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Microcausality in strongly interacting fields

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We study the properties of strongly interacting massive quantum fields in space-time as resulting from a parametric decay of the fields with a large decay width γ. The resulting imaginary part of the retarded and advanced propagators in this case is of Lorentzian form, and the theory conserves microcausality; i.e., the commutator between the fields vanishes for spacelike distances in space-time. However, when considering separately spacelike and timelike components of the spectral function in momentum space, we find microcausality to be violated for each component separately. This implies that the modeling of effective field theories for strongly interacting systems has to be considered with great care, and restrictions to timelike four-momenta in case of broad spectral functions have to be ruled out. Furthermore, when employing effective propagators with a width $\gamma(\mathbf{p}^2)$ depending explicitly on three-momentum $\mathbf{p}$, the commutator of the fields no longer vanishes for $r > t$, since the related field theory becomes nonlocal and violates microcausality.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is considered to be the theory of the strong interaction; however, it is accessible to perturbation theory only in the limits of short distance or high momentum transfer. Thermodynamical properties of hadronic or partonic matter at finite temperature T and/or chemical potential μ_q involve large-distance interactions and can rigorously only be addressed by lattice QCD (predominantly at vanishing chemical potential) in Euclidean space. Alternatively, one might employ effective field theories that share the symmetry properties of QCD and fix the couplings to reproduce field expectation values and correlators [1–11]. In fact, the knowledge about the phase diagram of strongly interacting hadronic/partonic matter has been increased substantially in the last decades. At vanishing (or low) chemical potentials, lattice QCD (lQCD) calculations have provided reliable results on the equation of state [12,13] and given a glance at the transport properties (or correlators [14–25]) in particular in the partonic phase.

Recent studies of "QCD matter" in equilibrium—using lattice QCD calculations [14,15] or partonic transport models in a finite box with periodic boundary conditions [26]—have demonstrated that the ratio of the shear viscosity to entropy density η/s should have a minimum close to the critical temperature T_c, similar to atomic and molecular systems [27]. On the other hand, the ratio of the bulk viscosity to the entropy density ζ/s should have a maximum close to T_c [26] or might even diverge at T_c [28–32]. Indeed, the minimum of η/s at $T_c \approx 160$ MeV is close to the lower bound of a perfect fluid with $\eta/s = 1/(4\pi)$ [33] for infinitely coupled supersymmetric Yang-Mills gauge theory (based on the AdS/CFT duality conjecture). This suggests the "hot QCD matter" to be the "most perfect fluid" [34–36]. On the other hand, the transport studies in Refs. [26,37,38] have provided results for the shear and bulk viscosity as well as the electric conductivity that are very close to lattice QCD results; however, they employ the notion of a strongly interacting gas of quasiparticles with a dynamically generated mass that is sufficiently larger than the width of their spectral functions. These studies have been based on the dynamical quasiparticle model (DQPM) [39,40] that incorporates effective propagators for the partons with a finite width of the spectral functions $A_i(\omega_i, \mathbf{p}_i)$; i.e., for scalar fields $[\tilde{p} = (\omega, \mathbf{p})]$,

$$A_i(\omega_i, \mathbf{p}_i) = \frac{\gamma_i}{2\tilde{E}_i}\left(\frac{1}{(\omega_i - \tilde{E}_i)^2 + \gamma_i^2} - \frac{1}{(\omega_i + \tilde{E}_i)^2 + \gamma_i^2}\right) = \frac{2\omega_i\gamma_i}{(\omega_i^2 - \mathbf{p}_i^2 - M_i^2)^2 + 4\gamma_i^2\omega_i^2}, \tag{1}$$

with $\tilde{E}_i^2(\mathbf{p}_i) = \mathbf{p}_i^2 + M_i^2 - \gamma_i^2$ and $i \in [g, q, \bar{q}]$. The spectral functions $A_i(\omega_i)$ are antisymmetric in ω_i and normalized as

$$\int_{-\infty}^{+\infty} \frac{d\omega_i}{2\pi} 2\omega_i A_i(\omega_i, \mathbf{p}) = 1, \tag{2}$$

where M_i and γ_i are the dynamical quasiparticle mass (i.e., pole mass) and the width of the spectral function for particle i, respectively. They are directly related to the real and imaginary parts of the related self-energy, e.g., $\Pi_i = M_i^2 - 2i\gamma_i\omega_i$ [40]. In the off-shell approach, ω_i is an independent variable and related to the "running mass" m_i by $\omega_i^2 = m_i^2 + \mathbf{p}_i^2$. In the case of vector fields or fermion fields, the following retarded propagators are employed that

