Extended Chaplygin gas as a unified fluid of dark components in varying gravitational constant theory

Jianbo Lu,[1,*] Lixin Xu,[2] Hongyan Tan,[1] and Shanshan Gao[1]

[1]*Department of Physics, Liaoning Normal University, Dalian 116029, People's Republic of China*
[2]*School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, People's Republic of China*

(Received 13 November 2013; published 24 March 2014)

Varying gravitational constant $G(t)$ (VG) cosmology is studied in this paper, where the modified Friedmann equation and the modified energy conservation equation are given with respect to the constant-G theory. Considering the extended Chaplygin gas (ECG) as background fluid (or thinking that ECG fluid is induced by the variation of G), the unified model of dark matter and dark energy is obtained in VG theory. The parameter spaces are investigated in the VG-ECG model by using the recent cosmic data. Constraint results show $\beta = -\frac{\dot{G}}{HG} = -0.003^{+0.021+0.034}_{-0.020-0.055}$ for the VG-GCG unified model and $\beta = -0.027^{+0.032+0.059}_{-0.032-0.066}$ for the VG-MCG unified model. Equivalently, they correspond to the limits on the current variation of Newton's gravitational constant at 95.4% confidence level $|\frac{\dot{G}}{G}|_{\mathrm{today}} \lesssim 4.1 \times 10^{-12}\ \mathrm{yr}^{-1}$ and $|\frac{\dot{G}}{G}|_{\mathrm{today}} \lesssim 6.6 \times 10^{-12}\ \mathrm{yr}^{-1}$. And for $z \leq 3.5$, bounds on the variation of $\frac{\dot{G}}{G}$ in the VG-ECG unified model are in accordance with the experiment explorations of varying G. In addition, in VG theory the used observational data point still cannot distinguish the VG-GCG and VG-MCG unified model from the most popular ΛCDM cosmology. Furthermore, to see the effects of varying G and physical properties for VG-ECG fluid, we discuss the evolutionary behaviors of cosmological quantities in VG theory, such as $\frac{\dot{G}}{G}$, $\frac{\ddot{G}}{G}$ and equation of state w, etc. For $\beta < 0$ a quintom scenario crossing over $w = -1$ can be realized in the VG-GCG model.

DOI: 10.1103/PhysRevD.89.063526 PACS numbers: 98.80.-k

I. INTRODUCTION

Several theories have been constructed to be responsible for the accelerating expansion of the Universe [1]. These include the study of $f(R)$ and $f(T)$ gravity theories [2] by modifying Einstein's theory, the study of RS and DGP higher-dimensional gravity theories [3] by investigating the nature of spacetime, and the construction of holographic and quintessence field dark energy models [4] from basic principles by introducing mysterious negative-pressure components in the Universe, etc. [5]. In addition, in standard dark-energy cosmology, current observations indicate that except the visible baryon matter and radiation, about 95% of the total energy density in the Universe are invisible, including dark matter (DM) and dark energy (DE). A type of interested model called extended Chaplygin gas (ECG) was introduced, which includes, for instance, the generalized Chaplygin gas (GCG) [6,7] and the modified Chaplygin gas (MCG) [8] models. An attractive property of this type of model is that two unknown dark sections in the Universe—DM and DE—can be unified in an exotic equation of state. ECG models have been studied widely. For examples, the quantum-cosmology studies of the ECG models were well investigated in Refs. [9,10], the behaviors of ECG fluid were discussed in Hořava-Lifshitz (HL) gravity [11,12] and extra-dimension theory [13,14], etc.

In general, cosmological models are studied with an assumption that the Newton gravity constant G is a constant. But some observations indicate that G may be variable with respect to the cosmic time t [15], such as observations of big bang nucleosynthesis [16], pulsating white dwarf stars [17,18], supernovae of type Ia [19], and binary pulsar PSR1913 [20]. Furthermore, a theory of varying gravity constant G (VG) could alleviate the dark matter problem [21] and the cosmic coincidence problem [22], and modify the main-sequence time of globular cluster stars [23], etc. In VG theory, HL gravity [24], the holographic dark energy model [25], and the interacting model [26] have been discussed. In this paper we investigate the unified model of dark matter and dark energy in the framework of varying gravitational constant, by considering the extended Chaplygin gas as the background fluid. Usually, VG cosmology were studied with using a standard Friedmann equation [25–27]. Here, we consider a modified Friedmann equation with the extra corrected term in VG theory. In addition, a different conservation equation is used with respect to the constant-G theory, too. Correspondingly, a modified expression of ECG energy density is given.

II. MAIN EQUATIONS IN VARYING GRAVITATIONAL CONSTANT THEORY

It is well known that action is expressed as $A = \int L_T d^4x$ with A being a scalar. We start with the total Lagrangian L_T

*lvjianbo819@163.com

of the system consisting of gravitational field $g_{\mu\nu}$, matter field ψ, and time-varying Newton gravitational constant G,

$$L_T = L_g + L_m = \sqrt{g}\left(\frac{R}{G(t)} + 16\pi\mathcal{L}_m\right). \tag{1}$$

Inspired by the studies on VG theory of Brans-Dicke (BD) in Refs. [28,29], the function G depending on the time t is usually written as $G = \phi^{-1} = G_0 a(t)^{-\beta}$ by considering the power-law form for both BD scalar field $\phi = \phi_0 t^{\xi}$ and scale factor $a = a_0 t^{\zeta}$, where ξ, ζ, and β are constant parameters. Still in Eq. (1), $g = -|g_{\mu\nu}|$ is defined by the determinant of the metric, $R = g^{\mu\nu}R_{\mu\nu}$ denotes the Ricci scalar with $R_{\mu\nu}$ being the Ricci tensor, and $\mathcal{L}_m$ is the Lagrangian density of the matter field. Using the variational principle to Eq. (1) with respect to gravitational field $g_{\mu\nu}$, we can get the gravitational field equation,

$$\begin{aligned} G_{\mu\nu} &\equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \\ &= 8\pi G T_{\mu\nu} + G(\nabla_\mu\partial_\nu G^{-1} - g_{\mu\nu}\nabla_\sigma\partial^\sigma G^{-1}), \end{aligned} \tag{2}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of universal matter which includes the photon, the pressureless baryon matter, and the unknown dark components, and it can be written as a perfect-fluid form $T^{\mu\nu} = (\rho + p)U^\mu U^\nu + pg^{\mu\nu}$ with U^μ being the four-velocity of the fluid.

One knows that modern cosmology is constructed from the gravitational field equation and the cosmological principle (i.e., the Robertson-Walker metric)

$$ds^2 = -dt^2 + a^2(t)dl^2, \tag{3}$$

where dl^2 is the metric on a maximally symmetric three-manifold. Then according to Eq. (2) and Eq. (3), the Friedmann equation in VG theory with a flat geometry can be derived by

$$(1+\beta)H^2 = \frac{8\pi G_0}{3}a^{-\beta}[\rho_r + \rho_b + \rho_{\mathrm{dark}}], \tag{4}$$

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble function. For $\beta = 0$, the constant-G standard Friedmann equation is recovered. ρ_r, ρ_b and ρ_{dark}, respectively, denote the energy density of radiation, baryon matter, and the dark component. These energy densities can be explicitly obtained by integrating the energy conservation equation. Due to variation of the Newton gravitational constant $G = G_0 a(t)^{-\beta}$, the energy conservation law as a fundamental equation playing an important role in the evolution of the Universe is modified. It has the following form:

$$\dot{\rho} + 3H\left(\rho + \frac{2+2\beta}{2+\beta}p\right) = \frac{\beta - \beta^2}{2+\beta}H\rho, \tag{5}$$

obtained by the Bianchi identity (more detail can be found in Appendix A). ρ and p represent the total density and the total pressure of the background fluid in the Universe, respectively. For $\beta = 0$, Eq. (5) reduces to the constant-G case. Then relative to the constant-G theory, in VG theory the energy densities of universal matter will be modified. The term on the right-hand side of Eq. (5) can be interpreted as the interaction between the cosmic fluids, and $\frac{\beta-\beta^2}{2+\beta}$ describes the interaction strength. Also, the energy conservation equation (5) can be available for each cosmic background fluid. Using Eq. (5), the energy densities of the radiation ($p_r = \frac{1}{3}\rho_r$) and the baryon ($p_b = 0$) are derived,

$$\rho_r = \rho_{0r}a^{\frac{-\beta^2-4\beta-8}{2+\beta}}, \tag{6}$$

$$\rho_b = \rho_{0b}a^{\frac{-\beta^2-2\beta-6}{2+\beta}}, \tag{7}$$

where subscript 0 denotes current values of parameters, and a is the cosmic scale factor which is related to the cosmic redshift by $z = \frac{1}{a} - 1$. For $\beta = 0$, they are reduced to the forms in the constant-G theory.

