, $a_\mu = (a_\mu)^\dagger$.

Let $M_{ij}^{(H)}$, $i < j$, $i, j = 1\ldots10$, be the Hermitian generators of SO(10) in the representation carried by $\varphi_I^{(H)}$, $I = 1\ldots dimH$. Then, the matrix gauge field $a_\mu^{(H)}$ is given by

$$a_\mu^{(H)} = \frac{1}{2}a_\mu^{ij}M_{ij}^{(H)}, \qquad (a_\mu^{(H)})^\dagger = a_\mu^{(H)}. \tag{2.4}$$

Let us now introduce the Becchi-Rouet-Stora (BRS) transformations that constitute the gauge symmetry of our GUT. Let $s$ denote the BRS operator, $c = \frac{1}{2}\Sigma^{ij}c^{ij}$ the ghost field associated to $a_\mu$ and $c^{(H)} = \frac{1}{2}M_{ij}^{(H)}c^{ij}$ the ghost field associated to $a_\mu^{(H)}$; then, we have the following BRS transformations:

$$\begin{aligned} &sc = -icc, \qquad sa_\mu = D_\mu c = \partial_\mu c + i[a_\mu, c], \\ &sc^{(H)} = -ic^{(H)}c^{(H)}, \\ &sa_\mu^{(H)} = D_\mu c^{(H)} = \partial_\mu c^{(H)} + i[a_\mu^{(H)}, c^{(H)}], \\ &s\psi_\alpha^{(16)f} = -ic\psi_\alpha^{(16)f}, \qquad s\varphi^{(H)} = -i[c, \varphi^{(H)}], \\ &s\phi^{(H)} = -ic^{(H)}\phi^{(H)}. \end{aligned} \tag{2.5}$$

Using the condition $(c^{ij})^* = c^{ij}$, one concludes that

$$\begin{aligned} &s(\psi_\alpha^{(16)f})^\dagger = i(\psi_\alpha^{(16)f})^\dagger c, \qquad s(\phi^{(H)})^\dagger = -i[c, (\phi^{(H)})^\dagger], \\ &s(\varphi^{(H)})^\dagger = i(\varphi^{(H)})^\dagger c^{(H)}. \end{aligned}$$

In the previous equations, and in the sequel, $\psi_\alpha^{(16)f}$ is viewed as the projection of a 32 Dirac spinor onto the 16 dimensional Weyl spinor subspace that carries 16 irrep of SO(10). This projection is carried out by the operator $P_+ = 1/2(1 + \Gamma_{11})$, $\Gamma_{11} = i^5\Gamma_1\Gamma_2\ldots\Gamma_{10}$.

To construct the Yukawa terms, one also introduces the following fermionic field:

$$\tilde{\psi}_\alpha^{(16)f} = (\psi_\alpha^{(16)f})^\top B, \qquad B = \prod_{i=\mathrm{odd}} \Gamma^i. \tag{2.6}$$

Taking into account that $(\Sigma^{ij})^\top B = -B\Sigma^{ij}$, one easily deduces that the BRS transformation of $\tilde{\psi}_\alpha^{(16)f}$ is given by

$$s\tilde{\psi}_\alpha^{(16)f} = i\tilde{\psi}_\alpha^{(16)f} c. \tag{2.7}$$

## III. INTRODUCING THE NONCOMMUTATIVE FIELDS OF THE NONCOMMUTATIVE SO(10) GUT

Within the enveloping-algebra framework of Refs. [4–6], one introduces at least a noncommutative field for each ordinary field. Each noncommutative field is a function—called the Seiberg-Witten map—of its ordinary counterpart, the ordinary gauge field and the noncommutativity matrix $\omega^{\mu\nu}$. This function—i.e., the Seiberg-Witten map—maps infinitesimal gauge orbits of the ordinary fields into noncommutative gauge orbits of their noncommutative counterparts. We shall assume—as suits the Feynman-diagram language—that the Seiberg-Witten map in momentum space is given by a formal power series expansion in the ordinary fields.

Let us first introduce the noncommutative gauge field, which we shall denote by $A_\mu[a_\nu;\omega]$, which is the counterpart of the ordinary field $a_\mu$ in (2.3). $A_\mu[a_\nu;\omega]$ is a solution to the following set of Seiberg-Witten map equations:

$$\begin{aligned}
&s_{nc}C[a_\mu,c;\omega] = sC[a_\mu,c;\omega],\\
&s_{nc}A_\mu[a_\nu;\omega] = sA_\mu[a_\nu;\omega], \qquad C[a_\mu,c;\omega=0]=c,\\
&A_\mu[a_\nu;\omega=0]=a_\mu, \qquad (C[a_\mu,c;\omega])^\dagger = C[a_\mu,c;\omega],\\
&(A_\mu[a_\nu;\omega])^\dagger = A_\mu[a_\nu;\omega],
\end{aligned} \tag{3.1}$$

where $C[a_\mu,c;\omega]$ is the noncommutative ghost field, $s$ is the ordinary BRS operator in (2.5) and $s_{nc}$ is the noncommutative BRS operator defined as follows:

$$s_{nc}C = -iC\star C, \qquad s_{nc}A_\mu = \partial_\mu C + i[A_\mu, C]_\star. \tag{3.2}$$

Here, $C = C[a_\mu,c;\omega]$ and $A_\mu = A_\mu[a_\nu;\omega]$. The noncommutative field $A_\mu[a_\nu;\omega]$ is an element of the universal enveloping algebra of the Lie algebra of S0(10) in the representation induced by the Dirac matrices $\Gamma^i$, $i = 1\ldots10$; $A_\mu[a_\nu;\omega]$ is, therefore, an element of $\mathbb{C}\mathrm{l}_{10}(\mathbb{C})$.

Next, $a_\mu^{(H)}$ in (2.4) gives rise to a noncommutative gauge field, which we shall denote by $A_\mu^{(H)}[a_\nu^{(H)};\omega]$. $A_\mu^{(H)}[a_\nu^{(H)};\omega]$ solves the following set of Seiberg-Witten map equations:

$$\begin{aligned}
&s_{nc}C^{(H)}[a_\mu^{(H)},c^{(H)};\omega] = sC^{(H)}[a_\mu^{(H)},c^{(H)};\omega],\\
&s_{nc}A_\mu^{(H)}[a_\nu^{(H)};\omega] = sA_\mu^{(H)}[a_\nu^{(H)};\omega],\\
&C^{(H)}[a_\mu^{(H)},c^{(H)};\omega=0] = c^{(H)},\\
&A_\mu^{(H)}[a_\nu^{(H)};\omega=0] = a_\mu^{(H)},\\
&(C^{(H)}[a_\mu^{(H)},c^{(H)};\omega])^\dagger = C^{(H)}[a_\mu^{(H)},c^{(H)};\omega],\\
&(A_\mu^{(H)}[a_\nu^{(H)};\omega])^\dagger = A_\mu^{(H)}[a_\nu^{(H)};\omega],
\end{aligned} \tag{3.3}$$

where $C^{(H)}[a_\mu^{(H)},c^{(H)};\omega]$ is the noncommutative ghost field and $s_{nc}$ is the noncommutative BRS operator defined in (3.2), but now $C = C^{(H)}[a_\mu^{(H)},c;\omega]$ and $A_\mu = A_\mu^{(H)}[a_\nu^{(H)};\omega]$. The noncommutative field $A_\mu^{(H)}[a_\nu^{(H)};\omega]$ is an element of the universal enveloping algebra of the Lie algebra of S0(10) in the representation induced the representation carried by $\varphi^{(H)}$. Recall that $H$ labels the representation and that $H = 210, 10, 45, \overline{126}$ and 120.

