Effective field theory of modified gravity with two scalar fields: Dark energy and dark matter

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We present a framework for discussing the cosmology of dark energy and dark matter based on two scalar degrees of freedom. An effective field theory of cosmological perturbations is employed. A unitary gauge choice renders the dark energy field into the gravitational sector, for which we adopt a generic Lagrangian depending on three-dimensional geometrical scalar quantities arising in the Arnowitt-Deser-Misner decomposition. We add to this dark energy–associated gravitational sector a scalar field ϕ and its kinetic energy X as dark matter variables. Compared to the single-field case, we find that there are additional conditions to obey in order to keep the equations of motion for linear cosmological perturbations at second order. For such a second-order multifield theory, we derive conditions under which ghosts and Laplacian instabilities of the scalar and tensor perturbations are absent. We apply our general results to models with dark energy emerging in the framework of the Horndeski theory and dark matter described by a k-essence Lagrangian $P(\phi, X)$. We derive the effective coupling between such an imperfect-fluid dark matter and the gravitational sector under the quasistatic approximation on subhorizon scales. By considering the purely kinetic Lagrangian $P(X)$ as a particular case, the formalism is verified to reproduce the gravitational coupling of a perfect-fluid dark matter.

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I. INTRODUCTION

The effective field theory (EFT) of cosmological perturbations has been widely studied in connection with inflation and dark energy to characterize the low-energy degree of freedom of a most general gravitational theory [1,2]. This approach allows for addressing all of the possible high-energy corrections to standard slow-roll inflation driven by a single scalar field [3]. Moreover, the EFT formalism of inflation is suitable for the parametrization of higher order correlation functions of cosmological perturbations, like primordial non-Gaussianities [4–6].

The EFT approach is also convenient for the unified description of dark energy because it can describe practically all single-field models proposed in the literature. The dynamics of dark energy has been investigated in the EFT formalism for scalar fields in both minimal and nonminimal couplings to gravity [7–17]. In this setup the background cosmology is governed by three free parameters supplementing other parameters associated with linear cosmological perturbations. The unified framework based on the EFT parametrization is useful both in imposing constraints on individual models and in providing model-independent constraints on the properties of dark energy and modified gravity [18,19].

In particular, Gleyzes *et al.* [15] described a most general single-field dark energy/modified gravity scenario in terms of a Lagrangian depending on the lapse function and some geometrical scalar quantities naturally emerging in the Arnowitt-Deser-Misner (ADM) decomposition on the flat Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological background. The choice of unitary gauge for the scalar field χ allows one to absorb the field perturbation $\delta\chi$ into the gravitational sector, so no explicit dependence on χ needs to be included in the Lagrangian. In this setup the time derivatives in the linear perturbation equations are at most of the second order, but spatial derivatives higher than second order could emerge. Gleyzes *et al.* [15] derived the conditions under which such higher order spatial derivatives are absent.

Recently the Horndeski gravitational theory [20] has also received much attention [21–23] as the most general scalar-tensor theory with second-order differential equations of motion. This interest is due to the generalization of covariant Galileons [24–26] allowing for the realizations of cosmic acceleration [27] and the Vainshtein screening of the fifth force [28]. The analysis of Ref. [15] shows that the Horndeski theory is accommodated in the framework of the EFT of dark energy as a special case. In fact, the Horndeski theory satisfies conditions for the absence of spatial derivatives higher than second order in the equations of linear cosmological perturbations. Gleyzes *et al.* [15] provided a convenient dictionary linking the variables between the Horndeski theory and the EFT of dark energy.

The EFT formalism advocated in Ref. [15] corresponds to a theory of a single scalar degree of freedom χ, which is responsible for cosmic acceleration. In a more general setup, another scalar field ϕ may be present. In fact, a scalar field described by the Lagrangian $P(\phi, X)$ [29] (where X is the kinetic energy of ϕ) could represent dark matter [30,31].

In this paper we study the EFT of dark energy and dark matter by including explicit dependencies of the second scalar field ϕ and of X into the Lagrangian, alongside the lapse N and other three-dimensional geometric scalars naturally emerging in the ADM formalism. We choose unitary gauge for the dark energy field, such that the field perturbation $\delta\chi$ is "eaten up" by the gravitational sector. Our analysis is based upon the expansion of the Lagrangian L up to second order in the cosmological perturbations, with coefficients involving the partial derivatives of L with respect to the scalar quantities (such as N and ϕ). As an additional motivation, we also note that the formalism can be applied to the models of multifield inflation such as those studied in Refs. [32].

With the increase of the number of degrees of freedom, there appear additional complications which need to be carefully considered. We show that in our formalism, in addition to the higher order spatial derivatives found in Ref. [15], combinations of spatial and time derivatives higher than second order also emerge in the linear perturbation equations of motion. Such terms need to be eliminated at the price of extra conditions supplementing those derived in Ref. [15]. Without imposing these, either the number of degrees of freedom would be further increased or some unwanted nonlocality (in the form of a truncated series expansion) would be introduced in the theory [33].

There are also additional conditions to obey. The Hamiltonian of the system could not be unbounded from below as then even an empty state could further decay; hence, the stability is lost. At a technical level, this no-ghost condition can be imposed as the positivity of the kinetic term in the Lagrangian [1,7]. For more degrees of freedom it is ensured by the positivity of the eigenvalues of the kinetic matrix. Similarly, the dispersion relation should not lead to ill-defined propagation speeds, in the sense that their square becomes negative, as such sign changes lead to Laplacian instabilities (Laplacian growth) on small scales. For the investigated multifield second-order theory, we obtain two conditions for the avoidance of scalar ghosts and two scalar propagation speeds in the ultraviolet limit. Finally we also derive conditions for the absence of tensor ghosts and of Laplacian instabilities. All of these conditions should be obeyed by viable models of dark energy and dark matter.

Our analysis covers the most general second-order scalar-tensor theories with a k-essence–type dark matter [30,31,34] as a specific case. On using the dictionary between the EFT parameters and the functions appearing in the Horndeski theory [15], we apply our results to a specific theory with dark energy given by the Horndeski Lagrangian and dark matter represented by the k-essence Lagrangian $P(\phi, X)$. In this case the field ϕ does not have a direct coupling to χ, so the no-ghost conditions and the propagation speeds of scalar perturbations are considerably simplified to reproduce results available in the literature [35,36]. We also derive the effective coupling $G_{\rm eff}$ between the field ϕ (which in the generic case can be interpreted as an imperfect fluid) and the gravitational sector, under the quasistatic approximation on subhorizon scales (see, e.g., Refs. [37–42]). Further, for the purely kinetic Lagrangian $P(X)$ [31], the field ϕ behaves as a perfect fluid [43], in which case $G_{\rm eff}$ previously derived in some modified gravitational models [38,40,44,45] could be reproduced.

Our paper is organized as follows.

In Sec. II we summarize the $3+1$ decomposition of spacetime and set up the framework for the EFT description of modified gravity by introducing a generic action depending both on the gravitational degrees of freedom and another independent scalar field ϕ.

In Sec. III we expand the action up to first order in cosmological perturbations and obtain the background equations of motion which involve the partial derivatives of the Lagrangian L with respect to scalar quantities.

In Sec. IV we derive the second-order action for perturbations and identify conditions under which the spatial and time derivatives higher than second order are absent. The conditions for the avoidance of ghosts and instabilities of scalar and tensor perturbations are also discussed here.

In Sec. V we apply our results to a theory described by the Horndeski Lagrangian and the field Lagrangian $P(\phi, X)$. The equations of matter perturbations as well as the effective gravitational coupling are derived for such a generic multifield system.

Sec. VI is devoted to conclusions.

Throughout the paper Greek and Latin indices denote components in spacetime and in a three-dimensional space-adapted basis, respectively. Quantities with an overbar are evaluated on the flat FLRW background. Only the scale factor a, the Hubble parameter $H = \dot{a}/a$, and the scalar fields χ, ϕ (also its energy-momentum tensor with its components), all referring to the background, do not carry the distinctive overbars, as the respective perturbed quantities will not require independent notation (rather, new notations for their perturbations are introduced). A dot represents a derivative with respect to the time t, a semicolon as a lower index the covariant derivative compatible with the 4-metric, while a bar as a lower index the covariant derivative compatible with the spatial 3-metric. A lower index of the Lagrangian L denotes the partial derivatives with respect to the scalar quantities represented in the index, e.g., $L_N \equiv \partial L/\partial N$ and $L_\phi \equiv \partial L/\partial\phi$.

II. 3 + 1 DECOMPOSITION OF SPACETIME AND THE EFT DESCRIPTION OF MODIFIED GRAVITY WITH TWO SCALAR FIELDS

We start with the generic ADM line element [46] given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (1)$$

which contains the lapse function N, the shift vector N^i, and the three-dimensional metric h_{ij}. The three-dimensional components are equivalent to those of the four-dimensional metric $g_{\mu\nu}$ as follows:

$$g_{00} = -N^2 + N_iN^i, \qquad g_{0i} = g_{i0} = N_i, \qquad g_{ij} = h_{ij}. \quad (2)$$

The inverse metric is then

$$g^{00} = -1/N^2, \qquad g^{0i} = g^{i0} = N^i/N^2, \qquad g^{ij} = h^{ij} - N^iN^j/N^2. \quad (3)$$

A unit normal to Σ_t is defined as $n_\mu = -Nt_{;\mu} = (-N, 0, 0, 0)$; hence, $n^\mu = (1/N, -N^i/N)$, and it satisfies the relation $g_{\mu\nu}n^\mu n^\nu = -1$.

The induced metric $h_{\mu\nu}$ on Σ_t can be expressed covariantly as $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$. The mixed indices form h^μ_ν of the induced metric acts as a projector operator to the tangent and cotangent spaces of the hypersurfaces Σ_t. The extrinsic curvature of the hypersurfaces is

$$K_{\mu\nu} = h^\lambda_\mu h^\sigma_\nu n_{\sigma;\lambda} = h^\lambda_\mu n_{\nu;\lambda} = n_{\nu;\mu} + n_\mu a_\nu, \quad (4)$$

where $a^\mu = n^\lambda n^\mu_{;\lambda}$ is the acceleration (the curvature) of the normal congruence n^μ. It is straightforward to confirm the property $n^\mu K_{\mu\nu} = 0$ so that $K_{\mu\nu}$ lives on the three-dimensional hypersurfaces. More explicitly it can be written in the form

$$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - N_{i|j} - N_{j|i}), \quad (5)$$

where $|i$ represents a covariant derivative with respect to the metric h_{ij}.

The four-dimensional and three-dimensional curvature scalars R and $\mathcal{R}$ (the latter being the trace of $\mathcal{R}_{\mu\nu} \equiv {}^{(3)}R_{\mu\nu}$, the Ricci tensor on Σ_t associated with $h_{\mu\nu}$) are related by the twice-contracted Gauss equation

$$R = \mathcal{R} + K_{\mu\nu}K^{\mu\nu} - K^2 + 2(Kn^\mu - a^\mu)_{;\mu}, \quad (6)$$

where K is the trace of the extrinsic curvature. Therefore, in a $3+1$ rewriting of the general relativistic Einstein-Hilbert action, only the above scalars of the intrinsic and extrinsic geometries appear.

In what follows, we discuss a modified gravitational dynamics, in which the Lagrangian describing the gravitational sector depends on the set of scalars [15]:

$$K \equiv K^\mu{}_\mu, \qquad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu}, \qquad \mathcal{R} \equiv \mathcal{R}^\mu{}_\mu, \qquad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}, \qquad \mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}. \quad (7)$$

We also allow for a dependence on the lapse function N, but not on the shift vector. Although a dependence of the magnitude square of the shift $\mathcal{N} = N^aN_a$ in principle could be introduced, we choose not to do so because the explicit dependence of $\mathcal{N}$ does not appear even in the most general scalar-tensor theories with second-order equations of motion.

In top of the gravitational sector we also include a scalar field ϕ whose kinetic term is denoted $X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Hence we consider a generalized action that depends on the scalar quantities (7), on ϕ, X, and the lapse N as

$$S = \int d^4x\sqrt{-g}L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}, \phi, X; t). \quad (8)$$

The action could also exhibit explicit time dependence for reasons to be discussed below.