A. Generalized Chaplygin gas as background fluid in varying gravitational constant theory

Besides the radiation and the baryon matter, the extended Chaplygin gas as a unification of dark matter and dark energy is assumed in the background. In a constant G theory, the Chaplygin gas (CG) as a unified model of dark component was constructed, with a relation between pressure and energy density, $p = -\frac{A}{\rho}$. This model can be associated with the parametrization invariant Nambu-Goto d-brane action in a $(d+1,\ 1)$ spacetime. However, this model is not favored by the cosmic observed data. So, several extended Chaplygin gas models are introduced, such as the GCG model and the MCG model. In this paper, we discuss the behaviors of ECG fluid in VG theory.

According to Ref. [6], one knows the GCG model ($p = -\frac{A}{\rho^\alpha}$) can be derived by a generalized Born-Infeld theory with a Lagrangian density,

$$\mathcal{L}_{\mathrm{GCG}} = -A^{\frac{1}{1+\alpha}}[1 - (g^{\mu\nu}\theta_{,\mu}\theta_{,\nu})^{\frac{1+\alpha}{2\alpha}}]^{\frac{\alpha}{1+\alpha}}, \tag{8}$$

with the relation $g^{\mu\nu}\theta_{,\mu}\theta_{,\nu} = V'(\varphi)$. Lagrangian quantity (8) can be regarded as a d-brane plus soft correcting terms [6]. In VG theory, this scalar field φ can be related to the time-varying gravitational constant, by replacing φ with $\varphi(t) = G(t)^{-1}$ i.e. the GCG fluid can be thought that it is induced from the variation of G. We consider the dark components in the Universe are performed by Lagrangian (8) with owing the same "potential" function form $V(\varphi) = V(\frac{1}{G})$ in Ref. [6], where potential has the power-law form relative to a with $\varphi = \varphi_0 a^\beta$. Then the energy density of GCG fluid with varying gravitational constant G (named as VG-GCG) can be derived as

$$\rho_{\mathrm{VG-GCG}} = \rho_{\mathrm{0VG-GCG}}\left[B_s + (1-B_s)a^{\left(-3+\frac{\beta-\beta^2}{2+\beta}\right)(1+\alpha)}\right]^{\frac{1}{1+\alpha}}, \tag{9}$$

where α, β and $B_s \equiv \frac{6+6\beta}{\beta^2+2\beta+6}\frac{A}{\rho_{\mathrm{0VG-GCG}}^{1+\alpha}}$ are three constant model parameters. For $\alpha = 0$ and $\beta = 0$, the standard ΛCDM model is recovered in above equation. And for $\alpha = 1$, Eq. (9) reduces to the standard Chaplygin gas scenario in VG theory (VG-CG). In addition, from Eq. (9) one can find that the VG-GCG fluid behaves like cosmological-constant type dark energy at late time (for $a \gg 1$, $\rho_{\mathrm{VG-GCG}} \approx \rho_{\mathrm{0VG-GCG}} B_s^{\frac{1}{1+\alpha}} \equiv \rho_{\mathrm{de}}$), and behaves as cold dark matter at early time (for $a \ll 1$, $\rho_{\mathrm{VG-GCG}} \approx \rho_{\mathrm{0VG-GCG}}(1-B_s)^{\frac{1}{1+\alpha}} \times a^{-3+\frac{\beta-\beta^2}{2+\beta}} \equiv \rho_{\mathrm{dm}}$, which is same to the result given by the cold dark matter density interacting with dark energy [30]). Fixing $a = 1$, at present the effective dark matter density can be expressed as $\rho_{\mathrm{0dm}} = \rho_{\mathrm{0VG-GCG}}(1-B_s)^{\frac{1}{1+\alpha}}$. Equivalently, it owns $\Omega_{\mathrm{0dm}} = \Omega_{\mathrm{0VG-GCG}}(1-B_s)^{\frac{1}{1+\alpha}}$, here $\Omega_{\mathrm{0VG-GCG}} = 1 + \beta - \Omega_b - \Omega_r$ with definitions of dimensionless energy densities $\Omega_r \equiv \frac{8\pi G_0 \rho_{0r}}{3H_0^2}$ and $\Omega_b \equiv \frac{8\pi G_0 \rho_{0b}}{3H_0^2}$. So, the VG-GCG fluid can be seen as a fixture composed by two components: the dark energy and the cold dark matter, i.e., $\rho_{\mathrm{VG-GCG}} = \rho_{\mathrm{de}} + \rho_{\mathrm{dm}}$, $p_{\mathrm{VG-GCG}} = p_{\mathrm{de}} + p_{\mathrm{dm}}$. Using Eq. (4), the dimensionless Hubble parameter for the GCG model in VG theory has the form

$$\begin{aligned} E^2(a) &= \frac{H^2(a)}{H_0^2} \\ &= \frac{1}{1+\beta}\{\Omega_{\mathrm{0VG-GCG}}[B_s a^{-\beta(1+\alpha)} \\ &\quad + (1-B_s)a^{-(3+\frac{2\beta^2+\beta}{2+\beta})(1+\alpha)}]^{\frac{1}{1+\alpha}} \\ &\quad + \Omega_b a^{\frac{-2\beta^2-4\beta-6}{2+\beta}} + \Omega_r a^{\frac{-2\beta^2-6\beta-8}{2+\beta}}\}, \end{aligned} \tag{10}$$

where H_0 is the Hubble constant. Again, Eq. (10) is recovered to the constant-G theory with standard Friedmann equation for $\beta = 0$.

B. Modified Chaplygin gas as background fluid in varying gravitational constant theory

A simple and popular generalization relative to the GCG model is the MCG model $p = B\rho - \frac{A}{\rho^\alpha}$, which we derive by adding a barotropic term. The MCG fluid can be seen as a unification of dark energy, cold dark matter and hot dark matter, as interpreted in the following. Also, this model can be derived from a scalar field φ with a power-law potential relative to a [31]. The energy density of MCG with varying gravitational constant is derived as

$$\begin{aligned} &\rho_{\mathrm{VG-MCG}} \\ &= \rho_{\mathrm{0VG-MCG}}[B_s + (1-B_s)a^{[-3(1+\frac{2+2\beta}{2+\beta}B)+\frac{\beta-\beta^2}{2+\beta}](1+\alpha)}]^{\frac{1}{1+\alpha}}, \end{aligned} \tag{11}$$

where α, β, B, and $B_s \equiv \frac{6(1+\beta)}{\beta^2+2\beta+6+6(1+\beta)B}\frac{A}{\rho_{\mathrm{0VG-MCG}}^{1+\alpha}}$ are four constant model parameters. From Eq. (11), we can see that at late time ($a \gg 1$) $\rho_{\mathrm{VG-MCG}} \approx \rho_{\mathrm{0VG-MCG}} B_s^{\frac{1}{1+\alpha}}$ describes a cosmological-constant-type dark energy, and at early time ($a \ll 1$) $\rho_{\mathrm{VG-MCG}} \approx \rho_{\mathrm{0VG-MCG}}(1-B_s)^{\frac{1}{1+\alpha}} a^{[-3(1+\frac{2+2\beta}{2+\beta}B)+\frac{\beta-\beta^2}{2+\beta}]}$ denotes a mixture of cold DM (state parameter $w \sim 0$) and hot DM ($w \sim \frac{1}{3}$) for $0 < \frac{2+2\beta}{2+\beta}B < \frac{1}{3}$, since the term $\frac{\beta-\beta^2}{2+\beta}$ is interpreted as the interaction. Concretely, the values of $\frac{2+2\beta}{2+\beta}B$ depend on a ratio: the hot DM component to the cold DM component. Thus for the physics of the MCG model, the MCG fluid can be seen as the mixture of the GCG and the hot DM, with GCG as the unification of cold dark matter and dark energy. The dimensionless Hubble parameter for the MCG model in VG theory (VG-MCG) can be expressed as

$$\begin{aligned} E^2(a) &= \frac{1}{1+\beta}\{\Omega_{\mathrm{0VG-MCG}}[B_s a^{-\beta(1+\alpha)} \\ &\quad + (1-B_s)a^{[-3(1+\frac{2+2\beta}{2+\beta}B)-\frac{2\beta^2+\beta}{2+\beta}](1+\alpha)}]^{\frac{1}{1+\alpha}} \\ &\quad + \Omega_b a^{\frac{-2\beta^2-4\beta-6}{2+\beta}} + \Omega_r a^{\frac{-2\beta^2-6\beta-8}{2+\beta}}\}, \end{aligned} \tag{12}$$

with $\Omega_{\mathrm{0VG-MCG}} = 1 + \beta - \Omega_b - \Omega_r$. Obviously, for $B = 0$ the VG-GCG scenario reappears.