We shall need the noncommutative field strengths, $F_{\mu\nu}[a_\mu;\theta]$ and $F_{\mu\nu}^{(H)}[a_\mu;\theta]$, to define the Yang-Mills action on noncommutative space-time for our noncommutative S0(10) GUT. We define

$$\begin{aligned}
&F_{\mu\nu}[a_\rho;\theta] = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]_\star,\\
&\qquad A_\mu = A_\mu[a_\nu;\omega],\\
&F_{\mu\nu}^{(H)}[a_\rho^{(H)};\theta] = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]_\star,\\
&\qquad A_\mu = A_\mu^{(H)}[a_\nu^{(H)};\omega].
\end{aligned} \tag{3.4}$$

Notice that $F_{\mu\nu}[a_\rho;\theta]$ belongs to $\mathbb{C}\mathrm{l}_{10}(\mathbb{C})$ and $F_{\mu\nu}^{(H)}[a_\rho;\theta]$ takes values in the universal enveloping algebra of de Lie algebra of SO(10) in the representation induced by the representation $H$ of the latter.

Using (3.2), one can show that

$$sF_{\mu\nu}[a_\rho;\theta] = i[F_{\mu\nu}[a_\rho;\theta], C]_\star = s_{nc}F_{\mu\nu}[a_\rho;\theta], \tag{3.5}$$

if $C = C[a_\mu,c;\omega]$. The same equation holds for $F_{\mu\nu}^{(H)}[a_\rho^{(H)};\theta]$, *mutatis mutandis*.

The noncommutative fermionic fields will be denoted by $\Psi_\alpha^{(16)f}[a_\mu,\psi_\alpha^{(16)f};\theta]$ and $\tilde{\Psi}_\alpha^{(16)f}[a_\mu,\tilde{\psi}_\alpha^{(16)f};\omega]$. These noncommutative fermionic fields satisfy the following equations:

$$\begin{aligned}
&s_{nc}\Psi_\alpha^{(16)f}[a_\mu,\psi_\alpha^{(16)f};\omega] = s\Psi_\alpha^{(16)f}[a_\mu,\psi_\alpha^{(16)f};\theta],\\
&\Psi_\alpha^{(16)f}[a_\mu,\tilde{\psi}_\alpha^{(16)f};\omega=0] = \psi_\alpha^{(16)f},\\
&s_{nc}\tilde{\Psi}_\alpha^{(16)f}[a_\mu,\tilde{\psi}_\alpha^{(16)f};\omega] = s\tilde{\Psi}_\alpha^{(16)f}[a_\mu,\tilde{\psi}_\alpha^{(16)f};\omega],\\
&\tilde{\Psi}_\alpha^{(16)f}[a_\mu,\tilde{\psi}_\alpha^{(16)f};\omega=0] = \tilde{\psi}_\alpha^{(16)f}.
\end{aligned} \tag{3.6}$$

The ordinary BRS operator, $s$, acts on the ordinary fermionic fields as given in (2.5) and (2.7).

The action of noncommutative BRS operator, $s_{nc}$, on $\Psi_\alpha^{(16)f}[a_\mu, \psi^{(16)f}; \omega]$ and $\tilde{\Psi}_\alpha^{(16)f}[a_\mu, \tilde{\psi}^{(16)f}; \omega]$ is given by the formulas:

$$s_{nc}\Psi_\alpha = -iC\star\Psi_\alpha, \qquad s_{nc}\tilde{\Psi}_\alpha = i\tilde{\Psi}_\alpha\star C, \tag{3.7}$$

with $C = C[a_\mu, c; \omega]$, $\Psi_\alpha = \Psi_\alpha^{(16)f}[a_\mu, \psi^{(16)f}; \omega]$ and $\tilde{\Psi}_\alpha = \tilde{\Psi}_\alpha^{(16)f}[a_\mu, \tilde{\psi}^{(16)f}; \omega]$.

The noncommutative Higgs field which is the noncommutative counterpart of the ordinary Higgs multiplet $\varphi^{(H)}$, $H = 210, 10, 45, \overline{126}, 120$, introduced below (2.1), will be denoted by $\hat{\phi}^{(H)}[a_\mu^{(H)}, \phi^{(H)}; \omega]$. $\hat{\phi}^{(H)}[a_\mu^{(H)}, \phi^{(H)}; \omega]$ solves the following equations:

$$\begin{aligned} &s_{nc}\hat{\phi}^{(H)}[a_\mu^{(H)}, \phi^{(H)}; \omega] = s\hat{\phi}^{(H)}[a_\mu^{(H)}, \phi^{(H)}; \omega], \\ &\hat{\phi}^{(H)}[a_\mu^{(H)}, \phi^{(H)}; \omega = 0] = \phi^{(H)}, \end{aligned} \tag{3.8}$$

where, by definition,

$$\begin{aligned} s_{nc}\hat{\phi}^{(H)}[a_\mu^{(H)}, \phi^{(H)}; \omega] &= -iC\star\hat{\phi}^{(H)}[a_\mu^{(H)}, \phi^{(H)}; \omega], \\ C &= C^{(H)}[a_\mu^{(H)}, c^{(H)}; \omega]. \end{aligned} \tag{3.9}$$

Next, $\varphi^{(H)}[a_\mu, \varphi^{(H)}; \omega]$, $H = 210, 10, 45, \overline{126}, 120$ will stand for the noncommutative counterparts of the ordinary Higgs fields $\phi^{(H)}$ in (2.2). The Seiberg-Witten map equations that solve $\varphi^{(H)}[a_\mu, \varphi^{(H)}; \omega]$ read

$$\begin{aligned} &s_{nc}\varphi^{(H)}[a_\mu, \varphi^{(H)}; \omega] = s\varphi^{(H)}[a_\mu, \varphi^{(H)}; \omega], \\ &\varphi^{(H)}[a_\mu, \varphi^{(H)}; \omega = 0] = \varphi^{(H)}, \end{aligned} \tag{3.10}$$

where $s_{nc}$ is given by

$$\begin{aligned} s_{nc}\varphi^{(H)}[a_\mu, \varphi^{(H)}; \omega] &= i[\varphi^{(H)}[a_\mu, \varphi^{(H)}; \omega], C]_\star, \\ C &= C[a_\mu, c; \omega]. \end{aligned} \tag{3.11}$$

