

Large-scale structure in superfluid Chaplygin gas cosmologyRongjia Yang^{*}*College of Physical Science and Technology, Hebei University, Baoding 071002, China;
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(Received 11 December 2013; revised manuscript received 13 February 2014; published 31 March 2014)

We investigate the growth of the large-scale structure in the superfluid Chaplygin gas (SCG) model. Both linear and nonlinear growth, such as σ_8 and the skewness S_3 , are discussed. We find the growth factor of SCG reduces to the Einstein–de Sitter case at early times while it differs from the cosmological constant model (Λ CDM) case in the large a limit. We also find there will be more structure growth on large scales in the SCG scenario than in Λ CDM and the variations of σ_8 and S_3 between SCG and Λ CDM cannot be discriminated.

DOI: 10.1103/PhysRevD.89.063014

PACS numbers: 95.36.+x, 98.65.Dx, 98.80.-k

I. INTRODUCTION

A wealth of astronomical observations indicated that the Universe is experiencing accelerated expansion. Assuming the validity of general relativity on large scales, an unknown energy component, the dark energy, is usually introduced to explain the accelerated expansion. The simplest candidate is the cosmological constant model (Λ CDM) that is consistent with most of the current astronomical observations but suffers from the cosmological constant problem [1] and age problem [2]. It is thus natural to study more complicated cases, such as models allowing a time evolution of the dark energy component: quintessence, phantom, k -essence, quintom, tachyon, and other scalar fields. Recently, superfluid Chaplygin gas (SCG), a model unified dark matter and dark energy, was proposed [3]. SCG involves the Bose-Einstein condensate as dark energy possessing the equation of state (EoS) of Chaplygin gas, and an excited state acts as dark matter. Though the condensate behaves like Chaplygin gas [4], the evolution of the Universe provided by SCG is different from that in the model with Chaplygin gas as dark energy as well as from that in the model with Chaplygin gas unified dark matter and dark energy [3]. In Chaplygin gas unified dark matter and dark energy, the negligible sound speed that produces unphysical oscillations and an exponential blowup in the dark matter power spectrum at present [5]. This problem was solved for the generalized Chaplygin gas model by decomposing the energy density into dark matter and dark energy [6]. Recently, observational constraints on the generalized Chaplygin model have been considered in [7]. SCG has been analyzed from the point of view of state finder [8] and has been constrained by using observational data [9].

Dark energy can affect the expansion rate causing geometrical effects that can be revealed through distance measurements, and can also affect structure formation that can be quantified by the growth factor. Thus, structure formation will be affected by the amount of dark energy and by its dynamical evolution over cosmological history. In this paper we study the large-scale structure growth in the SCG scenario from the point of view of gravitational collapse. The linear and nonlinear growth can be related to observations and hence lead to observational constraints, such as the skewness of the density field and the rms fluctuations on a sphere of $8 \text{ Mpc } h^{-1}$ [10]. We probe the significance of the SCG dark energy component throughout the epoch during which large-scale structures grow.

The rest of the paper is organized as follows. In Sec. II, we sketch the derivation of the spherical collapse formalism. In Sec. III, we review the SCG scenario. In Sec. IV, we investigate gravitational growth and the large structure in SCG cosmology. Conclusions and discussions are presented in Sec. IV.

II. GRAVITATIONAL COLLAPSE

To study the gravitational growth of the large-scale structure in SCG, we first briefly describe the usual formalism. The nonlinear equation for dust has been used in the context of structure formation [11,12] and for the study of the spherical and ellipsoidal collapse [13,14]. The linearized equation was presented in [15] for dust and relativistic matter and in [16] for a general model with constant EoS. The spherical collapse model has been considered in [17,18] for several dark energy scenarios and in [19,20] for nonstandard cosmologies and generalized Chaplygin gas (GCG) dark energy.

We consider a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) universe with scalar factor a ,

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$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 + Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (1)$$

where the spatial curvature constant $K = +1, 0,$ and -1 correspond to a closed, flat, and open universe, respectively. We define $k^2 = 8\pi G$ and use the units $c = 1$ throughout this paper.

