Six-dimensional standing-wave braneworld with normal matter as source

L. J. S. Sousa*
Instituto Federal de Educação, Ciência e Tecnologia do Ceará (IFCE), Campus Canindé, 62700-000 Canindé, Ceará, Brazil

W. T. Cruz†
Instituto Federal de Educação, Ciência e Tecnologia do Ceará (IFCE), Campus Juazeiro do Norte, 63040-000 Juazeiro do Norte, Ceará, Brazil

C. A. S. Almeida‡
Universidade Federal do Ceará, Departamento de Física, Campus do Pici, C.P. 6030, 60455-760, Fortaleza, Ceará, Brazil
(Received 26 November 2013; published 5 March 2014)

A six-dimensional standing-wave braneworld model has been constructed. It consists of an anisotropic four-brane generated by standing gravitational waves whose source is normal matter. In this model, the compact (on-brane) dimension is assumed to be sufficiently small in order to describe our Universe (hybrid compactification). The bulk geometry is nonstatic, unlike most of the braneworld models in the literature. The principal feature of this model is the fact that the source is not a phantomlike scalar field, as the original standing-wave model that was proposed in five dimensions and its six-dimensional extension recently proposed in the literature. Here, it was obtained a solution in the presence of normal matter what assures that the model is stable. Also, our model is the first standing-wave brane model in the literature that can be applied successfully to the hierarchy problem. Additionally, we have shown that the zero mode for the scalar and fermionic fields is localized around the brane. In particular, for the scalar field we show that it is localized on the brane, regardless of whether the warp factor is decreasing or increasing. This is in contrast to the case of the local stringlike defect, where the scalar field is localized for a decreasing warp factor only.

DOI: 10.1103/PhysRevD.89.064006 PACS numbers: 04.50.-h, 11.10.Kk, 11.27.+d, 12.60.-i

I. INTRODUCTION

The so-called braneworld models assume that our Universe is a membrane, or brane, embedded in a higher-dimensional space-time. The success of this idea between the physicists can be explained basically because these models have brought a solution for some insoluble problems in the Standard Model (SM) physics, such as the hierarchy problem. There are many theories that carry this basic idea, but the main theories in this context are the ones first proposed by Arkani-Hamed, Dimopoulos, and Dvali [1–3] and the so-called Randall-Sundrum (RS) model [4,5].

In these models, it is assumed *a priori* that all the matter fields are restricted to propagate only in the brane. The gravitational field is the only one which is free to propagate in all the bulk. However, some authors have argued that this assumption is not so obvious, and it is necessary to look for alternative theoretical mechanisms of field localization in such models [6,7]. Accordingly, before studying the cosmology of a braneworld model, it is convenient to analyze its capability to localize fields. Therefore, for a braneworld model to be indicated as a potential candidate of our Universe, it is necessary to be able to localize the Standard Model fields.

The Randall-Sundrum model was generalized to six dimensions by several interesting works [6–29]. A high number of the works in six dimensions refer to the scenarios where the brane has cylindrical symmetry, the so-called stringlike braneworlds, which are associated with topological defects. Some of these six-dimensional models are classified as global string [6,8,11], local string [12–14], thick string [16,17,19–22], and supersymmetric cigar universe [23] models. Also, the work proposed here is a generalization of the RS model for six-dimensional (6D) space-time. However, we treat the so-called standing-wave brane model, which will be discussed later.

On the other hand, studies of field localization are very common in the literature in five-dimensional (5D) [30–35] and 6D braneworlds [6,7,12,14,26–28]. In general, we find strategies of localization for all the Standard Model fields, but the way that this localization is possible varies in different works. In some of them, the localization is possible by means of gravitational interactions only [6,7]. In other works, it is necessary to consider the existence of auxiliary fields, like the dilaton [33,34]. As far as we know, there is not in the literature a purely analytical geometry that localizes all the SM fields by means of the gravitational field interaction

*luisjose@fisica.ufc.br
†wilami@fisica.ufc.br
‡carlos@fisica.ufc.br

only. Hence, to look for a model that must present both features namely, be analytical and able to localize all the SM fields, is an appropriate reason to study the localization of fields in different braneworld models.

The search for such a model has motivated the appearance of some braneworld scenarios with nonstandard transverse manifold. Randjbar-Daemi and Shaposhnikov obtained trapped massless gravitational modes and chiral fermions as well in a model that they called a Ricci-flat space or a homogeneous space [36]. Kehagias proposed an interesting model that drains the vacuum energy, through a conical tear-drop-like space, which forms a transverse space with a conical singularity. In this way, it was possible to explain the small value of the cosmological constant [37]. Another nontrivial geometry was proposed by Gogberashvili *et al*. They have found three generations for fermions on a three-brane whose transverse space has the shape of an apple [38]. It is still possible to cite other examples of space used like the torus [39], a space-time geometry with a football shape [40], and smooth versions of the conifold, classified as resolved conifold [41] and deformed conifold [42–44].

The standing-wave braneworld was first proposed in five dimensions by Gogberashvili and Singleton [45]. This is a completely anisotropic braneworld model whose source is a phantomlike scalar (a scalar field with a wrong sign in front of the kinetic term in the Lagrangian). To avoid the problem with instability, normally presented in theory with a phantom scalar, the model is embedded in a 5D Weyl geometry in such a way that the phantomlike scalar may be associated with the Weyl scalar [30,31,46], which is stable. About the Weyl scalar, we also should point out its presence in other braneworld scenarios, like pure gravity, which is an extension of the RS model. In the context of field localization in the standing-wave approach, it was possible to localize several fields in five dimensions, even though the right-handed fermions were not localized in either increasing or decreasing warp geometry [30,31]. It is worthwhile to mention that the models generated by phantomlike scalars are relevant phenomenologically since this exotic source is useful in different scenarios, like cosmology [25], where the phantom scalar is used to explain dark energy theories and the accelerated expansion of the Universe [47]. An extension for six dimensions of the standing-wave 5D model with a phantomlike scalar was first proposed by the authors of [48]. Additionally, the study of massive modes was not addressed in this model in five dimensions or even in its six-dimensional version.

In this paper, we do not specify *a priori* the source or the *stuff* from which the brane is formed. We consider a general matter source and look for a standing-wave solution. In contrast to the work of Gogberashvili and collaborators in the 5D model [30], in which the source is a phantomlike scalar field, here we have obtained standing gravitational wave solutions of Einstein equations in the presence of normal matter (we are using the classification for different types of matter given by M. Visser [49]). Since it is done by normal matter, the model constructed here is stable. Our model with normal matter as a source is the first 6D one, but quite recently Midodashvili *et al.* [50] constructed a 5D standing-wave braneworld model with a real field as a source.

The model built here consists of a 6D braneworld with an anisotropic four-brane, where the small, compact dimension belongs to the brane. The bulk is completely anisotropic, except for some points called the anti–de Sitter (AdS) islands [30,31]. The dynamics, as in the case of the works of Gogberashvili and collaborators and their extensions, represent a special feature in the sense that both metric and source are time dependent. We present two types of solutions: one with an isotropic cosmological constant where the source despite the fact that all its components are positive do not satisfy the dominant energy condition (DEC). This source may be classified as a not normal matter [49]. In the other case, we make use of a recently proposed approach that suggests an extension for the Randall-Sundrum model to higher dimensions in the presence of an anisotropic cosmological constant [51]. In this case, we find solutions in the presence of normal matter.