III. COSMIC CONSTRAINTS ON THE EXTENDED CHAPLYGIN GAS UNIFIED MODEL IN VARYING GRAVITATIONAL CONSTANT THEORY

Parameter spaces p_s of the ECG unified model in VG theory (VG-ECG) are studied by using the Markov Chain Monte Carlo method [32]. One can calculate χ^2 to obtain the confidence levels of VG-ECG (including VG-GCG and VG-MCG) model parameters: $\Omega_b h^2$, h, B_s, α, B and β. Here $\Omega_b h^2$ denotes the physical baryon density and h is a re-normalized quantity defined by $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. The used data includes the baryon acoustic oscillation (BAO) [33–35], the x-ray gas mass fraction (XGMF) [36], the cosmic microwave background (CMB) data [37], the Union2 data set of type supernovae Ia (SNIa) [38], and the Hubble data [39–43]. The methods of fitting data are introduced in Appendix B.

A. VG-GCG model

For this part, the first constraint on the VG-GCG model is obtained from the combination BAO + XGMF + CMB + SNIa + H. The calculation results on the model parameters are listed in Table II, where the mean values with 68.3% (1σ) and 95.4% (2σ) confidence levels (CL) are shown. Correspondingly, their 1σ and 2σ contours are

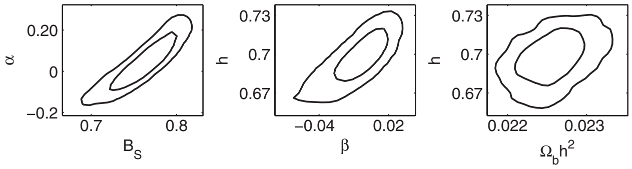

FIG. 1. The 68.3% and 95.4% confidence regions of the model parameters in varying-G frames including GCG background fluid as dark components.

plotted in Fig. 1. The 1σ and 2σ confidence levels of model parameters in given data sets give values of χ^2. Relative to the minimum value $\chi^2_{\rm min}$ they are defined by

$$\chi^2 - \chi^2_{\rm min} \leq \Delta\chi^2(p_s), \tag{13}$$

where $\Delta\chi^2(p_s)$ are taken the different values when the different numbers of free parameters p_s are included in the models. Several values of $\Delta\chi^2(p_s)$ are listed in Table I.

The value of parameter β reflects the property of G. Its constraint result is $\beta = -0.003^{+0.021+0.034}_{-0.020-0.055}$, i.e. the current value $-2.2\times10^{-12}\ {\rm yr}^{-1} \lesssim (\frac{\dot G}{G})_{\rm today} \lesssim 4.1\times10^{-12}\ {\rm yr}^{-1}$ (95.4% confidence level). Comparing with other combined constraint on parameter β by cosmological data, Ref. [44] shows that the holographic model with VG in the modified Friedmann equation has been constrained, where $-\beta = \alpha_G = \frac{\dot G}{HG} = 0.1647^{+0.3547}_{-0.2971}$ is found for a flat universe. Obviously, a larger error of β is given in Ref. [44] than in our case. For other VG-GCG model parameters, they satisfy $B_s = 0.763^{+0.036+0.062}_{-0.036-0.080}$ and $\alpha = 0.051^{+0.112+0.230}_{-0.112-0.204}$. Considering that parameter α is related to equation of state $w = \frac{p}{\rho} = -\frac{c_s^2}{\alpha}$ for VG-GCG fluid, α and w should have the contrary symbol, since adiabatic sound speed $c_s^2 = \frac{\delta p}{\delta\rho} \geq 0$. Thus, the values of $\alpha < 0$ should be ruled out for an accelerating universe with $w < 0$. In addition, in the VG-GCG model dimensionless matter density as a derived parameter can be calculated, $\Omega_{0m} \simeq 0.288^{+0.013+0.032}_{-0.013-0.025}$.

Fixing $B_s = 1$, $\alpha = 0$ and $\beta = 0$, the VG-GCG is reduced to the cosmological constant model, i.e. the ΛCDM model. The values of the ΛCDM model parameters are shown in Table II and Fig. 2. It can be seen that the values of α in the VG-GCG model is around zero, which indicates the VG-GCG model is close to the cosmological constant model. For other model-parameter values ($\Omega_b h^2$, h, Ω_{0m}) in the VG-GCG model, they are also consistent with the standard ΛCDM model with the larger errors. As a constraint result, in VG theory the observational data point still cannot distinguish the VG-GCG unified model from the most popular standard cosmology. But, a larger mean value for the derived parameter Ω_{0m} in the VG-GCG model is given than the standard ΛCDM model. Fixing $\alpha = 1$, the VG-GCG is reduced to the VG-CG model. The cosmic constraint on the VG-CG model parameters is listed in Table II, too. Relative to the above VG-GCG model ($\chi^2_{\rm min} = 601.031$), a larger value of $\chi^2_{\rm min} = 644.123$ reveals that the VG-CG model is not agreeing with cosmic observations.

TABLE I. The values of $\Delta\chi^2$ with different numbers of free parameter.

Numbers of free parameter	1	2	3	4	5	6
$\Delta\chi^2$ for 68.3% CL	1.00	2.30	3.53	4.72	5.89	7.04
$\Delta\chi^2$ for 95.4% CL	4.00	6.17	8.02	9.70	11.3	12.8

B. VG-MCG model

The calculation results on the model parameters with 68.3% and 95.4% confidence are listed in Table III for flat VG-MCG cosmology. And their contours are plotted in Fig. 3. The constraint result on parameter β is $-0.027^{+0.032+0.059}_{-0.032-0.066}$. Equivalently, it corresponds to the current value $-2.3\times10^{-12}\ {\rm yr}^{-1} \lesssim (\frac{\dot G}{G})_{\rm today} \lesssim 6.6\times10^{-12}\ {\rm yr}^{-1}$ (95.4% confidence level), which has a smaller confidence level than several cosmic experiments. For instance, exploration is from globular clusters, $-3.5\times10^{-11}\ {\rm yr}^{-1} \lesssim (\frac{\dot G}{G})_{\rm today} \lesssim 7\times10^{-12}\ {\rm yr}^{-1}$ [23], and survey is on the moon by using the laser-ranging method $|\frac{\dot G}{G}|_{\rm today} \lesssim 1.0\times10^{-11}\ {\rm yr}^{-1}$ [45]. Furthermore, according to the constraint result, in varying-G theory one finds that MCG is close to the

TABLE II. The mean values of free model parameters, and their marginalized limits by using the BAO + XGMF + CMB + SNIa + H data.

	VG-GCG	VG-CG	ΛCDM
$\chi^2_{\rm min}$	601.031	644.123	601.790
$100\Omega_b h^2$	$2.263^{+0.045+0.101}_{-0.045-0.088}$	$2.189^{+0.040+0.074}_{-0.039-0.075}$	$2.234^{+0.042+0.088}_{-0.042-0.081}$
h	$0.6989^{+0.0174+0.0336}_{-0.0167-0.0370}$	$0.7199^{+0.0126+0.0258}_{-0.0132-0.0247}$	$0.6956^{+0.0083+0.0165}_{-0.0083-0.0164}$
B_s	$0.763^{+0.036+0.062}_{-0.036-0.080}$	$0.924^{+0.007+0.013}_{-0.008-0.015}$	...
α	$0.051^{+0.112+0.230}_{-0.112-0.204}$	1	...
β	$-0.003^{+0.021+0.034}_{-0.020-0.055}$	$-0.002^{+0.009+0.018}_{-0.008-0.017}$	...
Ω_{0m}	...	...	$0.283^{+0.011+0.022}_{-0.011-0.021}$

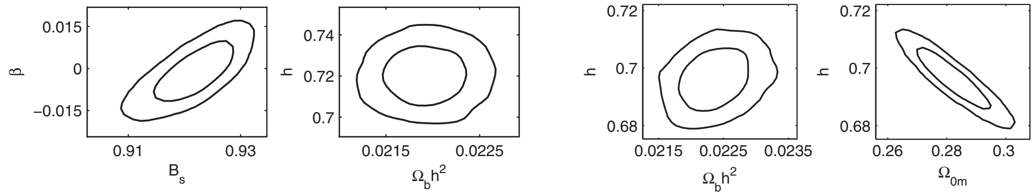

FIG. 2. The 68.3% and 95.4% confidence regions of the model parameters in the VG-CG model (left) and ΛCDM model (right).