To obtain the equation of motion for the density contrast, $\delta = \rho/\bar{\rho} - 1$ with $\bar{\rho}$ the mean background energy density, we resort to the Raychaudhuri equation, which in general takes the form

$$\dot{\Theta} + \frac{1}{3}\Theta = \omega_{\mu\nu}\omega^{\mu\nu} - \sigma_{\mu\nu}\sigma^{\mu\nu} + R_{\mu\nu}u^\mu u^\nu, \quad (2)$$

where u^μ is the fluid's 4-velocity, $\Theta \equiv \nabla^\mu u_\mu$, $\sigma_{\mu\nu}$ is the shear tensor, and $\omega_{\mu\nu}$ is the vorticity tensor. For a shear-free and nonrotating fluid, $\sigma_{\mu\nu} = 0$ and $\omega_{\mu\nu} = 0$, which is the case we are interested in here, and assuming that the geometry of the Universe is of the FLRW form and then $\Theta = 3H$ with H the local Hubble rate, we get

$$\dot{\Theta} + \frac{1}{3}\Theta^2 = 3(H^2 + \dot{H}). \quad (3)$$

Choosing the coordinate system such that the 4-velocity of the fluid is $u^\mu = (1, \dot{a}\vec{x} + \vec{v})$ with \vec{v} the peculiar velocity, we have

$$\Theta \equiv \nabla^\mu u_\mu = \frac{1}{a}\nabla \cdot (\dot{a}\vec{x} + \vec{v}) = 3\frac{\dot{a}}{a} + \frac{\theta}{a} = 3\bar{H} + \frac{\theta}{a}, \quad (4)$$

where $\theta \equiv \nabla \cdot \vec{v}$ and \bar{H} is the background Hubble rate. $\nabla_x = \frac{1}{a}\nabla_X$ with X the Friedman coordinate. In the FLRW spacetime therefore, the Raychaudhuri equation can be rewritten as

$$3(\bar{H}^2 + \dot{\bar{H}}) + \bar{H}\frac{\theta}{a} + \frac{\dot{\theta}}{a} + \frac{1}{3}\frac{\theta^2}{a^2} = 3(H^2 + \dot{H}). \quad (5)$$

Terms on the left hand side of Eq. (5) are composed of background quantities, while terms on the right hand side of Eq. (3) are composed of local perturbed quantities. Equation (5) can be reexpressed as

$$\begin{aligned} \bar{H}\frac{\theta}{a} + \frac{\dot{\theta}}{a} + \frac{1}{3}\frac{\theta^2}{a^2} &= 3(H^2 + \dot{H}) - 3(\bar{H}^2 + \dot{\bar{H}}) \\ &= -\frac{1}{2}k^2[(\rho - \bar{\rho}) + 3(P - \bar{P})]. \end{aligned} \quad (6)$$

To obtain Eq. (6), nothing but a FLRW spacetime filled with a perfect fluid is assumed; hence it can be applied to nonstandard cosmologies. In particular, no connection between the energy content and the geometry has been assumed, so we can study the evolution of the density perturbation by means of the continuity equation and the

Friedmann equation, while not needing to use Einstein's equations.

In a matter-dominated universe, the continuity equation for a nonrelativistic fluid ($\rho \gg P$) is given by [21]

$$\frac{d}{d\tau}\delta(\vec{x}, \tau) + \nabla \cdot [(1 + \delta(\vec{x}, \tau))\vec{v}(\vec{x}, \tau)] = 0, \quad (7)$$

where $d\tau = \frac{1}{a}dt$ is the conformal time. This continuity equation can also be written as

$$\frac{d\delta}{d\tau} + (1 + \delta)\theta = 0. \quad (8)$$

Then Eq. (6) can be rewritten as

$$\begin{aligned} \frac{d^2\delta}{d\tau^2} + \dot{a}\frac{d\delta}{d\tau} - \frac{4}{3}\frac{1}{1 + \delta}\left(\frac{d\delta}{d\tau}\right)^2 &= -3a^2(1 + \delta)[(H^2 + \dot{H}) \\ &\quad - (\bar{H}^2 + \dot{\bar{H}})]. \end{aligned} \quad (9)$$

Re-scaling the time variable, $\eta = \ln a$, we obtain

$$\begin{aligned} \frac{d^2\delta}{d\eta^2} + \left(2 + \frac{\dot{H}}{\bar{H}^2}\right)\frac{d\delta}{d\eta} - \frac{4}{3}\frac{1}{1 + \delta}\left(\frac{d\delta}{d\eta}\right)^2 \\ = -3(1 + \delta)\frac{(H^2 + \dot{H}) - (\bar{H}^2 + \dot{\bar{H}})}{\bar{H}^2}. \end{aligned} \quad (10)$$

Expand the $H^2 + \dot{H}$ term in terms of δ and then the whole right hand side of Eq. (10) as

$$3(1 + \delta)\frac{(H^2 + \dot{H}) - (\bar{H}^2 + \dot{\bar{H}})}{\bar{H}^2} \equiv 3(1 + \delta)\sum_{n=1} c_n \delta^n. \quad (11)$$