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differ from the "free" massive case only by the additional $(2\gamma_V\omega)^2$ or $\pm i\gamma_F$ in the denominator and the corresponding matrices in the numerator [41]:

$$A_V^{\mu\nu}(\omega,\mathbf{p},\gamma_V) = \gamma_V \frac{2\omega(g^{\mu\nu} - p^\mu p^\nu/M_V^2)}{(\omega^2 - \mathbf{p}^2 - M_V^2)^2 + 4\gamma_V^2\omega^2} \quad (3)$$

and

$$\begin{aligned} A_F(\omega,\mathbf{p},\gamma_F) \\ = \frac{1}{4E_F}\bigg(&\frac{E_F\gamma^0 + \mathbf{p}\cdot\boldsymbol{\gamma} + m_F}{\omega - E_F - i\gamma_F} - \frac{-E_F\gamma^0 + \mathbf{p}\cdot\boldsymbol{\gamma} + m_F}{\omega + E_F - i\gamma_F} \\ &- \frac{E_F\gamma^0 + \mathbf{p}\cdot\boldsymbol{\gamma} + m_F}{\omega - E_F + i\gamma_F} + \frac{-E_F\gamma^0 + \mathbf{p}\cdot\boldsymbol{\gamma} + m_F}{\omega + E_F + i\gamma_F}\bigg), \quad (4)\end{aligned}$$

with $E_F = \mathbf{p}^2 + m_F^2$ in obvious notation.

As is seen e.g., from the spectral function [Eq. (1)], it is nonvanishing for timelike ($\tilde{p}^2 > 0$) as well as for spacelike ($\tilde{p}^2 < 0$) four-momenta, such that the question emerges of whether the theoretical concept behind the DQPM (or other effective approaches) conserves microcausality; i.e., if the spectral function transformed to space-time only has support on and within the light cone. We recall that the Fourier transform of the spectral function $A(\omega,\mathbf{p})$ is proportional to the commutator of the fields at different space-time points, and its integration over energy ω ensures a proper quantization (see below). This is of particular importance, since a transport realization can only propagate "quasiparticles" within or on the light cone [42,43]. More importantly, in the DQPM, spectral contributions are separated into timelike ($\tilde{p}^2 > 0$) and spacelike ($\tilde{p}^2 < 0$) four-momentum parts, and the additional question arises of whether the contributions separately conserve microcausality.

From the analytic function theory of propagators, it is known that the propagation is causal and proceeds within the light cone if the retarded propagator is analytic in ω on the real axis and in the upper half-plane [44]. Furthermore, it should obey standard asymptotics in the high-frequency and momentum limit. As will become transparent in the following, the spectral function [Eq. (1)] is proportional to the imaginary part of a retarded propagator that is analytic on the real ω axis and in the upper half-plane, and accordingly should be causal. In fact, we will provide an explicit proof for that. However, this no longer holds for the retarded propagator when restricting to spacelike or timelike four-momenta. In this case, analyticity no longer holds, and one should expect microcausality to be violated. In this paper, we will explore quantitatively these formal expectations and demonstrate explicitly in space-time where such causality violations become dominant for spectral functions with $\gamma \ll M$, $\gamma \approx M$, and $\gamma > M$.

The layout of our study is as follows: In Sec. II, we briefly present the basic definitions and relations between retarded and advanced propagators and recall the analytic proof for microcausality in the case of the spectral functions [Eqs. (1), (3), and (4)]. In Sec. III, the actual problem is set up for timelike ($\tilde{p}^2 > 0$) and spacelike ($\tilde{p}^2 < 0$) four-momentum parts of the spectral function and its numerical realization. Furthermore, we present the actual numerical results for strong coupling and investigate the aperiodic limit as well as the case $\gamma > M$. A summary and discussion of results is given in Sec. IV.