TABLE III. The mean values of the VG-MCG model parameters, and their marginalized limits by using the BAO + XGMF + CMB + SNIa + H data.

	VG-MCG
$\chi^2_{\rm min}$	600.863
$100\Omega_b h^2$	$2.302^{+0.059+0.116}_{-0.058-0.110}$
h	$0.6959^{+0.0176+0.0339}_{-0.0172-0.0352}$
B_s	$0.746^{+0.040+0.076}_{-0.040-0.085}$
α	$0.039^{+0.110+0.235}_{-0.108-0.193}$
B	$0.0037^{+0.0033+0.0061}_{-0.0034-0.0070}$
β	$-0.027^{+0.032+0.059}_{-0.032-0.066}$

GCG model for very small values of B, and the VG-MCG model match the ΛCDM model ($\alpha = 0$, $\beta = 0$ and $B = 0$) well. In addition, as seen in Tables II and III, the uncertainties of model parameters for the VG-MCG model are larger than the VG-GCG case.

IV. EFFECTS OF VG ON BEHAVIORS OF COSMOLOGICAL QUANTITIES AND PHYSICAL PROPERTIES OF VG-ECG FLUID

For understanding the varying properties of G, we investigate the evolutionary behaviors of G and its derivatives with respect to time t,

$$\frac{\dot{G}}{G} = -\beta H, \tag{14}$$

$$\frac{\ddot{G}}{G} = \beta H^2[\beta + 1 + q]. \tag{15}$$

The shapes of $\frac{G}{G_0}$, $\frac{\dot{G}}{G}$, and $\frac{\ddot{G}}{G}$ in the VG-GCG (dot-dash lines) and VG-MCG (dot lines) models are illustrated in Fig. 4.

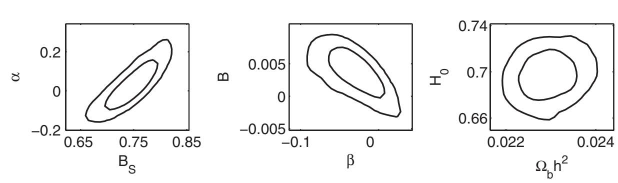

FIG. 3. The 68.3% and 95.4% confidence regions of model parameters in varying-G frames including MCG background fluid as unified dark components.

Referring to confidence levels of above combined constraint result from the data sets: BAO + XGMF + CMB+ SNIa + H, here parameter values $\beta = (-0.058, 0.031)$ for the VG-GCG model and $\beta = (-0.093, 0.032)$ for the VG-MCG model are fixed, respectively. For comparing with constant-G theory, $\beta = 0$ case is also plotted in Fig. 4 (solid lines). The behaviors of $\frac{G}{G_0}$ (or $\frac{\dot{G}}{G}$, $\frac{\ddot{G}}{G}$) are obviously different for taking different values of β. As one can see in Fig. 4, for $z \leq 3.5$, the VG-GCG model gives $-1.18 \times 10^{-11}\ \mathrm{yr}^{-1} \lesssim \frac{\dot{G}}{G} \lesssim 2.1 \times 10^{-11}\ \mathrm{yr}^{-1}$ and for the VG-MCG model, it indicates $-1.22 \times 10^{-11}\ \mathrm{yr}^{-1} \lesssim \frac{\dot{G}}{G} \lesssim 3.5 \times 10^{-11}\ \mathrm{yr}^{-1}$, in accordance with several cosmic experiments on $\frac{\dot{G}}{G}$. For example, measurements of the masses of young and old neutron stars in pulsar binaries limit $-4.8 \times 10^{-11}\ \mathrm{yr}^{-1} \leq \frac{\dot{G}}{G} \leq 3.6 \times 10^{-11}\ \mathrm{yr}^{-1}$ [46], explorations from PSR J0437-4715 yield $|\frac{\dot{G}}{G}| \leq 2.3 \times 10^{-11}\ \mathrm{yr}^{-1}$ [47], and cooling of white dwarfs limits $|\frac{\dot{G}}{G}| \leq 2 \times 10^{-11} yr^{-1}$ [48]. Graphs of $\frac{\ddot{G}}{G}$ shown in Fig. 4 indicate that $\frac{\ddot{G}}{G}$ have much smaller values (about $(-1.8 \sim 0.7) \times 10^{-20}$ for $z \leq 3.5$ at most and almost $\frac{\ddot{G}}{G} \sim 0$ at present) than $\frac{\dot{G}}{G}$. The reason for this smaller value is that $\frac{\ddot{G}}{G}$ relates to the quantity $H(z)^2$ with $H(z) = H_0 E(z)$ and the value of $H_0 = 100\mathrm{h\ km\,s^{-1}\,Mpc^{-1}} \simeq 1.02 \times 10^{-10}\mathrm{h\,yr^{-1}}$. So, it seems that it is harder to test the effects of $\frac{\ddot{G}}{G}$ by experiments, relative to $\frac{\dot{G}}{G}$.

Furthermore, as expressed in Eqs. (14) and (15), $\frac{\dot{G}}{G}$ and $\frac{\ddot{G}}{G}$ are related to the geometry quantities, the Hubble parameter $H = \frac{\dot{a}}{a}$ and deceleration parameter $q = -\frac{\ddot{a}}{aH^2} = (1 + z)\frac{1}{H}\frac{dH}{dz} - 1$, which are plotted in Fig. 5 for taking different values of β. As determined in Fig. 5, $H(z) > 0$ corresponds to an expanded universe and $q(z) > -(1 + \beta)$ for $z > 0$. Thus, from Eqs. (14) and (15) we can get that $\frac{\dot{G}}{G}$ and $\frac{\ddot{G}}{G}$ would have the opposite symbol for $z > 0$, and their symbols depend on the symbols of β. But in distance future ($z \ll 0$), for $\beta < 0$ case it has a different result, where $H(z) > 0$ and $q(z) \leq -(1 + \beta)$. In addition, Fig. 5 shows that the evolutions of $H(z)$ are almost the same for taking different values of β.

For using the mean value of parameter β, the shapes of $\frac{G}{G_0}$, $\frac{\dot{G}}{G}$, and $\frac{\ddot{G}}{G}$ are plotted in Fig. 6 for the VG-ECG model.

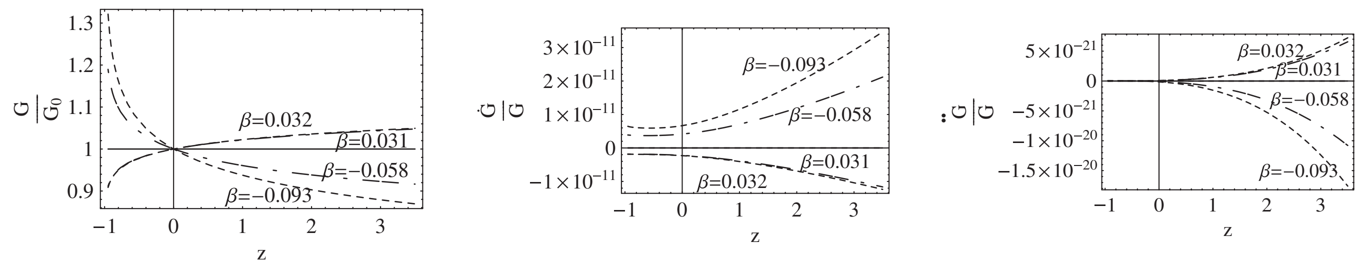

FIG. 4. The evolutions of $\frac{G}{G_0}$, $\frac{\dot{G}}{G}$ and $\frac{\ddot{G}}{G}$. Dot-dash lines denote the VG-GCG model for taking $\beta = (-0.058, 0.031)$ and dot lines denote the VG-MCG model for taking $\beta = (-0.093, 0.032)$.