To study how a small perturbation grows with time at different orders in perturbation theory, we expand δ as

$$\delta = \sum_{i=1}^{\infty} \delta^i = \sum_{i=1}^{\infty} \frac{D_i(\eta)}{i!} \delta_0^i, \quad (12)$$

where δ_0 is the small perturbation. According to this equation and expanding the perturbation in Eq. (11), we obtain the linear equation

$$D_1'' + \left(2 + \frac{\dot{H}}{\bar{H}^2}\right)D_1' + 3c_1 D_1 = 0, \quad (13)$$

where the prime denotes a derivative with respect to the natural logarithm of the scale factor. For Λ CDM, the linear equation (13) reduces to

$$D_1'' + \left[2 - \frac{3}{2}\frac{\Omega_{m0}}{\Omega_{m0} + \Omega_{\Lambda 0}a^3}\right]D_1' - \frac{3}{2}\frac{\Omega_{m0}}{\Omega_{m0} + \Omega_{\Lambda 0}a^3}D_1 = 0, \quad (14)$$

where $\Omega_{m0} \equiv 8\pi G\rho_0/(3H_0^2)$ and $\Omega_{\Lambda 0} \equiv \Lambda/(3H_0^2)$. At early times, $a \ll 1$, the Λ CDM universe reduces to the Einstein–de Sitter (EdS) universe ($\Omega_{\Lambda 0} = 0$) with solutions

$$D_1 = C_1 a + C_2 a^{-3/2}; \quad (15)$$

here and in the following content C_i are integral constants. These solutions reproduce the usual linear growth $D_1 \sim a$ and the decaying solutions $D_1 \sim a^{-3/2}$. The second order equation is found to be

$$D_2'' + \left(2 + \frac{\dot{H}}{H^2}\right) D_2' - \frac{8}{3} (D_1')^2 + 3c_1 D_2 + 6(c_1 + c_2) D_1^2 = 0. \quad (16)$$

One can go on to arbitrary order by using the solutions to the lower order equations. For Λ CDM, the second order equation (16) reduces to

$$D_2'' + \left[2 - \frac{3}{2} \frac{\Omega_{m0}}{\Omega_{m0} + \Omega_{\Lambda 0} a^3}\right] D_2' - \frac{8}{3} (D_1')^2 - \frac{3}{2} \frac{\Omega_{m0}}{\Omega_{m0} + \Omega_{\Lambda 0} a^3} D_2 - \frac{3\Omega_{m0}}{\Omega_{m0} + \Omega_{\Lambda 0} a^3} D_1^2 = 0. \quad (17)$$

For Gaussian initial conditions, the skewness of the density field at large scale is related to the second-order equations, defined as

$$S_3 = 3 \frac{D_2}{D_1^2}. \quad (18)$$

For an EdS universe the skewness can be calculated analytically: $S_3^{\text{EdS}} = \frac{34}{7}$.

III. SUPERFLUID CHAPLYGIN GAS MODEL WITH FLRW SPACETIME

It was argued in [3] that SCG does not depend on details of microscopic structure of the quantum liquid and then capitalizes on effective macroscopic quantities. SCG involves two independent flows: the coherent motion of the ground state named superfluid component, and a normal component produced by the quasiparticle gas. The particle number current and the energy-momentum tensor are, respectively, represented as

$$n^\mu = n_c V^\mu + n_n U^\mu, \quad (19)$$

$$T_{\mu\nu} = \mu n_c V_\mu V_\nu + W_n U_\mu U_\nu - P g_{\mu\nu}, \quad (20)$$

where μ is the chemical potential, n_c and n_n are the particle density of the superfluid and of the normal component, respectively, V^μ and U^μ are the unit 4-velocities of the superfluid and the normal component. In general, a

cosmological model in which two perfect fluids flow with distinct 4-velocities should give rise to anisotropic pressures [22–24], and it has been shown that the universe would acquire some anisotropic characteristics and its geometry will deviate from the standard FLRW one if there is a slight difference between the 4-velocities of the dark energy and dark matter [25]. However, if the two 4-velocities are parallel, which is the case we are interested in here, the spacetime can be homogeneous. Second, the observational data do not forbid the anisotropic evolution at the early stage but constraints are considerable, so in any realistic model the effect of the anisotropy at the early stage may disappear rapidly. The consideration of an anisotropic behavior is useful to study all sides of the model and to get constraints to its parameters, but this topic is beyond the scope of this paper. The generalized pressure is assumed to be with the form

$$P(\mu, \beta, \gamma) = p_c(\mu) + p_n(\mu, \beta, \gamma), \quad (21)$$

where $\gamma = V^\mu U_\mu$ is associated with the relative motion of the components, and β is the inverse temperature with respect to the reference frame comoving to the excitation gas. The excited state is described by the relations

$$\mu n_n = \gamma(1 - c_s^2) W_n, \quad p_n = \frac{c_s^2 W_n}{1 + \nu}, \quad (22)$$

where the adiabatic speed of sound is $c_s^2 = 4M^2/\rho_c^2$ with M a constant, and ν is a properly polytropic index. The assumption of $p_n \propto T^{\nu+1}$ is a generalization of the dependence $p_n \propto T^4$ followed by the relation $p_n = c_s^2 W_n/4$. The background superfluid obeys

$$n_c = \sqrt{\frac{\lambda}{M}} \sqrt{\rho_c^2 - 4M^2}, \quad p_c = -\frac{4M^2}{\rho_c}, \quad (23)$$