In addition to the field ϕ, we also allow for another scalar degree of freedom χ. This however can be absorbed into the gravitational sector by assuming unitary gauge in which the hypersurfaces of a constant value of this field coincide with the constant t hypersurfaces, i.e., $\chi = \chi(t)$ [15]. The time dependence of the quantities $\chi(t)$ and $\dot{\chi}(t)$ corresponds to the explicit temporal dependence included into the action. Moreover, defining the kinetic energy of the field χ as $Y \equiv g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$ for the ADM metric (1) with $N^i = 0$ (which can be safely assumed on the background), one obtains $Y = -\dot{\chi}^2/N^2$. Hence the kinetic term of χ depends only on N and the time. The field χ enters the equations of motion only in the form of the partial derivatives $L_N = \partial L/\partial N$ and $L_{NN} = \partial^2 L/\partial N^2$.

Due to the choice of unitary gauge for the field χ, the gauge freedom associated with the time component of the gauge-transformation vector has been used up, so the first field ϕ can be considered as independent of the gravitational sector. Hence the theory has two scalar degrees of freedom, i.e., the lapse N and the field ϕ.

In the context of the multifield Horndeski theory where not only the field χ but also ϕ has a nontrivial coupling to gravity [47], one would need to include in the action (8) the dependence on scalar quantities constructed from the second covariant derivative of ϕ, e.g., $(\Box\phi)^2$, $\phi^{;\mu\nu}\phi_{;\mu\nu}$, $R_{\mu\nu}\phi^{;\mu\nu}$, and $\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}$. Our interest lies however in a minimal extension of the single-field EFT of dark energy to the two-field case, so we do not include such terms in our analysis. In particular, we are interested in the possibility of describing scalar dark matter by the Lagrangian depending on ϕ and X.

III. COSMOLOGICAL PERTURBATIONS AND BACKGROUND EQUATIONS OF MOTION

In this section we start by defining the perturbations of the variables appearing in the action (8) and derive the background equations of motion.

A. Cosmological perturbations

In the cosmological setup, the flat FLRW spacetime with the line-element $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$ corresponds to $\bar{N} = 1$, $\bar{N}^i = 0$, and $\bar{h}_{ij} = a^2(t)\delta_{ij}$. At the background level, there is no shift vector N^i. Only when we consider cosmological perturbations, the shift appears at first order of the perturbations. Also, on the flat FLRW background, we have

$$\bar{K}_{\mu\nu} = H\bar{h}_{\mu\nu}, \qquad \bar{K} = 3H, \qquad \bar{\mathcal{S}} = 3H^2, \qquad \bar{\mathcal{R}}_{\mu\nu} = 0, \tag{9}$$

and hence $\bar{\mathcal{R}} = \bar{\mathcal{Z}} = \bar{\mathcal{U}} = 0$.

The general perturbed metric including four scalar metric perturbations A, ψ, ζ, and E can be expressed as [48,49]

$$ds^2 = -e^{2A}dt^2 + 2\psi_{|i}dx^i dt + a^2(t)(e^{2\zeta}\delta_{ij} + E_{|ij})dx^i dx^j. \tag{10}$$

We focus on scalar perturbations in most of our paper, but we discuss the second-order action for tensor perturbations in Sec. IV C. For the spatial derivatives of scalar quantities such as ψ, we use the notations $\partial_i\psi \equiv \psi_{|i} = \partial\psi/\partial x^i$ and $(\partial\psi)^2 \equiv (\partial_i\psi)(\partial_i\psi) = (\partial_1\psi)^2 + (\partial_2\psi)^2 + (\partial_3\psi)^2$, where same lower Latin indices are summed unless otherwise stated.

Under the transformation $t \to t + \delta t$ and $x^i \to x^i + \delta^{ij}\partial_j\delta x$, the perturbation $\delta\chi$ in the field χ and the metric perturbation E transform as [50]

$$\delta\chi \to \delta\chi - \dot{\chi}\delta t, \qquad E \to E - \delta x. \tag{11}$$

As we already mentioned, we choose unitary gauge

$$\delta\chi = 0, \tag{12}$$

in which the time slicing δt is fixed. We fix the spatial threading δx by choosing the gauge

$$E = 0. \tag{13}$$

Comparing the perturbed metric (10) with (1) in this case, we have the correspondence $N^2 - N^i N_i = e^{2A}$ and

$$N_i = \partial_i\psi, \tag{14}$$

$$h_{ij} = a^2(t)e^{2\zeta}\delta_{ij}. \tag{15}$$

Hence the metric perturbations ψ and ζ are related to the shift N_i emerging at first order and the perturbation of the spatial metric h_{ij}, respectively, while A combines with ψ to give the perturbation of N, hence of the scalar field χ. We also note that the gauge-invariant quantities such as $\zeta_{\rm GI} \equiv \zeta - H\delta\chi/\dot{\chi}$ and $\delta\phi_{\rm GI} \equiv \delta\phi - \dot{\phi}\delta\chi/\dot{\chi}$ reduce to $\zeta_{\rm GI} = \zeta$ and $\delta\phi_{\rm GI} = \delta\phi$ for the gauge choice (12).

We define the following perturbations:

$$\delta K^\mu_\nu = K^\mu_\nu - Hh^\mu_\nu, \qquad \delta K = K - 3H,$$
$$\delta\mathcal{S} = \mathcal{S} - 3H^2 = 2H\delta K + \delta K^\mu_\nu \delta K^\nu_\mu, \tag{16}$$

where the last equation arises from the first equation and the definition of $\mathcal{S}$. Note that the δ variations do not commute with raising and lowering of indices (hence $\delta K_{\mu\nu} \neq g_{\mu\rho}\delta K^\rho_\nu$). Since $\mathcal{R}$ and $\mathcal{Z}$ vanish on the background, they appear only as perturbations. They can be expressed up to second-order accuracy as

$$\delta\mathcal{R} = \delta_1\mathcal{R} + \delta_2\mathcal{R}, \qquad \delta\mathcal{Z} = \delta\mathcal{R}^\mu_\nu\delta\mathcal{R}^\nu_\mu, \tag{17}$$

where $\delta_1\mathcal{R}$ and $\delta_2\mathcal{R}$ are first-order and second-order perturbations in $\delta\mathcal{R}$, respectively. Note that the perturbation $\mathcal{Z}$ is higher than first order. The first equality (16) also implies

$$\mathcal{U} = H\mathcal{R} + \mathcal{R}^\mu_\nu\delta K^\nu_\mu, \tag{18}$$

where the second term on the rhs is a second-order quantity. Then the first-order perturbation $\delta_1\mathcal{U}$ is related to $\delta_1\mathcal{R}$, as $\delta_1\mathcal{U} = H\delta_1\mathcal{R}$.

We decompose the field ϕ into the background and perturbative components, as $\phi = \bar{\phi}(t) + \delta\phi(t, \boldsymbol{x})$. In the following, apart from the Lagrangian L, we omit the overbar for the background quantities. On using Eq. (3), the kinetic term $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ up to second order can be expressed as

$$X = -\dot{\phi}^2 + \delta_1 X + \delta_2 X, \tag{19}$$

where the first- and second-order perturbations are given by

$$\delta_1 X = 2\dot{\phi}^2\delta N - 2\dot{\phi}\,\dot{\delta\phi}, \tag{20}$$

$$\delta_2 X = -\dot{\delta\phi}^2 - 3\dot{\phi}^2\delta N^2 + 4\dot{\phi}\,\dot{\delta\phi}\,\delta N + \frac{2\dot{\phi}}{a^2}\partial_i\psi\partial_i\delta\phi + \frac{1}{a^2}(\partial\delta\phi)^2, \tag{21}$$

with the notation $(\partial\delta\phi)^2 \equiv \partial_i\delta\phi\partial_i\delta\phi$.

B. Background dynamics

We now expand the action (8) up to the second order in perturbations, as

$$L = \bar{L} + L_N\delta N + L_K\delta K + L_\mathcal{S}\delta\mathcal{S} + L_\mathcal{R}\delta\mathcal{R} + L_\mathcal{Z}\delta\mathcal{Z} + L_\mathcal{U}\delta\mathcal{U} + L_\phi\delta\phi + L_X\delta X + \frac{1}{2}\left(\delta N\frac{\partial}{\partial N} + \delta K\frac{\partial}{\partial K} + \delta\mathcal{S}\frac{\partial}{\partial\mathcal{S}} + \delta\mathcal{R}\frac{\partial}{\partial\mathcal{R}} + \delta\mathcal{Z}\frac{\partial}{\partial\mathcal{Z}} + \delta\mathcal{U}\frac{\partial}{\partial\mathcal{U}} + \delta\phi\frac{\partial}{\partial\phi} + \delta X\frac{\partial}{\partial X}\right)^2 L, \tag{22}$$

where $\bar{L}$ is the background value. Using the second and third relations of Eq. (16), it follows that

$$\begin{aligned} L_K\delta K + L_\mathcal{S}\delta\mathcal{S} &= \mathcal{F}(K-3H) + L_\mathcal{S}\delta K^\mu_\nu\delta K^\nu_\mu \\ &= -\dot{\mathcal{F}}/N - 3\mathcal{F}H + L_\mathcal{S}\delta K^\mu_\nu\delta K^\nu_\mu \\ &\simeq -\dot{\mathcal{F}} - 3\mathcal{F}H + \dot{\mathcal{F}}\delta N + L_\mathcal{S}\delta K^\mu_\nu\delta K^\nu_\mu - \dot{\mathcal{F}}\delta N^2, \end{aligned} \tag{23}$$

where

$$\mathcal{F} \equiv L_K + 2HL_\mathcal{S}. \tag{24}$$

In the second line of Eq. (23), we integrated the term $\mathcal{F}K$ by using $K = n^\mu_{;\mu}$, that is

$$\begin{aligned} \int d^4x\sqrt{-g}\mathcal{F}K &= -\int d^4x\sqrt{-g}n^\mu\mathcal{F}_{;\mu} \\ &= -\int d^4x\sqrt{-g}\frac{\dot{\mathcal{F}}}{N}, \end{aligned} \tag{25}$$

and we dropped the boundary term. In the third line of Eq. (23), we expanded the term $N^{-1} = (1+\delta N)^{-1}$ up to the second order.

As for the term $\mathcal{U}$, there is the relation $\lambda(t)\mathcal{U} = \lambda(t)\mathcal{R}K/2 + \dot{\lambda}(t)\mathcal{R}/(2N)$, valid up to boundary terms, where $\lambda(t)$ is an arbitrary function of t [15]. Since $\mathcal{U}$ is a perturbative quantity, the term $L_\mathcal{U}\delta\mathcal{U}$ in Eq. (22) reads

$$\begin{aligned} L_\mathcal{U}\delta\mathcal{U} &= \frac{1}{2}(\dot{L_\mathcal{U}} + 3HL_\mathcal{U})\delta_1\mathcal{R} + \frac{1}{2}(\dot{L_\mathcal{U}} + 3HL_\mathcal{U})\delta_2\mathcal{R} \\ &\quad + \frac{1}{2}(L_\mathcal{U}\delta K - \dot{L_\mathcal{U}}\delta N)\delta_1\mathcal{R}, \end{aligned} \tag{26}$$

with the first term on the rhs corresponding to a first-order quantity, while the rest is second order. We also note that the second-order terms including the perturbation $\delta\mathcal{U}$ are replaced by $H\delta_1\mathcal{R}$, e.g., $L_{N\mathcal{U}}\delta N\delta\mathcal{U} = HL_{N\mathcal{U}}\delta N\delta_1\mathcal{R}$.

