Dark energy in thermal equilibrium with the cosmological horizon?

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According to a generalization of black hole thermodynamics to a cosmological framework, it is possible to define a temperature for the cosmological horizon. The hypothesis of thermal equilibrium between the dark energy and the horizon has been considered by many authors. We find the restrictions imposed by this hypothesis on the energy transfer rate (Q_i) between the cosmological fluids, assuming that the temperature of the horizon has the form $T = b/2\pi R$, where R is the radius of the horizon. We more specifically consider two types of dark energy: Chaplygin gas (CG) and dark energy with a constant equation of state parameter (wDE). In each case, we show that for a given radius R, there is a unique term $Q_{\rm de}$ that is consistent with thermal equilibrium. We also consider the situation where, in addition to dark energy, other fluids (cold matter, radiation) are in thermal equilibrium with the horizon. We find that the interaction terms required for this will generally violate energy conservation ($\sum_i Q_i = 0$).

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I. INTRODUCTION

In the late 1990s, observations of supernovae [1–3] have suggested that the Universe is undergoing a state of accelerated expansion. Since then, additional evidence leading to the same conclusion have been found [4–12]. In the context of general relativity (GR), the equation of state (EOS) parameter of a fluid, defined as the ratio of its pressure over its energy density ($w \equiv p/\rho$), must be smaller than $-1/3$ in order to be able to drive the accelerated expansion of the Universe. Since normal matter satisfies the strong energy condition ($w \geq 0$), this condition is not fulfilled. Hence, two main approaches have been proposed to explain the acceleration: one of them consists to replace the GR by a modified gravity theory (see e.g. Refs. [13–15]) and the other, to keep GR while introducing a new cosmic fluid, known as dark energy (see Ref. [16] and references therein), endowed with a sufficiently large and negative EOS parameter ($w_{\rm de} < -1/3$). Alternatively, it has also been proposed that the (apparent) acceleration could be only an artifact caused by the spatial inhomogeneity of the Universe (see e.g. Refs. [17,18]).

The ΛCDM model is the simplest cosmological model which provides a reasonably good fit to the observational data. In this model, the two main components of the Universe are currently a form of dark energy provided by a cosmological constant (Λ) and a pressureless fluid known as cold dark matter (CDM). In addition to these two fluids, the Universe is also composed of ordinary matter (radiation, baryons). However, despite the excellent agreement with the observational data, the ΛCDM model is facing two theoretical difficulties. The most serious one concerns the value of the dark energy density and is known as the "cosmological constant problem" [19]. Indeed, there is a discrepancy of $\sim$123 orders of magnitude between the value expected from theoretical computations and the value inferred from observations ($\rho_{{\rm de}_{\rm obs}}/\rho_{{\rm de}_{\rm th}} \sim 10^{-123}$). The other one, dubbed the "coincidence problem" [20], relies on the observation that the values of the matter energy density and of the dark energy density are currently of the same order of magnitude. Unlike to the previous problem, this is not incompatible with the theory. However, since the matter energy density is diluted proportionally to the volume of the Universe as it is expanding ($\rho_m \propto a^{-3}$) while the dark energy density remains constant ($\rho_{\rm de} = {\rm const}$), the period of time during which $\rho_m/\rho_{\rm de} = \mathcal{O}(1)$ corresponds to a very narrow window in the Universe history. To currently lie in this window, a fine tuning of the initial conditions of the model is needed. However, it is worth mentioning that about a decade before the discovery of the accelerated expansion of the Universe, anthropic arguments were already addressing both problems [19,21].

A possible way to circumvent these problems, without the recourse to anthropic arguments, would be to allow the dark energy density to vary in time ($\rho_{\rm de} \neq {\rm const}$). It would then be possible for the dark energy density to decrease from an initial large value, consistent with the theoretical computation, to a smaller one, consistent with the current value inferred from the observations. Moreover, that could also extend duration of the period during which $\rho_m/\rho_{\rm de} = \mathcal{O}(1)$. A variable dark energy density could be obtained or by considering an EOS parameter $w_{\rm de}$ different from -1, either by allowing an energy transfer between the dark energy and another fluid (or by considering these two ways together). Several forms of dark energy models have been proposed, including quintessence [22], phantom fields [23], tachyon fields [24], Chaplygin gas [25], agegraphic dark energy [26] and holographic dark energy [27], to name few.

To study the thermodynamical implications of these models, the determination of the dark energy temperature is a question that must inevitably be addressed. A hypothesis often used [28,29] is that the dark energy temperature is proportional to that of the cosmological horizon ($T_{\rm de} \propto T_h$). Indeed, according to a generalization of black-hole thermodynamics to a cosmological framework, it is possible to define a temperature for the horizon which is related to its surface gravity (see Sec. III A for more details). A stronger hypothesis [30–42], albeit more motivated, consists in considering that the dark energy fluid and the horizon are in thermal equilibrium ($T_{\rm de} = T_h$). An argument presented in Ref. [30] and reused in Refs. [31–38] states that if this were not the case, then the "energy would spontaneously flow between the horizon and the fluid (or vice versa), something at variance with the FRW geometry." Following this argument, some authors [35–38] have even extended this hypothesis to the other fluids, assuming that the thermal equilibrium between the horizon and a given fluid must hold at least for late time.