In a spatially flat, homogeneous, and isotropic FLRW universe, the superfluid and the normal velocities are equal ($\gamma = 1$). Then Einstein equations take the forms

$$H^2 = \frac{8\pi G}{3} \rho_{\text{tot}}, \quad (24)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho_{\text{tot}} + p_{\text{tot}}), \quad (25)$$

where $\rho_{\text{tot}} = \rho_c + \rho_n$ with $\rho_n = W_n - p_n$, and $p_{\text{tot}} = p_c + p_n$. The local energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$ and the particle number conservation $\nabla_\mu n^\mu = 0$, respectively, lead to

$$\dot{\rho}_{\text{tot}} = -3H(\rho_{\text{tot}} + p_{\text{tot}}), \quad (26)$$

$$\dot{n}_{\text{tot}} = -3H n_{\text{tot}}. \quad (27)$$

Taking into account Eqs. (22) and (27), we can rewrite Eqs. (24) and (26) as the following forms:

$$H^2 = \frac{1}{3(1+\nu)} \left[\frac{1}{\rho} + \frac{k}{a^3} \left(\frac{\nu\rho}{\sqrt{\rho^2-1}} + \frac{\sqrt{\rho^2-1}}{\rho} \right) \right], \quad (28)$$

$$\begin{aligned} & 3H \left(1 + \nu - \frac{k}{a^3} \frac{1}{\sqrt{\rho^2-1}} \right) \\ & + \frac{\dot{\rho}}{\rho} \left[1 - \frac{k}{a^3} \left(\frac{1}{\sqrt{\rho^2-1}} - \frac{\nu\rho^2}{(\rho^2-1)^{3/2}} \right) \right] \\ & = 0, \end{aligned} \quad (29)$$

where $\rho = \rho_c/(2M)$ and $k = n_0/(2\sqrt{M\lambda})$. Given any value of the parameter ν , one can have a different solution for the fluid dynamics; see, for example, that the quasiparticle behaves like dust ($p_n = 0$) in the limit $\nu \rightarrow \infty$, which is the case we are interested in here. In this case, Eqs. (28) and (29) can be solved analytically and yield to

$$\rho_c = \sqrt{1 + \frac{k^2}{(a^3 + k_0)^2}}, \quad (30)$$

$$\rho_n = \frac{k_0}{a^3} \sqrt{1 + \frac{k^2}{(a^3 + k_0)^2}}. \quad (31)$$

With these two equations, Eq. (28) turns out to be

$$H^2 = \frac{8\pi G}{3} \left(1 + \frac{k_0}{a^3} \right) \sqrt{1 + \frac{k^2}{(a^3 + k_0)^2}}. \quad (32)$$

Equation (25) can be rewritten as

$$\begin{aligned} \dot{H} &= -4\pi G \left[1 + \frac{k_0}{a^3} - \frac{(a^3 + k_0)^2}{k^2 + (a^3 + k_0)^2} \right] \\ &\times \sqrt{1 + \frac{k^2}{(a^3 + k_0)^2}}. \end{aligned} \quad (33)$$

If we define $E^2 = H^2/H_0^2$ and $\Omega_c = 8\pi G\rho_{c0}/(3H_0^2)$, since $E(z=0) = 1$, there is a relation between Ω_c and k_0 : $\Omega_c = 1/(1 + k_0)$.

IV. GRAVITATIONAL GROWTH AND LARGE STRUCTURE IN SCG SCENARIO

In the following, we will discuss the gravitational growth and constraints coming from the linear and nonlinear aspects of structure formation in the SCG scenario, such as S_3 the skewness of the density field and σ_8 the rms fluctuations on a sphere of $8 \text{ Mpc } h^{-1}$ by using the analysis presented above.