Up to boundary terms the zeroth-order and first-order Lagrangians of (22) are given, respectively, by

$$L_0 = \bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F}, \tag{27}$$

$$L_1 = (\dot{\mathcal{F}} + L_N)\delta N + \mathcal{E}\delta_1\mathcal{R} + L_\phi\delta\phi + L_X\delta_1 X, \tag{28}$$

where

$$\mathcal{E} = L_\mathcal{R} + \frac{1}{2}\dot{L_\mathcal{U}} + \frac{3}{2}HL_\mathcal{U}. \tag{29}$$

The Lagrangian density is defined by $\mathcal{L} = \sqrt{-g}L = N\sqrt{h}L$, where h is the determinant of the three-dimensional metric h_{ij}. The zeroth-order term following from (27) is $\mathcal{L}_0 = a^3(\bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F})$. On using Eq. (20), the first-order Lagrangian density reads

$$\begin{aligned} \mathcal{L}_1 &= a^3(\bar{L} + L_N - 3H\mathcal{F} + 2L_X\dot{\phi}^2)\delta N \\ &\quad + (\bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F})\delta\sqrt{h} + a^3L_\phi\delta\phi \\ &\quad - 2a^3L_X\dot{\phi}\,\delta\dot{\phi} + a^3\mathcal{E}\delta_1\mathcal{R}. \end{aligned} \tag{30}$$

The last term becomes a total derivative, and hence, it can be dropped. Variations of the Lagrangian (30) with respect to δN, $\delta\sqrt{h}$, and $\delta\phi$ (the independent scalar field and the gravitational variables characterizing the background metric, which by unitary gauge fixing already include the other scalar field) lead to the following equations of motion, respectively:

$$\bar{L} + L_N - 3H\mathcal{F} + 2L_X\dot{\phi}^2 = 0, \tag{31}$$

$$\bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F} = 0, \tag{32}$$

$$\frac{d}{dt}(a^3L_X\dot{\phi}) + \frac{1}{2}a^3L_\phi = 0. \tag{33}$$

The zeroth-order Lagrangian (27) vanishes on account of Eq. (32).

Although Eq. (33) contains only derivatives related to the field ϕ, whenever the two fields are coupled in the Lagrangian, it becomes an equation containing both fields. We discuss such an example in Sec. IV, related to no-ghost conditions, which involves the term L_{NX}, corresponding to the coupling between two kinetic terms.

If the Lagrangian does not contain any interactions between χ and ϕ, then Eq. (33) becomes a continuity-type equation for the field ϕ alone. In the next subsection, we discuss an example exhibiting this property.

C. Noninteracting fields

Let us consider the following Lagrangian:

$$L = \frac{M_\mathrm{pl}^2}{2}R + f(\chi, Y) + P(\phi, X), \tag{34}$$

where M_pl is the reduced Planck mass. The function $f(\chi, Y)$ depends on the scalar field χ and its kinetic energy $Y = g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$, whereas the function $P(\phi, X)$ is dependent on ϕ and $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. The variables χ, Y are equivalent to N and an explicit time dependence, as argued before.

On using the property (6), the Lagrangian becomes

$$L = \frac{M_{\rm pl}^2}{2}(\mathcal{R} + \mathcal{S} - K^2) + f(\chi, Y) + P(\phi, X), \qquad (35)$$

where the total divergence term is dropped. Since $\bar{L} = -3M_{\rm pl}^2 H^2 + P$, $L_N = 2\dot{\chi}^2 f_Y$, $L_X = P_X$, $L_\phi = P_\phi$, and $\mathcal{F} = -2M_{\rm pl}^2 H$ on the flat FLRW background, Eqs. (31)–(33) read

$$3M_{\rm pl}^2 H^2 = -2f_Y\dot{\chi}^2 - 2P_X\dot{\phi}^2 - f - P, \qquad (36)$$

$$2M_{\rm pl}^2\dot{H} + 3M_{\rm pl}^2 H^2 = -f - P, \qquad (37)$$

$$\frac{d}{dt}(a^3 P_X\dot{\phi}) + \frac{1}{2}a^3 P_\phi = 0, \qquad (38)$$

which agree with those derived in Refs. [29,51] for a single-field case. When only the field χ is present, these reduce to the first two equations with $P = 0$. In the presence of the field ϕ alone, by eliminating H^2 and then $\dot{H}$ from the first two equations, one obtains the integrability condition

$$\frac{d}{dt}(a^3 P_X\dot{\phi}^2) + \frac{1}{2}a^3\dot{P} = 0, \qquad (39)$$

which reduces to Eq. (38) for $g_{00} = -1$ and $\phi = \phi(t)$. Therefore, on the flat FLRW background for $f = 0$ and for ϕ depending only on time at the background level, only two equations [(36) and (37)] are independent.

In the case where both fields are present, Eqs. (36) and (37) imply

$$\dot{\chi}\left[\frac{d}{dt}(a^3 f_Y\dot{\chi}) + \frac{1}{2}a^3 f_\chi\right] = \dot{\phi}\left[\frac{d}{dt}(a^3 P_X\dot{\phi}) + \frac{1}{2}a^3 P_\phi\right]. \qquad (40)$$

For χ dynamically changing in time, Eqs. (38) and (40) show that the field χ obeys a similar continuity equation as ϕ does. Hence both fields satisfy the continuity equations, which could also be derived from the vanishing of the covariant divergences of the individual energy-momentum tensors. In the above discussion, there was no need to impose these conditions by hand. The particular noninteracting structure of the Lagrangian combined with the equation of motion (33) leads to the continuity Eq. (38) for ϕ, while the integrability condition for Eqs. (36) and (37) leads to a similar one for the field χ. Both continuity equations thus emerge directly from the action.

IV. SECOND-ORDER LAGRANGIAN

In this section we expand the action (8) up to the second order in the perturbations in order to derive conditions for the avoidance of ghosts and of Laplacian instabilities for scalar and tensor perturbations. We also study the conditions under which the derivatives higher than second order are absent in our two-field setup.

A. Conditions for the absence of derivatives higher than second order

Up to second order of the scalar perturbations, the Lagrangian (22) reads

$$\begin{aligned} L = {} & \bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F} + (\dot{\mathcal{F}} + L_N)\delta N + \mathcal{E}\delta_1\mathcal{R} + L_\phi\delta\phi + L_X\delta_1 X \\ & + \left(\frac{1}{2}L_{NN} - \dot{\mathcal{F}}\right)\delta N^2 + \frac{1}{2}\mathcal{A}\delta K^2 + \mathcal{B}\delta K\delta N + \mathcal{C}\delta K\delta_1\mathcal{R} + \mathcal{D}\delta N\delta_1\mathcal{R} + \mathcal{E}\delta_2\mathcal{R} + \frac{1}{2}\mathcal{G}\delta_1\mathcal{R}^2 + L_{\mathcal{S}}\delta K^\mu_\nu\delta K^\nu_\mu \\ & + L_{\mathcal{Z}}\delta\mathcal{R}^\mu_\nu\delta\mathcal{R}^\nu_\mu + L_X\delta_2 X + \frac{1}{2}L_{\phi\phi}\delta\phi^2 + \frac{1}{2}L_{XX}\delta_1 X^2 + L_{\phi X}\delta\phi\delta_1 X + (L_{\mathcal{R}\phi} + HL_{\mathcal{U}\phi})\delta_1\mathcal{R}\delta\phi + L_{N\phi}\delta N\delta\phi \\ & + (L_{K\phi} + 2HL_{\mathcal{S}\phi})\delta K\delta\phi + (L_{\mathcal{R}X} + HL_{\mathcal{U}X})\delta_1\mathcal{R}\delta_1 X + L_{NX}\delta N\delta_1 X + (L_{KX} + 2HL_{\mathcal{S}X})\delta K\delta_1 X, \end{aligned} \qquad (41)$$

where

$$\mathcal{A} = L_{KK} + 4HL_{\mathcal{S}K} + 4H^2 L_{\mathcal{S}\mathcal{S}}, \qquad (42)$$

$$\mathcal{B} = L_{KN} + 2HL_{\mathcal{S}N}, \qquad (43)$$

$$\mathcal{C} = L_{K\mathcal{R}} + 2HL_{\mathcal{S}\mathcal{R}} + \frac{1}{2}L_{\mathcal{U}} + HL_{K\mathcal{U}} + 2H^2 L_{\mathcal{S}\mathcal{U}}, \qquad (44)$$

$$\mathcal{D} = L_{N\mathcal{R}} - \frac{1}{2}\dot{L_{\mathcal{U}}} + HL_{N\mathcal{U}}, \qquad (45)$$

$$\mathcal{G} = L_{\mathcal{R}\mathcal{R}} + 2HL_{\mathcal{R}\mathcal{U}} + H^2 L_{\mathcal{U}\mathcal{U}}. \tag{46}$$

The second-order Lagrangian density explicitly reads as

$$\begin{aligned}\mathcal{L}_2 = {}& \delta\sqrt{h}[(\dot{\mathcal{F}} + L_N)\delta N + \mathcal{E}\delta_1\mathcal{R} + L_\phi\delta\phi + L_X\delta_1 X] \\ & + a^3\bigg[\left(L_N + \frac{1}{2}L_{NN}\right)\delta N^2 + \mathcal{E}\delta_2\mathcal{R} + \frac{1}{2}\mathcal{A}\delta K^2 + \mathcal{B}\delta K\delta N + \mathcal{C}\delta K\delta_1\mathcal{R} + (\mathcal{D}+\mathcal{E})\delta N\delta_1\mathcal{R} + \frac{1}{2}\mathcal{G}\delta_1\mathcal{R}^2 \\ & + L_{\mathcal{S}}\delta K^\mu_\nu\delta K^\nu_\mu + L_{\mathcal{Z}}\delta\mathcal{R}^\mu_\nu\delta\mathcal{R}^\nu_\mu + L_X\delta_2 X + \frac{1}{2}L_{\phi\phi}\delta\phi^2 + \frac{1}{2}L_{XX}\delta_1 X^2 + L_{\phi X}\delta\phi\delta_1 X \\ & + (L_\phi + L_{N\phi})\delta N\delta\phi + (L_X + L_{NX})\delta N\delta_1 X + (L_{\mathcal{R}\phi} + HL_{\mathcal{U}\phi})\delta_1\mathcal{R}\delta\phi + (L_{\mathcal{R}X} + HL_{\mathcal{U}X})\delta_1\mathcal{R}\delta_1 X \\ & + (L_{K\phi} + 2HL_{\mathcal{S}\phi})\delta K\delta\phi + (L_{KX} + 2HL_{\mathcal{S}X})\delta K\delta_1 X\bigg]. \end{aligned}\tag{47}$$

Since h_{ij} is given by Eq. (15) in our gauge choice, it follows that

$$\delta\sqrt{h} = 3a^3\zeta, \qquad \delta\mathcal{R}_{ij} = -(\delta_{ij}\partial^2\zeta + \partial_i\partial_j\zeta), \qquad \delta_1\mathcal{R} = -4a^{-2}\partial^2\zeta, \qquad \delta_2\mathcal{R} = -2a^{-2}[(\partial\zeta)^2 - 4\zeta\partial^2\zeta], \tag{48}$$

where $\partial^2\zeta \equiv \partial_i\partial_i\zeta = [\partial^2/\partial(x^1)^2 + \partial^2/\partial(x^2)^2 + \partial^2/\partial(x^3)^2]\zeta$ and $(\partial\zeta)^2 = (\partial_i\zeta)(\partial_i\zeta)$. After integration by parts the perturbation $\delta_2\mathcal{R}$ reduces to $\delta_2\mathcal{R} = -10a^{-2}(\partial\zeta)^2$, up to a boundary term. From Eq. (5), the first-order extrinsic curvature is expressed as

$$\delta K^i_j = (\dot\zeta - H\delta N)\delta^i_j - \frac{1}{2a^2}\delta^{ik}(\partial_k N_j + \partial_j N_k), \tag{49}$$

where we used the fact that the Christoffel symbols Γ^k_{ij} are the first-order perturbations for nonzero k, i, j. Recalling that the shift N_i is related to the metric perturbation ψ via Eq. (14), the trace of Eq. (49) reads

$$\delta K = 3(\dot\zeta - H\delta N) - \frac{1}{a^2}\partial^2\psi. \tag{50}$$

Substituting Eqs. (20,21,48,49), and (50) into the Lagrangian density (47), it follows that