We shall need later the noncommutative covariant derivatives of the noncommutative matter fields, which are given by

$$\begin{aligned} D_\mu[A]\Psi_\alpha^{(16)f} &= \partial_\mu\Psi_\alpha^{(16)f} + iA_\mu\star\Psi_\alpha^{(16)f}, \\ D_\mu[A]\tilde{\Psi}_\alpha^{(16)f} &= \partial_\mu\tilde{\Psi}_\alpha^{(16)f} - i\tilde{\Psi}_\alpha^{(16)f}\star A_\mu, \\ D_\mu[A^{(H)}]\hat{\varphi}^{(H)} &= \partial_\mu\hat{\varphi}^{(H)} + iA_\mu^{(H)}\star\hat{\varphi}^{(H)}, \\ D_\mu[A^{(H)}]\varphi^{(H)} &= \partial_\mu\varphi^{(H)} + i[A_\mu, \varphi^{(H)}]_\star, \end{aligned} \tag{3.12}$$

with $\Psi_\alpha^{(16)f} = \Psi_\alpha^{(16)f}[a_\mu, \psi^{(16)f}; \omega]$, $\tilde{\Psi}_\alpha^{(16)f} = \tilde{\Psi}_\alpha^{(16)f}[a_\mu, \tilde{\psi}^{(16)f}; \omega]$, $\hat{\phi}^{(H)} = \hat{\phi}^{(H)}[a_\mu^{(H)}, \phi^{(H)}; \omega]$, $\varphi^{(H)} = \varphi^{(H)}[a_\mu, \varphi^{(H)}; \omega]$, $A_\mu^{(H)} = A_\mu^{(H))}[a_\nu^{(H)}; \omega]$ and $A_\mu = A_\mu[a_\nu; \omega]$.

## IV. THE PECCEI-QUINN CHARGES OF THE NONCOMMUTATIVE FIELDS

The need to solve the strong CP problem and to explain why dark matter exists in the observed amount demands [1] that the ordinary SO(10) GUT of Ref. [1] should have a spontaneously broken Peccei-Quinn symmetry—see also Refs. [46,47]. This spontaneously broken global symmetry gives rise to a particle, called the axion, that may constitute the dark matter of the Universe.

The Peccei-Quinn symmetry is a global U(1) symmetry of the action of the ordinary theory. The invariance of the ordinary Yukawa terms under the Peccei-Quinn symmetry imposes the following transformation laws on the ordinary fields $\psi_\alpha^{(16)}$, $\phi^{(10)}$, $\phi^{(\overline{126})}$ and $\phi(120)$:

$$\begin{aligned} &\psi_\alpha^{(16)} \to e^{iQ\theta}\psi_\theta^{(16)}, \qquad \phi^{(10)} \to e^{-i2Q\theta}\phi^{(10)}, \\ &\phi^{(\overline{126})} \to e^{-i2Q\theta}\phi^{(\overline{126})}, \qquad \phi^{(120)} \to e^{-i2Q\theta}\phi^{(120)}, \end{aligned}$$

where we have chosen the Peccei-Quinn charge, $Q$, of the fermionic multiplet as the unit of the Peccei-Quinn charge. Notice that the Higgs fields $\phi^{(10)}$, $\phi^{(120)}$ must be chosen to be non-Hermitian, thus giving rise to two irreps of SO(10) each. In Ref. [1], it has been shown that $\phi^{(45)}$ cannot be neutral under the Peccei-Quinn U(1), but with charge $Q' \neq Q$:

$$\phi^{(45)} \to e^{iQ'\theta}\phi^{(45)}.$$

All the other fields of the SO(10) GUT are chosen to be neutral under the Peccei-Quinn symmetry.

We shall be conservative and impose that global symmetries of the action are not modified by the noncommutative character of space-time. Hence, we shall choose Seiberg-Witten maps such that the Peccei-Quinn charge of each noncommutative field is well defined and agrees with that of its ordinary counterpart, i.e.,

$$\begin{aligned} \Psi_\alpha^{(16)}[a_\mu, e^{iQ\theta}\psi_\alpha^{(16)}; \omega] &= e^{iQ\alpha}\Psi_\alpha^{(16)}[a_\mu, \psi_\alpha^{(16)}; \omega], \\ \varphi^{(10)}[a_\mu, e^{-i2Q\theta}\phi^{(10)}; \omega] &= e^{-i2Q\theta}\varphi^{(10)}[a_\mu, \phi^{(10)}; \omega], \\ \varphi^{(\overline{126})}[a_\mu, e^{-i2Q\theta}\phi^{(\overline{120})}; \omega] &= e^{-i2Q\theta}\varphi^{(\overline{126})}[a_\mu, \phi^{(\overline{126})}; \omega], \\ \varphi^{(120)}[a_\mu, e^{-i2Q\theta}\phi^{(120)}; \omega] &= e^{-i2Q\alpha}\varphi^{(120)}[a_\mu, \phi^{(120)}; \omega], \\ \varphi^{(45)}[a_\mu, e^{iQ'\theta}\phi^{(45)}; \omega] &= e^{i2Q'\alpha}\varphi^{(45)}[a_\mu, \phi^{(45)}; \omega], \\ \hat{\varphi}^{(10)}[a_\mu^{(10)}, e^{-i2Q\theta}\varphi^{(10)}; \omega] &= e^{-i2Q\theta}\hat{\varphi}^{(10)}[a_\mu^{(10)}, \varphi^{(10)}; \omega], \\ \hat{\varphi}^{(\overline{126})}[a_\mu^{(\overline{126})}, e^{-i2Q\theta}\varphi^{(\overline{120})}; \omega] &= e^{-i2Q\theta}\hat{\varphi}^{(\overline{126})}[a_\mu^{(\overline{126})}, \varphi^{(\overline{126})}; \omega], \\ \hat{\varphi}^{(120)}[a_\mu^{(120)}, e^{-i2Q\theta}\varphi^{(120)}; \omega] &= e^{-i2Q\theta}\hat{\varphi}^{(120)}[a_\mu^{(120)}, \varphi^{(120)}; \omega], \\ \hat{\varphi}^{(45)}[a_\mu^{(45)}, e^{iQ'\theta}\varphi^{(45)}; \omega] &= e^{i2Q'\theta}\hat{\varphi}^{(45)}[a_\mu^{(45)}, \varphi^{(45)}; \omega]. \end{aligned} \tag{4.1}$$

That there are Seiberg-Witten maps satisfying the transformation laws in (4.1) is a consequence of the fact that the

Seiberg-Witten map for the matter fields can always be chosen so that it is linear in the corresponding ordinary fields. See Ref. [45] and the Appendix, for further details.

## V. THE ACTION OF THE NONCOMMUTATIVE SO(10) GUT

Let us first point out that we shall assume that Lorentz indices are raised and lowered with the help of the Minkowski metric $(-,+,+,+)$.

The action, $S$, which gives the dynamics of our noncommutative GUT, will be sum of integrated monomials with regard to the $\star$ product of the noncommutative fields, introduced in the previous section, and their derivatives. We shall restrict the mass dimension of these monomials to be less than or equal to 4, since we are interested in constructing the noncommutative counterpart of a renormalizable ordinary SO(10) GUT. So, not considering monomials with mass dimension bigger than 4 is the simplest choice to start with. For the sake of simplicity, we shall also assume that the dependence on $\omega^{\mu\nu}$ of the noncommutative action only occurs through the Seiberg-Witten map and the $\star$ product. We shall demand that the noncommutative action be invariant under the noncommutative BRS transformations defined in (3.2), (3.7) and (3.11) and the Peccei-Quinn transformations in (4.1). We shall break the action into four parts:

$$S = S_{\mathrm{YM}} + S_{\mathrm{fermionic}} + S_{\mathrm{Yukawa}} + S_{\mathrm{Higgs}} \tag{5.1}$$

and discuss each part separately below.