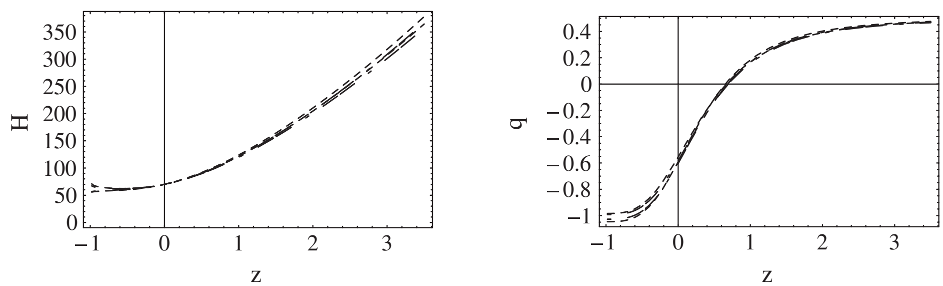

FIG. 5. The evolutions of $q(z)$ and $H(z)$. Dot-dash lines denote the VG-GCG model for taking $\beta = (-0.058, 0.031)$ and dot lines denote the VG-MCG model for taking $\beta = (-0.093, 0.032)$.

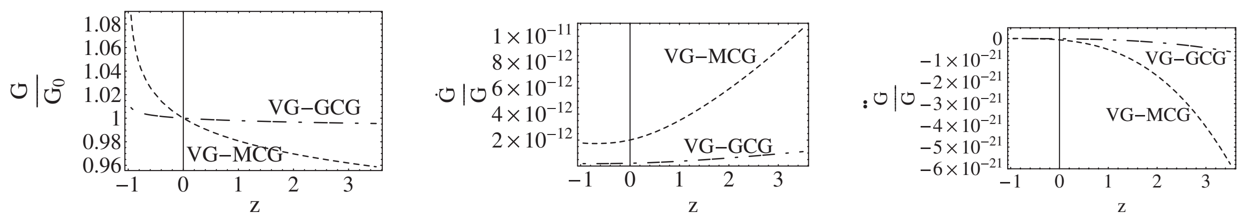

FIG. 6. The evolutions of $\frac{G}{G_0}$, $\frac{\dot{G}}{G}$ and $\frac{\ddot{G}}{G}$ in the VG-GCG and VG-MCG models by using the mean values of combined constraint results.

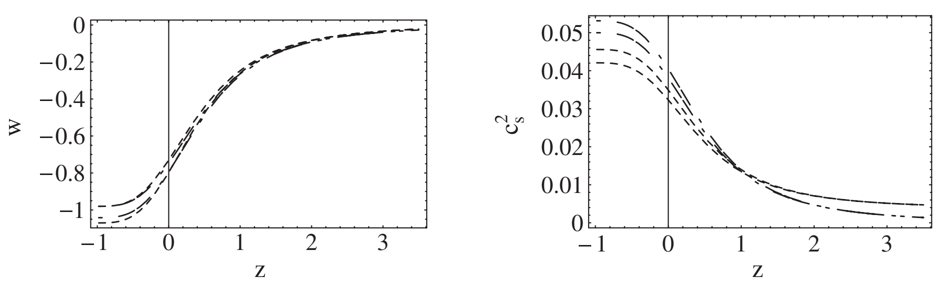

FIG. 7. The evolutions of $w(z)$ and $c_s^2(z)$. Dot-dash lines denote the VG-GCG fluid for taking $\beta = (-0.058, 0.031)$ and dot lines denote the VG-MCG fluid for taking $\beta = (-0.093, 0.032)$.

From Fig. 6, one can see that (i) $\frac{G}{G_0}$ and $\frac{\ddot{G}}{G}$ are the monotone functions and $\frac{\dot{G}}{G} > 0$ (an increasing G with the time), and (ii) in the future ($z < 0$), $\frac{\ddot{G}}{G}$ ($\simeq 0$), and $\frac{\dot{G}}{G}$ (> 0) are almost constants.

The physical properties of the VG-ECG fluid are described by its equation of state (EOS) $w = \frac{p}{\rho}$ and adiabatic sound speed $c_s^2 = \frac{\delta p}{\delta \rho}$. For the VG-GCG and VG-MCG fluid, w is expressed as

$$w_{\mathrm{VG-GCG}}(z) = \frac{-\beta^2 - 2\beta - 6}{6(1+\beta)} \frac{B_s}{B_s + (1-B_s)(1+z)^{\left[3 - \frac{\beta - \beta^2}{2+\beta}\right](1+\alpha)}} \tag{16}$$

$$w_{\mathrm{VG-MCG}}(z) = B + \frac{-\beta^2 - 2\beta - 6 - 6(1+\beta)B}{6(1+\beta)} \times \frac{B_s}{B_s + (1-B_s)(1+z)^{\left[3(1+\frac{2+2\beta}{2+\beta}B) - \frac{\beta-\beta^2}{2+\beta}\right](1+\alpha)}}. \tag{17}$$

From the left panel of Fig. 7, one sees that both VG-GCG and VG-GCG behave like cold dark matter with almost zero EOS at early time, and behaves like constant-EOS-type dark energy in the distant future. And for $\beta < 0$ or $\beta > 4$ (nonphysical), the quintom model crossing $w = -1$ can be realized in the VG-GCG model. The VG-MCG model, however, requires the relation $-\beta^2 + 4\beta < 6B(1+\beta)$ to obtain the quintom scenario.

Adiabatic sound speed for the VG-GCG and VG-MCG fluid are described

$$c^2_{s,\mathrm{VG-GCG}} = -\alpha w_{\mathrm{VG-GCG}} \tag{18}$$

$$c^2_{s,\mathrm{VG-MCG}} = -\alpha w_{\mathrm{VG-MCG}} + (1+\alpha)B. \tag{19}$$

The right panel of Fig. 7 shows that the value of the sound speed of ECG in the VG theory is a small positive number at high redshift. The small values of c_s^2 make it possible to form large-scale structures.

V. CONCLUSIONS

Many observations indicate that the gravitational constant G may be variable, not a constant. In this paper we investigate the unified model of dark matter and dark energy in the framework of varying gravitational constant, by considering the extended Chaplygin gas as the background fluid which can be induced from the variation of G. Usually, VG cosmology were studied with using a standard Friedmann equation. Here, we consider a modified Friedmann equation with a corrected extra term βH^2. In addition, a different conservation equation is used with respect to the constant-G theory. Correspondingly, a modified expression of ECG energy density is given in this paper, too. Including the unified background fluid of dark matter and dark energy, we apply recently observed data to constrain the varying gravitational constant theory. It is shown that the evolutionary behaviors of $\frac{\dot{G}}{G}$ and the current value of variable gravitational constant $(\frac{\dot{G}}{G})_{\mathrm{today}}$ are comparable with several cosmic experiment explorations. In varying-G theory, one finds that the MCG model is close to the GCG model for a very small values of B. Also, the VG-GCG and VG-MCG models match the ΛCDM model ($\alpha = 0$, $\beta = 0$ and $B = 0$) well; i.e., in the VG theory the observational data points still cannot distinguish the VG-GCG and VG-MCG models from the standard ΛCDM model. The VG-CG model is not favored by the cosmic data, according to the calculation value of χ^2.

By analyzing the behaviors of geometry quantities—the Hubble parameter and deceleration parameter—one can receive the result that $\frac{\dot{G}}{G}$ and $\frac{\ddot{G}}{G}$ would have the opposite symbol for $z > 0$, and their symbols depend on the symbols of β. For using the mean values of the VG-ECG model, behaviors of $\frac{\dot{G}}{G}$ exhibit an increasing G with the time. And in future ($z < 0$), $\frac{\ddot{G}}{G}$ ($\simeq 0$) and $\frac{\dot{G}}{G}$ (> 0) are almost constants. The physical properties of the VG-ECG fluid are described by its equation of state w and adiabatic sound speed c_s^2. For w, we can see that VG-ECG behaves like cold dark matter with almost zero EOS at early time, and behaves like constant-EOS-type dark energy in the distant future. For $\beta < 0$, the quintom model crossing $w = -1$ can be realized in the VG-GCG model. And adiabatic sound speed for the VG-ECG fluid shows that the value of the sound speed is a small positive number at the early universe. This small value of c_s^2 makes it possible to form large-scale structures in our Universe.