Although this assumption may be questionable (especially in regard to its extension to other fluids), the objective of this paper is not to directly discuss of its validity. Instead, we will demonstrate that in order to maintain thermal equilibrium between a given fluid and the horizon, a specific energy transfer rate is required, which constitutes a restrictive condition for its application.

II. DYNAMICS

A. Interacting fluids

In a Friedmann-Robertson-Walker (FRW) spacetime, the continuity equations for a model allowing interactions between the different cosmic fluids (dark energy, dark matter, baryonic matter and radiation) are given by

$$\dot{\rho}_i + 3H(1+w_i)\rho_i = Q_i. \tag{1}$$

If we treat the curvature as fictitious fluid, this equation can also be used to describe the evolution of its energy density $\rho_k \equiv -3k/8\pi G a^2$.[1] Since the Hubble term is defined as $H = \dot{a}/a$, where a is the scale factor, in absence of interaction ($Q_i = 0$), the solution to this equation is

$$\rho_i = \rho_{i_0} a^{-3(1+w_i)}. \tag{2}$$

Here, we have set $a_0 = 1$ (in this paper, the subscript 0 refers to the current value of a variable). The Friedmann equations can be written as

$$H^2 = \frac{M_p^{-2}}{3}\sum_i \rho_i, \tag{3}$$

$$\dot{H} = -\frac{M_p^{-2}}{2}\sum_i (1+w_i)\rho_i, \tag{4}$$

[1]The curvature parameter k, whose dimensions are $(\text{length})^{-2}$, is negative for an open universe and positive for a closed one.

where $M_p = (8\pi G)^{-1/2}$ is the reduced Planck mass (throughout this paper, we will use a unit system where $\hbar = k_B = c = 1$). The lhs of Eq. (1) has the same form as in the noninteracting case, where $H \equiv \dot{a}/a$ (a is the scale factor) stands for the Hubble term, ρ_i, for the energy density of a given fluid and $w_i \equiv p_i/\rho_i$ (p_i is the pressure), for the equation of state (EOS) parameter of this fluid. The values of these parameters are the usual ones for radiation ($w_r = 1/3$), for curvature ($w_k = -1/3$) and, in absence of interaction (see Sec. III B 3), for dark and baryonic matter ($w_{\rm dm} = w_b = 0$). For dark energy, $w_{\rm de}$ is not necessarily fixed to -1 as in the ΛCDM model and could even be variable. The rhs of the equation represents the possible interactions between the fluids. A positive value ($Q_i > 0$) represents a gain of energy for the fluid, and negative value ($Q_i < 0$), a loss. The ensemble of these terms is subject to the energy conservation condition $\sum_i Q_i = 0$. It is to be noticed that the interaction is allowed only between the *real* fluids. For the curvature, the interaction term Q_k must be zero, otherwise it would imply that the curvature parameter k is variable, which would be inconsistent with the FRW metric.

B. Types of dark energy

The exact nature of dark energy is not known and several models have been proposed. In Sec. III B, we will obtain an expression for the form of the interaction term which is required to have a thermal equilibrium between a generic type of dark energy and the cosmological horizon. To provide a specific example, we will consider the case of the Chaplygin gas (CG)[2] which was the first form of dark energy for which the hypothesis of the thermal equilibrium with the horizon was considered [30]. A Chaplygin gas [25] is a fluid for which its pressure and energy density are related through

$$p_{\rm cg} = -\frac{\rho_{\rm cg_\infty}^2}{\rho_{\rm cg}}, \tag{5}$$

where the constant $\rho_{\rm cg_\infty}$ is the late-time value of $\rho_{\rm cg}$. In absence of interaction ($Q_{\rm cg} = 0$), the solution to the continuity equation is

$$\rho_{\rm cg} = \sqrt{\rho_{\rm cg_\infty}^2 + (\rho_{\rm cg_0}^2 - \rho_{\rm cg_\infty}^2)a^{-6}}, \tag{6}$$

[2]We had previously also considered the example of holographic dark energy (HDE), but the hypothesis of thermal equilibrium with the cosmological horizon is actually not consistent for this form of dark energy. The entropy of the HDE is related to the Bekenstein-Hawking entropy associated with the horizon through $S_{\rm hde} = S_{\rm BH}^{3/4} \propto R^{3/2}$ [43,44], where R is the horizon radius. We also know that in a volume $V \sim R^3$, the thermal entropy of an effective quantum field theory is given by $S_{\rm hde} \propto R^3 T_{\rm hde}^3$ [43]. Thus, the temperature of the HDE should scale as $T_{\rm hde} \propto R^{-1/2}$ which is not consistent with the hypothesis of thermal equilibrium since $T_{\rm hde} \neq T_h \propto R^{-1}$ (see Eq. (16).