$$\begin{aligned}\mathcal{L}_2 = a^3\bigg\{ & \left[\frac{1}{2}(2L_N + L_{NN} + 9\mathcal{A}H^2 - 6\mathcal{B}H + 6L_{\mathcal{S}}H^2) + \mathcal{Q}_1\right]\delta N^2 \\ & + \left[(\mathcal{B} - 3\mathcal{A}H - 2L_{\mathcal{S}}H)\left(3\dot\zeta - \frac{\partial^2\psi}{a^2}\right) + 4(3H\mathcal{C} - \mathcal{D} - \mathcal{E})\frac{\partial^2\zeta}{a^2} + \mathcal{Q}_2\right]\delta N \\ & - (3\mathcal{A} + 2L_{\mathcal{S}})\dot\zeta\frac{\partial^2\psi}{a^2} - 12\mathcal{C}\dot\zeta\frac{\partial^2\zeta}{a^2} + \left(\frac{9}{2}\mathcal{A} + 3L_{\mathcal{S}}\right)\dot\zeta^2 + 2\mathcal{E}\frac{(\partial\zeta)^2}{a^2} + \mathcal{Q}_3 \\ & + \frac{1}{2}(\mathcal{A} + 2L_{\mathcal{S}})\frac{(\partial^2\psi)^2}{a^4} + 4\mathcal{C}\frac{(\partial^2\psi)(\partial^2\zeta)}{a^4} + 2(4\mathcal{G} + 3L_{\mathcal{Z}})\frac{(\partial^2\zeta)^2}{a^4}\bigg\}, \end{aligned}\tag{51}$$

where the terms $\mathcal{Q}_1$, $\mathcal{Q}_2$, and $\mathcal{Q}_3$, which appear in the presence of the field ϕ, are given by

$$\mathcal{Q}_1 = \dot\phi^2[2\dot\phi^2 L_{XX} - L_X + 2L_{NX} - 6H(L_{KX} + 2HL_{\mathcal{S}X})], \tag{52}$$

$$\begin{aligned}\mathcal{Q}_2 = {}& [L_\phi + L_{N\phi} + 2\dot\phi^2 L_{\phi X} - 3H(L_{K\phi} + 2HL_{\mathcal{S}\phi})]\delta\phi - 2\dot\phi(2\dot\phi^2 L_{XX} - L_X + L_{NX})\dot{\delta\phi} \\ & + 2\dot\phi(L_{KX} + 2HL_{\mathcal{S}X})\left(3\dot\phi\,\dot\zeta + 3H\dot{\delta\phi} - \dot\phi\frac{\partial^2\psi}{a^2}\right) - 8\dot\phi^2(L_{\mathcal{R}X} + HL_{\mathcal{U}X})\frac{\partial^2\zeta}{a^2}, \end{aligned}\tag{53}$$

$$\mathcal{Q}_3 = \frac{1}{2}L_{\phi\phi}\delta\phi^2 + (2\dot{\phi}^2 L_{XX} - L_X)\delta\dot{\phi}^2 - 2L_{\phi X}\dot{\phi}\delta\phi\delta\dot{\phi} + 3\zeta(L_\phi\delta\phi - 2\dot{\phi}L_X\delta\dot{\phi}) - 2\dot{\phi}L_X\delta\phi\frac{\partial^2\psi}{a^2} + L_X\frac{(\partial\delta\phi)^2}{a^2}$$
$$- \left[(L_{K\phi} + 2HL_{\mathcal{S}\phi})\left(\frac{\partial^2\psi}{a^2} - 3\dot{\zeta}\right) + 4(L_{\mathcal{R}\phi} + HL_{\mathcal{U}\phi})\frac{\partial^2\zeta}{a^2}\right]\delta\phi$$
$$+ 2\dot{\phi}\left[(L_{KX} + 2HL_{\mathcal{S}X})\left(\frac{\partial^2\psi}{a^2} - 3\dot{\zeta}\right) + 4(L_{\mathcal{R}X} + HL_{\mathcal{U}X})\frac{\partial^2\zeta}{a^2}\right]\delta\dot{\phi}. \quad (54)$$

There is a term $3a^3(L_N + \dot{\mathcal{F}} + 2\dot{\phi}^2 L_X)\zeta\delta N$ in $\mathcal{L}_2$, but it disappears due to the background equations of motion (31) and (32). In Eq. (54), the term $-2\dot{\phi}L_X\delta\phi\partial^2\psi/a^2$ originates from $2\dot{\phi}L_X\partial_i\psi\partial_i\delta\phi/a^2$ after integration by parts.

The Lagrangian density (51) contains the terms δN and $\partial^2\psi$ but not their time derivatives. Varying the second-order action $S_2 = \int d^4x\mathcal{L}_2$ with respect to δN and $\partial^2\psi$, we obtain the following *Hamiltonian constraint and momentum constraint*, respectively,

$$\begin{aligned}&[2L_N + L_{NN} - 6H\mathcal{W} - 3H^2(3\mathcal{A} + 2L_{\mathcal{S}})\\ &\quad + 2\dot{\phi}^2(2L_{NX} - L_X + 2\dot{\phi}^2 L_{XX})]\delta N - \mathcal{W}\frac{\partial^2\psi}{a^2} + 3\mathcal{W}\dot{\zeta}\\ &\quad + 4[3H\mathcal{C} - \mathcal{D} - \mathcal{E} - 2\dot{\phi}^2(L_{\mathcal{R}X} + HL_{\mathcal{U}X})]\frac{\partial^2\zeta}{a^2}\\ &\quad + [L_\phi + 2\dot{\phi}^2 L_{\phi X} + L_{N\phi} - 3H(L_{K\phi} + 2HL_{\mathcal{S}\phi})]\delta\phi\\ &\quad + 2\dot{\phi}[L_X - 2\dot{\phi}^2 L_{XX} - L_{NX} + 3H(L_{KX} + 2HL_{\mathcal{S}X})]\delta\dot{\phi} = 0,\end{aligned} \quad (55)$$

$$\begin{aligned}&\mathcal{W}\delta N - (\mathcal{A} + 2L_{\mathcal{S}})\frac{\partial^2\psi}{a^2} + (3\mathcal{A} + 2L_{\mathcal{S}})\dot{\zeta} - 4\mathcal{C}\frac{\partial^2\zeta}{a^2}\\ &\quad + (L_{K\phi} + 2HL_{\mathcal{S}\phi} + 2\dot{\phi}L_X)\delta\phi\\ &\quad - 2\dot{\phi}(L_{KX} + 2HL_{\mathcal{S}X})\delta\dot{\phi} = 0,\end{aligned} \quad (56)$$

where we have denoted

$$\mathcal{W} \equiv \mathcal{B} - 3\mathcal{A}H - 2L_{\mathcal{S}}H + 2\dot{\phi}^2(L_{KX} + 2HL_{\mathcal{S}X}). \quad (57)$$

From Eqs. (55) and (56), we can express δN and $\partial^2\psi/a^2$ in terms of the four quantities $\dot{\zeta}$, $\partial^2\zeta/a^2$, $\delta\phi$, and $\delta\dot{\phi}$. By substituting these relations into Eq. (51), the Lagrangian density $\mathcal{L}_2$ obeys a simpler functional dependence

$$\begin{aligned}\mathcal{L}_2 = a^3\Big[&\mathcal{C}_1\dot{\zeta}^2 + \mathcal{C}_2\frac{(\partial\zeta)^2}{a^2} + \mathcal{C}_3\dot{\zeta}\frac{\partial^2\zeta}{a^2} + \mathcal{C}_4\frac{(\partial^2\zeta)^2}{a^4}\\ &+ \mathcal{C}_5\delta\phi^2 + \mathcal{C}_6\delta\dot{\phi}^2 + \mathcal{C}_7\frac{(\partial\delta\phi)^2}{a^2} + \mathcal{C}_8\delta\phi\delta\dot{\phi}\\ &+ \mathcal{C}_9\zeta\delta\phi + \mathcal{C}_{10}\zeta\delta\dot{\phi} + \mathcal{C}_{11}\dot{\zeta}\delta\phi + \mathcal{C}_{12}\dot{\zeta}\,\delta\dot{\phi}\\ &+ \mathcal{C}_{13}\frac{\partial^2\zeta}{a^2}\delta\phi + \mathcal{C}_{14}\frac{\partial^2\zeta}{a^2}\delta\dot{\phi}\Big],\end{aligned} \quad (58)$$

where $\mathcal{C}_i$'s ($i = 1, 2, \ldots$) are time-dependent coefficients. The contribution to the action corresponding to the third term of Eq. (58) can be rewritten, up to a boundary term, as

$$\int d^4x a\mathcal{C}_3\dot{\zeta}\partial^2\zeta = \int d^4x\frac{1}{2}\frac{d}{dt}(a\mathcal{C}_3)(\partial\zeta)^2, \quad (59)$$

which generates an additional contribution to the second term of Eq. (58).

The fourth term of Eq. (58) gives rise to the equations of motion for ζ with spatial derivatives higher than second order. This contribution comes from the last three terms in Eq. (51); hence, provided the three conditions

$$\mathcal{A} + 2L_{\mathcal{S}} = 0, \qquad \mathcal{C} = 0, \qquad 4\mathcal{G} + 3L_{\mathcal{Z}} = 0 \quad (60)$$

are satisfied, the coefficient $\mathcal{C}_4$ vanishes. Even in the absence of the scalar field ϕ, the *conditions for the avoidance of spatial derivatives higher than second order* (60) are equivalent to those derived in Ref. [15].

The last term of Eq. (58) corresponds to the mixture of time and spatial derivatives higher than second order. Under the conditions (60), the coefficient $\mathcal{C}_{14}$ reduces to

$$\begin{aligned}\mathcal{C}_{14} = -\frac{8\dot{\phi}}{\mathcal{W}}[&(L_{KX} + 2HL_{\mathcal{S}X})\{\mathcal{D} + \mathcal{E} + 2\dot{\phi}^2(L_{\mathcal{R}X} + HL_{\mathcal{U}X})\}\\ &-(L_{\mathcal{R}X} + HL_{\mathcal{U}X})\mathcal{W}].\end{aligned} \quad (61)$$

The two combinations $L_{KX} + 2HL_{\mathcal{S}X}$ and $L_{\mathcal{R}X} + HL_{\mathcal{U}X}$ originate from the terms on the third line of Eq. (54) as well as from other contributions. If the conditions

$$L_{KX} + 2HL_{\mathcal{S}X} = 0, \qquad L_{\mathcal{R}X} + HL_{\mathcal{U}X} = 0 \quad (62)$$

are satisfied, it follows that $\mathcal{C}_{14} = 0$. In the context of two scalar fields, we require that the *conditions for the avoidance of time and spatial derivatives of combined order higher than two* (62) also hold, complementing the conditions (60) such that any combinations of time and spatial derivatives higher than second order are eliminated.