### A. The noncommutative Yang-Mills action

In view of (3.5) and following Ref. [10], we shall define the noncommutative Yang-Mills action, $S_{\mathrm{YM}}$, as follows:

$$\begin{aligned} S_{\mathrm{YM}} = &-\kappa_c \int d^4x\, \mathrm{Tr} F_{\mu\nu}[a_\rho;\omega]\star F^{\mu\nu}[a_\rho;\omega] \\ &- \sum_H \kappa^{(H)} \int d^4x\, \mathrm{Tr} F^{(H)}_{\mu\nu}[a^{(H)}_\rho;\omega]\star F^{(H)\mu\nu}[a^{(H)}_\rho;\omega]. \end{aligned} \tag{5.2}$$

$F_{\mu\nu}[a_\rho;\omega]$ and $F^{(H)}_{\mu\nu}[a^{(H)}_\rho;\omega]$ are given in (3.4) and the real constants $\kappa_c$, $\kappa_H$ are constrained by the following equation:

$$\frac{1}{g^2_{\mathrm{YM}}} = 32k_c + \sum_H 4I_2(H)\kappa_H. \tag{5.3}$$

$g_{\mathrm{YM}}$ is the tree-level Yang-Mills coupling constant and $I_2(H)$ is the second order Dynkin index of the irrep $H$ of SO(10), i.e.,

$$\mathrm{Tr} M^{(H)}_{ij} M^{(H)}_{kl} = I_2(H)(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}).$$

Notice that (5.3) comes from the need to have the right normalization of the free kinetic term of the gauge field. Positivity of $S_{\mathrm{YM}}$ for Euclidean signature puts further constraints on the $\kappa$'s, which are automatically satisfied if the $\kappa$'s are positive or vanish.

We should like to point out that there is no reason to set to zero from the beginning any of the $\kappa$'s in (5.2), for one-loop Higgs radiative corrections generate contributions—see Ref. [16]—to the effective action of gauge field that are of the type

$$\int d^4x\, \mathrm{Tr} F^{(H)}_{\mu\nu}[a^{(H)}_\rho;\omega]\star F^{(H)\mu\nu}[a^{(H)}_\rho;\omega].$$

### B. The fermionic part of action

Furnished with the noncommutative fermionic fields defined by (3.6) and their covariant derivatives in (3.12), one constructs the fermionic part of the action, $S_{\mathrm{fermionic}}$, of our noncommutative GUT. We shall assume that $S_{\mathrm{fermionic}}$ is quadratic in the noncommutative fermionic fields and linear in the noncommutative gauge field. This is the simplest choice for an $S_{\mathrm{fermionic}}$. $S_{\mathrm{fermionic}}$ reads

$$\begin{aligned} S_{\mathrm{fermionic}} = \sum_f \Big( &\kappa^f \int d^4x i \Psi^{f\dagger}_{\dot\alpha} \bar\sigma^{\dot\alpha\alpha}_\mu D^\mu[A]\Psi^f_\alpha \\ &- \tilde\kappa^f \int d^4x i D_\mu[A]\tilde\Psi^{f\alpha}\sigma^\mu_{\alpha\dot\alpha}\tilde\Psi^{f\dagger\dot\alpha}\Big), \end{aligned}$$

where $\Psi^f_\alpha = \Psi^{(16)f}_\alpha[a_\mu,\psi^{(16)f}_\alpha;\omega]$, $\tilde\Psi^f_\alpha = \tilde\Psi^{(16)f}_\alpha[a_\mu,\tilde\psi^{(16)f}_\alpha;\omega]$, $\Psi^{f\dagger}_{\dot\alpha} = (\Psi^f_\alpha)^\dagger$ and $\tilde\Psi^{f\dagger}_{\dot\alpha} = (\tilde\Psi^f_\alpha)^\dagger$. The conventions on dotted and undotted indices that we use are those of Ref. [48]. The proper normalization of the free propagator of the fermions demands that

$$\kappa^f + \tilde\kappa^f = 1.$$

### C. The Yukawa terms

Let us recall—see, for instance, Ref. [47]—that the most general Yukawa terms in a renormalizable ordinary SO(10) GUT with only fermionic multiplets in the 16 irrep reads

$$\begin{aligned} \sum_{ff'} \int d^4x (&Y^{(10)}_{ff'} \tilde\psi^{(16)f\alpha}\varphi^{(10)}\psi^{(16)f'}_\alpha \\ &+ Y^{(\overline{126})}_{ff'} \tilde\psi^{(16)f\alpha}\varphi^{(\overline{126})}\psi^{(16)f'}_\alpha \\ &+ Y^{(120)}_{ff'} \tilde\psi^{(16)f\alpha}\varphi^{(120)}\psi^{(16)f'}_\alpha + \mathrm{H.c.}). \end{aligned}$$

Recall that $\phi^{(10)}$, $\varphi^{(\overline{126})}$ and $\phi^{(120)}$ have been defined in (2.2) and $\tilde\psi^{(16)f\alpha}$ has been introduced in (2.6). It is apparent that the simplicity and beauty of the previous expression comes from the fact that Higgs fields which occur in it can be interpreted as elements of $\mathbb{C}\mathrm{l}_{10}(\mathbb{C})$. It is also this feature of the Higgs fields in the ordinary Yukawa terms above that enables us to introduce, in a natural way, the following noncommutative Yukawa terms:

$$S_{\mathrm{Yukawa}} = \sum_{ff'} \int d^4x (Y^{(10)}_{ff'} \tilde{\Psi}^{(16)f\alpha} \star \varphi^{(10)} \star \Psi^{(16)f'}_{\alpha} + Y^{(\overline{126})}_{ff'} \tilde{\Psi}^{(16)f\alpha} \star \varphi^{(\overline{126})} \star \Psi^{(16)f'}_{\alpha} + Y^{(120)}_{ff'} \tilde{\Psi}^{(16)f\alpha} \star \varphi^{(120)} \star \Psi^{(16)f'}_{\alpha} + \mathrm{H.c.}), \tag{5.4}$$

where

$$\tilde{\Psi}^{(16)f}_{\alpha} = \tilde{\Psi}^{(16)f}_{\alpha}[a_\mu, \tilde{\Psi}^{(16)f}_{\alpha}; \omega],$$
$$\Psi^{(16)f}_{\alpha} = \Psi^{(16)f}_{\alpha}[a_\mu, \Psi^{(16)f}_{\alpha}; \omega],$$
$$\varphi^{(H)} = \varphi^{(H)}[a^{(H)}_\mu, \varphi^{(H)}; \omega],$$
$$H = 10, \overline{126}, 120$$

are Seiberg-Witten maps that solve (3.6) and (3.10). By construction, $S_{\mathrm{Yukawa}}$, in (5.4), is invariant under BRS transformations of the ordinary fields and the corresponding noncommutative BRS transformations in (3.7) and (3.11).