II. PROPAGATORS AND SPECTRAL FUNCTIONS

In this work, we will concentrate on the model case of a massive scalar field coupled, e.g., to an external fermion field [$\sim\partial_\mu\Phi(x)\bar{\Psi}(x)\gamma^\mu\Psi(x)$] with a vanishing three-current; i.e., the field equation

$$\left(\frac{\partial^2}{\partial t^2} - \Delta + M^2 + 2\gamma\frac{\partial}{\partial t}\right)\Phi(x) = 0, \quad (5)$$

where γ stands for the strength of the coupling (e.g., $g_s\langle\Psi^\dagger\Psi\rangle/2$). Equation (5) has the algebraic solution

$$\tilde{G}(\mathbf{p}) = \frac{-1}{\omega^2 - \mathbf{p}^2 - M^2 + 2i\gamma\omega}, \quad (6)$$

which leads to the retarded Green function $G_{\rm ret}$ obeying

$$G_{\rm ret}(x-y) = 0 \quad \text{for } x^0 - y^0 < 0 \quad (7)$$

by a four-dimensional Fourier transformation of Eq. (6),

$$G_{\rm ret}(x) = \int \frac{d^4\tilde{p}}{(2\pi)^4}\tilde{G}(\tilde{p})\exp(-i\tilde{p}x). \quad (8)$$

We point out that $\Im\tilde{G}(\tilde{p})$ is identical to Eq. (1). We recall, furthermore, that solutions of the Kadanoff-Baym equations [45] for Φ^4 theory in $2+1$ dimensions [46] have led to spectral functions that are very close to Eq. (1) also for strong coupling.

A. Analytical results

The integration over $d\omega = dp^0$ in Eq. (8) can be carried out by contour integration, and the angular integration in three-momentum is straightforward. With $\mu = \sqrt{M^2-\gamma^2}$, the remaining integral kernel reads ($p = |\mathbf{p}|$)

$$\begin{aligned} K(x) &:= \frac{1}{|\boldsymbol{x}|}\int_0^\infty p\frac{\sin(t\sqrt{\mu^2+p^2})}{\sqrt{\mu^2+p^2}}\sin(|\boldsymbol{x}|p){\rm d}p \\ &= \frac{1}{2|\boldsymbol{x}|}\int_{-\infty}^\infty p\frac{\sin(t\sqrt{\mu^2+p^2})}{\sqrt{\mu^2+p^2}}\sin(|\boldsymbol{x}|p){\rm d}p, \quad (9)\end{aligned}$$

which has a singular contribution on the light cone and a regular part on and within the light cone. The remaining integration over dp gives for the retarded Green function [using $x = (t, \boldsymbol{x}) = (x^0, \boldsymbol{x})$]

$$G_{\text{ret}}(x) = \frac{e^{-\gamma t}\Theta(t)}{2\pi}\delta(t^2 - \boldsymbol{x}^2) - \frac{e^{-\gamma t}\Theta(t)}{4\pi}\Theta(t^2 - \boldsymbol{x}^2)\frac{\mu}{\sqrt{t^2 - \boldsymbol{x}^2}}J_1(\mu\sqrt{t^2 - \boldsymbol{x}^2}) \tag{10}$$

for $\mu^2 \geq 0$. With

$$\delta(t^2 - \boldsymbol{x}^2)\Theta(t) = \delta((t - |\boldsymbol{x}|)(t + |\boldsymbol{x}|))\Theta(t) = \frac{\delta(t - |\boldsymbol{x}|)}{2|\boldsymbol{x}|}, \tag{11}$$

one arrives at the final result [47]

$$G_{\text{ret}}(\boldsymbol{x}) = \left(\frac{\delta(t - |\boldsymbol{x}|)}{4\pi|\boldsymbol{x}|} - R(t^2 - \boldsymbol{x}^2)\right)e^{-\gamma t}\Theta(t), \tag{12}$$

with

$$R(t^2 - \boldsymbol{x}^2) = \Theta(t^2 - \boldsymbol{x}^2)\frac{\mu}{4\pi\sqrt{t^2 - \boldsymbol{x}^2}}J_1(\mu\sqrt{t^2 - \boldsymbol{x}^2}), \tag{13}$$

where J_1 is the Bessel function. In the actual calculations, the δ-distribution term on the light cone will be subtracted, and we will address the regular part [Eq. (13)] including the overall exponential decay in time, i.e.,

$$\tilde{R}(t^2 - \boldsymbol{x}^2) = R(t^2 - \boldsymbol{x}^2)e^{-\gamma t}\Theta(t). \tag{14}$$

We note in passing that the related results for massive vector fields and Dirac fields read [48]

$$G^{\mu\nu}_{\text{ret}}(x) = \left(g^{\mu\nu} + \frac{1}{M_V^2}\partial^\mu\partial^\nu\right)G_{\text{ret}}(x) \tag{15}$$

and

$$G^F_{\text{ret}}(x) = (m_F \cdot 1_4 + i\gamma^\mu\partial_\mu)G_{\text{ret}}(x). \tag{16}$$

Equations (15) and (16) demonstrate that it is sufficient to investigate microcausality for the scalar case, since microcausality for the scalar field implies microcausality for the corresponding vector and fermion fields.