We have obtained an analytical solution for the warp factor, which corresponds to a thin brane, for both decreasing and increasing warp factor. The bulk is smooth everywhere and converges asymptotically to an AdS_6 manifold. We have considered a minimally coupled scalar field, and we have shown that it is localized in this model. Here, we have obtained results that are more general that those encountered for the stringlike defect and the 5D and 6D versions of the standing-wave approach. Indeed, here the scalar field is trapped for both decreasing and increasing warp factor whereas in the stringlike is a localized mode for a decreasing warp factor only. In addition, in 5D and 6D versions of the standing-wave models the scalar field is localized for increasing warp factor only.

Furthermore, our six-dimensional standing-wave braneworld with physical source is an interesting scenario in order localize fermions fields. Indeed, we show that right-handed fermions can be localized in this brane.

We organize this work as follows: in Sec. II the model is described and the Einstein equations are solved in order to obtain the general expressions for the source and the function that characterizes the anisotropy. In Sec. III we have found the standing-wave solutions, and we have discussed its main features. We still show that the energy-momentum components are all positive. In the case of an anisotropic cosmological constant, they obey all the energy conditions that characterize a normal matter source. The localization of the zero mode of scalar and fermionic fields has been done in Secs. IV and V, respectively. Some remarks and conclusions are outlined in Sec. VI.

II. THE MODEL

Our intent is to derive a standing-wave solution of the Einstein equations by considering normal matter as source. So, we consider the standard Einstein-Hilbert action in six-dimensional space-time added by a matter field action that may be time dependent, namely,

$$S = \frac{1}{2\kappa_6^2}\int d^6x\sqrt{-g}[(R - 2\Lambda_6) + L_m], \tag{1}$$

where κ_6 is the six-dimensional gravitational constant, Λ_6 is the bulk cosmological constant, and L_m is any matter field Lagrangian.

From the action (1) we derive the Einstein equations

$$R_{MN} - \frac{1}{2}g_{MN}R = -\Lambda g_{MN} + \kappa_6^2 T_{MN}, \tag{2}$$

where M, N, … denote D-dimensional space-time indices and the T_{MN} is the energy-momentum tensor defined as

$$T_{MN} = -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{MN}}\int d^6x\sqrt{-g}L_m. \tag{3}$$

The general ansatz for the metric considered in this work is given as follows

$$ds^2 = e^A(-dt^2 + e^u dx^2 + e^u dy^2 + e^{-3u}dz^2) + dr^2 + R_0^2 e^{B+u}d\theta^2, \tag{4}$$

where the functions $A(r)$ and $B(r)$ depend only on r and the function u depends on r and t variables. For this metric ansatz, (4), the Einstein equations (2) may be rewritten as

$$G_{xx} = G_{yy} = \left(\frac{1}{4}e^{A+u}\right)(6A'^2 + B'^2 + 3A'B' + 6A'' + 2B'' + 6(u'^2 - e^{-A}\dot{u}^2) + 2e^{-A}\ddot{u} - 5A'u' - 2u'') = \kappa_6^2 T_{xx} - e^{A+u}\Lambda_6, \tag{5}$$

$$G_{zz} = \left(\frac{1}{4}e^{A-3u}\right)(6A'^2 + B'^2 + 3A'B' + 6A'' + 2B'' + 6(u'^2 - e^{-A}\dot{u}^2) - 6e^{-A}\ddot{u} + (11A' + 4B')u' + 6u'') = \kappa_6^2 T_{zz} - e^{A-3u}\Lambda_6, \tag{6}$$

$$G_{tt} = \left(\frac{1}{4}e^{A}\right)(-6A'^2 - B'^2 - 3A'B' - 6A'' - 2B'' - 6(u'^2 + e^{-A}\dot{u}^2) + (A' - B')u') = \kappa_6^2 T_{tt} + e^{A}\Lambda_6, \tag{7}$$

$$G_{rt} = \frac{1}{4}\dot{u}(A' - B' - 12u') = \kappa_6^2 T_{rt}, \tag{8}$$

$$G_{rr} = \left(\frac{1}{4}\right)(6A'^2 + 4A'B' - 6(u'^2 + e^{-A}\dot{u}^2) + (A' - B')u') = \kappa_6^2 T_{rr} - \Lambda_6, \tag{9}$$

and

$$G_{\theta\theta} = \left(\frac{1}{4}R_0^2 e^{B+u}\right)(10A'^2 + 8A'' + 6(u'^2 - e^{-A}\dot{u}^2) + 2e^{-A}\ddot{u} - 5A'u' - 2u'') = \kappa_6^2 T_{\theta\theta} - R_0^2 e^{B+u}\Lambda_6. \tag{10}$$

The case $A = B = 2ar$ was treated in a previous work [48]. As a matter of fact, in this case, it is possible to find a standing gravitational wave solution in the presence of a phantomlike scalar field, similar to the one first found in five dimensions. Another 6D standing-wave braneworld has been recently proposed [50]. However, in that case the metric is quite different from the one considered here as given by Eq. (4). In this last model the spatial metric components x, y, z are all multiplied by the same factor e^{2ar+u}, while the compact extra dimension is multiplied by e^{2ar-3u}. The solution, in this case, is similar to the one found in Ref. [48] and the source is still a phantomlike scalar field. The two models are still similar in the results of field localization.

As was mentioned above, the two 6D standing-wave braneworld models recently proposed in the literature have shown interesting results in field localization, but both are generated by exotic matter, a phantomlike scalar. Here, we are interested in studying the possibility to have a standing-wave braneworld generated by normal matter whereas maintains the efficiency in localizing fields. So we will consider the case where $A \neq B$, $A(r) = 2cr$, and $B(r) = c_1 r$. In this case, the set of equations (5)–(16) will be simplified to

$$\left(\frac{1}{4}\right)(24c^2 + c_1^2 + 6cc_1 + 6(u'^2 - e^{-2cr}\dot{u}^2) + 2e^{-2cr}\ddot{u} - 10cu' - 2u'') = \kappa_6^2 T^x_x - \Lambda_6, \tag{11}$$

$$\left(\frac{1}{4}\right)(24c^2 + c_1^2 + 6cc_1 + 6(u'^2 - e^{-2cr}\dot{u}^2) - 6e^{-2cr}\ddot{u} + (22c + 4c_1)u' + 6u'') = \kappa_6^2 T^z_z - \Lambda_6, \tag{12}$$

$$-\left(\frac{1}{4}\right)(-24c^2 - c_1^2 - 6cc_1 - 6(u'^2 + e^{-2cr}\dot{u}^2) + (2c - c_1)u') = \kappa_6^2 T^t_t - \Lambda_6, \tag{13}$$

$$\frac{1}{4}\dot{u}(2c - c_1 - 12u') = T_{rt}, \tag{14}$$

$$\left(\frac{1}{4}\right)(24c^2 + 8cc_1 - 6(u'^2 + e^{-2cr}\dot{u}^2) + (2c - c_1)u') = \kappa_6^2 T^r_r - \Lambda_6, \tag{15}$$

and

$$\left(\frac{1}{4}\right)(40c^2 + 6(u'^2 - e^{-2cr}\dot{u}^2) + 2e^{-2cr}\ddot{u} - 10cu' - 2u'') = \kappa_6^2 T^\theta_\theta - \Lambda_6. \tag{16}$$