Equivalently, we can write

$$w_{\rm cg} = -\left(\frac{\rho_{{\rm cg}_\infty}}{\rho_{\rm cg}}\right)^2 = -\frac{1}{1+(\rho^2_{{\rm cg}_0}/\rho^2_{{\rm cg}_\infty}-1)a^{-6}}. \qquad (7)$$

At early times ($a \ll 1$), the CG behaves like cold matter ($w_{\rm cg} \approx 0$) and at late times ($a \gg 1$) like a cosmological constant ($w_{\rm cg} \approx -1$) providing a unified form of dark matter and dark energy.

Since Eq. (7) is a function of the scale factor, as a complement to CG, we will consider a second type of dark energy for which the EOS parameter has a fixed value (wDE).

III. THERMODYNAMICS

A. Cosmological horizon temperature

Since the seminal works of Hawking [45] and Bekenstein [46] in the seventies, the thermodynamical properties of black holes have been widely studied. One of the most well known feature is that, as consequence of the existence of an event horizon, the stationary (or quasistationary) black holes behave like black bodies emitting thermal radiation with a temperature proportional to the value of the surface gravity evaluated on the horizon,

$$T_h = \frac{\kappa}{2\pi}. \qquad (8)$$

A first extension of black hole thermodynamics to a cosmological framework was done by Gibbons and Hawking in Ref. [47] by considering de Sitter space. In this case, the surface gravity on the event horizon is given by the inverse of the horizon radius ($R_E = a\int_t^{t_{\rm end}}\frac{dt}{a}$),[3] $\kappa = 1/R_E = \sqrt{3/\Lambda}$, thus the temperature is given by

$$T_h = \frac{1}{2\pi R_E}. \qquad (9)$$

Unlike to de Sitter space, the event horizon is not always well defined for FRW spacetime. However, it has been argued [48,49] that it is actually the apparent horizon ($R_A = 1/\sqrt{H^2 - \frac{1}{3}M_p^{-2}\rho_k}$), and not the event horizon, that is responsible for Hawking radiation (in the case of de Sitter space, the two horizons coincide). It worth mentioning that for de Sitter space, the event horizon radius has a constant value, while for a FRW spacetime, the value of the apparent horizon radius varies. To compute the surface gravity, this could be problematic. Indeed, this quantity is usually defined in terms of Killing horizons, which work well in stationary (or quasistationary) situations. For the dynamical situations where no such horizons exist, several definitions have been proposed (see [50,51] for a review). If we consider a generic spherically symmetric spacetime, the line element is given by

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega^2, \qquad (10)$$

where $x^0 = t$, $x^1 = r$, $\tilde{r} = a(t)r$ and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. For the FRW spacetime, the 2-dimensional metric h_{ab} is given by ${\rm diag}(-1, a^2/(1-kr^2))$. A frequently used definition of the surface gravity has been proposed by Hayward in Ref. [52]:

$$\kappa = \frac{1}{2}\nabla\cdot\nabla\tilde{r} = \frac{1}{2\sqrt{-h}}\partial_a(\sqrt{-h}h^{ab}\partial_b\tilde{r}). \qquad (11)$$

Here, the divergence and gradient refer to the two-dimensional space normal to the spheres of symmetry. An evaluation of this expression at $\tilde{r} = R_A$ gives $\kappa = (1-\epsilon)/R_A$, where $\epsilon \equiv \dot{R}_A/(2HR_A)$. Thus the horizon temperature is given by

$$T_h = \frac{1-\epsilon}{2\pi R_A}. \qquad (12)$$

An alternative definition [29] for the dynamical surface gravity is

$$\kappa = -\frac{1}{2}\partial_{\tilde{r}}\chi = \frac{\tilde{r}}{R_A^2}, \qquad (13)$$

where $\chi \equiv h^{ab}\partial_a\tilde{r}\partial_b\tilde{r}$.[4] At $\tilde{r} = R_A$, the surface gravity is then given by $\kappa = 1/R_A$, and the horizon temperature by

$$T_h = \frac{1}{2\pi R_A}. \qquad (14)$$

Among the papers where a thermal equilibrium between the horizon and the dark energy is considered, both Eq. (12) [37–40] and Eq. (14) [32–35,41] are commonly used as a definitions of the horizon temperature. Although it has been argued [53] that the ϵ term can be neglected in certain situations, these two expressions are generally different and one can wonder whether one definition is better motivated than the other. In favor of Eq. (14), it was shown in Ref. [54], using the tunneling approach, that an observer inside the apparent horizon of a FRW universe will see a thermal spectrum with a temperature given by $T_h = 1/(2\pi R_A)$, without the extra ϵ term. It is also interesting to notice that using this expression for the temperature, it is possible to

[3]The upper integration limit is given by $t_{\rm end} = \infty$ in an eternally expanding model and by the time of the big crunch in a recollapsing model. This expression may also be computed as $R_E = a\int_a^{a_{\rm end}}\frac{da}{H^2 a}$, where $a_{\rm end} = a(t_{\rm end})$.