B. Conditions for the avoidance of scalar ghosts and instabilities

In the following, we focus on the theories in which the conditions (60) and (62) are satisfied. Then the Lagrangian density (58) can be expressed in the form

$$\mathcal{L}_2 = a^3\left(\dot{\vec{\mathcal{X}}}^t\boldsymbol{K}\dot{\vec{\mathcal{X}}} - \frac{1}{a^2}\partial_i\vec{\mathcal{X}}^t\boldsymbol{G}\partial_i\vec{\mathcal{X}} - \vec{\mathcal{X}}^t\boldsymbol{B}\dot{\vec{\mathcal{X}}} - \vec{\mathcal{X}}^t\boldsymbol{M}\vec{\mathcal{X}}\right), \quad (63)$$

where the vector $\vec{\mathcal{X}}$ is composed from two dimensionless gauge-invariant quantities ζ and $\delta\phi/M_{\rm pl}$, as

$$\vec{\mathcal{X}}^t = (\zeta, \delta\phi/M_{\rm pl}). \tag{64}$$

The 2×2 matrices $\boldsymbol{K}$, $\boldsymbol{G}$, $\boldsymbol{B}$, and $\boldsymbol{M}$ are defined in terms of the coefficients appearing in Eq. (58). We note that the term $a\mathcal{C}_{13}\partial^2\zeta\delta\phi$ reduces to $-a\mathcal{C}_{13}\partial\zeta\partial\delta\phi$ after integration by parts. The components of the four matrices are given, respectively, by

$$K_{11} = \frac{2L_{\mathcal{S}}}{\mathcal{W}^2}[g_2 + 8L_{\mathcal{S}}\dot{\phi}^2(g_1 + 2L_{NX})], \qquad K_{12} = K_{21} = -\frac{4L_{\mathcal{S}}\dot{\phi}M_{\rm pl}}{\mathcal{W}}(g_1 + L_{NX}), \qquad K_{22} = g_1 M_{\rm pl}^2, \tag{65}$$

$$G_{11} = -\frac{1}{2}(\dot{\mathcal{C}}_3 + H\mathcal{C}_3 + 4\mathcal{E}), \qquad G_{12} = G_{21} = -\frac{\mathcal{C}_3 M_{\rm pl}}{8L_{\mathcal{S}}}g_3 - 2g_4 M_{\rm pl}, \qquad G_{22} = -L_X M_{\rm pl}^2, \tag{66}$$

$$\begin{aligned} &B_{11} = 0, \qquad B_{12} = 6\dot{\phi}L_X M_{\rm pl}, \\ &B_{21} = \frac{2M_{\rm pl}}{\mathcal{W}^2}[2L_{\mathcal{S}}g_3\{g_5 + 2\dot{\phi}^2(g_1 + 2L_{NX})\} - 2L_{\mathcal{S}}\mathcal{W}(g_6 + 3Hg_3)] + 6\dot{\phi}L_X M_{\rm pl}, \\ &B_{22} = \frac{2\dot{\phi}M_{\rm pl}^2}{\mathcal{W}}[L_{\phi X}\mathcal{W} - g_3(g_1 + L_{NX})], \end{aligned} \tag{67}$$

$$\begin{aligned} &M_{11} = 0, \qquad M_{12} = M_{21} = -\frac{3}{2}L_\phi M_{\rm pl}, \\ &M_{22} = -\frac{M_{\rm pl}^2}{2\mathcal{W}^2}[g_3^2\{g_5 + 2\dot{\phi}^2(g_1 + 2L_{NX})\} - 2g_3 g_6\mathcal{W} + L_{\phi\phi}\mathcal{W}^2], \end{aligned} \tag{68}$$

where

$$\begin{aligned} &g_1 \equiv 2\dot{\phi}^2 L_{XX} - L_X, \qquad g_2 \equiv 4L_{\mathcal{S}}(2L_N + L_{NN}) + 3(L_{KN} + 2HL_{\mathcal{S}N})^2, \\ &g_3 \equiv L_{K\phi} + 2HL_{\mathcal{S}\phi} + 2\dot{\phi}L_X, \qquad g_4 \equiv L_{\mathcal{R}\phi} + HL_{\mathcal{U}\phi}, \\ &g_5 \equiv 2L_N + L_{NN} + 12H^2 L_{\mathcal{S}}, \qquad g_6 \equiv L_\phi + L_{N\phi} + 2\dot{\phi}^2 L_{\phi X} + 6H\dot{\phi}L_X. \end{aligned} \tag{69}$$

The coefficient $\mathcal{C}_3$ is given by

$$\mathcal{C}_3 = -\frac{16L_{\mathcal{S}}}{\mathcal{W}}(\mathcal{D} + \mathcal{E}), \tag{70}$$

where

$$\begin{aligned} \mathcal{W} &= L_{KN} + 2HL_{\mathcal{S}N} + 4HL_{\mathcal{S}}, \\ \mathcal{D} + \mathcal{E} &= L_{\mathcal{R}} + L_{N\mathcal{R}} + 3HL_{\mathcal{U}}/2 + HL_{N\mathcal{U}}. \end{aligned} \tag{71}$$

Note that there is the relation $g_2 = 3\mathcal{W}(\mathcal{W} - 8HL_{\mathcal{S}}) + 4L_{\mathcal{S}}g_5$.

The *conditions for the avoidance of scalar ghosts* are fulfilled if the two eigenvalues λ_1 and λ_2 of the kinetic matrix $\boldsymbol{K}$ are positive:

$$\begin{aligned} \lambda_1 + \lambda_2 = \frac{1}{\mathcal{W}^2}[&(16\dot{\phi}^2 L_{\mathcal{S}}^2 + M_{\rm pl}^2\mathcal{W}^2)g_1 + 2L_{\mathcal{S}}g_2 \\ &+ 32\dot{\phi}^2 L_{\mathcal{S}}^2 L_{NX}] > 0, \end{aligned} \tag{72}$$

$$\lambda_1\lambda_2 = \frac{2M_{\rm pl}^2 L_{\mathcal{S}}}{\mathcal{W}^2}(g_1 g_2 - 8\dot{\phi}^2 L_{\mathcal{S}} L_{NX}^2) > 0. \tag{73}$$

As we prove in Sec. IV C, the tensor ghost is absent for $L_{\mathcal{S}} > 0$. Taking into account this constraint, the conditions (72) and (73) read

$$(16\dot{\phi}^2 L_{\mathcal{S}}^2 + M_{\rm pl}^2\mathcal{W}^2)g_1 + 2L_{\mathcal{S}}g_2 + 32\dot{\phi}^2 L_{\mathcal{S}}^2 L_{NX} > 0, \tag{74}$$

$$g_1 g_2 > 8\dot{\phi}^2 L_{\mathcal{S}} L_{NX}^2. \tag{75}$$

In the absence of couplings between the kinetic terms X and Y we have $L_{NX} = 0$. In this case the conditions (74) and (75) are satisfied for $g_1 > 0$ and $g_2 > 0$.

Let us derive *conditions for the avoidance of Laplacian instabilities* for the modes with a wave number k and a frequency ω in the large k limit. The dispersion relation following from the Lagrangian density (63) is given by

$$\det(\omega^2\boldsymbol{K} - k^2\boldsymbol{G}/a^2) = 0. \tag{76}$$

Introducing the scalar propagation speed c_s as $\omega^2 = c_s^2 k^2/a^2$, it follows that

$$\det(c_s^2\boldsymbol{K} - \boldsymbol{G}) = 0. \tag{77}$$

This can be written in the form

$$c_s^4 - \frac{\mu_1}{\mu_0} c_s^2 + \frac{\mu_2}{\mu_0} = 0, \tag{78}$$

where

$$\mu_0 = \lambda_1 \lambda_2 = \frac{2M_{\rm pl}^2 L_{\mathcal{S}}}{\mathcal{W}^2}(g_1 g_2 - 8\dot{\phi}^2 L_{\mathcal{S}} L_{NX}^2), \tag{79}$$

$$\mu_1 = -\frac{M_{\rm pl}^2}{2\mathcal{W}^2}\Big[(\dot{\mathcal{C}}_3 + H\mathcal{C}_3 + 4\mathcal{E})g_1\mathcal{W}^2 + 2(\mathcal{C}_3 g_3 + 16L_{\mathcal{S}}g_4)\dot{\phi}(g_1 + L_{NX})\mathcal{W} + 4L_{\mathcal{S}}L_X\{g_2 + 8\dot{\phi}^2 L_{\mathcal{S}}(g_1 + 2L_{NX})\}\Big], \tag{80}$$

$$\mu_2 = \frac{M_{\rm pl}^2}{64L_{\mathcal{S}}^2}[32(\dot{\mathcal{C}}_3 + H\mathcal{C}_3 + 4\mathcal{E})L_{\mathcal{S}}^2 L_X - (\mathcal{C}_3 g_3 + 16L_{\mathcal{S}}g_4)^2]. \tag{81}$$

The solution to Eq. (78) is given by

$$c_s^2 = \frac{\mu_1}{2\mu_0}\left[1 \pm \sqrt{1 - \frac{4\mu_0\mu_2}{\mu_1^2}}\right]. \tag{82}$$

Since $\mu_0 > 0$ under the no-ghost condition (73), the two solutions of c_s^2 are positive for $\mu_1 > 0$ and $\mu_2 > 0$, which translate to

$$(\dot{\mathcal{C}}_3 + H\mathcal{C}_3 + 4\mathcal{E})g_1\mathcal{W}^2 + 2(\mathcal{C}_3 g_3 + 16L_{\mathcal{S}}g_4)\dot{\phi}(g_1 + L_{NX})\mathcal{W} + 4L_{\mathcal{S}}L_X\{g_2 + 8\dot{\phi}^2 L_{\mathcal{S}}(g_1 + 2L_{NX})\} < 0, \tag{83}$$

$$32(\dot{\mathcal{C}}_3 + H\mathcal{C}_3 + 4\mathcal{E})L_{\mathcal{S}}^2 L_X - (\mathcal{C}_3 g_3 + 16L_{\mathcal{S}}g_4)^2 > 0. \tag{84}$$

In the absence of the field ϕ, the condition (83) reads $(\dot{\mathcal{C}}_3 + H\mathcal{C}_3 + 4\mathcal{E})g_1\mathcal{W}^2 < 0$. Since $g_1 > 0$ to avoid scalar ghosts, we have that $\dot{\mathcal{C}}_3 + H\mathcal{C}_3 + 4\mathcal{E} < 0$. This condition agrees with the one derived in Ref. [15] for a single scalar field.

At the end of this subsection we present *the master equations for scalar perturbations in the two-field scenario* satisfying the conditions (60) and (62). First, the Hamiltonian and momentum constraint Eqs. (55) and (56) read

$$[g_5 + 2\dot{\phi}^2(g_1 + 2L_{NX}) - 6H\mathcal{W}]\delta N - \mathcal{W}\frac{\partial^2\psi}{a^2} + 3\mathcal{W}\dot{\zeta} - 4(\mathcal{D} + \mathcal{E})\frac{\partial^2\zeta}{a^2} + (g_6 - 3Hg_3)\delta\phi - 2\dot{\phi}(g_1 + L_{NX})\dot{\delta\phi} = 0, \tag{85}$$

$$\mathcal{W}\delta N - 4L_{\mathcal{S}}\dot{\zeta} + g_3\delta\phi = 0. \tag{86}$$

Variations of the Lagrangian density (63) with respect to ζ and $\delta\phi$ lead to

$$\frac{d}{dt}(2M_{\rm pl}K_{11}\dot{\zeta} + 2K_{12}\dot{\delta\phi} - B_{21}\delta\phi) + 3H(2M_{\rm pl}K_{11}\dot{\zeta} + 2K_{12}\dot{\delta\phi} - B_{21}\delta\phi) - 2M_{\rm pl}G_{11}\frac{\partial^2\zeta}{a^2} - 2G_{12}\frac{\partial^2\delta\phi}{a^2} + B_{12}\dot{\delta\phi} + 2M_{12}\delta\phi = 0, \tag{87}$$

$$\frac{d}{dt}(2M_{\rm pl}K_{12}\dot{\zeta} + 2K_{22}\dot{\delta\phi} - B_{22}\delta\phi) + 3H(2M_{\rm pl}K_{12}\dot{\zeta} + 2K_{22}\dot{\delta\phi} - B_{22}\delta\phi) - 2M_{\rm pl}G_{12}\frac{\partial^2\zeta}{a^2} - 2G_{22}\frac{\partial^2\delta\phi}{a^2} + B_{22}\dot{\delta\phi} + 2M_{22}\delta\phi + M_{\rm pl}(B_{21} - B_{12})\dot{\zeta} = 0. \tag{88}$$

In deriving Eq. (88), we used the property

$$\dot{B}_{12} + 3HB_{12} - 2M_{12} = 0, \tag{89}$$

which follows from the background Eq. (33). From Eqs. (85) and (86) we have

$$2M_{\rm pl}K_{11}\dot{\zeta} + 2K_{12}\dot{\delta\phi} - B_{21}\delta\phi = M_{\rm pl}\left(4L_{\mathcal{S}}\frac{\partial^2\psi}{a^2} - \mathcal{C}_3\frac{\partial^2\zeta}{a^2}\right) - 6M_{\rm pl}L_X\dot{\phi}\delta\phi. \tag{90}$$

Substituting this relation into Eq. (87) and using Eqs. (33) and (86), we obtain

$$-\frac{\mathcal{C}_3\mathcal{W}}{16L_{\mathcal{S}}}\delta N + L_{\mathcal{S}}\dot{\psi} + (\dot{L}_{\mathcal{S}} + HL_{\mathcal{S}})\psi + \mathcal{E}\zeta + g_4\delta\phi = 0, \tag{91}$$

which corresponds to the traceless part of the gravitational field equations.