In order to have a standing-wave solution, we will choose

$$-e^{-2cr}\ddot{u} + \frac{1}{6}(22c + 4c_1)u' + u'' = 0. \tag{17}$$

In this case, the energy-momentum components have to satisfy the relations

$$\kappa_6^2 T^x_x = \kappa_6^2 T^y_y = \frac{1}{4}\left(6(u'^2 - e^{-2cr}\dot{u}^2) - \frac{4}{3}(2c - c_1)u' + 6cc_1\right), \tag{18}$$

$$\kappa_6^2 T^z_z = \frac{1}{4}(6(u'^2 - e^{-2cr}\dot{u}^2)) + 6cc_1), \tag{19}$$

$$\kappa_6^2 T^t_t = -\frac{1}{4}(-6(u'^2 + e^{-2cr}\dot{u}^2) + (2c - c_1)u' - 6cc_1), \tag{20}$$

$$\kappa_6^2 T^r_r = \frac{1}{4}(-6(u'^2 + e^{-2cr}\dot{u}^2) + (2c - c_1)u' - c_1^2 + 8c_1 c), \tag{21}$$

and

$$\kappa_6^2 T^\theta_\theta = \frac{1}{4}\left(6(u'^2 - e^{-2cr}\dot{u}^2) - \frac{4}{3}(2c - c_1)u' + 16c^2 - c_1^2\right). \tag{22}$$

The component $\kappa_6^2 T_{rt}$ must be equal to G_{rt}. This energy-momentum component, as will be seen, does not influence the general results here, since we will consider only time-averaged features of the above quantities. This will be better explained later in Sec. IV.

Finally, the bulk cosmological constant will assume the relation

$$\Lambda_6 = -\frac{1}{4}(24c^2 + c_1^2). \tag{23}$$

This will imply $\Lambda_6 < 0$, which allows us to obtain relations between c, c_1, and $|\Lambda_6|$, namely,

$$c_1 = \pm\sqrt{4|\Lambda_6| - 24c^2}, \tag{24}$$

where

$$c^2 \leq \frac{1}{6}|\Lambda_6|. \tag{25}$$

It may be useful to highlight that for the configurations $A(r) = 2cr$ and $B(r) = c_1 r$, the metric (4) will assume the simpler form,

$$ds^2 = e^{2cr}(dt^2 - e^u dx^2 - e^u dy^2 - e^{-3u}dz^2) - dr^2 - R_0^2 e^{c_1 r + u} d\theta^2. \tag{26}$$

Here c and $c_1 \in R$ are real constants. The range of the variables r and θ are $0 \leq r < \infty$ and $0 \leq \theta < 2\pi$, respectively. The function $u = u(r,t)$ depends only on the variables r and t. The compact dimension θ, different from the stringlike defect model, lives on the brane; i.e., θ is a brane coordinate for $r = 0$. This particular feature is called hybrid compactification [52].

The metric ansatz (26) is a combination of the 6D warped braneworld model through the e^{2cr} and $e^{c_1 r}$ terms (particularly this is similar to the global stringlike defect) [6–8,25] plus an anisotropic (r, t)-dependent warping of the brane coordinates, x, y, and z, through the terms $e^{u(t,r)}$ and $e^{-3u(t,r)}$. This may be seen as a six-dimensional generalization of the 5D standing-wave braneworld model [29–31,45,46] and a 6D generalization of the six-dimensional standing-wave braneworld [48,50]. Still, we can see our model as an extension of the global stringlike defect [6,7]. Therefore, for $u = 0$, the metric (26) is the same of the thin global stringlike defects [6,7]. As will be seen, there is more than one point where $u = 0$. In these points the geometry is the so-called AdS island.

In addition, we can consider the exponential of the function $u(r,t)$ as a correction of the stringlike models, resulting in an anisotropic, time-dependent braneworld.

III. STANDING WAVE SOLUTION

In order to obtain a standing-wave solution, we rewrite here the differential equation for the $u(r,t)$ function (17) as

$$e^{-2cr}\ddot{u}(r,t) - au'(r,t) - u''(r,t) = 0, \tag{27}$$

where prime and dots mean differentiation with respect to r and t, respectively, and $a = \frac{1}{6}(22c + 4c_1)$. In order to solve Eq. (27), we proceed as in Ref. [45] by choosing $u(r,t) = \sin(\omega t)\rho(r)$. The general solution to the equation for the variable $\rho(r)$ is given by

$$\rho(r) = D_1 e^{-\frac{a}{2}r} J_{-\frac{a}{2c}}\left(\frac{\omega}{c}e^{-cr}\right) + D_2 e^{-\frac{a}{2}r} J_{\frac{a}{2c}}\left(\frac{\omega}{c}e^{-cr}\right), \tag{28}$$

where $D_1 = C_1(\omega/2c)^{a/2c}\Gamma(1 - a/2c)$ and $D_2 = C_2(\omega/2c)^{a/2c}\Gamma(1 + a/2c)$. C_1 and C_2 are integration constants. $J_{-\frac{a}{2c}}$ and $J_{\frac{a}{2c}}$ are the first types of Bessel functions of orders $-\frac{a}{2c}$ and $\frac{a}{2c}$, respectively, and Γ represents the Gamma function. Now that we found the solution (28), we have the so-called standing-wave solution, which generalizes the 5D work [45] and the 6D works [48,50]. Depending on the values of c and a, one can obtain solutions similar to that in six dimensions. If one has $a = 5c$ and $D_1 = 0$, the solution will depend on the Bessel function $J_{\frac{5}{2}}$, which is the case in the works in six dimensions. So these present solutions are more general then those obtained in the works cited above.

Some features of the function $u(r,t)$ can be derived from the above solution. The first one is the fact that both functions $J_{-\frac{a}{2c}}$ and $J_{\frac{a}{2c}}$ are regular at the origin and at infinity ($r \to \infty$), given the possibility to maintain the general solution (28). Depending on the relation between ω, c and a the functions $J_{-\frac{a}{2c}}$ and $J_{\frac{a}{2c}}$ converge for both $c > 0$ or $c < 0$ enabling solutions with decreasing and increasing warp factor. Furthermore, we require that the function u is zero on the brane, i.e., at $r = 0$ [45]. This assumption may be expressed by

$$\frac{\omega}{c} = X_{\pm\frac{a}{2c},n}, \tag{29}$$

where $X_{\pm\frac{a}{2c},n}$ represents the nth zero of $J_{-\frac{a}{2c}}$ or $J_{\frac{a}{2c}}$, depending on whether C_1 or C_2 is equal to zero in (28). The boundary condition (29) quantizes the ω frequency.

By this consideration the u function will assume the value zero in some specific r values, namely, r_m. For these r_m values our model may be identified with other 6D braneworld models [6–8,12–14,23,25], as one can see in the metric (26). For $c > 0$ the convergence of the function (28) for $C_1 = 0$ or $C_2 = 0$ will depend essentially on the value of the ratio ω/c. The quantity of zeros, or AdS islands, will depend on the value of c and mainly on the value of this ratio. For the case discussed here, we have a finite number of zeros. For $c < 0$ (with either $C_1 = 0$ or $C_2 = 0$), the solution will present infinite zeros.

Once we know u, we may obtain the components of the energy-momentum tensor. This will be done for the cases where a and c have the same sign and for the case where they have opposite sign.