A. Gravitational growth

Assume that the dark energy component of SCG affects large-scale structure growth only through its effect on the background evolution, while the fluctuations of the dark matter component of SCG are responsible for the large-scale structure. According to Eq. (11), we get

$$c_1 = -\frac{4\pi G \bar{\rho}_n}{3 \bar{H}^2} = -\frac{1}{2} \frac{k_0}{a^3 + k_0}, \quad (34)$$

$$c_2 = 0. \quad (35)$$

From Eqs. (32) and (33), the term appearing in front of the δ' in the perturbation equation (10) is found to be

$$2 + \frac{\dot{H}}{H^2} = \frac{1}{2} + \frac{3}{2} \frac{a^3(a^3 + k_0)}{k^2 + (a^3 + k_0)^2}. \quad (36)$$

At early times ($a \ll 1$), SCG reduces to the EdS (also Λ CDM) case with the solution given by Eq. (15). In the large a limit, the linear equation (13) in SCG reduces to

$$D_1'' + 2D_1' - \frac{3k_0}{2a^3} D_1 = 0. \quad (37)$$

Accordingly, the solution to this equation is

$$D_1 = \frac{c_3}{a} F_1\left(\frac{2}{3}, \xi(a)\right) + \frac{c_4}{a} F_2\left(\frac{2}{3}, \xi(a)\right), \quad (38)$$

where F_1 and F_2 are the Bessel functions of the first and second kinds with $\xi(a) = \frac{1}{3} i\sqrt{6k_0} a^{-3/2}$, respectively.

In Figs. 1 and 2, we plot numerically the linear growth factor $D = D_1(z)/D_1(0)$ of the perturbations for different values of k_0 (with $k = 0.173$ obtained in [9]) and k (with

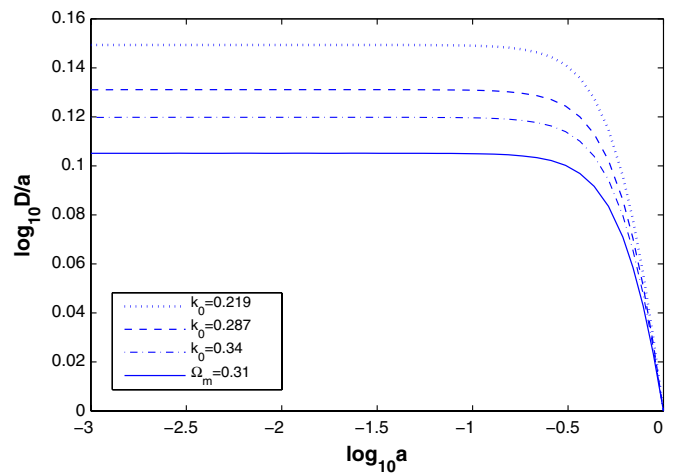


FIG. 1 (color online). Linear growth D (normalized with the scale factor a) for $\Omega_{m0} = 0.31$ [26] in Λ CDM and different values of k_0 with $k = 0.173$ in SCG.

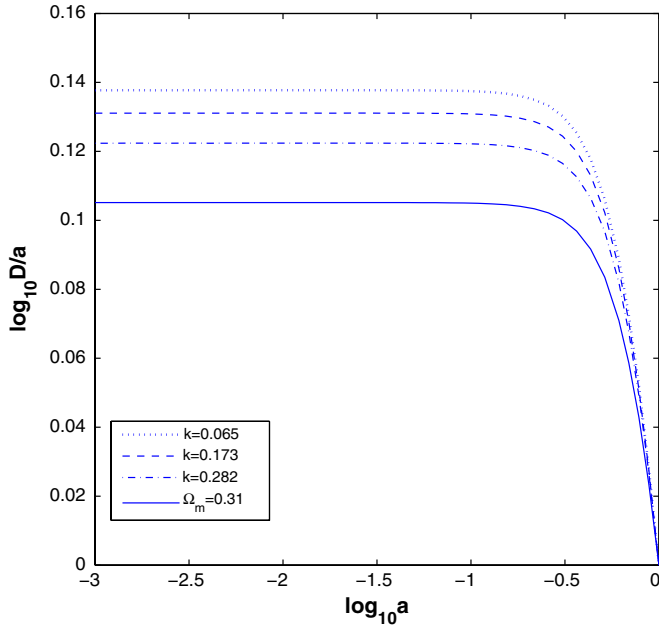


FIG. 2 (color online). Linear growth D for $\Omega_{m0} = 0.31$ in Λ CDM and different values of k with $k_0 = 0.287$ in SCG.

$k_0 = 0.287$ obtained in [9]). The linear growth is normalized with the scale factor, a , the growth rate in the EdS universe (it is also equal to the growth rate in the Λ CDM universe at early times). The differential equations are solved numerically in the region of cosmological interest $a = 0.001$ to $a = 1$. The initial condition is chosen such that at $a = 0.001$ the standard exponential solution, $D_1 \sim a$, is reached. The form of the linear growth factor is similar to the case of Λ CDM. The smaller the value of k_0 (or k) is, the larger the deviation of the growth factor between SCG and Λ CDM is, but the variation is observationally insignificant, being at best less than 4%. There will be more structure growth on the large scale in the SCG scenario than in Λ CDM.