The retarded Green function [Eq. (12)] is close to the solution of the free massive Klein-Gordon field except for the factor $e^{-\gamma t}$ describing the decay of the propagator in time and the reduced mass $\mu = \sqrt{M^2 - \gamma^2}$ that incorporates a downward shift of the mass M as in the case of the damped harmonic oscillator.

Similar relations hold for the advanced Green function, which is obtained by replacing $\gamma \to -\gamma$ and a multiplication by -1 due to the opposite contour integration:

$$G_{\text{av}}(x) = \left(-\frac{\delta(t + |\boldsymbol{x}|)}{4\pi|\boldsymbol{x}|} + R(t^2 - \boldsymbol{x}^2)\right)e^{+\gamma t}\Theta(-t), \tag{17}$$

following $G_{\text{av}}(x-y) = 0$ for $x^0 - y^0 > 0$. In four-momentum space, the advanced propagator is given by Eq. (6), replacing γ with $-\gamma$. Accordingly, $\tilde{G}_{ad}(\tilde{p}) - \tilde{G}_{\text{ret}}(\tilde{p}) = -2iA(\tilde{p})$ is purely imaginary and equal to twice the spectral function [Eq. (1)].

B. Spectral functions

Of central interest in our study is the scalar spectral function $A(\omega, \mathbf{p})$ [Eq. (1)], i.e., the imaginary part of the retarded propagator. The commutator between the fields at different space-time points can also be written as the difference of advanced and retarded propagators (due to opposite signs of the imaginary parts in the propagators [49]):

$$[\Phi(x), \Phi^\dagger(\mathbf{0})] = i\Delta^\star(x) = i(G_{\text{av}}(x) - G_{\text{ret}}(x)) =: C(x). \tag{18}$$

Except for a factor $\exp(-\gamma t)$, the quantity Δ^* is identical to the Schwinger Δ function $\Delta(x, \mu)$ with effective mass μ,

$$\Delta^\star(x) = \Delta(x, \mu) \cdot e^{-\gamma|t|}, \tag{19}$$

$$\Delta(x, \mu) = -\frac{i}{(2\pi)^3}\int \epsilon(\tilde{p})\delta(\tilde{p}^2 - \mu^2)e^{-i\tilde{p}\cdot x}\mathrm{d}^4\tilde{p}, \tag{20}$$

with $\epsilon(\tilde{p}) = 1$ for $\omega > 0$ and $\epsilon(\tilde{p}) = -1$ for $\omega < 0$. Since $\Delta(x, \mu)$ vanishes for spacelike distances $x^2 < 0$ [49], we find that microcausality is fulfilled also in the interacting case (cf. Ref. [48]). Since the DQPM—as an effective approach to QCD—employs spectral functions of the type in Eq. (12) [or Eqs. (15) and (16)], we may conclude that the model approach conserves microcausality strictly [50].

III. SPACELIKE AND TIMELIKE MOMENTUM CONTRIBUTIONS

We now come to the central question of our study: Is microcausality fulfilled in the four-momentum integral

$$C(x) = \int \frac{d\omega}{2\pi}\frac{d^3p}{(2\pi)^3}\Im(G_{\text{ret}}(\omega, \mathbf{p}))\exp(-i(\omega t - \mathbf{p}\cdot\mathbf{x})) \tag{21}$$

when restricting to timelike $\Theta(\omega^2 - \mathbf{p}^2)$ or spacelike $\Theta(\mathbf{p}^2 - \omega^2)$ four-momenta?

To answer this question, we can no longer perform the contour integration over $d\omega$ due to the Θ functions in four-momentum and have to evaluate the integrals in Eq. (21) numerically, exploiting the antisymmetry of the integrand [Eq. (1)] in ω and carrying out the angular integration in the three-momentum. This leads to

$$C(x) = -\frac{i\gamma}{2\pi^3|\boldsymbol{x}|}\int_0^\infty dp\int_0^\infty d\omega \sin(\omega t)\sin(|\boldsymbol{x}|p) \times \frac{p}{\tilde{E}}\left(\frac{1}{(\omega-\tilde{E})^2+\gamma^2} - \frac{1}{(\omega+\tilde{E})^2+\gamma^2}\right), \tag{22}$$