A. Case A: The same sign for *a* and *c*

In this case we will choose $a = 4c$, which will imply $c_1 = \frac{c}{2}$. Here we will consider only the time average of the energy-momentum tensor components. This option will be better explained in the section about field localization. In the case $a = 4c$ and $D_1 = 0$, the solution (28) will depends on J_2, so our energy-momentum components will be done in terms of this function.

In the figures below we plot the quantities $\langle T^x_x\rangle = \langle T^y_y\rangle = \langle T^z_z\rangle$, $\langle T^t_t\rangle$, $\langle T^r_r\rangle$, and $\langle T^\theta_\theta\rangle$ for $D_2 = \kappa_6 = c = 1$; $\omega = 5.13$. In Fig. 1 the dot-dashed line represents $\langle T^x_x\rangle = \langle T^y_y\rangle = \langle T^z_z\rangle$, the dotted one represents $\langle T^r_r\rangle$, the dashed line represents $\langle T^\theta_\theta\rangle$, and finally the filled line

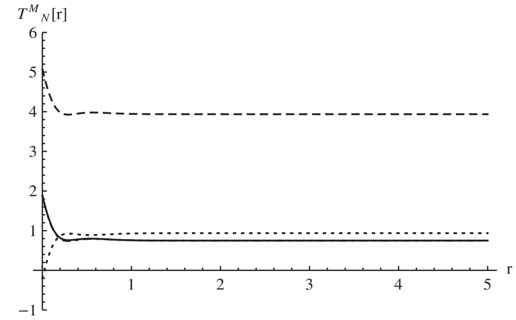

FIG. 1. $\langle T^M_N\rangle$ profile.

represents the energy density $\langle T^t_t\rangle$. As one can see, all these quantities are positive (part of T^r_r is negative but $|\langle T^r_r\rangle| < |\langle T^r_r\rangle|$), which is an advantage when one compares it with the other works in this context [45,48,50]. But it is not possible to say that this is a normal matter once the DEC is violated. However, it is not an exotic source once the null (NEC), strong (SEC), and weak (WEC) energy conditions are satisfied. By following the matter classification given in [49], this is a "not normal matter."

But we are interested in a solution generated by normal matter. In order to have a normal matter solution it is necessary that $\rho \geq p$. In order to treat this unique possibility, we have to consider an anisotropic cosmological constant. As a matter of fact, recently, a higher-dimensional Randall-Sundrum toy model was proposed by Archer and Huber [51], which contains a bulk with anisotropic cosmological constant given by

$$\Lambda = \begin{pmatrix} \Lambda\eta_{\mu\nu} & & \\ & \Lambda_5 & \\ & & \Lambda_6 \end{pmatrix},$$

where $\eta_{\mu\nu}$ is the metric of the brane.

Following this procedure it is possible to find our solution in the presence of normal matter. For an anisotropic cosmological constant where its brane part is given by $\Lambda = -\frac{1}{4}(c_1^2 + 6cc_1)$, $\Lambda_5 = -\frac{1}{4}(8cc_1)$, and $\Lambda_6 = -\frac{1}{4}(16c^2)$, the components of the energy-momentum tensor (18)–(22) will assume the form

$$\kappa_6^2\langle T^x_x\rangle = \kappa_6^2\langle T^y_y\rangle = \kappa_6^2\langle T^z_z\rangle = \kappa_6^2\langle T^\theta_\theta\rangle = \frac{1}{4}(6(u'^2 - e^{-2cr}\dot{u}^2) + 24c^2), \tag{30}$$

$$\kappa_6^2 T^t_t = -\frac{1}{4}(-6(u'^2 + e^{-2cr}\dot{u}^2) - 24c^2), \tag{31}$$

and

$$\kappa_6^2\langle T^r_r\rangle = \frac{1}{4}(-6(u'^2 + e^{-2cr}\dot{u}^2) + 24c^2). \tag{32}$$

We plot these quantities in Fig. 2 as in Fig. 1, with the same values for the constants. The dotted line represents the

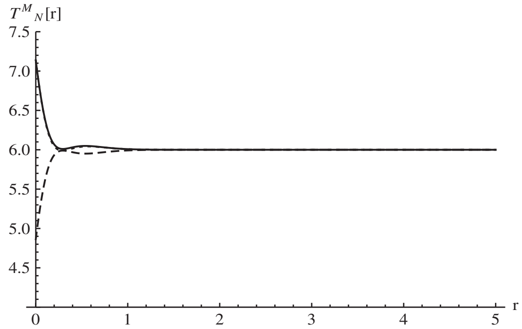

FIG. 2. $\langle T^M_N \rangle$ profile.

spatial components, except the r component which is represented by the shaded line. The filled line represents the temporal component of the energy-momentum tensor. As one can see all these quantities are positive and all the energy conditions (particularly DEC) are satisfied. This is sufficient to assure that our source is a normal matter and that our model is stable. Of course it is possible to choose the value of the cosmological constant in a different way and still keep the normal matter solution.

B. Case B: *a* and *c* have opposite signs

For this case we will consider $a = -4c$ which will give $c_1 = -\frac{23}{2}c$. As in the other case, if the cosmological constant is isotropic, it is possible to find a solution with all energy-momentum tensor components positive, but it would not be possible to obey the dominant energy condition, as in the case above. In fact, once we choose $\Lambda_6 = -\frac{1}{4}(24c^2 + 6cc_1)$, which is positive for $a = -4c$ (meaning that the bulk is asymptotically de Sitter), then we obtain not normal matter as in case A above. But the principal interest consists in a source which corresponds to normal matter. So we will once again look for a solution with an anisotropic cosmological constant. There are several ways to choose the energy-momentum components and cosmological constant in order to have a solution in the presence of normal matter. Here we assume $\Lambda = -\frac{1}{4}(24c^2 + 6cc_1)$, $\Lambda_5 = -\frac{1}{4}(24c^2 + 8cc_1 - c_1^2)$, and $\Lambda_\theta = -\frac{1}{4}(40c^2 - c_1^2)$. Since we know the relation between c and c_1, it is easy to see that the components of the anisotropic cosmological constant are all positive. The time-averaged components of the energy-momentum tensor are

$$\kappa_6^2\langle T^x_x\rangle = \kappa_6^2\langle T^y_y\rangle = \kappa_6^2\langle T^z_z\rangle = \kappa_6^2\langle T^\theta_\theta\rangle = \frac{1}{4}(6(u'^2 - e^{-2cr}\dot{u}^2) + c_1^2), \tag{33}$$

$$\kappa_6^2 T^t_t = -\frac{1}{4}(-6(u'^2 + e^{-2cr}\dot{u}^2) - c_1^2), \tag{34}$$

and

$$\kappa_6^2\langle T^r_r\rangle = \frac{1}{4}(-6(u'^2 + e^{-2cr}\dot{u}^2) + c_1^2). \tag{35}$$

For $D_2 = 0$ and $a = -4c$ in (28), we plot the time-averaged components of the energy-momentum tensor (34)–(35) in Fig. 3. As in Fig. 2 the filled line represents the energy density, the dotted one gives $\kappa_6^2\langle T^x_x\rangle = \kappa_6^2\langle T^y_y\rangle = \kappa_6^2\langle T^z_z\rangle = \kappa_6^2\langle T^\theta_\theta\rangle$, and the dashed line represents the $\langle T^r_r\rangle$ component. As one can see, all these quantities are positive and $\rho \geq p$, which assures the dominant energy condition. Therefore, we again obtained a standing-wave solution generated by normal matter.