ACKNOWLEDGMENTS

We thank the reviewer for very instructive comments, which have helped us greatly and improved our paper. The research work is supported by the National Natural Science Foundation of China (No. 11205078, No. 11275035, No. 11175077), the Natural Science Foundation of Liaoning Province in China (No. L2011189, No. 905035, No. 905061, No. 901142).

APPENDIX A: THE CONSERVATION EQUATION OF ENERGY DENSITY IN VARIABLE GRAVITY CONSTANT THEORY

The contraction of Bianchi identity gives the result $\nabla_\mu(R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu}) = 0$, i.e., the vanishing of the covariant divergence of the Einstein tensor $G^{\mu\nu}_{;\mu} = 0$, which is a natural result from theory of the curvature geometry. According to the Einstein equation (2), for a variable gravity constant G the energy conservation equation can be given by

$$(GT^{\mu\nu} + W^{\mu\nu})_{;\mu} = 0, \tag{A1}$$

where $W^{\mu\nu} \equiv G(\nabla^\mu\partial^\nu G^{-1} - g^{\mu\nu}\nabla_\sigma\partial^\sigma G^{-1})$. The energy-momentum tensor of perfect fluid is written as $T^{\mu\nu} = pg^{\mu\nu} + (p+\rho)U^\mu U^\nu$. Using the formulas $T^{\alpha\cdots\beta}_{\delta\cdots\sigma;\eta} = T^{\alpha\cdots\beta}_{\delta\cdots\sigma,\eta} + \Gamma^\alpha_{\tau\eta}T^{\tau\cdots\beta}_{\delta\cdots\sigma}\cdots + \Gamma^\beta_{\tau\eta}T^{\alpha\cdots\tau}_{\delta\cdots\sigma} - \Gamma^\tau_{\delta\eta}T^{\alpha\cdots\beta}_{\tau\cdots\sigma}\cdots - \Gamma^\tau_{\sigma\eta}T^{\alpha\cdots\beta}_{\delta\cdots\tau}$, and $\Gamma^\sigma_{\mu\nu} = \frac{1}{2}g^{\sigma\lambda}(g_{\lambda\mu,\nu} + g_{\nu\lambda,\mu} - g_{\mu\nu,\lambda})$, we have

$$\dot{\rho} + 3H\left(\rho + \frac{2+2\beta}{2+\beta}p\right) = \frac{\beta-\beta^2}{2+\beta}H\rho. \tag{A2}$$

For $\beta = 0$, conservation equation (A2) is reduced to the standard form in constant G theory. Comparing with the standard form, one can see that the variation of G introduces two additional terms. The term $\frac{\beta-\beta^2}{2+\beta}H\rho$ on the right-hand side in Eq. (A2) can be interpreted as the interaction between universal matters, and $\frac{\beta-\beta^2}{2+\beta}$ denotes the interacting strength. Equation (A2) is also equivalent to the following expressions,

$$\dot{\rho}_{\mathrm{eff}} + 3H(\rho_{\mathrm{eff}} + p_{\mathrm{eff}}) = 0, \tag{A3}$$

with effective density and effective pressure as follows,

$$\rho_{\rm eff} = \frac{a^{-\beta}}{1+\beta}\rho \tag{A4}$$

$$p_{\rm eff} = \frac{2a^{-\beta}}{2+\beta}p + \frac{2\beta^2+\beta}{3(2+\beta)}\rho_{\rm eff}. \tag{A5}$$

APPENDIX B: METHODS OF FITTING DATA

Here we describe the cosmic data used, including the BAO, $f_{\rm gas}$, CMB, SNIa, and H(z) data. First, we introduce three distance parameters in the following. $D_A(z)$ is the proper angular diameter distance,

$$D_A(z) = \frac{c}{(1+z)\sqrt{|\Omega_k|}}\,{\rm sinn}\left[\sqrt{|\Omega_k|}\int_0^z \frac{dz'}{H(z')}\right]. \tag{B1}$$

It relates to two other distance quantities D_L and D_V by

$$D_L(z) = \frac{H_0}{c}(1+z)^2 D_A(z) \tag{B2}$$

$$D_V(z) = \left[(1+z)^2 D_A^2(z)\frac{cz}{H(z;p_s)}\right]^{1/3} = H_0\left[\frac{z}{E(z;p_s)}\left(\int_0^z \frac{dz'}{E(z';p_s)}\right)^2\right]^{\frac{1}{3}}. \tag{B3}$$

p_s denotes the theoretical model parameters, $\sin n(\sqrt{|\Omega_k|}x)$, respectively, denotes $\sin(\sqrt{|\Omega_k|}x)$, $\sqrt{|\Omega_k|}x$, and $\sinh(\sqrt{|\Omega_k|}x)$ for $\Omega_k<0$, $\Omega_k=0$ and $\Omega_k>0$.

1. BAO

BAO data are extracted from the WiggleZ Dark Energy Survey (WDWS) [33], the Two Degree Field Galaxy Redshift Survey (2dFGRS) [34] and the Sloan Digitial Sky Survey (SDSS) [35]. $\chi^2_{\rm BAO}(p_s)$ is [49]

$$\chi^2_{\rm BAO}(p_s) = X^t V^{-1} X, \tag{B4}$$

with

$$V^{-1} = \begin{pmatrix} 4444 & 0 & 0 & 0 & 0 & 0\\ 0 & 30318 & -17312 & 0 & 0 & 0\\ 0 & -17312 & 87046 & 0 & 0 & 0\\ 0 & 0 & 0 & 23857 & -22747 & 10586\\ 0 & 0 & 0 & -22747 & 128729 & -59907\\ 0 & 0 & 0 & 10586 & -59907 & 125536 \end{pmatrix},\quad X = \begin{pmatrix} \frac{r_s(z_d)}{D_V(0.106)} - 0.336\\ \frac{r_s(z_d)}{D_V(0.2)} - 0.1905\\ \frac{r_s(z_d)}{D_V(0.35)} - 0.1097\\ \frac{r_s(z_d)}{D_V(0.44)} - 0.0916\\ \frac{r_s(z_d)}{D_V(0.6)} - 0.0726\\ \frac{r_s(z_d)}{D_V(0.73)} - 0.0592 \end{pmatrix}. \tag{B5}$$

V^{-1} is the inverse covariance matrix [35,50]. X is a column vector formed from theoretical values minus observational values, and X^t denotes its transpose. $r_s(z)$ is the comoving sound horizon size $r_s(z) = c\int_0^t \frac{c_s dt}{a} = \frac{c}{\sqrt{3}}\int_0^{1/(1+z)} \times \frac{da}{a^2H(a)\sqrt{1+3a\Omega_b/(4\Omega_\gamma)}}$. c_s is the sound speed of the photon -baryon fluid, $c_s^{-2} = 3+\frac{4}{3}\times(\frac{\Omega_b}{\Omega_\gamma})a$ with $\Omega_\gamma = 2.469\times 10^{-5}h^{-2}$. z_d denotes the drag epoch, $z_d = \frac{1291(\Omega_{0m}h^2)^{-0.419}}{1+0.659(\Omega_{0m}h^2)^{0.828}}\times[1+b_1(\Omega_b h^2)^{b_2}]$ with $b_1 = 0.313(\Omega_{0m}h^2)^{-0.419}[1+0.607(\Omega_{0m}h^2)^{0.674}]$ and $b_2 = 0.238(\Omega_{0m}h^2)^{0.223}$.