Let us point out that the noncommutative Higgses $\hat{\varphi}^{(H)}_I$, $I = 1 \ldots dimH$, $H = 10, \overline{126}, 120$ defined by (3.8) are of no help for constructing integrated cubic monomials, with regard to the $\star$ product, of the noncommutative fields $\tilde{\Psi}^{(16)f}_{\alpha}$, $\Psi^{(16)f}_{\alpha}$ and $\hat{\varphi}^{(H)}_I$. Indeed, let $\mathcal{T}^{rIr'}$ be complex numbers, then, the fact that the $\star$ product is not commutative and that $C[a_\mu, c; \omega]$ and $C^{(H)}[a^{(H)}_\mu, c^{(H)}; \omega]$ take values in the enveloping algebra of SO(10), prevents the term

$$\int d^4x \mathcal{T}^{rIr'} \tilde{\Psi}^{(16)f\alpha}_{r} \star \hat{\varphi}^{(H)}_I \star \Psi^{(16)f}_{\alpha r'} \tag{5.5}$$

from being invariant under the noncommutative BRS transformations in (3.7) and (3.9). This result holds whatever the ordering of the fields in (5.5).

In view of the previous discussion the reader may ask why we have introduced the noncommutative Higgs fields $\hat{\varphi}^{(H)}_I$. We shall answer this question in the next subsection.

### D. The Higgs action

$S_{\mathrm{Higgs}}$ in (5.1) contains only Higgs fields and gauge fields. Let us begin by introducing the kinetic terms. These we take to be quadratic in the noncommutative Higgs fields and their covariant derivatives. We also assume that—as in the ordinary field theory case—these terms are semipositive definite after a Wick rotation has been performed. We thus end up with the following gauge covariant kinetic terms for the noncommutative Higgses:

$$\mathcal{K}_{\mathrm{Higgs}} = -\sum_{H} \int d^4x (s_H (D_\mu[A^{(H)}]\hat{\phi}^{(H)})^\dagger D_\mu[A^{(H)}]\hat{\phi}^{(H)} + t_H \,\mathrm{Tr}((D_\mu[A]\varphi^{(H)})^\dagger D_\mu[A]\varphi^{(H)})), \tag{5.6}$$

where $D_\mu[A^{(H)}]\hat{\phi}^{(H)}$ and $D_\mu[A]\varphi^{(H)}$ are given in (3.12) and $H = 210, 10, 45, \overline{126}$ and 120. The parameters $s_H$ and $t_H$ are positive real numbers such that each free kinetic term has the right normalization.

It is plain that the second summand of (5.6) is needed, for the noncommutative Yukawa terms in (5.4) involve $\varphi^{(H)}$. To provide the rationale for the first summand of (5.6), we must discuss why the construction of a phenomenologically sensible noncommutative Higgs potential seems to require that the noncommutative fields $\hat{\varphi}$ be added to the pool of noncommutative Higgs fields.

In Ref. [49], it has been analyzed the classical vacuum structure of an ordinary S0(10) GUT with a Higgs in the 45 and another in the 16. It has been shown there that if the monomial $\mathrm{Tr}((\varphi^{(45)})^\dagger \varphi^{(45)} (\varphi^{(45)})^\dagger \varphi^{45)})$ occurs in the Higgs potential, then the monomial $(\mathrm{Tr}((\varphi^{(45)})^\dagger \varphi^{(45)}))^2$ must be also a summand of the Higgs potential. This result comes from demanding boundedness from below of the Higgs potential and absence of tachyons.

Now, it is true that the ordinary GUT corresponding to our noncommutative GUT has a Higgs content more involved than the one used in Ref. [49] and that no analysis similar to that in the latter paper has been carried out for an ordinary GUT with our Higgs content; so it cannot be claimed that both types of monomials must necessarily occur in the Higgs potential. However, until such a complicated analysis is carried out in the ordinary case, we shall play it safe and include in the noncommutative Higgs potential noncommutative counterparts of both $\mathrm{Tr}((\varphi^{(H)})^\dagger \varphi^{(H)} (\varphi^{(H)})^\dagger \varphi^{H)})$ and $(\mathrm{Tr}((\varphi^{(H)})^\dagger \varphi^{(H)}))^2$, $H = 210, 10, 45, \overline{126}$ and 120.

It is plain that

$$\int d^4x \,\mathrm{Tr}(\varphi^{(H)})^\dagger \star \varphi^{(H)} \star (\varphi^{(H)})^\dagger \star \varphi^{H)})) \quad \text{and}$$
$$\int d^4x \,\mathrm{Tr}(\varphi^{(H)})^\dagger \star (\varphi^{(H)})^\dagger \star \varphi^{(H)} \star (\varphi^{(H)})$$

are invariant under the noncommutative BRS transformations in (3.11), provided $\varphi^{(H)}$ is given by a Seiberg-Witten map. However,

$$\int d^4x (\mathrm{Tr}((\varphi^{(H)})^\dagger \star \varphi^{(H)}))^2$$

is not invariant under the noncommutative BRS transformations in (3.11), for—unlike the ordinary case—the unintegrated monomial $(\mathrm{Tr}((\varphi^{(H)})^\dagger \star \varphi^{(H)}))^2)$ is not invariant under BRS transformations and, what is worse, its BRS variation is not a total derivative. Hence, to construct the noncommutative counterpart of the ordinary integrated monomial

$$\int d^4x (\mathrm{Tr}((\phi^{(H)})^\dagger \phi^{(H)}))^2, \tag{5.7}$$

we shall take into account that $\mathrm{Tr}((\phi^{(H)})^\dagger\phi^{(H)}) = s_H(\varphi^{(H)})^{\dagger I}\varphi_I^{(H)})$, where $s_H$ is a real number—$s_{210} = 32\times 4!, s_{10} = 32, s_{45} = 16, \ldots$—whose actual value is irrelevant to our discussion, and use the noncommutative scalar $\hat\varphi_I^{(H)}$ in (3.8), rather than $\varphi^{(H)}$, to define the noncommutative counterpart of the integrated ordinary monomial in (5.7) as follows:

$$\int d^4x((\hat\phi^{(H)})^{\dagger I}\star\hat\phi_I^{(H)}))^2.$$

Let $Q^{(H)}$ stand for either $\varphi^{(H)}$ or its Hermitian conjugate $(\varphi^{(H)})^\dagger$. Then, we are now ready to introduce the noncommutative Higgs-potential term, $\hat V_{\mathrm{Higgs}}$, of our noncommutative SO(10) GUT:

$$\begin{aligned}\hat V_{\mathrm{Higgs}} = &\sum_H \alpha_H(\hat\phi^{(H)})^{\dagger I}\star\hat\phi_I^{(H)} + \sum_{H_1H_2}\beta_{H_1,H_2}((\hat\phi^{(H_1)})^{\dagger I}\star\hat\phi_I^{(H_1)})\star((\hat\phi^{(H_2)})^{\dagger I}\star\hat\phi_I^{(H_2)}) + \gamma_{210}\,\mathrm{Tr}\varphi^{(210)}\\ &+\sum_{H_1H_2}\kappa_{H_1H_2}\,\mathrm{Tr}(Q^{(H_1)}\star Q^{(H_2)}) + \sum_{H_1H_2H_3}\gamma_{H_1H_2H_3}\,\mathrm{Tr}(Q^{(H_1)}\star Q^{(H_2)}\star Q^{(H_3)})\\ &+\sum_{H_1H_2H_3H_4}\lambda_{H_1H_2H_3H_4}\,\mathrm{Tr}(Q^{(H_1)}\star Q^{(H_2)}\star Q^{(H_3)}\star Q^{(H_4)}),\end{aligned} \tag{5.8}$$

where $\alpha_H$, $\beta_{H_1H_2}$, $\kappa_{H_1H_2}$, $\gamma_{H_1H_2H_3}$ $\lambda_{H_1H_2H_3H_4}$ are numbers; which are real, if the monomial they go with is real. Boundedness from below of $\hat V_{\mathrm{Higgs}}$ put constraints on the couplings $\beta_H$'s and $\lambda_{H_1H_2H_3H_4}$'s. $H$, $H_1$, $H_2$, $H_3$ and $H_4$ run over the set $210, 10, 45, \overline{126}, 126, 120$. The monomials $\mathrm{Tr}(Q^{(H_1)}\star Q^{(H_2)})$, $\mathrm{Tr}(Q^{(H_1)}\star Q^{(H_2)}\star Q^{(H_3)})$ and $\mathrm{Tr}(Q^{(H_1)}\star Q^{(H_2)}\star Q^{(H_3)}\star Q^{(H_4)})$ satisfy the following conditions: (i) The sum of the Peccei-Quinn charges of the fields entering the monomial must vanish—thus, the action will have a Peccei-Quinn symmetry; (ii) if a monomial occurs, so does its Hermitian conjugate multiplied by the appropriate complex conjugate coupling constant; and (iii) monomials obtained by cyclic permutations of the fields of a given monomial are dropped. Notice that, by setting to zero the ordinary gauge fields in $Q^{(H)}$ and $\hat\varphi_I^{(H)}$, $\hat V_{\mathrm{Higgs}}$ in (5.8) yields the Higgs potential for the ordinary Higgses on noncommutative space-time. This noncommutative potential further reduces to the appropriate—a quartic polynomial—Higgs potential on ordinary Minkowski space-time by setting $\omega^{\mu\nu} = 0$.

Finally, the BRS invariant noncommutative Higgs action, $S_{\mathrm{Higgs}}$, reads

$$S_{\mathrm{Higgs}} = \mathcal{K}_{\mathrm{Higgs}} - \int d^4x\hat V_{\mathrm{Higgs}},$$

where $\mathcal{K}_{\mathrm{Higgs}}$ and $\hat V_{\mathrm{Higgs}}$ are given in (5.6) and (5.8), respectively.

Before closing this section we would like to point out that there are monomials in $V_{\mathrm{Higgs}}$ that vanish—due to SO (10) invariance—at $\omega^{\mu\nu} = 0$, but are nonvanishing otherwise. These monomials yield intrinsically noncommutative interactions—since they vanish at $\omega^{\mu\nu} = 0$—between Higgses of several species and the gauge field. Let us give just one example:

$$\begin{aligned}\int d^4x\,\mathrm{Tr}(\varphi^{(10)})^\dagger\star\varphi^{(10)}\star\varphi^{(210)} = 4!\times 16\times\int\prod_{i=1}^4\frac{d^4p_i}{(2\pi)^4}(2\pi)^4\delta(-p_1+p_2+p_3+p_4)(\phi^{(10)})^*_{i_1}(p_1)\phi^{(10)}_{j_1}(p_2)a^{k_1k_2}_{\mu_1}\\ \times(p_3)\phi^{(210)}_{i_1j_1k_1k_2}(p_4)\Big[-e^{\frac{i}{2}(p_1-p_3)\wedge p_2)}\omega^{\mu_1\mu_2}p_{1\mu_2}\frac{\sin(\frac{1}{2}p_3\wedge p_1)}{p_3\wedge p_1} + e^{\frac{i}{2}p_1\wedge(p_2+p_3)}\omega^{\mu_1\mu_2}p_{2\mu_2}\\ \times\frac{\sin(\frac{1}{2}p_3\wedge p_2)}{p_3\wedge p_2} + e^{\frac{i}{2}p_1\wedge p_2}\omega^{\mu_1\mu_2}p_{4\mu_2}\frac{\sin(\frac{1}{2}p_3\wedge p_4)}{p_3\wedge p_4}\Big] + O(a_\mu^2).\end{aligned}$$

The right-hand side (rhs) of the previous equation has been derived with the help of the results presented in the Appendix. Let us stress that terms like the one in the previous equation describe the tree-level coupling between different species of ordinary Higgses, and the ordinary gauge field, as they move around in noncommutative space-time. Tree-level couplings such as this are not possible in ordinary Minkowski space-time, so its eventual experimental detection will give a clear hint of the noncommutative character of space-time. This strategy to experimentally probe the possible noncommutative character of space-time was pioneered by the authors of Ref. [50].

Another peculiarity of the noncommutative Higgs-potential term in (5.8) is that it contains a contribution that is proportional to $\mathrm{Tr}\varphi^{(210)}$. This term is not forbidden neither by gauge invariance nor by the Peccei-Quinn symmetry. Using the results presented in the Appendix one shows that $\mathrm{Tr}\varphi^{(210)}$ vanishes at $\omega^{\mu\nu}=0$ and that the first nontrivial contribution coming from it occurs at order $(a_\mu)^2$, thus giving rise to a non-Lorentz invariant coupling between two gauge fields and the 210 Higgs. Notice that similar terms for the other Higgses of the GUT will explicitly break the Peccei-Quinn symmetry and this would not do.

## VI. CONCLUSIONS AND OUTLOOK

In this paper we have successfully formulated the classical action of a noncommutative SO(10) GUT which is the counterpart of the phenomenologically relevant ordinary SO(10) GUT of Ref. [1]. The noncommutative model presented here has two immediate distinct qualitative phenomenological consequences: one is the possibility of breaking particle Lorentz invariance by having noncommutative coordinates while there are grand unification effects at work; the other is the existence of tree-level interactions between several Higgs species that do not occur in the corresponding ordinary SO(10) of Ref. [1]: those interactions vanish as $\omega^{\mu\nu}\to 0$. The detection of these noncommutative interactions will strongly support the idea that space-time is not a smooth manifold at high enough energies. A quantitative study of these two consequences requires a separate paper.

An important issue that should be tackled without delay is the analysis of the UV/IR mixing effects in this noncommutative theory. We would like to stress that another physical consequence of our formulation is that those UV/IR mixing effects do not generally occur in the same type of terms as in noncommutative U(N) theories. Indeed, for U(N) theories, the UV/IR mixing phenomenon makes the two point function of the effective action of the ordinary field develop the following well-known IR divergences [51]:

$$\mathrm{Tr}a_\mu(p)\,\frac{\tilde p^\mu\tilde p^\nu}{\tilde p^4}\,\mathrm{Tr}a_\nu(-p),$$
$$\mathrm{Tr}a_\mu(p)\ln(-p^2\tilde p^2)(p^2\eta^{\mu\nu}-p^\mu p^\nu)\,\mathrm{Tr}a_\nu(-p).$$

But these types of terms do not occur for simple gauge groups since now $\mathrm{Tr}a_\mu=0$. Recall that $\tilde p^\mu=\omega^{\mu\nu}p_\nu$. Of course, in view of the calculations presented in Ref. [36] for the U(1) case, the detailed computation of the UV/IR structure of the model formulated here will involve very lengthy and complicated computations.