using Eq. (1), which can be "solved" on a numerical grid as well as by analytical integration (cf. Sec. II). As mentioned before, the integral in Eq. (22) has a singular part $[\delta(t-r)/(4\pi r)$ using $r=|\boldsymbol{x}|]$, as well as a regular part given by Eq. (14). The singular part can be subtracted in the integral [Eq. (22)]—to achieve a better convergence—by considering

$$C(x) - \frac{\delta(t-r)}{4\pi r}e^{-\gamma t} = -\frac{i}{2\pi^3 r}\int_0^\infty dp\int_0^\infty d\omega \sin(\omega t)\sin(rp) \times \frac{8p\omega(M^2-\gamma^2)[\omega^2-p^2-(\gamma^2+M^2)/2]}{[(\omega^2-p^2-M^2)^2+4\omega^2\gamma^2][(\omega^2-p^2-\gamma^2)^2+4\omega^2\gamma^2]}. \tag{23}$$

In this way, the δ distribution on the light cone (decaying exponentially in time) is subtracted explicitly on the same computational grid. In order to demonstrate the validity of this numerical subtraction scheme, we show in Fig. 1 a comparison of the analytical result [Eq. (14)] with the corresponding numerical evaluation of Eq. (23) for $M = 1$ GeV and $\gamma = 0.3$ GeV at $t = 15$ GeV. Indeed, both results agree within the linewidth and are identical to zero for $r > t$.

In Fig. 2, we show the regular part [Eq. (14)] for the widths $\gamma = 0.045$ GeV [Fig. 2(a)] and $\gamma = 0.3$ GeV [Fig. 2(b)]. The signal decays exponentially in time [$\sim\exp(-\gamma t)$] and shows hyperbolic oscillations within the light cone while being zero outside the light cone.

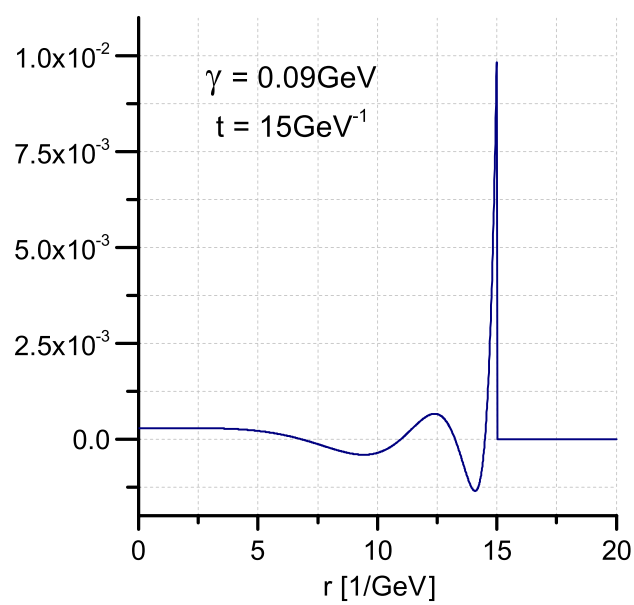

FIG. 1 (color online). The regular part of the Green function [Eq. (14)] at time $t = 15\ \mathrm{GeV}^{-1}$ as a function of the distance r. The analytical and numerical results from the integration on the grid are identical within the linewidth.