On the other hand, one important feature of the brane-world models is the possibility to solve the hierarchy problem. In other standing-wave braneworld works, this feature was not explored. Here we are interested in showing that it is possible to readdress this solution in this context. The condition to solve the hierarchy problem in this context is that the integral below be convergent, namely,

$$M_4^2 = 2\pi M_6^4 \int_0^\infty dr e^{(2c+\frac{c_1}{2})r}. \tag{36}$$

For $c_1 = c = 2a$, where a is a positive constant, as in the two 6D standing-wave braneworld models cited above, the integral is not convergent and the hierarchy problem is not solvable. The same is valid for the first case presented in this work, where $c_1 = \frac{1}{2}c$ and $c > 0$. But in this second case where $c_1 = -\frac{23}{2}c$ and c is positive, we obtain the hierarchy problem solution. So this is another advantage of the model presented here in relation to the other ones done in this same context.

After we obtain the solutions for the standing-wave braneworld and after we demonstrate that one of our solutions is able to solve the hierarchy problem, we are now interested in the potential of our model to localize the Standard Model (SM) fields.

In the other 6D standing-wave braneworld models [48,50], the localization issues of scalar, vector, and fermion fields were already exhaustively studied. Therefore, since from our model we may obtain these other 6D standing-wave braneworlds, here it is sufficient to assure that the scenario presented here is convenient for

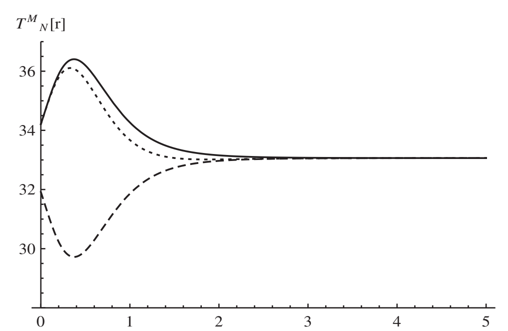

FIG. 3. $\langle T^M_N \rangle$ profile.

localizing Standard Model fields. However, we will briefly present the study of localization for the scalar and fermion fields. This last one is interesting because, in five dimensions, it was not possible to localize the right fermion.

IV. SCALAR FIELD LOCALIZATION

This section is devoted to the study of the localization of the bulk scalar field. We will follow again the proceedings given in Refs. [30,31,45]. Then, considering the general metric (4), we have that $\sqrt{-g} = R_0 e^{2A+B/2}$. Therefore, the equation for the scalar field may be written as

$$\left[\partial_t^2 - e^{-u}(\partial_x^2 + \partial_y^2) - e^{3u}\partial_z^2 - \frac{e^{-u}}{R_0^2}\partial_\theta^2\right]\Phi = e^{-A-B/2}(e^{2A+B/2}\Phi')'. \tag{37}$$

Next, we consider a solution of the form

$$\Phi(x^M) = \Psi(r,t)\chi(x,y)\zeta(z)e^{il\theta}. \tag{38}$$

If one separates the variables r and t by making $\Psi(r,t) = e^{iEt}\bar{\rho}(r)$, the equation for the r variable will assume the form

$$(e^{2A+B/2}\bar{\rho}(r)')' - e^{A+B/2}G(r)\bar{\rho}(r) = 0, \tag{39}$$

where

$$G(r) = (p_x^2 + p_y^2)(e^{-u} - 1) + p_z^2(e^{3u} - 1) + \frac{l^2}{R_0^2}e^{-u}. \tag{40}$$

It will be convenient to write (39) as an analogue nonrelativistic quantum mechanic problem. So we will assume the change of variable $\bar{\rho}(r) = e^{-(A+B/4)}\bar{\Psi}(r)$. With this change we will find

$$\bar{\Psi}''(r) - V(r)\bar{\Psi}(r) = 0, \tag{41}$$

where

$$V(r) = \frac{1}{2}\left(2A'' + \frac{B''}{2}\right) + \frac{1}{4}(2A' + \frac{B'}{2})^2 + e^{-A}G(r). \tag{42}$$

From now on, we will consider $A = 2cr$, $B = c_1 r$, and the simplified metric (26). Next, we will obtain the rt-dependent function Ψ, in order to analyze the localization of the scalar field. In other words it is necessary to solve Eq. (41), but this will be done only for the zero mode scalar and s-wave. This case is obtained when we assume $(l = 0)$ and $E = p_x^2 + p_y^2 + p_z^2$. Additionally, it is considered that $\omega \gg E$, which justifies performing the time-averaging of $V(r)$, reducing the number of independent variables to one, namely r. By applying this simplification we will find the following expansion,

$$\langle e^{bu}\rangle = 1 + \sum_{n=1}^{+\infty}\frac{(b)^{2n}}{2^{2n}(n!)^2}\left[D_1 e^{-\frac{a}{2}r}J_{-\frac{a}{2c}}\left(\frac{\omega}{c}e^{-cr}\right) + D_2 e^{-\frac{a}{2}r}J_{\frac{a}{2c}}\left(\frac{\omega}{c}e^{-cr}\right)\right]^{2n}, \tag{43}$$

or

$$\langle e^{bu}\rangle = I_0(b\rho(r)), \tag{44}$$

where I_0 is the modified Bessel function of the first kind. It is evident from the expression above that our problem is still very complex. As can be seen from the expression (43), the approach to analytically solving Eq. (41) is hard work. Our strategy consists in considering simplification and asymptotic approximations for the above expression.

Let us begin the approximations by making $D_1 = 0$ in (28). Once we do this, the $u(r,t)$ will depend on the first kind of Bessel function $J_{\frac{a}{2c}}$. The expansion (43) will be given by

$$\langle e^{bu}\rangle = 1 + \sum_{n=1}^{+\infty}\frac{(bD_2)^{2n}e^{-anr}}{2^{2n}(n!)^2}\left[J_{\frac{a}{2c}}\left(\frac{\omega}{c}e^{-cr}\right)\right]^{2n}. \tag{45}$$

It is evident that our solution is still very general, since the order of the function J is $a/2c$. This gives us the advantage of choosing the order of the function J, which is more convenient for our interest, since our choosing is in accordance with the relations (24) and (25). If one chooses $a = 4c$, the order of J will be 2, as in case A discussed above. This solution is very similar to the one first proposed in five dimensions for the localization of the scalar field, with the difference being that the authors there considered the second kind of Bessel function, Y_2, rather than J_2 [45].

After applying the above, let us study (41) by considering asymptotic approximation far from and near the brane. For the first case, $r \to +\infty$, the expression $J_{\frac{a}{2c}} = J_2$ goes to zero ($(\omega/c)e^{-cr} \to 0$) and the relation (45) will be approximated as $\langle e^{bu}\rangle \approx 1$. This will result in the following simpler form for Eq. (41), namely,

$$\bar{\Psi}''(r) - \frac{289}{64}c^2\bar{\Psi}(r) = 0, \tag{46}$$

whose solution is $e^{\pm\frac{17}{8}cr}$. We choose $\bar{\Psi} = e^{-\frac{17}{8}cr}$ and $c > 0$, which is convergent for all r values. This solution is similar to the one found in the 5D standing-wave context in the case of scalar field localization for this same asymptotic limit assumed here [31].