B. Amplitude normalization: σ_8

Another important parameter related to the growth is the σ_8 parameter, which is the rms matter density contrast in a sphere with a comoving radius of $8h^{-1}$ Mpc at present, where h is the usual dimensionless Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The rms mass fluctuation σ_8 is defined as

$$\sigma^2(R, z) = \int_0^\infty \frac{dk}{k} W^2(kR) \Delta^2(k, z), \quad (39)$$

where $W(kR)$ is the window function defined as

$$W(kR) = 3 \left[\frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2} \right] r, \quad (40)$$

and

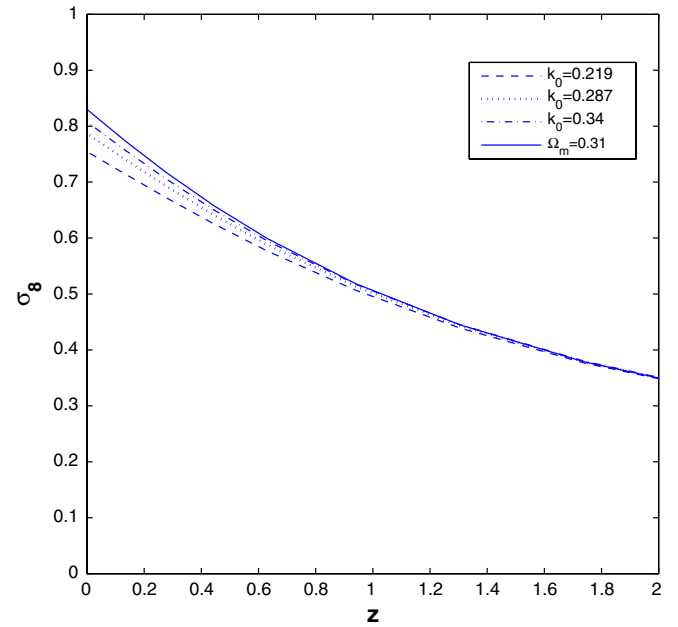


FIG. 3 (color online). The evolution of $\sigma_8(z)$ with different values of k_0 and $k = 0.173$ in SCG, comparing with the Λ CDM case.

$$\Delta^2(k, z) = 4\pi k^3 P_\delta(k, z), \quad (41)$$

where $P_\delta(k, z)$ is the primordial matter power spectrum. The function σ_8 is related to $D(z)$ as

$$\sigma_8(z) = D(z) \sigma_8(0). \quad (42)$$

Here the value of $\sigma_8(0)$ is normalized to the Λ CDM model according to $\sigma_8(0) = \frac{D_1(0)}{D_{1,\Lambda\text{CDM}}(0)} \sigma_{8,\Lambda\text{CDM}}(0)$ with $\sigma_{8,\Lambda\text{CDM}}(0) = 0.83$ [26].

In Figs. 3 and 4, the evolution of $\sigma_8(z)$ in the SCG scenario is plotted numerically. We can see that the value

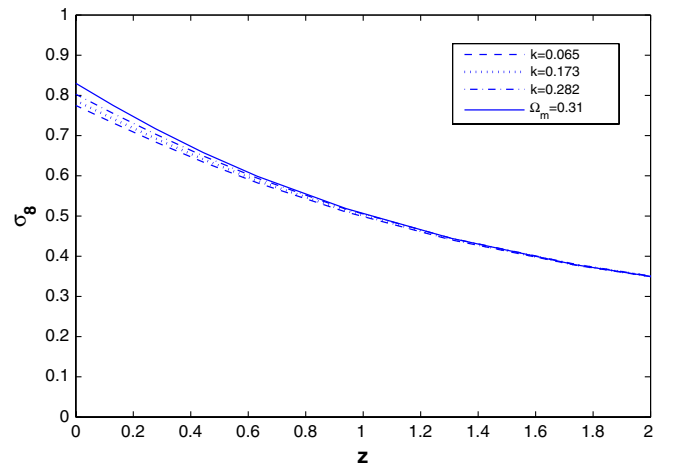


FIG. 4 (color online). The evolution of $\sigma_8(z)$ with different values of k and $k_0 = 0.287$ in SCG, comparing with the Λ CDM case.

of $\sigma_8(0)$ is very close to 0.8 when the value of k_0 (or k) increases. We also plot $\sigma_{8,\Lambda\text{CDM}}(z)$ for comparison. The variation between $\sigma_8(z)$ and $\sigma_{8,\Lambda\text{CDM}}(z)$ almost cannot be discriminated at $z \sim 1$. The deviation is observationally insignificant even at $z = 0$, being at best less than 6% when k_0 (or k) takes values changing from the best-fit value in the 1σ range obtained in [9]. In other words, the value of $\sigma_8(0)$ is compatible with that of ΛCDM , and we cannot use $\sigma_8(z)$ to distinguish the SCG model from the ΛCDM model at 68.3% confidence level. We can conclude, however, that $\sigma_8(0)$ in the SCG scenario is smaller than that of the ΛCDM model at 68.3% confidence level. Significantly smaller σ_8 have been found at low redshift by velocity fields [27] and some weak lensing studies

[28], which can be more easily accounted for by using the SCG cosmological model.