[4]It is to be noticed that the radius of the apparent horizon, R_A, is defined as the value of $\tilde{r}$ for which the scalar χ vanishes (which implies that the vector $\nabla\tilde{r}$ is null on the apparent horizon surface).

recover the second Friedmann equation [Eq. (4)] from the first law of thermodynamics [53]. Some authors still consider the event horizon as the relevant one and use Eq. (9) to define the horizon temperature [30,31,36] (see however Ref. [42] where Eq. (13) is evaluated at $\tilde{r} = R_E$, which leads to $T_h = R_E/(2\pi R_A^2)$). In Refs. [29,34,41], the horizon temperature is assumed to be proportional to its de Sitter value, i.e.

$$T_h = \frac{b}{2\pi R_H}, \tag{15}$$

where b is a constant parameter and R_H the Hubble radius ($R_H = 1/|H|$). It would be interesting to consider all these different definitions, but for the sake of conciseness we will restrict our attention (while keeping in mind that there is no clear consensus on how the horizon temperature should be defined and which horizon should be considered) to the case where the temperature has the dependence on the horizon radius given by

$$T_h = \frac{b}{2\pi R}. \tag{16}$$

Here R could stand for, with $b = 1$, the event horizon radius [Eq. (9)] and the apparent horizon radius [Eq. (14)], as well for the Hubble radius [Eq. (15)].

B. Conditions for thermal equilibrium

To find the form of the energy transfer rate Q_i required to maintain thermal equilibrium between a fluid, whose the continuity equation is given by Eq. (1), and the cosmological horizon, we will first derive an equation for the temperature evolution for this fluid. Our derivation is similar to that presented in Ref. [55]. The starting point is the Gibbs equation, $T_i dS_i = dE_i + p_i dV$. For simplicity we will consider a comoving volume of unit coordinate volume and hence a physical volume of $V = a^3$. Since the energy of the fluid is given by $E_i = \rho_i V$, we can rearrange the Gibbs equation as

$$dS_i = \frac{\rho_i + p_i}{T_i} dV + \frac{V}{T_i} d\rho_i. \tag{17}$$

From this expression for the entropy, we can show that the integrability condition,

$$\left[\frac{\partial}{\partial V}\left(\frac{\partial S_i}{\partial T_i}\right)_{N_i,V}\right]_{N_i,T_i} = \left[\frac{\partial}{\partial T_i}\left(\frac{\partial S_i}{\partial V}\right)_{N_i,T_i}\right]_{N_i,V}, \tag{18}$$

implies that

$$T_i\left(\frac{\partial p_i}{\partial T_i}\right)_{N_i,V} = (\rho_i + p_i) + V\left(\frac{\partial \rho_i}{\partial V}\right)_{N_i,T_i}. \tag{19}$$

Except for the cases where the derivatives vanish or are ill defined (e.g. for the DE in ΛCDM model), this equation is equivalent to

$$T_i\left(\frac{\partial p_i}{\partial \rho_i}\right)_{N_i,V} = (\rho_i + p_i)\left(\frac{\partial T_i}{\partial \rho_i}\right)_{N_i,V} - V\left(\frac{\partial T_i}{\partial V}\right)_{N_i,\rho_i}. \tag{20}$$

Since we can express the temperature as a function of the volume and the energy density ($T_i = T_i(\rho_i, V)$), its time derivative may be expressed as $\dot{T}_i = (\partial T_i/\partial \rho_i)\dot{\rho}_i + (\partial T_i/\partial V)\dot{V}$. The time derivative of the physical volume $V = a^3$ is $\dot{V} = 3HV$; then using also Eq. (1) to replace $\dot{\rho}_i$, we get

$$\dot{T}_i = -3H\left[(\rho_i + p_i)\left(\frac{\partial T_i}{\partial \rho_i}\right)_{N_i,V} - V\left(\frac{\partial T_i}{\partial V}\right)_{N_i,\rho_i}\right] + Q_i\left(\frac{\partial T_i}{\partial \rho_i}\right)_{N_i,V}. \tag{21}$$

The expression in the square brackets is identical to the rhs of Eq. (20); hence, we can write the temperature evolution equation as

$$\frac{\dot{T}_i}{T_i} = -3H\left(\frac{\partial p_i}{\partial \rho_i}\right)_{N_i,V} + \frac{Q_i}{T_i}\left(\frac{\partial T_i}{\partial \rho_i}\right)_{N_i,V}. \tag{22}$$