The other case to be considered here for asymptotic approximation is the case where $r \to 0$. In this case Eq. (41) may be approximated as

$$\bar{\Psi}''(r) - (8dc^2r^2 - 6dcr + d')\bar{\Psi}(r) = 0. \tag{47}$$

This equation is more general than the equivalent equation considered in five dimensions [31]; there it was considered only as a first-order approximation. The constants d and d' are given, respectively, by

$$d = \left(\frac{D_2}{4}\right)^2 \left(\frac{\omega}{c}\right)^4 (p_x^2 + p_y^2 + 9p_z^2), \tag{48}$$

and

$$d' = \frac{9}{4}c^2 + d. \tag{49}$$

The solution of Eq. (47) is given by

$$\bar{\Psi}(r) = E_1 D_\mu \left(-\frac{3}{2^{7/4}}\sqrt{\frac{\sqrt{d}}{c}} + 2\left(\sqrt{c\sqrt{2d}}\right)r\right) + E_2 D_\nu \left(-i\frac{3}{2^{7/4}}\sqrt{\frac{\sqrt{d}}{c}} + 2i\left(\sqrt{c\sqrt{2d}}\right)r\right), \tag{50}$$

where D is the parabolic cylinder function, and E_1 and E_2 are integration constants. We see that E_2 must be zero in order to have a real solution. The μ, ν indexes are given, respectively, by

$$\mu = -\frac{18c^2 + 16\sqrt{2d}c - d}{32\sqrt{2d}c}, \tag{51}$$

and

$$\nu = \frac{18\sqrt{2}c^2 - 32\sqrt{d}c - \sqrt{2}d}{64\sqrt{d}c}. \tag{52}$$

For $E_2 = 0$ and $\omega/a = 5.13$, which corresponds to the first zero of J_2, and requiring $\mu = 0$, it is possible to show that this solution is convergent for either $c > 0$ or $c < 0$, as can be seen in the figures below. We see that the extra part of the scalar zero-mode wave function $\bar{\rho}(r)$ has a minimum at $r = 0$, increases and then falls off, for the case $c = 1$, as can be seen in Fig. 4. For $c = -1$ the function has a maximum at $r = 0$, and it rapidly falls off as we move away from the brane, as can be seen in Fig. 5. On the other hand, for $r \to \infty$, it assumes the asymptotic form $e^{-(17/8)cr}$ which is in accordance with [31] only for $c > 0$. In general, however, for other relations between a and c, it is possible to have localization for c positive or negative, i.e., for increasing or decreasing warp factor.

The results of this section show that we have the localization of the zero-mode scalar field in the model considered in this work. This is an expected result since the study of localization of the scalar field was performed in simpler 5D and 6D models than the one considered here. It is relevant to stress the fact that the localization here is possible for both increasing or decreasing warp factor, whereas in the thin stringlike brane the localization of the zero-mode scalar field is obtained only for a decreasing warp factor, and in other standing-wave braneworlds, this result was obtained only for the case of an increasing warp factor [6,7,48,50].

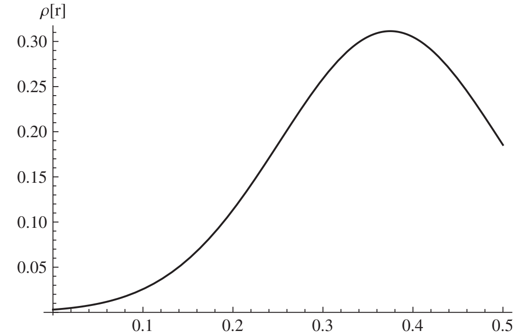

FIG. 4. $\bar{\rho}$ profile. $c = 1$

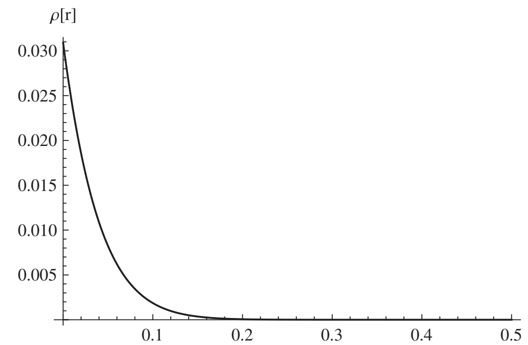

FIG. 5. $\bar{\rho}$ profile. $c = -1$

V. LOCALIZATION OF SPIN 1/2 FERMIONIC ZERO MODE

The study of localization of zero-mode spin 1/2 fermions is interesting in this context since, in the 5D standing-wave braneworld, it was not possible to localize the zero-mode right fermion. However in the six-dimensional models cited above, it was demonstrated that this field is localized. Once the model presented here is more general than that, it is natural that we find the same results here. As a matter of fact, our results, as will be seen in this section, are very similar to the ones found in Refs. [48,50], except that there, the Bessel function considered is $J_{\frac{5}{2}}$, and here we are using J_2. Therefore, we begin with the action for the massless spin 1/2 fermion in six dimensions, which may be written as

$$S = \int d^6x\sqrt{-g}\bar{\Psi}i\Gamma^M D_M \Psi. \tag{53}$$

From this action we derive the respective equation of motion, namely,

$$(\Gamma^\mu D_\mu + \Gamma^r D_r + \Gamma^\theta D_\theta)\Psi(x^M) = 0. \quad (54)$$

In this expression Γ^M represents the curved gamma matrices which relate to the flat ones as

$$\Gamma^M = h^M_{\bar{M}}\gamma^{\bar{M}}, \quad (55)$$

where the vielbein $h^M_{\bar{M}}$ is defined as follows:

$$g_{MN} = \eta_{\bar{M}\bar{N}} h^{\bar{M}}_M h^{\bar{N}}_N. \quad (56)$$

The covariant derivative assumes the classical form,

$$D_M = \partial_M + \frac{1}{4}\Omega^{\bar{M}\bar{N}}_M \gamma_{\bar{M}}\gamma_{\bar{N}}. \quad (57)$$

The spin connection $\Omega^{\bar{M}\bar{N}}_M$ in this case is defined as

$$\Omega^{\bar{M}\bar{N}}_M = \frac{1}{2}h^{N\bar{M}}(\partial_M h^{\bar{N}}_N - \partial_N h^{\bar{N}}_M) + -\frac{1}{2}h^{N\bar{N}}(\partial_M h^{\bar{M}}_N - \partial_N h^{\bar{M}}_M) - \frac{1}{2}h^{P\bar{M}}h^{Q\bar{N}}h^{\bar{R}}_M(\partial_P h_{Q\bar{R}} - \partial_Q h_{P\bar{R}}). \quad (58)$$

In order to find the relations between the curved gamma matrices and the flat gamma matrices, we refer to the metric ansatz (26) and use the relation (55), which will give us the nonzero results,

$$\Gamma^t = e^{-cr}\gamma^{\bar{t}};\quad \Gamma^x = e^{-cr-\frac{u}{2}}\gamma^{\bar{x}};\quad \Gamma^y = e^{-cr-\frac{u}{2}}\gamma^{\bar{y}};$$
$$\Gamma^z = e^{-cr+\frac{3u}{2}}\gamma^{\bar{z}};\quad \Gamma^r = \gamma^{\bar{r}};\quad \Gamma^\theta = R_0^{-1}e^{-\frac{c_1}{2}r-\frac{u}{2}}\gamma^{\bar{\theta}}. \quad (59)$$