C. The skewness S_3

Now we investigate the nonlinear aspects of the large-scale structure formation, such as the skewness of the density field. In the large a limit, the second order equation (16) in SCG reads

$$D_2'' + 2D_2' - \frac{8}{3}(D_1')^2 - \frac{3k_0}{2a^3}D_2 - \frac{3k_0}{a^3}D_1^2 = 0. \quad (43)$$

Substituting Eq. (39), the solution to this equation is

$$\begin{aligned} D_2 = & \frac{c_5}{a} F_1\left(\frac{2}{3}, \xi(a)\right) + \frac{c_6}{a} F_2\left(\frac{2}{3}, \xi(a)\right) + \frac{\pi k_0}{3a} \left[F_1\left(\frac{2}{3}, \xi(a)\right) \int \left(-\frac{4}{a^4} F_2\left(\frac{2}{3}, \xi(a)\right) \left(c_3^2 F_1^2\left(-\frac{1}{3}, \xi(a)\right) \right. \right. \right. \\ & - \frac{3c_3^2}{4} F_1^2\left(\frac{2}{3}, \xi(a)\right) + 2c_3c_4 F_1\left(-\frac{1}{3}, \xi(a)\right) F_2\left(-\frac{1}{3}, \xi(a)\right) \\ & - \frac{3}{2}c_3c_4 F_1\left(\frac{2}{3}, \xi(a)\right) F_2\left(\frac{2}{3}, \xi(a)\right) + c_4^2 F_2^2\left(-\frac{1}{3}, \xi(a)\right) - \frac{3}{4}c_4^2 F_2^2\left(\frac{2}{3}, \xi(a)\right) \left. \left. \left. \right) \right) dx \right. \\ & - F_2\left(\frac{2}{3}, \xi(a)\right) \int \left(-\frac{4}{a^4} F_1\left(\frac{2}{3}, \xi(a)\right) \left(-\frac{3c_3^2}{4} F_1^2\left(\frac{2}{3}, \xi(a)\right) \right. \right. \\ & - \frac{3}{2}c_3c_4 F_1\left(\frac{2}{3}, \xi(a)\right) F_2\left(\frac{2}{3}, \xi(a)\right) + c_3^2 F_1^2\left(-\frac{1}{3}, \xi(a)\right) \\ & \left. \left. \left. + 2c_3c_4 F_1\left(-\frac{1}{3}, \xi(a)\right) F_2\left(-\frac{1}{3}, \xi(a)\right) + c_4^2 \left(F_2^2\left(-\frac{1}{3}, \xi(a)\right) - \frac{3}{4} F_2^2\left(\frac{2}{3}, \xi(a)\right) \right) \right) \right) dx \right], \end{aligned} \quad (44)$$

From the first and the second order equations, we see that a SCG universe behaves fundamentally differently from the ΛCDM and GCG universes: at early times ($a \ll 1$), SCG and ΛCDM reduce to the EdS case, whereas GCG does not; at very late times ($a \gg 1$), GCG reduces to the ΛCDM case, whereas SCG does not.

In Figs. 5 and 6, we plot the evolution of the skewness S_3 in the SCG scenario. To solve the second order equation (43) numerically, we chose the initial condition as $S_3 = 34/7$ at $a = 0.001$; namely, the standard solution, the EdS universe, is recovered at the early time. It is obvious that the deviation of skewness $S_3(0)$ between the SCG and ΛCDM models is very small, being at best less than 1% when k_0 (or k) takes values changing from the best-fit value in the 1σ range obtained in [9]. For $a \leq 0.1$, the evolution of S_3 in the SCG and ΛCDM models is almost the same. Current estimations for S_3 agree with the standard predictions but with large uncertainties, of order 20%–30% [29]. Thus current observations on S_3 cannot be used to distinguish different cosmological models. Future measurement of S_3 , after removing biasing uncertainties, could be used to constrain the parameters of cosmological models [30].