Now to find the form of the energy transfer rate required to have thermal equilibrium ($\tilde{Q}_i$) between the cosmic fluid and the cosmological horizon ($T_i = T_h \equiv T$), we must simply solve the preceding equation for Q_i and replace the temperature by the expression given by Eq. (16),

$$\tilde{Q}_i = \frac{b}{2\pi}\left(\frac{\partial \rho_i}{\partial T}\right)_{N_i,V}\left[\frac{3HRp_i' - \dot{R}}{R^2}\right], \tag{23}$$

where $p_i' \equiv (\partial p_i/\partial \rho_i)_{N_i,V}$. In the peculiar case where the energy density of the fluid depends only on the temperature, we can replace the first partial derivative in Eq. (23) by a total derivative and write $d\rho_i/dT = \dot{\rho}_i/\dot{T}$. This leads, after some simple manipulations, to

$$\tilde{Q}_i = \dot{\rho}_i\left[1 - 3p_i' H\frac{R}{\dot{R}}\right]. \tag{24}$$

Using the continuity equation [Eq. (1)] to replace $\tilde{Q}_i$, we obtain the following differential equation,

$$\frac{\dot{\rho}_i}{\rho_i} = -\left(\frac{1+w_i}{p'_i}\right)\frac{\dot{R}}{R}, \tag{25}$$

which relates the evolution of the energy density to that of the horizon radius. In the following sections, we will evaluate $\tilde{Q}_i$ for the two different types of dark energy (CG and wDE), as well for relativistic and nonrelativistic matter. The energy density of the relativistic matter depend only on the temperature. We will also assume that is the case for the Chaplygin gas and the wDE. The only fluid for which we will not use Eqs. (24) and (25) is the non-relativistic matter.

1. Chaplygin gas

For the Chaplygin gas, the EOS parameter is given by $w_{\rm cg} = -\rho^2_{{\rm cg}_\infty}/\rho^2_{\rm cg}$ and the derivative of the pressure by $p'_{\rm cg} = -w_{\rm cg}$. Inserting these expressions in Eqs. (25) and (24), we find that the energy density is given by

$$\rho_{\rm cg} = \rho_{{\rm cg}_\infty}\left[\frac{1}{1+(\rho^2_{{\rm cg}_\infty}/\rho^2_{{\rm cg}_0}-1)(R_0/R)^2}\right]^{\frac{1}{2}} \tag{26}$$

and the interaction term by

$$\tilde{Q}_{\rm cg} = \left[\frac{3(1+w_{\rm cg})HR + (1+w_{\rm cg}^{-1})\dot{R}}{R}\right]\rho_{\rm cg}. \tag{27}$$

The expression that we got for the energy density is different from that obtained in absence of interaction (Eq. (7), but still consistent with an unified form of dark energy and dark matter. Indeed, at late times (when R is large), the value of the energy density approaches $\rho_{{\rm cg}_\infty}$, and thus the Chaplygin gas behaves like a cosmological constant ($w_{\rm cg} \to -1$, $\tilde{Q}_{\rm cg} \to 0$) and could drive the accelerated expansion of the Universe. For $\rho_{{\rm cg}_\infty} > \rho_{{\rm cg}_0}$, the value of the energy density is increasing in time (as R is increasing) and $\rho_{{\rm cg}_\infty}$ represents the maximum value that can be reached. In this case, the Chaplygin gas cannot play the role of dark matter ($w_{\rm cg} \approx 0$) since $w_{\rm cg} \leq -1$ for all time. However, for $\rho_{{\rm cg}_\infty} < \rho_{{\rm cg}_0}$, the energy density decreases in time and $\rho_{{\rm cg}_\infty}$ represents the minimum value that can be reached, which means that $w_{\rm cg} \geq -1$ for all time. Actually, for radii smaller than $R(t_{\rm min}) \equiv R_0\sqrt{1-(\rho_{{\rm cg}_\infty}/\rho_{{\rm cg}_0})^2}$, the energy density becomes imaginary. Hence, we must conclude that a thermal equilibrium between the CG and the cosmological horizon is impossible at early time ($t < t_{\rm min}$). At $R_{\rm min}$, the value of the EOS parameter is $w_{\rm cg} = 0$. Hence, providing that thermal equilibrium is established after $t_{\rm min}$, the solution that we found is consistent with an unified form of dark energy and dark matter and the thermal equilibrium hypothesis.