It is still necessary to make evident the nonvanishing components of the spin connection (58), namely,

$$\Omega^{\bar{t}\bar{x}}_x = \Omega^{\bar{t}\bar{y}}_y = \frac{1}{R_0}\Omega^{\bar{t}\bar{\theta}}_\theta = -\frac{\dot{u}}{2}e^{u/2};\quad \Omega^{\bar{t}\bar{z}}_z = \frac{3\dot{u}}{2}e^{-3u/2};$$
$$\Omega^{\bar{r}\bar{x}}_x = \Omega^{\bar{r}\bar{y}}_y = \left(c + \frac{u'}{2}\right)e^{cr+u/2};$$
$$\Omega^{\bar{r}\bar{z}}_z = \left(c - \frac{3u'}{2}\right)e^{cr-3u/2};$$
$$\frac{1}{R_0}\Omega^{\bar{r}\bar{\theta}}_\theta = \left(\frac{c_1}{2} + \frac{u'}{2}\right)e^{\frac{c_1}{2}r+u/2};\quad \Omega^{\bar{r}\bar{t}}_t = ce^{cr}. \quad (60)$$

On the other hand, in order to solve Eq. (54), it will be necessary to consider the case where $\omega \gg E$, as in the last section. In addition, a time average of the equation of motion must be performed. Furthermore, we assume the decomposition $\Psi(x^A) = \psi(x^\mu)\rho(r)e^{il\theta}$ for the wave function. This will allow us to write the equation of motion 54)) as

$$\left[\mathcal{D} + \gamma^r\left(\frac{3c + c_1}{2} + \partial_r\right) - e^{-cr}R_0^{-1}l^2\langle e^{-u/2}\rangle\right]\psi(x^\mu)\rho(r) = 0, \quad (61)$$

where the operator $\mathcal{D}$ is given as

$$\mathcal{D} = e^{-ar}[(\langle e^{-u/2}\rangle - 1)(\gamma^x\partial_x + \gamma^y\partial_y) + (\langle e^{3u/2}\rangle - 1)\gamma^z\partial_z]. \quad (62)$$

Once again it is not possible to analytically solve the equation above, and then we have to study this equation in the limits $r \to 0$ and $r \to \infty$. It is relevant to note that the operator $\mathcal{D}$ may be approximated as $\mathcal{D} \approx 0$ in these two distinct regions. This is a consequence of the fact that $\langle e^{bu}\rangle \approx 1$ for $r \to 0$. This result is shown schematically in Fig. 6 below. The constant b assumes the values $b = -0.5$ and $b = 1.5$, represented by the filled and dotted lines, respectively. As one can see from this figure, the quantity $\langle e^{bu}\rangle$ is approximately one for both values of the constant b. Therefore, in both cases, Eq. (61) for the s-wave ($l = 0$) may be simplified as

$$\left(\frac{3c + c_1}{2} + \partial_r\right)\rho(r) = 0. \quad (63)$$

It is very easy to solve this equation resulting in $\rho(r) \propto e^{-\frac{3c+c_1}{2}r}$. This solution shows that for $r \to 0$, the function ρ has a maximum at the origin and that it decays as $e^{-\frac{3c+c_1}{2}r}$ for $a > 0$ when $r \to \infty$. To show the localization, we insert this solution in the action (53). By doing this, the resultant integral in variable r will assume the form $I \propto \int_0^\infty dre^{-ar}$. It is evident that this integral is convergent for $a > 0$. This is sufficient to assure that the spin 1/2 fermion zero mode is localized in this model. Therefore, similar to the other results in six dimensions, we show that the geometry is important to localized fields in contrast to the 5D standing-wave braneworld, where it is not possible to find localization for the fermion. This still shows that the model presented here is more general that the other six-dimensional standing-wave braneworld model and

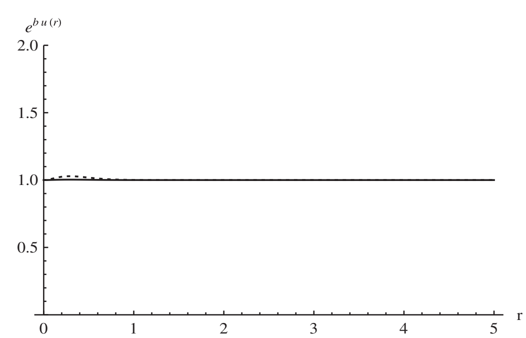

FIG. 6. $\langle e^{bu}\rangle$ profile. The filled line represents $b = -0.5$ and the dotted line represents $b = 1.5$

allows the localization of fields for different Bessel functions.

VI. REMARKS AND CONCLUSIONS

In this paper, we have obtained the standing gravitational wave solution for the six-dimensional Einstein equation in the presence of an anisotropic brane generated by normal matter. The compact dimension belongs to the brane and is small enough to assure that our model is realistic. Our metric ansatz is anisotropic and nonstatic, unlike most models considered in the braneworld literature. We find a solution for the warp factor which represents a thin brane, and in this case the bulk may be seen as a generalization of the stringlike defect [6,7]. Apart from having a physical field as the source, our model is more general than other six-dimensional standing-wave braneworlds recently considered in the literature [48,50], since their solutions may be derived from our solution as special cases. Our metric ansatz has two different warp factors e^{2cr} and $e^{2c_1 r}$, similar to the global stringlike defect, and it also has general warp factors $e^{u(r,t)}$ and $e^{-3u(r,t)}$ similar to the 6D standing-wave braneworld with an exotic source. The different warp factors permit us to find a solution for the function u for increasing or decreasing warp factor $e^{\pm cr}$, which is an advantage when compared with the similar model.

We find the standing-wave solution for the function $u(r,t)$, and we show that this solution is more general and comprises the other six-dimensional standing-wave braneworld solutions that have recently appeared in the literature [48,50]. In fact, our solutions depend on the Bessel function $J_{\pm\frac{a}{2c}}$, which for the special case $a = \pm 5c$ coincides with the models cited above.

From the $u(r,t)$ function, it was possible to choose the type of matter that generates the brane. We have found two types of matter depending on whether we consider the isotropic or anisotropic cosmological constant. In the first case, for a negative cosmological constant, and $a = 4c$, we have demonstrated that the energy density and pressure components are all positive and the energy conditions NEC, WEC, and SEC are satisfied, although the dominant energy condition is violated in this case. If one considers $a = -4c$, similar results are found for the matter, but in this case, the cosmological constant is positive which means that in this case the bulk is asymptotically de Sitter, while in the first case we have a 6D AdS space-time. It is interesting to note that an asymptotically AdS space-time has been considered in the other 5D and 6D standing-wave braneworlds, but a de Sitter geometry was found for the first time here. For the case of the anisotropic cosmological constant, we have considered the situation recently suggested in the literature for the higher-dimensional Randall-Sundrum model [51]. So we consider that the cosmological constant on the brane has a fixed value λ, but the extra components Λ_5 and Λ_6 may assume different values. This is reasonable in our case since we are dealing with an anisotropic bulk. Therefore, for $a = 4c$ all the "components" of the cosmological constant are negative and for $a = -4c$ they are positive, in line with the findings for the case of the isotropic cosmological constant discussed above. In the two cases, we have obtained the components of the energy-momentum tensor and shown that all of them are positive and that all the energy conditions are satisfied. This means that we have constructed a 6D standing-wave braneworld that is generated by normal matter and, therefore, stable.