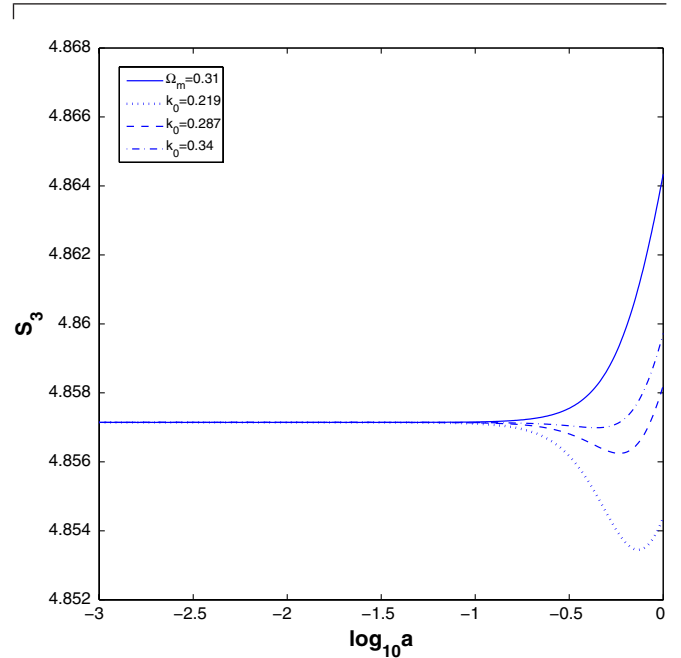


FIG. 5 (color online). The evolution of $S_3(z)$ with different values of k_0 and $k = 0.173$ in SCG, comparing with the ΛCDM case.

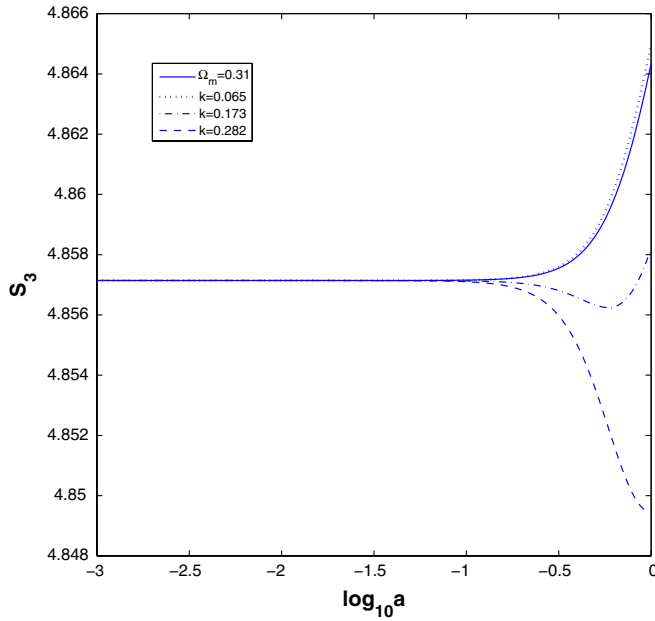


FIG. 6 (color online). The evolution of $S_3(z)$ with different values of k and $k_0 = 0.287$ in SCG, comparing with the Λ CDM case.

V. SUMMARY AND CONCLUSIONS

The growth of the large-scale structure in SCG cosmology has been studied. SCG with two free parameters is a phenomenological viable model in which the first and the second order perturbation equations have been obtained. The linear growth factor, σ_8 , and S_3 are important quantities in studies of the large-scale structure, and all of them were investigated here.

The form of the linear growth factor is similar to the Λ CDM case. There is more stricture growth on large scales in the SCG model than in Λ CDM. The smaller the values of parameters are, the larger the deviation of the growth factor between SCG and Λ CDM is, but the variation is at best less than 4%.

The variation between $\sigma_8(z)$ and $\sigma_{8,\Lambda\text{CDM}}(z)$ cannot be discriminated, being at best less than 6% when parameters of SCG take values changing from the best-fit value in the 1σ range obtained in [9]. The $\sigma_8(0)$ in the SCG scenario is smaller than that of the Λ CDM model, which can more easily account for the smaller σ_8 found at low redshift by some weak lensing studies and velocity fields.

We also have found that the deviation of skewness S_3 between the SCG and Λ CDM models is very small, being at best less than 1% when the parameters change from the best-fit value in the 1σ range obtained in [9]. Thus current observations of S_3 cannot be used to distinguish the SCG model from Λ CDM.

ACKNOWLEDGMENTS

I thank Y. Liu for helpful discussions. I am grateful to the Center for High Energy Physics of Peking University and the Kavli Institute for Theoretical Physics China for hospitality where this work was completed during a program on Cosmology after Planck. This study is supported in part by National Natural Science Foundation of China (Grants No. 11147028 and No. 11273010) and Hebei Provincial Natural Science Foundation of China (Grants No. A2011201147 and No. A2014201068).

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