2. XGMF

In observation of XGMF, for the reference model ΛCDM, parameter f_{gas} is presented as [36]

$$f_{\rm gas}^{\Lambda{\rm CDM}}(z) = \frac{KA\gamma b(z)}{1+s(z)}\left(\frac{\Omega_b}{\Omega_{0m}}\right)\left[\frac{D_A^{\Lambda{\rm CDM}}(z)}{D_A(z)}\right]^{1.5}. \tag{B6}$$

A is the angular correction factor, $A = (\frac{H(z)D_A(z)}{[H(z)D_A(z)]^{\Lambda{\rm CDM}}})^\eta$. Index η is the slope of $f_{\rm gas}(r/r_{2500})$ data with $\eta = 0.214\pm0.022$ [36]. Parameter γ denotes permissible departures from the assumption of hydrostatic equilibrium. Bias factor $b(z) = b_0(1+\alpha_b z)$ accounts for uncertainties in the cluster depletion factor. $s(z) = s_0(1+\alpha_s z)$ accounts for uncertainties of the baryonic mass fraction in stars and a Gaussian prior for s_0 is employed, with $s_0 = (0.16\pm0.05)h_{70}^{0.5}$ [36]. Factor K is used to describe the combined effects of the residual uncertainties, such as the instrumental calibration and certain x-ray modeling issues, and a Gaussian prior for the "calibration" factor is considered by $K = 1.0\pm0.1$ [36]. Adopting the data points published in Ref. [36] and following the method in Ref. [36], χ^2 for the x-ray gas mass fraction analysis is expressed as

$$\chi^2_{f_{\rm gas}} = \sum_{i=1}^{42}\frac{[f_{\rm gas}^{\Lambda{\rm CDM}}(z_i) - f_{gas}(z_i)]^2}{\sigma^2_{f_{\rm gas}}(z_i)} + \frac{(s_0-0.16)^2}{0.0016^2} + \frac{(K-1.0)^2}{0.01^2} + \frac{(\eta-0.214)^2}{0.022^2}, \tag{B7}$$

TABLE IV. H(z) data with their errors at different redshifts (in units [$\mathrm{km\,s^{-1}\,Mpc^{-1}}$]) [39–43].

z	0.09	0.17	0.179	0.199	0.27	0.352	0.4	0.48	0.593	0.68	0.78	0.875	0.88	0.9	1.037	1.3	1.43	1.53	1.75	0.24	0.34	0.43
H	69	83	75	75	77	83	95	97	104	92	105	125	90	117	154	168	177	140	202	79.69	83.8	86.45
σ	12	8	4	5	14	14	17	62	13	8	12	17	40	23	20	17	18	14	40	2.32	2.96	3.27

where $\sigma_{f_{\rm gas}}(z_i)$ is the statistical uncertainties. As pointed out in [36], the acquiescent systematic uncertainties have been considered according to the parameters η, $b(z)$, $s(z)$ and K.

3. CMB

For CMB constraint, $\chi^2_{\rm CMB}$ is [51]

$$\chi^2_{\rm CMB}(p_s) = \Delta d_i[{\rm Cov}^{-1}(d_i(p_s), d_j(p_s))][\Delta d_i]^t, \quad \text{(B8)}$$

where $\Delta d_{i,{\rm CMB}}(p_s) = d^{\rm theory}_{i,{\rm CMB}}(p_s) - d^{\rm obs}_{i,{\rm CMB}}$ is a row vector. According to the 9-year WMAP survey the value of $d^{\rm obs}_{i,{\rm CMB}} = [l_A(z_*) = 302.04; R(z_*) = 1.7246; z_* = 1090.88]$, and the corresponding inverse covariance matrix is [37]

$${\rm Cov}^{-1} = \begin{pmatrix} 3.182 & 18.253 & -1.429 \\ 18.253 & 11887.879 & -193.808 \\ -1.429 & -193.808 & 4.556 \end{pmatrix}. \quad \text{(B9)}$$

Concretely, z_* is the redshift at decoupling epoch of photons given by $z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}] \times [1 + g_1(\Omega_{0m}h^2)^{g_2}]$ with $g_1 = 0.0783(\Omega_b h^2)^{-0.238} \times (1 + 39.5(\Omega_b h^2)^{0.763})^{-1}$ and $g_2 = 0.560(1 + 21.1(\Omega_b h^2)^{1.81})^{-1}$; $l_A(p_s; z_*)$ is the acoustic scale $l_A(p_s; z_*) = (1 + z_*)\frac{\pi D_A(p_s; z_*)}{r_s(z_*)}$; $R(p_s; z_*)$ is the CMB shift parameter $R(p_s; z_*) = \sqrt{\Omega_{0m} H_0^2}(1 + z_*)D_A(p_s; z_*)/c$.

4. SNIa

Cosmic constraint from SNIa observation can be determined by a calculation on the likelihood [52–62]

$$\chi^2_{\rm SNIa}(p_s) \equiv \sum_{i=1}^{557} \frac{\{\mu_{\rm th}(p_s, z_i) - \mu_{\rm obs}(z_i)\}^2}{\sigma^2_{\mu_i}}. \quad \text{(B10)}$$

$\mu_{\rm th}(z)$ is the theoretical distance modulus $\mu_{\rm th}(z) = 5\log_{10}[D_L(z)] + \frac{15}{4}\log_{10}\frac{G}{G_0} + \mu_0$ with the Hubble-free luminosity distance $D_L(z)$ and $\mu_0 = 5\log_{10}(\frac{H_0^{-1}}{\rm Mpc}) + 25 = 42.38 - 5\log_{10}h$. $\mu_{\rm obs}(z_i)$ is the observed distance moduli at different redshift z_i, which can be given by SNIa observation data sets [38].

5. H(z) data

Using the H(z) data listed in Table IV, the model parameters are determined by minimizing [63–74]

$$\chi^2_H(H_0, p_s) = \sum_{i=1}^{24} \frac{[H_{\rm th}(H_0, p_s; z_i) - H_{\rm obs}(z_i)]^2}{\sigma^2_H(z_i)}, \quad \text{(B11)}$$

where $H_{\rm th}$ is the predicted value for the Hubble parameter and $H_{\rm obs}$ is the observed value.

[1] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).

[2] J. D. Barrow and S. Cotsakis, Phys. Lett. B **214**, 515 (1988); P. X. Wu and H. W. Yu, Phys. Lett. B **703**, 223 (2011).

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000); L. X. Xu and Y. T. Wang, Phys. Rev. D **82**, 043503 (2010).

[4] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988); M. Li, Phys. Lett. B **603**, 1 (2004).

[5] R. Nakamura, M. Hashimoto, S. Gamow, and K. Arai, Astron. Astrophys. **448**, 23 (2006); S. Das and N. Banerjee, Phys. Rev. D **78**, 043512 (2008); X. Li, C. Sun, and P. Xi, J. Cosmol. Astropart. Phys. 04 (2009) 015; H. Farajollahi, M. Farhoudi, A. Salehi, and H. Shojaie, Astrophys. Space Sci. **337**, 415 (2012); H. W. Lee, K. Y. Kim, and Y. S. Myung, Eur. Phys. J. C **71**, 1585 (2011); L. X. Xu, J. B. Lu, and W. B. Li, Eur. Phys. J. C **64**, 89 (2009); R. G. Cai, Phys. Lett. B **657**, 228 (2007); C. Feng, B. Wang, Y. Gong, and R. K. Su, J. Cosmol. Astropart. Phys. 09 (2007) 005; C. J. Feng, Phys. Lett. B **670**, 231 (2008).

[6] M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **66**, 043507 (2002).

[7] L. X. Xu, J. B. Lu, and Y. T. Wang, Eur. Phys. J. C **72**, 1883 (2012); Z. H. Zhu, Astron. Astrophys. **423**, 421 (2004); P. X. Wu and H. W. Yu, Phys. Lett. B **644**, 16 (2007).

[8] J. Lu, L. Xu, Y. Wu, and M. Liu, Gen. Relativ. Gravit. **43**, 819 (2011).

[9] R. Jackiw, arXiv:physics/0010042; P. Pedram and S. Jalalzadeh, arXiv:0711.1996.

[10] M. Bouhmadi-Lopez and P. V. Moniz, Phys. Rev. D **71**, 063521 (2005).

[11] A. Ali, S. Dutta, E. N. Saridakis, and A. A. Sen, Gen. Relativ. Gravit. **44**, 657 (2012).

[12] B. C. Paul and P. Thakur, arXiv:1205.2796.