An important feature of some braneworld models is their ability to solve the so-called hierarchy problem, but in the context of the standing-wave braneworld, this problem was never treated. Here, we have demonstrated that it is possible to solve the hierarchy problem in this context. This is another advantage of the model proposed in this work over other standing-wave braneworld models.

Finally, we have considered the localization of fields in our model. Since it generalizes our previous work in this subject [48], it is reasonable to expect that the localization of the fields studied there is also possible here. Indeed, here we have considered $a = 4c$ and we studied only the zero-mode scalar and fermion field localization. As was expected, we have shown that there is a zero-mode localization for both scalar and fermion fields. The solution found here for the scalar field is in accordance with the ones that were encountered in five dimensions [31] and six dimensions [48,50].

It is known that quantum effects may play important roles in braneworld models, for example, in the mechanism of generating 4D Newtonian gravity in a static three-brane [53,54]. Moreover, quantum corrections in warped backgrounds may lead to gravity delocalization [55]. Therefore, although we are dealing with normal matter, it is interesting to study how quantum fluctuation could interfere in the stability of our ansatz metric. This also will be left for a future work.

ACKNOWLEDGMENTS

The authors thank the Fundação Cearense de apoio ao Desenvolvimento Científico e Tecnológico (FUNCAP), the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), and the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for financial support.

[1] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436**, 257 (1998).
[2] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **429**, 263 (1998).
[3] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Rev. D **59**, 086004 (1999).
[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[6] I. Oda, Phys. Lett. B **496**, 113 (2000).
[7] I. Oda, Phys. Rev. D **62**, 126009 (2000).
[8] R. Gregory, Phys. Rev. Lett. **84**, 2564 (2000).
[9] J. W. Chen, M. A. Luty, and E. Ponton, J. High Energy Phys. 09 (2000) 012.
[10] A. G. Cohen and D. B. Kaplan, Phys. Lett. B **470**, 52 (1999).
[11] I. Olasagasti and A. Vilenkin, Phys. Rev. D **62**, 044014 (2000).
[12] T. Gherghetta and M. E. Shaposhnikov, Phys. Rev. Lett. **85**, 240 (2000).
[13] E. Ponton and E. Poppitz, J. High Energy Phys. 02 (2001) 042.
[14] M. Giovannini, H. Meyer, and M. E. Shaposhnikov, Nucl. Phys. **B619**, 615 (2001).
[15] P. Tinyakov and K. Zuleta, Phys. Rev. D **64**, 025022 (2001).
[16] S. Kanno and J. Soda, J. Cosmol. Astropart. Phys. 07 (2004) 002.
[17] J. Vinet and J. M. Cline, Phys. Rev. D **70**, 083514 (2004).
[18] J. M. Cline, J. Descheneau, M. Giovannini, and J. Vinet, J. High Energy Phys. 06 (2003) 048.
[19] E. Papantonopoulos, A. Papazoglou, and V. Zamarias, Nucl. Phys. **B797**, 520 (2008).
[20] I. Navarro, J. Cosmol. Astropart. Phys. 09 (2003) 004.
[21] I. Navarro and J. Santiago, J. High Energy Phys. 02 (2005) 007.
[22] E. Papantonopoulos and A. Papazoglou, J. Cosmol. Astropart. Phys. 07 (2005) 004.
[23] B. de Carlos and J. M. Moreno, J. High Energy Phys. 11 (2003) 040.
[24] M. Gogberashvili and P. Midodashvili, Europhys. Lett. **61**, 308 (2003).
[25] R. Koley and S. Kar, Classical Quantum Gravity **24**, 79 (2007).
[26] R. S. Torrealba, Phys. Rev. D **82**, 024034 (2010).
[27] J. E. G. Silva and C. A. S. Almeida, Phys. Rev. D **84**, 085027 (2011).
[28] L. J. S. Sousa, W. T. Cruz, and C. A. S. Almeida, Phys. Lett. B **711**, 97 (2012).
[29] Localization of Matter Fields in the 5D Standing Wave Braneworld, M. Gogberashvili, J. High Energy Phys. 09 (2012) 056.
[30] M. Gogberashvili, P. Midodashvili, and L. Midodashvili, Phys. Lett. B **707**, 169 (2012).
[31] M. Gogberashvili, P. Midodashvili, and L. Midodashvili, Phys. Lett. B **702**, 276 (2011).
[32] A. Kehagias and K. Tamvakis, Phys. Lett. B **504**, 38 (2001).
[33] W. T. Cruz, M. O. Tahim, and C. A. S. Almeida, Europhys. Lett. **88**, 41001 (2009).
[34] W. T. Cruz, A. R. Gomes, and C. A. S. Almeida, Europhys. Lett. **96**, 31001 (2011).
[35] M. O. Tahim, W. T. Cruz, and C. A. S. Almeida, Phys. Rev. D **79**, 085022 (2009).
[36] S. Randjbar-Daemi and M. E. Shaposhnikov, Phys. Lett. B **491**, 329 (2000).
[37] A. Kehagias, Phys. Lett. B **600**, 133 (2004).
[38] M. Gogberashvili, P. Midodashvili, and D. Singleton, J. High Energy Phys. 08 (2007) 033.
[39] Y.-S. Duan, Y.-X. Liu, and Y.-Q. Wang, Mod. Phys. Lett. A **21**, 2019 (2006).
[40] J. Garriga and M. Porrati, J. High Energy Phys. 08 (2004) 028.
[41] J. F. Vazquez-Poritz, J. High Energy Phys. 09 (2002) 001.
[42] H. Firouzjahi and S. H. Tye, J. High Energy Phys. 01 (2006) 136.
[43] T. Noguchi, M. Yamaguchi, and M. Yamashita, Phys. Lett. B **636**, 221 (2006).
[44] F. Brummer, A. Hebecker, and E. Trincherini, Nucl. Phys. **B738**, 283 (2006).
[45] M. Gogberashvili and D. Singleton, Mod. Phys. Lett. A **25**, 2131 (2010).
[46] M. Gogberashvili, A. Herrera-Aguilar, and D. Malagon-Morejon, Classical Quantum Gravity **29**, 025007 (2012).
[47] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003); S. M. Carroll, M Hoffman, and M. Trodden, Phys. Rev. D **68**, 023509 (2003); P. Frampton, Phys. Lett. B **555**, 139 (2003); V. Sahni and Y. Shtanov, J. Cosmol. Astropart. Phys. 11 (2003) 014.
[48] L. J. S. Sousa, J. E. G. Silva, and C. A. S. Almeida, arXiv:1209.2727.
[49] M. Visser, Phys. Rev. D **56**, 7578 (1997).
[50] P. Midodashvili, arXiv:1211.0206v1.
[51] P. R. Archer and S. J. Huber, J. High Energy Phys. 03 (2011) 018.
[52] B. Carter, A. Nielsen, and D. Wiltshire, J. High Energy Phys. 07 (2006) 034.
[53] G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000).
[54] G. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001).
[55] Z. Kakushadze, Phys. Lett. B **497**, 125 (2001).