The explicit dynamics of scalar perturbations emerges as a solution of Eqs. (85)–(88) and (91) for any given Lagrangian.

C. Tensor perturbations

Tensor perturbations (gravitational waves) are outside the general framework of our paper, but they provide useful conditions for the avoidance of ghosts and of Laplacian instabilities which have to hold together with those previously derived. For this purpose, let us derive the second-order action for tensor perturbations γ_{ij} under the conditions (60). We express the three-dimensional metric in the form

$$h_{ij} = a^2(t)e^{2\zeta}\hat{h}_{ij}, \qquad \hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{il}\gamma_{lj}, \qquad \det\hat{h} = 1, \tag{92}$$

where $\gamma_{ii} = \partial_i\gamma_{ij} = 0$. The second-order term $\gamma_{il}\gamma_{lj}/2$ has been introduced for the simplification of calculations [52].

We substitute the expression (92) into the Lagrangian (22) and set all of the scalar perturbations to be zero. In doing so, we use the following properties of tensor perturbations:

$$\delta K = 0, \qquad \delta K_{ij}^2 = \frac{1}{4}\dot{\gamma}_{ij}^2, \qquad \delta_1 \mathcal{R} = 0,$$
$$\delta_2 \mathcal{R} = -\frac{1}{4a^2}(\partial_k \gamma_{ij})^2. \tag{93}$$

Then the second-order action for gravitational waves reads

$$S_h^{(2)} = \int d^4x a^3 [L_{\mathcal{S}}(\delta K^\nu_\mu \delta K^\mu_\nu - \delta K^2) + \mathcal{E}\delta_2\mathcal{R}]$$
$$= \int d^4x \frac{a^3}{4} L_{\mathcal{S}} \left[\dot{\gamma}_{ij}^2 - \frac{\mathcal{E}}{L_{\mathcal{S}}}\frac{1}{a^2}(\partial_k\gamma_{ij})^2\right]. \tag{94}$$

This shows that the *no-ghost condition for tensor perturbations* corresponds to

$$L_{\mathcal{S}} > 0. \tag{95}$$

The tensor propagation speed square is given by

$$c_t^2 = \frac{\mathcal{E}}{L_{\mathcal{S}}}. \tag{96}$$

Provided that the condition (95) holds, the *condition for the avoidance of the Laplacian instability for tensor perturbations* is

$$\mathcal{E} = L_{\mathcal{R}} + \frac{1}{2}\dot{L_{\mathcal{U}}} + \frac{3}{2}HL_{\mathcal{U}} > 0. \tag{97}$$

In addition to the conditions for the absence of scalar ghosts and of Laplacian instabilities derived in the previous section, the theory needs to respect the two conditions (95) and (97).

V. A PARTICULAR FAMILY OF DARK ENERGY AND DARK MATTER MODELS

In this section we apply our results derived in the previous sections to a family of models describing both dark energy and dark matter. We use both N and χ, depending on the circumstances, as representing the dark energy scalar field, while ϕ will play the role of dark matter.

A. Horndeski-type dark energy and *k*-essence–type dark matter

For dark energy we consider a scalar degree of freedom χ in the framework of the Horndeski theory, whereas for dark matter we pick a k-essence–like scalar field ϕ without a direct coupling to gravity. Such a theory is described by the Lagrangian

$$L = \sum_{i=2}^{5} L_i, \tag{98}$$

where

$$L_2 = G_2(\chi, Y, \phi, X), \tag{99}$$

$$L_3 = G_3(\chi, Y)\Box\chi, \tag{100}$$

$$L_4 = G_4(\chi, Y)R - 2G_{4Y}(\chi, Y)[(\Box\chi)^2 - \chi^{;\mu\nu}\chi_{;\mu\nu}], \tag{101}$$

$$L_5 = G_5(\chi, Y)G_{\mu\nu}\chi^{;\mu\nu} + \frac{1}{3}G_{5Y}(\chi, Y)[(\Box\chi)^3$$
$$-3(\Box\chi)\chi_{;\mu\nu}\chi^{;\mu\nu} + 2\chi_{;\mu\nu}\chi^{;\mu\sigma}\chi^{;\nu}{}_{;\sigma}]. \tag{102}$$

Here G_2 to G_5 are arbitrary functions of the indicated variables. Note that L_2 is the only contribution to the Lagrangian directly affected by the scalar field ϕ. In the Horndeski theory with a perfect-fluid dark matter, the equations of linear perturbations and the resulting bispectrum associated with large-scale structures have been derived in Refs. [36,42,53,54]. We also caution that the definition of Y is different from that used in Refs. [36,42] (the factor -2 multiplied), but it is the same as the notation of Ref. [15].

Since we have chosen unitary gauge ($\delta\chi = 0$), the unit vector n_μ orthogonal to constant χ hypersurfaces is given by

$$n_\mu = -\gamma\chi_{;\mu}, \qquad \gamma = \frac{1}{\sqrt{-Y}}. \tag{103}$$

From this it follows that

$$\chi_{;\mu\nu} = -\frac{1}{\gamma}(K_{\mu\nu} - n_\mu a_\nu - n_\nu a_\mu) + \frac{\gamma^2}{2}\chi^{;\sigma}Y_{;\sigma}n_\mu n_\nu, \tag{104}$$

$$\Box\chi = -\frac{1}{\gamma}K + \frac{\chi^{;\sigma}Y_{;\sigma}}{2Y}. \tag{105}$$

Using these relations and Eq. (6), the three Lagrangians L_3, L_4, and L_5 can be expressed as [15]

$$L_3 = 2(-Y)^{3/2}F_{3Y}K - YF_{3\chi}, \tag{106}$$

$$L_4 = G_4\mathcal{R} + (G_4 - 2YG_{4Y})(\mathcal{S} - K^2) - 2\sqrt{-Y}G_{4\chi}K, \tag{107}$$

$$L_5 = \sqrt{-Y}F_5\left(\frac{1}{2}K\mathcal{R} - \mathcal{U}\right)$$
$$- H(-Y)^{3/2}G_{5Y}(2H^2 - 2KH + K^2 - \mathcal{S})$$
$$+ \frac{1}{2}Y(G_{5\chi} - F_{5\chi})\mathcal{R} + \frac{1}{2}YG_{5\chi}(K^2 - \mathcal{S}), \tag{108}$$

where $F_3(\chi, Y)$ and $F_5(\chi, Y)$ are auxiliary fields defined by $G_3 \equiv F_3 + 2XF_{3X}$ and $G_{5Y} \equiv F_{5Y} + F_5/(2Y)$. We note that Y depends on N through the relation $Y = -\dot{\chi}^2/N^2$, valid on the FLRW background and also to linear order as the unitary gauge is imposed. For the Lagrangian (98) with (99) and (106)–(108), one can show that the conditions (60) and (62) are satisfied, so this theory does not have derivatives higher than second order.

B. No-ghost conditions and propagation speeds

For the theories described by the Lagrangian (98), the conditions for the avoidance of tensor ghosts and of Laplacian instabilities become

$$L_{\mathcal{S}} = G_4 - 2YG_{4Y} - H\dot{\chi}YG_{5Y} - \frac{1}{2}YG_{5\chi} > 0, \quad (109)$$

$$\mathcal{E} = G_4 + \frac{1}{2}YG_{5\chi} - YG_{5Y}\ddot{\chi} > 0, \quad (110)$$

which agree with those derived for the single-field Horndeski theory [23,55,56]. Note that in the presence of the Lagrangians L_4 and L_5 the tensor propagation speed square $c_t^2 = \mathcal{E}/L_{\mathcal{S}}$ is generally different from 1.

The term L_{NX} in Eqs. (74) and (75) is given by

$$L_{NX} = 2\dot{\chi}^2 G_{2YX}. \quad (111)$$

If the two kinetic terms Y and X do not have a direct coupling, it follows that $L_{NX} = 0$. In the following we shall focus on the theories obeying $L_{NX} = 0$. Then, the no-ghost conditions (74) and (75) translate to

$$g_1 = 2\dot{\phi}^2 G_{2XX} - G_{2X} > 0, \quad (112)$$

$$g_2 = (8L_{\mathcal{S}}w + 9\mathcal{W}^2)/3 > 0, \quad (113)$$

where

$$\begin{aligned} w \equiv\, & 3L_N + 3L_{NN}/2 - 9H(L_{KN} + 2HL_{\mathcal{S}N}) - 18L_{\mathcal{S}}H^2 \\ =\, & -18H^2G_4 + 3(YG_{2Y} + 2Y^2G_{2YY}) - 18H\dot{\chi}(2YG_{3Y} + Y^2G_{3YY}) - 3Y(G_{3\chi} + YG_{3\chi Y}) \\ & + 18H^2(7YG_{4Y} + 16Y^2G_{4YY} + 4Y^3G_{4YYY}) - 18H\dot{\chi}(G_{4\chi} + 5YG_{4\chi Y} + 2Y^2G_{4\chi YY}) \\ & + 6H^3\dot{\chi}(15YG_{5Y} + 13Y^2G_{5YY} + 2Y^3G_{5YYY}) + 9H^2Y(6G_{5\chi} + 9YG_{5\chi Y} + 2Y^2G_{5\chi YY}), \end{aligned} \quad (114)$$

$$\begin{aligned} \mathcal{W} =\, & 4HG_4 + 2\dot{\chi}YG_{3Y} - 16H(YG_{4Y} + Y^2G_{4YY}) \\ & + 2\dot{\chi}(G_{4\chi} + 2YG_{4\chi Y}) - 2H^2\dot{\chi}(5YG_{5Y} + 2Y^2G_{5YY}) \\ & - 2HY(3G_{5\chi} + 2YG_{5\chi Y}). \end{aligned} \quad (115)$$

The conditions (112) and (113) correspond to the no-ghost conditions for the scalar fields ϕ and χ, respectively. The latter condition coincides with the one derived in Refs. [23,55,56] in the single-field Horndeski theory.[1]

For the Lagrangian (98), we have the relation

$$L_{\mathcal{S}} = \mathcal{D} + \mathcal{E} = L_{\mathcal{R}} + L_{N\mathcal{R}} + \frac{3}{2}HL_{\mathcal{U}} + HL_{N\mathcal{U}}, \quad (116)$$

so that the coefficient $\mathcal{C}_3$ in Eq. (70) reads

$$\mathcal{C}_3 = -\frac{16L_{\mathcal{S}}^2}{\mathcal{W}}. \quad (117)$$

Using this property, the squares of the two scalar propagation speeds (82) yield

$$c_{s1}^2 = \frac{G_{2X}}{G_{2X} - 2\dot{\phi}^2 G_{2XX}}, \quad (118)$$

$$c_{s2}^2 = \frac{16L_{\mathcal{S}}^2(H\mathcal{W} + 2\dot{\phi}^2G_{2X}) - \mathcal{W}^2(\dot{\mathcal{C}}_3 + 4\mathcal{E})}{4g_2L_{\mathcal{S}}}, \quad (119)$$

where $L_{\mathcal{S}}$, $\mathcal{E}$, g_2, and $\mathcal{W}$ are given by Eqs. (109,110,113), and (115), respectively. The result (118) matches the propagation speed derived for the single-field k inflation [57]. In the particular case $\dot{\phi} = 0$ the second propagation speed (119) reproduces the one derived in the Horndeski theory [56], but the presence of the field ϕ modifies the single-field result. This latter property is consistent with the result of Ref. [36] derived for a perfect-fluid dark matter. Under the no-ghost conditions (109,112), and (113), the instability of scalar perturbations can be avoided for

$$G_{2X} < 0, \quad (120)$$

$$16L_{\mathcal{S}}^2(H\mathcal{W} + 2\dot{\phi}^2G_{2X}) - \mathcal{W}^2(\dot{\mathcal{C}}_3 + 4\mathcal{E}) > 0. \quad (121)$$

C. Equations of dark-matter perturbations

In the following we study the theories where the Lagrangian L_2 describes noninteracting scalar fields

$$L_2 = f(\chi, Y) + P(\phi, X), \quad (122)$$

[1] Compared to the quantities $w_{1,2,3,4}$ introduced in Ref. [56], there are the correspondences $w_1 = 2L_{\mathcal{S}}$, $w_2 = \mathcal{W}$, $w_3 = w$, and $w_4 = 2\mathcal{E}$.

while the Lagrangians $L_{3,4,5}$ are still given by Eqs. (100)–(102). In this case the field ϕ does not directly couple to χ, but the latter field has a coupling to gravity through the Lagrangians L_4 and L_5. The dark-matter field ϕ indirectly feels the change of the gravitational law through the modified Poisson equation.