[13] C. Ranjit, P. Rudra, and S. Kundu, Astrophys. Space Sci. **347**, 423 (2013).
[14] S. Ghose, A. Saha, and B. C. Paul, arXiv:1203.2113.
[15] K. I. Umezu, K. Ichiki, and M. Yahiro, Phys. Rev. D **72**, 044010 (2005); S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **73**, 103511 (2006).
[16] C. J. Copi, A. N. Davis, and L. M. Krauss, Phys. Rev. Lett. **92**, 171301 (2004).
[17] M. Biesiada and B. Malec, Mon. Not. R. Astron. Soc. **350**, 644 (2004).
[18] O. G. Benvenuto, E. Garcia-Berro, and J. Isern, Phys. Rev. D **69**, 082002 (2004).
[19] E. Gaztanaga, E. Garcia-berro, J. Isern, E. Bravo, and I. Dominguez, Phys. Rev. D **65**, 023506 (2001).
[20] T. Damour, G. W. Gibbons, J. H. Taylor, Phys. Rev. Lett., **61**, 1151 (1988).
[21] I. Goldman, Phys. Lett. B **281**, 219 (1992).
[22] M. Jamil, F. Rahaman, and M. Kalam, Eur. Phys. J. C **60**, 149 (2009)
[23] S. DeglInnocenti, G. Fiorentini, G. G. Raffelt, B. Ricci, and A. Weiss, Astron. Astrophys. **312**, 345 (1996).
[24] K. Saaidi, A. Aghamohammadi, and M. R. Setare, Astrophys. Space Sci. **332**, 503 (2011).
[25] M. Jamil, E. N. Saridakis, and M. R. Setare, Phys. Lett. B **679**, 172 (2009).
[26] F. Adabi, K. Karami, F. Felegary, and Z. Azarmi, Res. Astron. Astrophys. **12**, 26 (2012).
[27] J. B. Lu, E. N. Saridakis, M. R. Setare, and L. X. Xu, J. Cosmol. Astropart. Phys. 03 (2010) 031.
[28] N. Banerjee and D. Pavon, Phys. Rev. D **63**, 043504 (2001).
[29] A. A. Sen, S. Sen, and S. Sethi, Phys. Rev. D **63**, 107501 (2001); J. C. Fabris, S. V. B. Goncalves, and R. de Sa Ribeiro, Gravitation Cosmol. **12**, 49 (2006); S. Sen and T. R. Seshadri, Int. J. Mod. Phys. D **12**, 445 (2003); B. K. Sahoo, and L. P. Singh, Mod. Phys. Lett. A **18**, 2725 (2003); O. Bertolami and P. J. Martins, Phys. Rev. D **61**, 064007 (2000).
[30] Y. Pan, S. Cao, Y. G. Gong, K. Liao, and Z. H. Zhu, Phys. Lett. B **718**, 699 (2013); M. Sharif and A. Jawad, Eur. Phys. J. C **72**, 2097 (2012); H. Amirhashchi, A. Pradhan, and H. Zainuddin, Res. Astron. Astrophys. **13**, 129 (2013); J. B. Lu, S. S. Gao, Y. Y. Zhao, and Y. B. Wu, Eur. Phys. J. Plus **127**, 154 (2012).
[31] B. C. Paul, P. Thakur, and A. Saha, arXiv:0809.3491.
[32] A. Lewis and S. Bridle, Phys. Rev. D **66**, 103511 (2002).
[33] C. Blake *et al.*, Mon. Not. R. Astron. Soc. **418**, 1707 (2011).
[34] F. Beutler, C. Blake, M. Colless, D. Heath Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders, and F. Watson, Mon. Not. R. Astron. Soc. **416**, 3017 (2011).
[35] W. J. Percival *et al.*, Mon. Not. R. Astron. Soc. **401**, 2148 (2010).
[36] S. W. Allen, D. A. Rapetti, R. W. Schmidt, H. Ebeling, R. G. Morris, and A. C. Fabian, Mon. Not. R. Astron. Soc. **383**, 879 (2008).
[37] G. Hinshaw *et al.*, Astrophys. J. Suppl. Ser. **208**, 19 (2013).
[38] R. Amanullah *et al.* (Supernova Cosmology Project Collaboration), Astrophys. J. **716**, 712 (2010).
[39] J. Simon, L. Verde, and R. Jimenez, Phys. Rev. D **71**, 123001 (2005).
[40] D. Stern, R. Jimenez, L. Verde, M. Kamionkowski, and S. A. Stanford, arXiv:0907.3149.
[41] J. Simon, L. Verde, and R. Jimenez, Phys. Rev. D **71**, 123001 (2005).
[42] M. Moresco, A. Cimatti, R. Jimenez *et al.*, J. Cosmol. Astropart. Phys. 08 (2012) 006.
[43] E. Gaztañaga, A. Cabré, and L. Hui, Mon. Not. R. Astron. Soc. **399**, 1663 (2009).
[44] H. Alavirad and M. Malekjani, Astrophys. Space Sci. **349**,967 (2014).
[45] J. Muller, M. Schneider, M. Soffel, and H. Ruder, Astrophys. J. **382**, L101 (1991).
[46] S. E. Thorsett, Phys. Rev. Lett. **77**, 1432 (1996).
[47] J. P. W. Verbiest, M. Bailes, W. van Straten, G. B. Hobbs, R. T. Edwards, R. N. Manchester, N. D. R. Bhat, J. M. Sarkissian, B. A. Jacoby, and S. R. Kulkarni, Astrophys. J. **679**, 675 (2008).
[48] E. Garcia-Berro *et al.*, Mon. Not. R. Astron. Soc. **277**, 801 (1995).
[49] Y. G. Gong, B. Wang, and R. G. Cai, J. Cosmol. Astropart. Phys. 04 (2010) 019.
[50] S. Nesseris and J. Garcia-Bellido, J. Cosmol. Astropart. Phys. 11 (2012) 033.
[51] R. G. Cai, Q. Su, and H. B. Zhang, arXiv:1001.2207.
[52] V. Acquaviva and L. Verde, J. Cosmol. Astropart. Phys. 12 (2007) 001.
[53] R. G. Cai, Z. L. Tuo, Y. B. Wu, and Y. Y. Zhao, Phys. Rev. D **86** (2012) 023511.
[54] Y. G. Gong, R. G. Cai, Y. Chen, and Z. H. Zhu, J. Cosmol. Astropart. Phys. 01 (2010) 019.
[55] A. C. C. Guimaraes, J. V. Cunha, and J. A. S. Lima, J. Cosmol. Astropart. Phys. 10 (2009) 010.
[56] M. Szydlowski and W. Godlowski, Phys. Lett. B **633**, 427 (2006).
[57] L. X. Xu, Phys. Rev. D **87**, 043503 (2013).
[58] L. X. Xu, Phys. Rev. D **87**, 043525 (2013).
[59] J. B. Lu, Y. B. Wu, M. L. Liu, and S. S. Gao, Gen. Relativ. Gravit. **45**, 2023 (2013).
[60] R. Gannouji and D. Polarski, J. Cosmol. Astropart. Phys. 05 (2008) 018.
[61] R. G. Cai, Q. Su, and H. B. Zhang, J. Cosmol. Astropart. Phys. 04 (2010) 012.
[62] U. Alam and V. Sahni, Phys. Rev. D **73**, 084024 (2006).
[63] R. Lazkoz and E. Majerotto, J. Cosmol. Astropart. Phys. 07 (2007) 015.
[64] R. G. Cai and Q. P. Su, Phys. Rev. D **81**, 103514 (2010).
[65] J. B. Lu, L. X. Xu, and M. L. Liu, Phys. Lett. B **699**, 246 (2011).
[66] M. Moresco, L. Verde, L. Pozzetti, R. Jimenez, and A. Cimatti, J. Cosmol. Astropart. Phys. 07 (2012) 053.
[67] G. Chen and B. Ratra, Publ. Astron. Soc. Pac. **123**, 1127 (2011).
[68] E. Calabrese, M. Archidiacono, A. Melchiorri, and B. Ratra, Phys. Rev. D **86**, 043520 (2012).

[69] L. P. Chimento and M. G. Richarte, Phys. Rev. D **86**, 103501 (2012).

[70] L. P. Chimento, M. I. Forte, and M. G. Richarte, Eur. Phys. J. C **73**, 2285, (2013).

[71] L. P. Chimento and M. G. Richarte, Eur. Phys. J. C **73**, 2352 (2013).

[72] L. P. Chimento, R. Lazkoz, and I. Sendra, Gen. Relativ. Gravit. **42**, 1189 (2010).

[73] O. Farooq and B. Ratra, Astrophys. J. **766**, L7 (2013).

[74] J. B. Lu, L. X. Xu, Y. B. Wu, M. L. Liu, and T. Q. Li, Int. J. Mod. Phys. D **22**, 1350059 (2013).