An issue which should also be addressed is a comprehensive study of the set of classical vacua of our SO(10) GUT and how it is modified at the quantum level. Of course, the phenomenology that the SO(10) GUT presented here gives rise to should be studied. In this regard the analysis—perhaps, along the lines of Refs. [37–40]—of the neutrino physics that our GUT yields looks particularly interesting.

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## APPENDIX

It has been discussed in Ref. [45] how to obtain systematically $\theta$-exact solutions to the Seiberg-Witten map equations in (3.1), (3.3), (3.6) and (3.8). Here, we shall show how to construct a $\theta$-exact solution to (3.10).

Let us first point out that (3.1) holds in any number of space-time dimensions whatever the value of noncommutativity matrix $\omega^{ij}$. Now, assume that we are in $4+1$ space-time dimensions and that $\omega^{ij}$, $i,j=0,1,2,3,4$ is such that $\omega^{\mu 4}=0$, $\mu=0,1,2,3$. It was shown in Refs. [42,43] that the following "evolution" equations give a solution to (3.1)

$$\begin{aligned}
&\frac{d}{dh}C(h\omega)=\frac{1}{4}\omega^{\rho\sigma}\{\partial_\rho C(h\omega),A_\rho(h\omega)\}_{\star_h},\qquad C(h=0)=c,\\
&\frac{d}{dh}A_\mu(h\omega)=\frac{1}{2}\omega^{\rho\sigma}\{A_\rho(h\omega),\partial_\sigma A_\mu(h\omega)\}_{\star_h}-\frac{1}{4}\omega^{\rho\sigma}\{A_\rho(h\omega),\partial_\mu A_\sigma(h\omega)\}_{\star_h}+\frac{i}{4}\omega^{\rho\sigma}\{A_\rho(h\omega),[A_\sigma(h\omega),A_\mu(h\omega)]_{\star_h}\}_{\star_h},\\
&A_\mu(h=0)=a_\mu,\\
&\frac{d}{dh}A_4(h\omega)=\frac{1}{2}\omega^{\rho\sigma}\{A_\rho(h\omega),\partial_\sigma A_4(h\omega)\}_{\star_h}-\frac{1}{4}\omega^{\rho\sigma}\{A_\rho(h\omega),\partial_4 A_\sigma(h\omega)\}_{\star_h}+\frac{i}{4}\omega^{\rho\sigma}\{A_\rho(h\omega),[A_\sigma(h\omega),A_4(h\omega)]_{\star_h}\}_{\star_h},\\
&A_4(h=0)=a_4,
\end{aligned}\tag{A1}$$

where the Greek indices run over 0,1,2 and 3 and $\star_h$ denotes the Moyal product where $h\omega^{\mu\nu}$ has replaced $\omega^{\mu\nu}$.

Now, neither $A_4$ nor $a_4$ enter the first two equations in (A1), so these two equations give Seiberg-Witten maps $C[a_\rho,c;\omega]$ and $A_\mu[a\rho;\omega]$ which do not depend on $a_4$. On the other hand, the last equation in (A1) yields a Seiberg-Witten map $A_4[a_\mu,a_4;\omega]$ which depends on both $a_\mu$ and $a_4$. Let us particularize (A1) to ordinary fields $a_i=(a_\mu,a_4)$, $\mu=0,1,2,3$, which do not depend on $x^4$ and ordinary ghost fields which do not depend on $x^4$, either. For these ordinary field configurations we have that $C[a_\rho,c;\omega]$, $A_\mu[a_\rho;\omega]$ and $A_4[a_\mu,a_4;\omega]$ solving (A1) do not depend on $x^4$, so they are actually noncommutative fields which live in $3+1$ space-time dimensions. From this four-dimensional point of view, $A_\mu[a_\rho;\omega]$ and $C[a_\rho,c;\omega]$ are, respectively, the noncommutative gauge field and the corresponding ghost field—i.e., $A_\mu[a_\rho;\omega]$ and $C[a_\rho,c;\omega]$ solve (3.1) in $3+1$ dimensions, whereas $A_4[a_\mu,a_4;\omega]$ is a noncommutative field solving

$$\begin{aligned}sA_4[a_\mu,a_4;\omega]&=\partial_4 C[a_\mu,c;\omega]+i[[A_4[a_\mu,a_4;\omega],C[a_\mu,c;\omega]]_\star\\&=-i[C[a_\mu,c;\omega],A_4[a_\mu,a_4;\omega]]_\star,\\A_4[\omega=0]&=a_4,\end{aligned}$$

since $\partial_4 C[a_\mu,c;\omega]=0$. If, in the previous equation one substitutes $\varphi^{(H)}[a_\rho,\varphi^{(H)};\omega]$ for $A_4[a_\rho,\varphi^{(H)};\omega]$, one obtains (3.10). Hence, by replacing $A_4$ with $\varphi^{(H)}$ and $a_4$ with $\phi^{(H)}$ in the last equation of (A1), a solution to (3.10) will be produced, if the term involving $\partial_4$ is dropped. We have this shown that the evolution problem that yields the Seiberg-Witten map which defines $\Phi^{(H)}$ reads

$$\begin{aligned}\frac{d}{dh}\Phi^{(H)}(h\omega)&=\frac{1}{2}\omega^{\rho\sigma}\{A_\rho(h\omega),\partial_\sigma\Phi^{(H)}(h\omega)\}_{\star_h}\\&\quad+\frac{i}{4}\omega^{\rho\sigma}\{A_\rho(h\omega),[A_\sigma(h\omega),\Phi^{(H)}(h\omega)]_{\star_h}\}_{\star_h},\\\Phi^{(H)}(h=0)&=\Phi^{(H)}.\end{aligned}\tag{A2}$$