The energy-momentum tensor of the field ϕ is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}P(\phi,X))}{\delta g^{\mu\nu}} = -2P_X\partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}P. \tag{123}$$

From this, the background energy density arises as $\rho = -T^0_0 = 2XP_X - P$. The isotropic pressure, defined as the coefficient of δ^i_j in T^i_j, is exactly the Lagrangian P of the scalar field ϕ [57]. We note that Eq. (33) is equivalent to the continuity equation $\dot{\rho} + 3H(\rho+P) = 0$.

The perturbation of the energy density reads

$$\delta\rho = -\delta T^0_0 = (P_X + 2XP_{XX})\delta X - (P_\phi - 2XP_{\phi X})\delta\phi, \tag{124}$$

where $\delta X = 2\dot{\phi}^2\delta N - 2\dot{\phi}\,\dot{\delta\phi}$. The pressure perturbation δP, defined by $\delta T^i_j = \delta P\delta^i_j$, is

$$\delta P = P_X\delta X + P_\phi\delta\phi. \tag{125}$$

At the level of the background, the momentum $q_i = T^0_i$ vanishes. The momentum perturbation δq, defined by $\delta T^0_i = \partial_i\delta q$, becomes

$$\delta q = 2P_X\dot{\phi}\delta\phi. \tag{126}$$

Anisotropic stresses are not included, as they arise only at second order (they are bilinear in $\delta\partial_i\phi$ due to the fact that on the background $\phi = \phi(t)$; hence, $\partial_i\phi$ vanishes to leading order).

Since the field ϕ does not directly couple to χ, the energy-momentum tensor T^μ_ν obeys the continuity equation $T^\mu_{\nu;\mu} = 0$. The $\nu = 0$ component of the linearized energy-momentum tensor satisfies

$$\delta T^\mu_{0;\mu} = \dot{\delta T^0_0} + \partial_i\delta T^i_0 + \delta\Gamma^i_{0i}T^0_0 + \Gamma^i_{0i}\delta T^0_0 - \delta\Gamma^i_{0i}T^i_i - \Gamma^i_{0i}\delta T^i_i, \tag{127}$$

where the lhs denotes the variation of the covariant 4-divergence and the first term on the rhs is the time derivative of the variation. On using the properties $\delta T^i_0 = a^{-2}(2XP_X\partial_i\psi - \partial_i\delta q)$, $\Gamma^i_{0j} = H\delta^i_j$, and $\delta\Gamma^i_{0j} = \dot{\zeta}\delta^i_j$ for the metric (10) with the gauge choice $E = 0$, it follows that

$$\dot{\delta\rho} + 3H(\delta\rho + \delta P) + (\rho + P)\left(3\dot{\zeta} - \frac{\partial^2\psi}{a^2}\right) + \frac{1}{a^2}\partial^2\delta q = 0. \tag{128}$$

From Eqs. (85) and (86) we can express $\partial^2\zeta/a^2$ in terms of $\dot{\zeta}$, $\partial^2\psi/a^2$, $\delta\phi$, and $\dot{\delta\phi}$. Substituting this relation into Eq. (88), rewriting $\delta\phi$ and $\dot{\delta\phi}$ in terms of $\delta\rho$ and δP, and using the properties $g_3 = 2\dot{\phi}P_X$, $g_4 = 0$, $g_6 = P_\phi + 2\dot{\phi}^2P_{\phi X} + 6H\dot{\phi}P_X$, and (117), we can also derive Eq. (128).[22]

Similarly, from the continuity equation $\delta T^\mu_{i\,;\mu} = 0$, we obtain

$$\dot{\delta q} + 3H\delta q + (\rho + P)\delta N + \delta P = 0. \tag{129}$$

One can easily confirm that δq given in (126) satisfies Eq. (129) by using the background Eq. (33).

From the perturbations $\delta\rho$ and δq we can construct the following gauge-invariant variables

$$\hat{\delta\rho} \equiv \delta\rho - 3H\delta q, \qquad \hat{\delta} \equiv \frac{\hat{\delta\rho}}{\rho} = \delta - 3Hv, \tag{130}$$

where $\delta \equiv \delta\rho/\rho$ and $v \equiv \delta q/\rho$. We define the "adiabatic sound speed" c_a of the field ϕ, as

$$c_a^2 \equiv \frac{\dot{P}}{\dot{\rho}} = w - \frac{\dot{w}}{3H(1+w)}, \tag{131}$$

where $w \equiv P/\rho$ is the equation of state. We also introduce the "general sound speed" c_x, as

$$c_x^2 \equiv \frac{\delta P}{\delta\rho}. \tag{132}$$

For a perfect fluid, c_x^2 is identical to c_a^2, but for an imperfect fluid like a scalar field, c_x^2 is generally different from c_a^2. In order to address this difference, we define the following entropy perturbation [49,58]:

$$\delta s \equiv (c_x^2 - c_a^2)\delta = \frac{\delta P}{\rho} - c_a^2\frac{\delta\rho}{\rho}. \tag{133}$$

In the scalar-field rest frame, we have $\delta q = 0$ and $\hat{\delta} = \delta$, so that the entropy perturbation reads $\hat{\delta s} = (\hat{c}_x^2 - c_a^2)\hat{\delta}$. Here $\hat{c}_x^2 = \hat{\delta P}/\hat{\delta\rho}$ can be obtained by setting $\delta\phi = 0$ in Eqs. (124) and (125), that is

$$\hat{c}_x^2 = c_{s1}^2 = \frac{P_X}{P_X + 2XP_{XX}}, \tag{134}$$

where c_{s1}^2 is given in Eq. (118). Using the property that the entropy perturbation (133) is gauge invariant, the pressure perturbation can be generally expressed as

[22]It is convenient to notice the following relation:

$$2M_{\rm pl}K_{12}\dot{\zeta} + 2K_{22}\dot{\delta\phi} - B_{22}\delta\phi = (\delta\rho + P_\phi\delta\phi)M_{\rm pl}^2/\dot{\phi}.$$

$$\delta P = c_{s1}^2 \delta\rho - 3H(c_{s1}^2 - c_a^2)\delta q = c_{s1}^2 \hat{\delta\rho} + 3Hc_a^2 \delta q. \quad (135)$$

Using the quantities $\hat{\delta}$, v, c_{s1}^2, c_a^2, and w, the perturbation Eqs. (128) and (129) in Fourier space read

$$\dot{\hat{\delta}} + 3H(c_{s1}^2 - w)\hat{\delta} + \left[9H^2(c_a^2 - w) + 3\dot{H} - \frac{k^2}{a^2}\right]v + 3H\dot{v} + (1+w)\left(3\dot{\zeta} + \frac{k^2}{a^2}\psi\right) = 0, \quad (136)$$

$$\dot{v} + 3H(c_a^2 - w)v + (1+w)\delta N + c_{s1}^2 \hat{\delta} = 0. \quad (137)$$

D. Effective gravitational couplings for subhorizon perturbations

For perturbations related to large-scale structures, we are interested in the subhorizon modes with $k^2/a^2 \gg \{H^2, |\dot{H}|\}$. Let us consider cold dark matter obeying the conditions $|w| \ll 1$ and $|\dot{w}/H| \ll 1$. The k-essence dark-matter model with the Lagrangian $P(X) = F_0 + F_2(X - X_0)^2$ [31] can satisfy these conditions in the early matter era. Taking the time derivative of Eq. (136) and using Eq. (137), the matter perturbation on subhorizon scales approximately obeys the following equation:

$$\ddot{\hat{\delta}} + 2H\dot{\hat{\delta}} + c_{s1}^2 \frac{k^2}{a^2}\hat{\delta} + \frac{k^2}{a^2}\Psi \simeq 0, \quad (138)$$

where $\Psi \equiv \delta N + \dot{\psi}$ is the gauge-invariant gravitational potential [48]. For the theories with $c_{s1}^2 > 0$, the gravitational growth of $\hat{\delta}$ is prevented by the pressure perturbation.

Substituting the relation (117) into Eq. (91) and using the fact that $g_4 = 0$ for the theories we are studying now, it follows that

$$\Psi = -\left(\frac{\dot{L}_\mathcal{S}}{L_\mathcal{S}} + H\right)\psi - \frac{\mathcal{E}}{L_\mathcal{S}}\zeta. \quad (139)$$

Since the first two terms of Eq. (138) are at most of the order of $H^2\hat{\delta}$, the gravitational potential Ψ can be estimated as $\Psi \sim (aH/k)^2\hat{\delta}$. For the modes deep inside the Hubble radius ($k \gg aH$), it follows that $|\Psi| \ll |\hat{\delta}|$. In the following we use the quasistatic approximation on subhorizon scales, under which the contributions of metric perturbations in field equations are neglected unless they are multiplied by the factor k^2/a^2.

From Eq. (128) the order of the momentum perturbation can be estimated as $H\delta q \simeq (aH/k)^2\delta\rho$ so that $|H\delta q| \ll |\delta\rho|$ and $\hat{\delta\rho} \simeq \delta\rho$ for $k \gg aH$. From Eq. (126) the momentum perturbation δq is proportional to $\delta\phi$, whereas the density perturbation $\delta\rho$ in Eq. (124) involves both $\dot{\delta\phi}$ and $\delta\phi$. Under the subhorizon approximation the dominant contribution to $\delta\rho$ comes from the $\dot{\delta\phi}$-dependent terms. From Eq. (86) the metric perturbation δN inside the term δX of Eq. (20) does not contain terms involving $\dot{\delta\phi}$. Hence the gauge-invariant density perturbation is approximately given by

$$\hat{\delta\rho} \simeq \delta\rho \simeq -2\dot{\phi}(P_X + 2XP_{XX})\dot{\delta\phi} = 2\dot{\phi}g_1\dot{\delta\phi}. \quad (140)$$

Under the quasistatic approximation on subhorizon scales, Eq. (85) reads

$$\mathcal{W}\frac{k^2}{a^2}\psi + 4L_\mathcal{S}\frac{k^2}{a^2}\zeta - \rho\hat{\delta} \simeq 0. \quad (141)$$

Neglecting the variation of ζ in Eqs. (87) and (88), it follows that

$$2M_{\rm pl}G_{11}\frac{k^2}{a^2}\zeta + \left(2G_{12}\frac{k^2}{a^2} + 2M_{12} - \dot{B}_{21} - 3HB_{21}\right)\delta\phi + (B_{12} - B_{21} + 2\dot{K}_{12} + 6HK_{12})\dot{\delta\phi} + 2K_{12}\ddot{\delta\phi} \simeq 0, \quad (142)$$

$$2M_{\rm pl}G_{12}\frac{k^2}{a^2}\zeta + \left(2G_{22}\frac{k^2}{a^2} + 2M_{22} - \dot{B}_{22} - 3HB_{22}\right)\delta\phi + 2(\dot{K}_{22} + 3HK_{22})\dot{\delta\phi} + 2K_{22}\ddot{\delta\phi} \simeq 0. \quad (143)$$

Instead of the curvature perturbation ζ, we can also employ the gauge-invariant Mukhanov-Sasaki variable $\delta\chi_\zeta \equiv \delta\chi - \dot{\chi}\zeta/H$ [59] ($\delta\chi_\zeta = -\dot{\chi}\zeta/H$ in unitary gauge). If we rewrite Eqs. (87) and (88) in terms of $\delta\chi_\zeta$, there appears a term associated with the mass m_χ of the dark energy field χ. By neglecting the time derivatives of ζ in Eqs. (142) and (143), we also drop the contribution of such a mass term. This approximation is valid for a light scalar field with m_χ much smaller than the physical wave number k/a of interest. For the models in which the dark energy field becomes heavy in the past, we need to take into account such a mass term (along the line of Refs. [40,42]). Since such a heavy field merely recovers the general relativistic behavior in the past, our treatment of a nearly massless dark energy field is sufficient to understand the modification of gravity at the late cosmological epoch.