2. Dark energy with a constant EOS parameter

For a form of dark energy with a constant EOS parameter, $p'_{wde} = w_{wde}$. The expressions for the energy density and the interaction term follow directly from Eqs. (25) and (24)

$$\rho_{wde} = \rho_{{\rm de}_0}\left(\frac{R}{R_0}\right)^{-\frac{1+w_{wde}}{w_{wde}}}, \tag{28}$$

$$\tilde{Q}_{wde} = \left[\frac{3(1+w_{wde})HR - (1+w_{wde}^{-1})\dot{R}}{R}\right]\rho_{wde}. \tag{29}$$

This expression is valid for any constant EOS parameter except $w_{wde} = -1$. We recover the dark energy of the ΛCDM model for this value ($\rho_{wde} = {\rm const}$ and $Q_{wde} = 0$), but we cannot conclude that thermal equilibrium with the horizon is possible for this type of dark energy since, as was pointed after Eq. (19) our derivation is not valid for a fluid whose energy density and pressure are intrinsically constant (in this case, we can even ask whether a temperature can be meaningfully defined).

3. Other fluids

As mentioned above, some authors [35–41] considered the possibility that, in addition to dark energy, other fluids could also be in thermal equilibrium with the horizon. We will now consider the implications of this hypothesis. For an ultra-relativistic fluid (photons, neutrinos) the energy density and the pressure are given by

$$\rho_r = 4\sigma T_r^4, \tag{30}$$

$$p_r = \frac{\rho_r}{3}, \tag{31}$$

where σ is the Stefan-Boltzmann constant. From Eq. (23), the interaction term needed to maintain thermal equilibrium follows immediately:

$$\tilde{Q}_r = \left[\frac{4HR - 4\dot{R}}{R}\right]\rho_r. \tag{32}$$

We note that by replacing the variables associated with dark energy in Eq. (29) by those associated with radiation, we get the same expression. This is not surprising since to obtain Eq. (29), we considered a fluid with a constant EOS parameter and whose energy density depends only on the temperature, as is the case for radiation ($\rho_r = 4\sigma T_r^4$, $w_r = 1/3$). More generally, all the results of Sec. III B 2 hold for any fluid fulfilling these two conditions, which excludes however nonrelativistic matter. In particular, Eq. (28) becomes for radiation

$$\rho_r = \rho_{r_0}\left(\frac{R}{R_0}\right)^{-4}. \tag{33}$$

For a nonrelativistic fluid, such as dark matter or baryonic matter, the energy density and the pressure are given by

$$\rho_m = n_m m + \frac{3}{2} n_m T_m, \tag{34}$$

$$p_m = n_m T_m, \tag{35}$$

where $n_m \equiv N_m/V$ is the particle number density. Here we consider a single particle species of mass m, but the generalization to many species is straightforward. Inserting Eqs. (34) and (35) into Eq. (23) leads to

$$\tilde{Q}_m = \left[\frac{3w_m HR - \frac{3}{2}w_m \dot{R}}{R}\right]\rho_m, \tag{36}$$

where the EOS parameter is given by

$$w_m \equiv \frac{p_m}{\rho_m} = \frac{T_m}{m + \frac{3}{2}T_m}. \tag{37}$$

Assuming that the rest-energy of the fluid is much larger than its kinetic energy ($m \gg T_m$), the EOS parameter may be approximated by $w_m \approx T_m/m$. Since $w_m \ll 1$, cold matter is usually considered to be pressureless ($w_m = 0$). However, we cannot use this approximation here since that would imply, according to Eq. (36), that $\tilde{Q}_m = 0$. Using Eq. (16), the EOS parameter may be written more conveniently as a function of the horizon radius

$$w_m = w_{m_0}\frac{R_0}{R}, \tag{38}$$

where $w_{m_0} \equiv b/(2\pi m R_0)$. Inserting the interaction term $\tilde{Q}_m$ into the continuity equation (1) and solving it yields

$$\rho_m = \rho_{m_0} a^{-3} \exp\left[\frac{3}{2}w_{m_0}\left(\frac{R_0 - R}{R}\right)\right]. \tag{39}$$

Now we must check whether the interaction terms found are consistent with the energy conservation condition $\sum Q_i = \sum \tilde{Q}_{i_{\rm eq}} + \sum Q_{i_{\rm neq}} = 0$. The summation indices $i_{\rm eq}$ and $i_{\rm neq}$ refer respectively to the fluids that are in thermal equilibrium with the horizon, and to those that are not. In the case where at least one of the interacting fluid is not in equilibrium, we can set $\sum Q_{i_{\rm neq}} = -\sum \tilde{Q}_{i_{\rm eq}}$ in order to fulfill the energy conservation condition. However, when all the interacting fluids are assumed to be in thermal equilibrium we must have $\sum \tilde{Q}_{i_{\rm eq}} = 0$, from which we get the following expression for the Hubble rate

$$H = \left[\frac{\sum \beta_{i_{\rm eq}}\rho_{i_{\rm eq}}}{3\sum(1 + w_{i_{\rm eq}} - \delta^m_{i_{\rm eq}})\rho_{i_{\rm eq}}}\right]\frac{\dot{R}}{R}, \tag{40}$$

where $\beta_i = -(1 + w_{\rm cg})^{-1}$, $(1 + w_{wde})^{-1}$, 4 and $\frac{3}{2}w_m$ respectively for CG, wDE, radiation and cold matter. The value of δ^m_i is 1 when $i = m$ and 0 otherwise. The energy density of the fluids in thermal equilibrium [Eqs. (26), (28), (33) and (39)] depends only on the horizon radius R and on the scale factor a (for cold matter); hence, Eq. (40) can be integrated (at least numerically) in order to find the relationship between these two variables. However, the function $R(a)$ thus obtained does not necessarily coincide with one of the three radii (R_H, R_A, R_E) considered in Sec. III A.