The $\omega$-exact solution to (A2) that is a formal series expansion in power of the ordinary fields is obtained recursively. Let us express $A_\mu[a_\rho;h\omega]$ and $\Phi^{(H)}[a_\rho,\phi^{(H)};h\omega]$ as follows:

$$\begin{aligned}A_\mu[a_\rho;h\omega]&=\sum_{n>0}A_\mu^{(n)}[a_\mu;h\omega],\\\Phi^{(H)}[a_\rho,\phi^{(H)};h\omega]&=\sum_{n\geq 0}\Phi^{(H,n)}[a_\rho,\phi^{(H)};h\omega],\end{aligned}$$

where $A_\mu^{(n)}[a_\rho;h\omega]$ and $\Phi^{(H,n)}[a_\rho,\phi^{(H)};h\omega]$ are monomials of degree $n$ with regard to $A_\mu$. Then substituting them in (A2), one obtains the infinite set of equations,

$$\begin{aligned}\Phi^{(H,0)}[a_\rho,\phi^{(H)};h\omega]&=\phi^{(H)},\\\Phi^{(H,1)}[a_\rho,\phi^{(H)};\omega]&=\int_0^1 dh\left(\frac{1}{2}\omega^{\rho\sigma}\{A_\rho^{(1)}(h\omega),\partial_\sigma\phi^{(H)}\}_{\star_h}\right),\\\Phi^{(H,2)}[a_\rho,\phi^{(H)};\omega]&=\int_0^1 dh\left(\frac{1}{2}\omega^{\rho\sigma}\{A_\rho^{(2)}(h\omega),\partial_\sigma\phi^{(H)}\}_{\star_h}+\frac{1}{2}\omega^{\rho\sigma}\{A_\rho^{(1)}(h\omega),\partial_\sigma\Phi^{(H,1)}(h\omega)\}_{\star_h}\right.\\&\quad\left.+\frac{i}{4}\omega^{\rho\sigma}\{A_\rho^{(1)}(h\omega),[A_\sigma^{(1)}(h\omega),\phi^{(H)}]_{\star_h}\}_{\star_h}\right),\\\Phi^{(H,n)}[a_\rho,\phi^{(H)};\omega]&=\int_0^1 dh\left(\sum_{m=0}^{n-1}\frac{1}{2}\omega^{\rho\sigma}\{A_\rho^{(n-m)}(h\omega),\partial_\sigma\Phi^{(H,m)}(h\omega)\}_{\star_h}\right.\\&\quad\left.+\sum_{m_1+m_2+m_3=n}\frac{i}{4}\omega^{\rho\sigma}\{A_\rho^{(m_1)}(h\omega),[A_\sigma^{(m_2)}(h\omega),\phi^{(H,m_3)}(h\omega)]_{\star_h}\}_{\star_h}\right),\qquad n\geq 3,\end{aligned}\tag{A3}$$

where $m_1>0$, $m_2>0$ and $m_3\geq 0$. We would like to stress that each $\Phi^{(H,n)}[a_\rho,\phi^{(H)};\omega]$ in (A3) is linear in the ordinary field $\phi^{(H)}$. Hence, the corresponding equation in (4.1) holds for this Seiberg-Witten map.

Next, with the help of the results presented in Ref. [45], one may work out the rhs of each equality in (A3) recursively; we shall display below the explicit expressions that we have obtained for $\Phi^{(H,1)}[a_\rho,\phi^{(H)};\omega]$ and $\Phi^{(H,2)}[a_\rho,\phi^{(H)};\omega]$:

$$\begin{aligned}\Phi^{(H,1)}[a_\rho,\phi^{(H)};\omega]&=\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}e^{i(p_1+p_2)x}\omega^{\mu_1\mu_2}p_{2\mu_2}\left(\frac{e^{\frac{i}{2}p_1\wedge p_2}-1}{p_1\wedge p_2}\phi^{(H)}(p_2)a_{\mu_1}(p_1)\right.\\&\quad\left.-\frac{e^{-\frac{i}{2}p_1\wedge p_2}-1}{p_1\wedge p_2}a_{\mu_1}(p_1)\phi^{(H)}(p_2)\right),\end{aligned}$$

where $p_1\wedge p_2=\omega^{\mu_1\mu_2}p_{1\mu_1}p_{2\mu_2}$, and

$$\Phi^{(H,2)}[a_\rho,\phi^{(H)};\omega]=\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}e^{i(p_1+p_2+p_3)x}\Big\{\frac{1}{2}\omega^{\mu\nu}\omega^{\rho\sigma}[2(p_{2\sigma}\delta^{\mu_1}_\rho\delta^{\mu_2}_\mu+p_{1\sigma}\delta^{\mu_1}_\mu\delta^{\mu_2}_\rho)$$
$$-(p_2-p_1)_\mu\delta^{\mu_1}_\rho\delta^{\mu_2}_\sigma]p_{3\nu}[\mathcal{G}(-p_3;p_1,p_2;\omega)a_{\mu_1}(p_1)a_{\mu_2}(p_2)\phi^{(H)}(p_3)$$
$$+\mathcal{G}(p_3;p_1,p_2;\omega)\phi^{(H)}(p_3)a_{\mu_1}(p_1)a_{\mu_2}(p_2)]+\omega^{\mu\nu}\omega^{\rho\sigma}(p_2+p_3)_\nu p_{3\sigma}\delta^{\mu_1}_\mu\delta^{\mu_2}_\rho$$
$$\times[\mathcal{G}(p_1;p_2,p_3;\omega)a_{\mu_1}(p_1)a_{\mu_2}(p_2)\phi^{(H)}(p_3)+\mathcal{G}(-p_1;p_2,p_3;\omega)a_{\mu_2}(p_2)\phi^{(H)}(p_3)a_{\mu_1}(p_1)$$
$$+\bar{\mathcal{G}}(p_1;p_2,p_3;\omega)a_{\mu_1}(p_1)a_{\mu_2}(p_2)\phi^{(H)}(p_3)+\bar{\mathcal{G}}(-p_1;p_2,p_3;\omega)a_{\mu_1}(p_1)\phi^{(H)}(p_3)a_{\mu_2}(p_2)]$$
$$-\frac{1}{2}\omega^{\mu_1\mu_2}[\mathcal{F}(p_1;p_2,p_3;\omega)a_{\mu_1}(p_1)a_{\mu_2}(p_2)\phi^{(H)}(p_3)+\mathcal{F}(-p_1;p_2,p_3;\omega)a_{\mu_2}(p_2)\phi^{(H)}(p_3)a_{\mu_1}(p_1))$$
$$+\bar{\mathcal{F}}(p_1;p_2,p_3;\omega)\phi^{(H)}(p_3)a_{\mu_2}(p_2)a_{\mu_1}(p_1)+\bar{\mathcal{F}}(-p_1;p_2,p_3;\omega)a_{\mu_1}(p_1)\phi^{(H)}(p_3)a_{\mu_2}(p_2)]\Big\}.$$

In the previous equation, $\bar{\mathcal{G}}$ and $\bar{\mathcal{F}}$ are the complex conjugates of the functions $\mathcal{G}$ and $\mathcal{F}$, respectively. The functions $\mathcal{G}$ and $\mathcal{F}$ are defined as follows:

$$\mathcal{G}(p_1;p_2,p_3;\omega)=\frac{1}{p_2\wedge p_3}\left[\frac{e^{-\frac{i}{2}(p_1\wedge p_2+p_1\wedge p_3+p_2\wedge p_3)}-1}{p_1\wedge p_2+p_1\wedge p_3+p_2\wedge p_3}-\frac{e^{-\frac{i}{2}(p_1\wedge p_2+p_1\wedge p_3)}-1}{p_1\wedge p_2+p_1\wedge p_3}\right],$$
$$\mathcal{F}(p_1;p_2,p_3;\omega)=\frac{e^{-\frac{i}{2}(p_1\wedge p_2+p_1\wedge p_3+p_2\wedge p_3)}-1}{p_1\wedge p_2+p_1\wedge p_3+p_2\wedge p_3}.$$


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