From Eqs. (142) and (143) we can express ζ in terms of $\dot{\delta\phi}$ and $\delta\phi$, as

$$\frac{k^2}{a^2}\zeta \simeq \frac{(B_{12} - B_{21} + 2\dot{K}_{12})K_{22} - 2K_{12}\dot{K}_{22}}{2M_{\rm pl}(G_{12}K_{12} - G_{11}K_{22})}\dot{\delta\phi} + \frac{(\dot{B}_{22} + 3HB_{22} - 2M_{22})K_{12} - (\dot{B}_{21} + 3HB_{21} - 2M_{12})K_{22}}{2M_{\rm pl}(G_{12}K_{12} - G_{11}K_{22})}\delta\phi, \quad (144)$$

where we used the property $G_{22}K_{12} = G_{12}K_{22}$. On using Eqs. (126) and (140), it follows that

$$\frac{k^2}{a^2}\zeta \simeq \frac{(B_{12}-B_{21}+2\dot{K}_{12})K_{22}-2K_{12}\dot{K}_{22}}{4g_1\dot{\phi}M_{\rm pl}(G_{12}K_{12}-G_{11}K_{22})}\delta\rho + \frac{(\dot{B}_{22}+3HB_{22}-2M_{22})K_{12}-(\dot{B}_{21}+3HB_{21}-2M_{12})K_{22}}{4P_X\dot{\phi}M_{\rm pl}(G_{12}K_{12}-G_{11}K_{22})}\delta q. \tag{145}$$

The second term on the rhs of Eq. (145) is much smaller than the first term for the modes deep inside the Hubble radius, and hence,

$$\frac{k^2}{a^2}\zeta \simeq \frac{(B_{12}-B_{21}+2\dot{K}_{12})K_{22}-2K_{12}\dot{K}_{22}}{4g_1\dot{\phi}M_{\rm pl}(G_{12}K_{12}-G_{11}K_{22})}\rho\hat{\delta}. \tag{146}$$

Substituting Eq. (146) into Eq. (141), we have

$$\frac{k^2}{a^2}\psi \simeq \frac{g_1\dot{\phi}M_{\rm pl}(G_{12}K_{12}-G_{11}K_{22})+2L_{\mathcal{S}}K_{12}\dot{K}_{22}-L_{\mathcal{S}}K_{22}(B_{12}-B_{21}+2\dot{K}_{12})}{g_1\dot{\phi}\mathcal{W}M_{\rm pl}(G_{12}K_{12}-G_{11}K_{22})}\rho\hat{\delta}. \tag{147}$$

Finally, plugging the relations (146) and (147) into Eq. (139), we obtain

$$\frac{k^2}{a^2}\Psi \simeq -\left[\frac{\dot{L}_{\mathcal{S}}+HL_{\mathcal{S}}}{\mathcal{W}L_{\mathcal{S}}}+\frac{\{4L_{\mathcal{S}}(\dot{L}_{\mathcal{S}}+HL_{\mathcal{S}})-\mathcal{E}\mathcal{W}\}\{2K_{12}\dot{K}_{22}-K_{22}(B_{12}-B_{21}+2\dot{K}_{12})\}}{4g_1\dot{\phi}L_{\mathcal{S}}\mathcal{W}M_{\rm pl}(G_{12}K_{12}-G_{11}K_{22})}\right]\rho\hat{\delta}. \tag{148}$$

The rhs of Eq. (148) works as a driving force for the growth of the density perturbation $\hat{\delta}$ in Eq. (138).

Let us first consider the theory described by the Lagrangian (34), i.e., two minimally coupled scalar fields in the framework of general relativity (GR). Since this Lagrangian reduces to (35), we have that $L_{\mathcal{S}}=\mathcal{E}=M_{\rm pl}^2/2$ and $\mathcal{W}=2HM_{\rm pl}^2$. Then the second term in the square bracket of Eq. (148) vanishes so that

$$\frac{k^2}{a^2}\Psi \simeq -\frac{1}{2M_{\rm pl}^2}\rho\hat{\delta} = -4\pi G\rho\hat{\delta}, \tag{149}$$

where $G=1/(8\pi M_{\rm pl}^2)$ is the Newton's gravitational constant. For the models with $c_{s1}^2 \ll 1$, the matter perturbation grows as $\hat{\delta} \propto a$ during the deep-matter era.

In modified gravitational theories, the second term in the square bracket of Eq. (148) does not generally vanish so that the Poisson equation is subject to change. We note that the result (148) has been derived for a scalar-field dark matter, whereas in a number of past works [38,40,42,44], the modified Poisson equation was obtained for a pressureless perfect fluid. If we consider a purely kinetic scalar Lagrangian $P(X)$ [31], then $c_{s1}^2=P_X/(P_X+2XP_{XX})$ is equivalent to the adiabatic sound speed square c_a^2. In this case the scalar field ϕ behaves as a perfect fluid [43] with the limit $c_{s1}^2 \to 0$ for cold dark matter.

In order to confirm that the result (148) can reproduce the effective gravitational coupling derived for some modified gravity models, let us study the model described by the Lagrangian

$$L=\frac{1}{2}M_{\rm pl}\chi R-\frac{M_{\rm pl}\omega_{\rm BD}}{2\chi}Y+P(X). \tag{150}$$

This is the Brans-Dicke (BD) theory [60] (with the BD parameter $\omega_{\rm BD}$) in the presence of a dark energy field χ coupled to R and a purely kinetic dark matter. From Eq. (107), the Lagrangian (150) can be expressed as

$$L=\frac{1}{2}M_{\rm pl}\chi(\mathcal{R}+\mathcal{S}-K^2)-M_{\rm pl}\sqrt{-Y(N)}K - \frac{M_{\rm pl}\omega_{\rm BD}}{2\chi}Y(N)+P(X). \tag{151}$$

From the background equations of motion (31)–(33) we obtain

$$\ddot{\chi}=-2\dot{H}\chi+H\dot{\chi}-\omega_{\rm BD}\dot{\chi}^2/\chi+2P_X\dot{\phi}^2/M_{\rm pl}, \tag{152}$$

$$\ddot{\phi}=3HP_X\dot{\phi}/g_1. \tag{153}$$

The quantities such as $L_{\mathcal{S}}$, $\mathcal{E}$, and $\mathcal{W}$ depend on the field χ, as $L_{\mathcal{S}}=\mathcal{E}=M_{\rm pl}\chi/2$ and $\mathcal{W}=M_{\rm pl}(\dot{\chi}+2H\chi)$. Evaluating other quantities in Eq. (148), using Eqs. (152) and (153), and taking the limit $P_X/(XP_{XX}) \to 0$ (i.e., $c_{s1}^2 \to 0$), Eq. (148) reduces to

$$\frac{k^2}{a^2}\Psi \simeq -4\pi G_{\rm eff}\rho\hat{\delta}, \qquad G_{\rm eff}=\frac{4+2\omega_{\rm BD}}{3+2\omega_{\rm BD}}\frac{M_{\rm pl}}{\chi}G. \tag{154}$$

The effective gravitational coupling agrees with the one derived for a pressure-less perfect-fluid dark matter [38,40,42,44]. In the limit that $\omega_{\rm BD} \to \infty$ and $\chi \to M_{\rm pl}$, we recover the GR behavior $G_{\rm eff} \to G$. For the general BD parameter, the gravitational coupling differs from G, which modifies the growth rate of $\hat{\delta}$ through Eq. (138).

VI. CONCLUDING REMARKS

The EFT of cosmological perturbations is a powerful tool to deal with a variety of dark energy and modified gravity models in a unified way. The starting Lagrangian depends on the lapse function N and all the possible geometric scalar quantities constructed by the $3+1$ decomposition of the ADM formalism [15]. In this setup there is one scalar degree of freedom χ whose perturbation can be absorbed into the gravitational sector by choosing unitary gauge. The field χ manifests itself in the perturbation equations of motion through the lapse dependence of the kinetic energy $Y = g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$ and also through a possible explicit time dependence.

In this paper we have extended the single-field EFT of dark energy to the case in which another scalar field ϕ is present. In the Lagrangian, we have included the explicit dependences on ϕ and its kinetic energy X, in addition to the scalar quantities of geometric type which arise in the single field case (with origin in the ADM decomposition). Our interest is the application of the multifield EFT of cosmological perturbations to a joint description of dark matter and dark energy. The second field ϕ plays the role of scalar dark matter, whereas the first scalar degree of freedom χ is responsible for the late-time cosmic acceleration. Our formalism can be applied to multifield inflation as well.

In such a two-field system we expanded the action up to second order in the perturbations around the flat FLRW background. Despite the original Lagrangian containing several gravitational variables, their geometrical origin implies that some of the variables in the first-order Lagrangian density are interrelated, leaving only three of them as independent. The first-order Lagrangian density (30) gives rise to the background Eqs. (31)–(33). When the fields are noninteracting, an integrability condition of these equations ensures that each of them obeys a continuity equation—a natural requirement, which however in this case should not be imposed by hand, as it already follows at the level of the action.

We derived the second-order perturbed Lagrangian density (51), which contains the new contributions $\mathcal{Q}_i$ $(i = 1, 2, 3)$ generated by the field ϕ. By employing the Hamiltonian and momentum constraints, we reduce the Lagrangian density $\mathcal{L}_2$ to the simpler form (58). The sufficient conditions to eliminate the spatial derivatives higher than second order are given by Eq. (60), whose result coincides with those derived in Ref. [15]. In the multifield system, however, the Lagrangian $\mathcal{L}_2$ generally contains the term $\mathcal{C}_{14}\partial^2\zeta\delta\dot{\phi}/a^2$, which is the product of temporal and spatial derivatives at combined order higher than two. The sufficient conditions for the absence of this new term are presented in Eq. (62).

We proceeded by investigating such second-order theories satisfying the conditions (60) and (62). The no-ghost conditions for scalar perturbations were obtained as Eqs. (72) and (73). In the small-scale limit we also derived the squares of two scalar propagation speeds, given in Eq. (82), both required to be positive in order to avoid Laplacian-type instabilities. The additional conditions (95) and (97) associated with the absence of tensor ghosts and of Laplacian instabilities further restrict the viable model parameter space.

In Sec. V we applied our results to the Horndeski theory augmented by the scalar field ϕ with the Lagrangian (99). In the absence of the coupling between the two kinetic terms ($L_{NX} = 0$), the no-ghost conditions agree with those derived in earlier works. In this case one of the propagation speeds c_{s1} is associated with dark matter, whereas another speed c_{s2} carries the information on the modification of gravity. We note that c_{s2} is also affected by the presence of the field ϕ, exhibiting properties consistent with the findings of Ref. [36] for a perfect-fluid dark matter.

For the two-field system described by the Lagrangian $P(\phi, X)$ plus the Horndeski Lagrangian, we have also derived the equations of gauge-invariant perturbations of dark matter. Under the quasistatic approximation on subhorizon scales we have obtained the modified Poisson Eq. (148), associated with the growth rate of matter perturbations. This is valid for an imperfect-fluid dark matter described by the k-essence Lagrangian $P(\phi, X)$. Dark matter with a purely kinetic Lagrangian $P(X)$ behaves as a perfect fluid, in which case the effective gravitational coupling in the presence of a Brans-Dicke scalar field χ reduces to the one known in the literature.

Since we have derived the full linear perturbation equations of motion in this general multifield setup, our formalism is useful for constructing realistic scalar-field dark matter and modified gravity models, compatible with observations. We leave the detailed analysis of the evolution of matter perturbations and the confrontation of these models with observational constraints for future work.

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