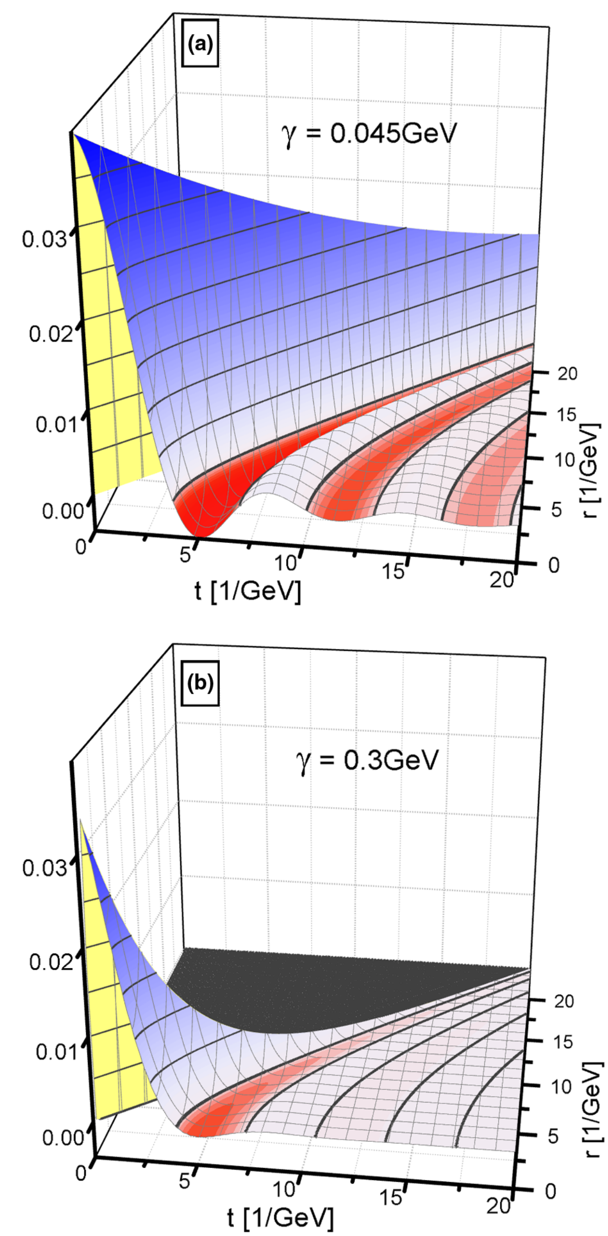

FIG. 2 (color online). The regular part of the commutator [Eq. (14)] as a function of the distance r and time t for (a) $\gamma = 0.045$ GeV and (b) $\gamma = 0.3$ GeV.

The numerical results and the analytical expression [Eq. (14)] are identical on the level of three digits for both cases.

We now turn to the numerical results for strong coupling when gating on timelike and spacelike four-momenta in Eq. (23) separately. The results are displayed in Fig. 3 for $\gamma = 0.3$ GeV when including only timelike momenta [Fig. 3(a)] or only spacelike momenta [Fig. 3(b)]. Note that the numerical results in Fig. 3 have been multiplied by $\exp(\gamma t)$ in order to compensate for the exponential decay in time.

It is seen that both results do not vanish for $r > t$, and thus they violate microcausality. This is shown more explicitly in Fig. 4(a) as a function of time t and $r - t$ in the spacelike region and demonstrates that both contributions are nonvanishing but of opposite sign, such that their sum becomes identically zero. In Fig. 4(b), we display the same quantities as in Fig. 4(a) but multiplied by $\exp(\gamma t)$, which demonstrates that both contributions do not decay exponentially in time as seen from the full analytical solution [Eq. (12)]. This clearly demonstrates that a restriction to either spacelike or timelike momentum parts of the spectral function violates causality, while both parts together conserve microcausality in line with the analytical result in Sec. II. The violation of microcausality is tiny in the case of $\gamma \ll M$ but becomes sizeable for $\gamma > 0.1$ GeV.

When considering the "aperiodic" limit $\gamma \to M$, i.e., $\mu \to 0$, we find from Eq. (13) that $\tilde{R}(t^2 - r^2)$ vanishes identically for $\mu = 0$, and the commutator [Eq. (18)] only has support on the light cone. Furthermore, in the limit $\mu \to 0$, the oscillations in $R(t^2 - r^2)$ vanish, as can be