To illustrate the previous statement, we will consider the case where wDE and radiation are in thermal equilibrium and are the only two interacting fluids. This example is among the simpler to consider because Eq. (40), which becomes

$$H = \left[\frac{4\rho_r + (1 + w_{wde}^{-1})\rho_{wde}}{4\rho_r + 3(1 + w_{wde})\rho_{wde}}\right]\frac{\dot{R}}{R}, \tag{41}$$

can be integrated analytically. Inserting the expressions found for ρ_r and ρ_{wde} (Eqs. (28) and (33)) gives

$$H = \left[\frac{4r_{r_0} + (1 + w_{wde}^{-1})\tilde{R}^{3-w_{wde}^{-1}}}{4r_{r_0} + 3(1 + w_{wde})\tilde{R}^{3-w_{wde}^{-1}}}\right]\frac{\dot{\tilde{R}}}{\tilde{R}}. \tag{42}$$

Here, we have introduced the dimensionless radius $\tilde{R} = R/R_0$ and the radiation to dark energy density ratio at t_0 ($r_{r_0} \equiv \rho_{r_0}/\rho_{wde_0}$). Integration of Eq. (42) yields

$$a = \left[\frac{4r_{r_0} + 3(1 + w_{wde})}{4r_{r_0} + 3(1 + w_{wde})\tilde{R}^{3-w_{wde}^{-1}}}\right]^{\frac{1}{3}}\tilde{R}. \tag{43}$$

By differentiating this equation, we find that the scale factor reaches a maximum value $a_{\max}$ at

$$\tilde{R}_{a_{\max}} = \left(-\frac{4r_{r_0}w_{wde}}{1 + w_{wde}}\right)^{\frac{1}{3-w_{wde}^{-1}}}. \tag{44}$$

Consistently, the expression for the Hubble rate given by Eq. (41) is zero at $\tilde{R} = \tilde{R}_{a_{\max}}$. The expression for the Hubble rate given by the first Friedmann equation [Eq. (3)] must also be zero at this point. This condition reduces by one the number of free parameters in the model. For instance, we can express the value of the energy density of the spatial curvature as

$$\rho_{k_0} = -\sum\nolimits_{i\neq k}\rho_i a^2|_{\tilde{R}=\tilde{R}_{a_{\max}}}, \tag{45}$$

where the energy density of the noninteracting fluids ($i \neq wde$, r) is given by Eq. (2). Not surprisingly for a cosmic scenario involving recollapse, we find that the spatial curvature is positive ($\rho_{k_0} < 0$). The value of the remaining parameters can be chosen freely (provided that

$\sum_{i\neq k}\rho_{i_0}+\rho_{k_0}\geq 0$, in order to have $H_0\in\mathbb{R}$) and leads to a self-consistent cosmology where the radiation and wDE and are in thermal equilibrium with a cosmological horizon whose radius is implicitly defined in Eq. (43). Now, we want to verify whether this radius coincides either with the Hubble radius, the apparent radius or the event horizon radius. By solving the equation $\tilde{R}=\tilde{R}_H(\tilde{R})=1/|H(\tilde{R})|$ for the constant ρ_{r_0}, we get

$$\rho_{r_0}=-\Big[\sum_{i\neq r,wde}\rho_{i_0}a(\tilde{R})^{-3(1+w_i)}+\rho_{wde_0}\tilde{R}^{-(1+w_{wde}^{-1})}-3M_p^2\tilde{R}^{-2}\Big]\tilde{R}^4. \tag{46}$$

Solving $\tilde{R}=\tilde{R}_A(\tilde{R})$ for ρ_{r_0} leads to the same expression, except that now, the spatial curvature is excluded from the summation ($i\neq k,\ r,\ wde$). In both cases, we obtain an expression for the constant ρ_{r_0} which is actually a function of $\tilde{R}$. This inconsistency shows that $\tilde{R}\neq\tilde{R}_H$ and $\tilde{R}\neq\tilde{R}_A$. For the event horizon radius, we cannot directly compare $\tilde{R}_E$ to $\tilde{R}$ by reason of the integral involved in the definition of this radius. However, we can compare its time derivative, which is

$$\dot{\tilde{R}}_E=\frac{\dot{R}_E}{R_{E_0}}=H\tilde{R}_E-R_{E_0}^{-1}, \tag{47}$$