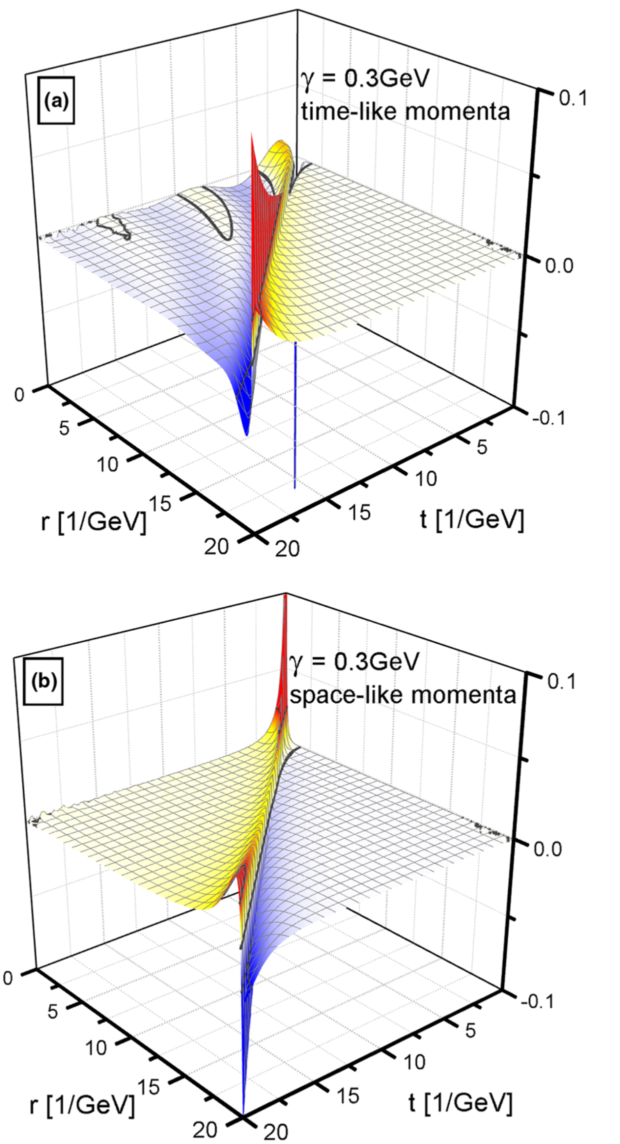

FIG. 3 (color online). The regular part of the commutator [Eq. (14)] [multiplied by $\exp(\gamma t)$] as a function of the distance r and time t for $\gamma = 0.3$ GeV for (a) timelike four-momenta and (b) spacelike four-momenta.

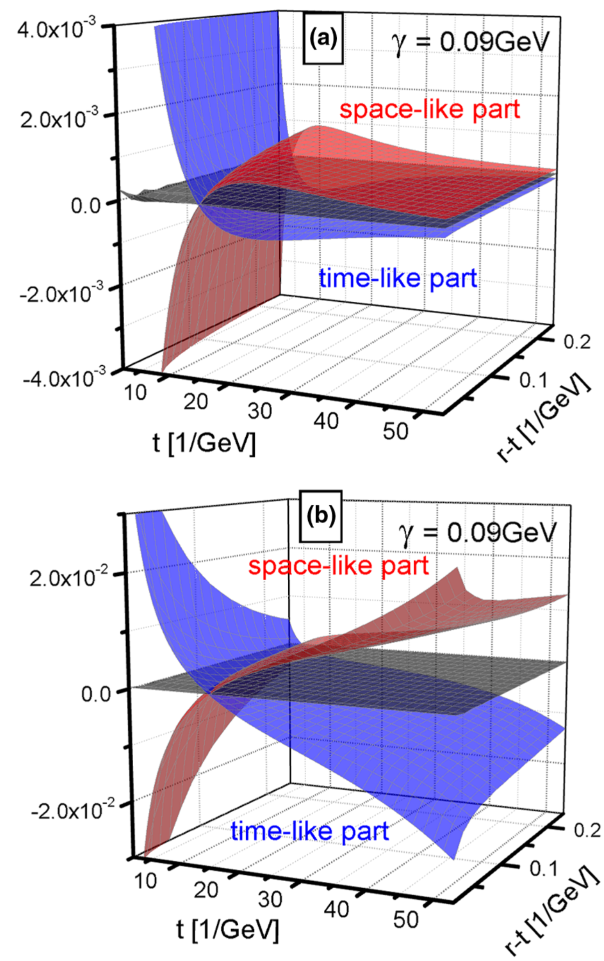

FIG. 4 (color online). (a) The regular part of the commutator [Eq. (14)] as a function of the time t and $r - t$ in the spacelike region for $\gamma = 0.09$ GeV. The contribution from spacelike momenta is given (for $t > 20\ \mathrm{GeV}^{-1}$) by the upper (red) area, while the contribution from timelike momenta is given by the lower (blue) area, which is of opposite sign. (b) Same as (a), but multiplied by $\exp(\gamma t)$.

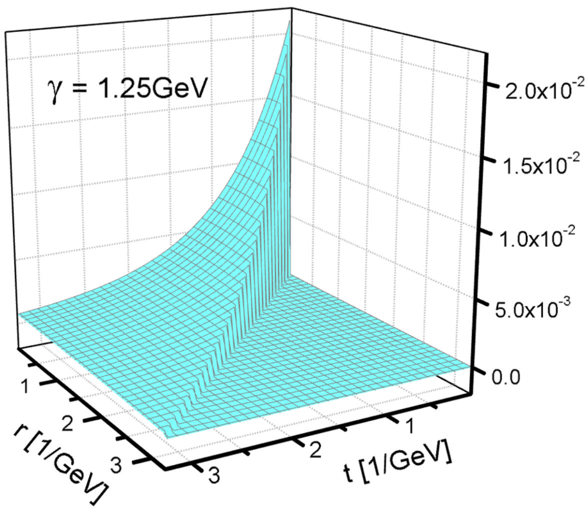

FIG. 5 (color online). The regular part of the commutator [Eq. (14)] as a function of the distance r and time t for $\gamma = 1.25$ GeV.

extracted from a Taylor expansion of Eq. (14) [with $J_1(z) \approx z/2 \pm \cdots$]:

$$R(t^2 - r^2) \approx \Theta(t^2 - r^2) \frac{\mu}{8\pi\sqrt{t^2 - r^2}} (\mu\sqrt{t^2 - r^2}) \cdots$$
$$= \Theta(t^2 - r^2) \frac{\mu^2}{8\pi} \cdots. \quad (24)$$

For $\gamma > M$ (overdamped fields), we no longer find oscillations of the regular part [Eq. (14)] within the light cone but only an exponentially decaying signal as seen from Fig. 5 for $\gamma = 1.25$ GeV.

We close by pointing out that our numerical scheme allows us to employ almost arbitrary spectral functions in Eq. (22) and to check if microcausality holds. Without explicit representation, we note that using a three-momentum width $\gamma(\mathbf{p}^2)$ in the spectral function [Eq. (1)], the commutator [Eq. (22)] no longer vanishes for $r > t$, since the individual momentum modes decay on different time scales, and the field equation (5) becomes nonlocal in this case. Nevertheless, the normalization condition [Eq. (2)] still is fulfilled. Furthermore, the commonly adopted form,

$$A(\omega, \mathbf{p}) = \frac{2M\gamma}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\gamma^2 M^2}, \quad (25)$$

also violates microcausality. In fact, this finding is not too surprising, since Eq. (25) is the imaginary part of the retarded propagator

$$\tilde{G}(\mathbf{p}) = \frac{-1}{\omega^2 - \mathbf{p}^2 - M^2 + 2i\gamma M}, \quad (26)$$

which has a pole in ω in the upper half-plane and thus is no longer analytic on the real axis and above. Note, however, that in interacting (causal) systems, the width γ may well show an explicit three-momentum dependence as demonstrated within Φ^4 theory [46]. Although the spectral function might be approximated by Eq. (25), it is illegitimate to replace the full retarded propagator with Eq. (26).

IV. SUMMARY

In this study, we have examined effective propagators of the type in Eq. (6) as used, e.g., in the dynamical quasiparticle model (DQPM) [39,40] for an approximation to QCD propagators at temperatures above the critical temperature T_c for deconfinement. It could be shown analytically that their spectral functions (or imaginary parts) do not lead to a violation of microcausality, i.e., to a vanishing commutator of the interacting fields outside the light cone. This result is in accord with analytic function theory, since the related retarded propagator [Eq. (6)] is analytic in ω on the real axis and in the upper half-plane, thus ensuring microcausality [44]. However, when restricting to only spacelike or timelike four-momentum contributions of the spectral function, a violation of microcausality is found which becomes severe in the case of strong coupling. This finding is in line with analytic function theory, since the restricted spectral functions no longer correspond to analytic retarded propagators. Moreover, the spacelike or timelike four-momentum contributions separately no longer decay exponentially in time, as in the case of the full solution [Eq. (12)]. Furthermore, we have found that using a three-momentum-dependent width $\gamma(\mathbf{p}^2)$ in the spectral function [Eq. (1)], the commutator [Eq. (22)] no longer vanishes for $r > t$, since the individual momentum modes decay on different time scales, and the field equation (5) becomes nonlocal in space in this case. This also holds for the spectral function [Eq. (25)], which is often employed in phenomenological models. This is also expected from analytic function theory, since the related retarded propagator [Eq. (26)] has a pole in ω in the upper half-plane.

Our findings imply that the modeling of effective field theories for strongly interacting systems has to be considered with great care, and restrictions to timelike four-momenta in the case of broad spectral functions have to be ruled out.

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[50] Note also that the spectral function [Eq. (1)] corresponds to a Breit-Wigner representation of the integrand $\epsilon(\tilde{p})\delta(\tilde{p}^2 - \mu^2)$ in Eq. (20) for a finite width γ.