to the expression for $\dot{\tilde{R}}$ obtained from Eq. (42). Solving $\dot{\tilde{R}}=\dot{\tilde{R}}_E$ for R_{E_0} and replacing $\tilde{R}_E$ by $\tilde{R}$ yields

$$R_{E_0}=\left[\frac{\sqrt{3}M_p}{1-2w_{wde}-3w_{wde}^2}\right]\left[\frac{4r_0w_{wde}\tilde{R}^{w_{wde}^{-1}-4}+(1+w_{wde})\tilde{R}^{-1}}{\sqrt{\sum_{i\neq r,wde}\rho_{i_0}a(\tilde{R})^{-3(1+w_i)}+\rho_{r_0}\tilde{R}^{-4}+\rho_{wde_0}\tilde{R}^{-(1+w_{wde}^{-1})}}}\right] \tag{48}$$

Once again, we obtain an inconsistent equation where a constant is equal to a function of $\tilde{R}$, showing that $\tilde{R}\neq\tilde{R}_E$. Here we have shown that none of the three radius definitions considered in Sec. III A could lead to thermal equilibrium between the cosmological horizon, radiation and wDE if the other fluids are not interacting. More generally, when a different combination of fluids is considered, we should proceed similarly to this example and verify whether the radius obtained from Eq. (40) is meaningful or not.

IV. SUMMARY

When the thermodynamical properties of dark energy are studied, the hypothesis of (late time) thermal equilibrium between the cosmological horizon and the dark energy fluid is frequently assumed [30–42] and, in some cases, even extended to other cosmological fluids [35–41]. The aim of this paper was to find the restriction imposed by this hypothesis on the energy transfer rate (Q_i) between the fluids.

A first difficulty occurs in defining the temperature of the horizon. In a dynamical spacetime, such as the FRW spacetime, there is no consensus for which horizon (if any) should emit Hawking radiation and, for a given choice, what should be the temperature associated with this radiation. In order to recover different expressions used in the literature, we have considered a temperature of the form $T_h=b/2\pi R$, where R could stand for the Hubble radius (R_H) [29,34,41], for the apparent radius (R_A) [32–35,41] or for the event horizon radius (R_E) [30,31,36].

A second difficulty is the unknown nature of dark energy. We considered a generic fluid to find the interaction term required to maintain thermal equilibrium [Eq. (23)], but to go further in our analysis, we specialized to two specific types of dark energy, namely Chaplygin gas (CG) and dark energy with a constant EOS parameter (wDE). In both cases, we assumed that the energy density depends only on the temperature. This leads to interaction terms given by Eq. (27) for CG and Eq. (29) for wDE. These results illustrate that if we assume thermal equilibrium between the dark energy and a horizon of radius R, we cannot choose the interaction term Q_i freely (if an other type of dark energy is considered, its interaction term can be derived from Eq. (23), just as we did for CG and wDE). Conversely, if we impose a specific choice for the interaction term, the radius R will be determined by inverting these equations, which will not necessarily correspond to a physically meaningful horizon.

Finally, we found the interaction terms for which radiation [Eq. (32)] and cold matter [Eq. (36)] are in thermal equilibrium with the horizon. Since the ensemble of the interaction terms must satisfy $\sum_i Q_i=0$, it is nontrivial to propose a cosmological model for which all the interacting fluids are in thermal equilibrium with the horizon. Indeed, in this case, the horizon radius will be determined by Eq. (40) and will not necessarily be physically meaningful. With this regard, the hypothesis where the dark energy is the only fluid in thermal equilibrium with the horizon is better motivated. Moreover, since the baryons and the photons densities are tightly bound by the big bang nucleosynthesis (BBN) constraints and by the CMB constraints, an interaction between dark energy and dark matter is more likely to be consistent with the observational data. In this case, the interaction terms will be given by $Q_{\rm de}=\tilde{Q}_{\rm de}$ and $Q_{\rm dm}=-\tilde{Q}_{\rm de}$, where $\tilde{Q}_{\rm de}$ is given by Eq. (27) for CG, by Eq. (29) for wDE or by an analogous expression derived from Eq. (23) if an other type of dark energy is considered.

To conclude, we can remind to the reader that it is possible to obtain dynamical dark energy without the recourse of an interaction with an other fluid if its EOS parameter is different from -1. In particular, certain noninteracting models involve a variable EOS parameter $w_{\rm de}(t)$ (see [10,56–60] and references therein). In this case, the temperature evolution equation [Eq. (22)] becomes $\dot{T}_{\rm de}/T_{\rm de} = -3H(w_{\rm de} + \rho_{\rm de}\partial w_{\rm de}/\partial\rho_{\rm de})$. As a future perspective, it would be interesting to find under which conditions a thermal equilibrium with the horizon ($T_{\rm de} = T_h$) is possible for these kind of models.

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