

**Realization of effective supersymmetry with strong unification**Chun Liu<sup>\*</sup> and Zhen-hua Zhao<sup>†</sup>*Institute of Theoretical Physics, Chinese Academy of Sciences, and State Key Laboratory of Theoretical Physics, P.O. Box 2735, Beijing 100190, China*

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A natural model of realizing effective supersymmetry is presented. Two sets of the Standard Model-like gauge group  $G_1 \times G_2$  are introduced, where  $G_i = \text{SU}(3)_i \times \text{SU}(2)_i \times \text{U}(1)_i$ , which break diagonally to the Standard Model gauge group at the energy scale  $M \sim 10^7$  GeV. Gauge couplings in  $G_1$  are assumed to be much larger than those in  $G_2$ . Gauge-mediated supersymmetry breaking is adopted. The first two generations (third generation) are only charged only under  $G_1$  ( $G_2$ ). The effective supersymmetry spectrum is obtained. The reproduction of realistic Yukawa couplings is studied. The fine-tuning for a 126 GeV Higgs is reduced by the large  $A$  term due to direct Higgs-messenger interaction. Finally,  $G_2$  is found to be a nontrivial realization of the strong unification scenario in which case we can predict  $\alpha_s(M_Z)$  without real unification.

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**I. INTRODUCTION**

A Standard Model (SM)-like Higgs particle of 126 GeV has been discovered at the LHC [1]. If we are insisting on the naturalness of the SM, this discovery strengthens motivation for low-energy supersymmetry (SUSY) which stabilizes the Higgs mass at the electroweak (EW) scale. However, there is yet no definite sign of sparticles after an integrated luminosity of  $20 \text{ fb}^{-1}$  at  $\sqrt{s} = 8$  TeV. This null result for the SUSY search sets a lower bound for the first two generations of squarks,  $m_{\tilde{Q}_{1,2}} > 1$  TeV, while top squarks/sbottoms with a mass of about 500 GeV are still allowed [2]. We also note that naturalness sets an upper bound for sparticle masses of 1 TeV.

SUSY, if it is relevant to EW physics, should be beyond its simplest version. Actually it was noted a long time ago that naturalness only requires the third generation sfermions and particles (gauginos and Higgsinos) that interact significantly with the Higgs to have sub-TeV masses, while the first two generations of sparticles can be heavy, up to 20 TeV [3–5]. Such sparticle spectra also alleviate the SUSY flavor-changing neutral current problem. This phenomenological scenario is dubbed effective SUSY by Cohen *et al.* [5]. Nowadays this effective SUSY has become one of the main scenarios to reconcile naturalness with the null SUSY search [6].

In this paper, we realize effective SUSY through modifying the models of Refs. [7,8]. In those models it was introduced two sets of SM-like gauge groups  $G_1 \times G_2$ , where  $G_i = \text{SU}(3)_i \times \text{SU}(2)_i \times \text{U}(1)_i$ . At the TeV scale,  $G_1$  is strongly interacting and  $G_2$  is weakly interacting, respectively. They break diagonally to the SM gauge group. SUSY breaking is due to  $G_1 \times G_2$  gauge mediation. Furthermore,  $G_2$  exhibits strong unification [8], namely, its gauge coupling constants have a common Landau pole

at the unification scale [9–11]. In that model, all three generations are put in  $G_2$ , which does not result in any sparticle splitting. To produce sparticle splitting, the first two generations have to be treated differently than the third one. In this work, the first two generations are put in  $G_1$  and the third in  $G_2$ .

Among other things, we need to solve the following problems. First, we need to generate Yukawa interactions between the first two generations and the third generation. This is because the three generations are in different gauge sectors from the beginning. Then, we wish to reduce the fine-tuning of the 126 GeV Higgs. In conventional gauge-mediated SUSY breaking (GMSB) [12], fine-tuning seemed unavoidable for a 126 GeV Higgs. Furthermore, we wish to re-examine strong unification. This is needed due to all the changes in the particle content, and the conditions of strong unification are quite subtle.

We notice that a similar model was proposed by Craig *et al.* [13,14] in which the first two generations were also put in  $G_1$  and the third one in  $G_2$ . However, there are several main differences. The first involves  $G_1$ . In the model of Ref. [14],  $G_1$  gauge interactions are weakly coupled at the TeV scale, whereas our  $G_1$  is superstrong. The second difference involves  $G_2$ . Their  $G_2$  gauge interactions unify weakly in the sense of ordinary grand unification, whereas our  $G_2$  is of strong unification [9–11]. The third difference involves GMSB. They only used a messenger for  $G_1$ , and we have messengers for both  $G_1$  and  $G_2$ . And finally, we need to use a direct Higgs-messenger interaction to reduce the fine-tuning of a 126 GeV Higgs. These differences make this model qualitatively different.

There are a few other ways to realize effective SUSY [15,16]. Some of them use an extra  $\text{U}(1)$  gauge group which contributes larger masses to the first two generations of sparticles than to the third generation ones. Usually, this  $\text{U}(1)$  symmetry suppresses the Yukawa couplings for the first two generations compared to that for the third generation, giving an explanation for the fermion mass

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hierarchy. Some other works have assumed particular boundary conditions at a high scale, then employed the renormalization group technique [16].

The paper is organized as follows. Our model will be given in the next section, where flavor physics for fermions and the problem of naturalness in light of a 126 GeV Higgs will be discussed. After all particle contents and the mass spectrum have been fixed, the prediction for  $\alpha_s(M_Z)$  will be calculated by means of strong unification in Sec. III. The final section summarizes our results and gives discussions.

## II. THE MODEL

We consider a SUSY model with two sets of the SM-like gauge groups  $G_1$  and  $G_2$  where  $G_i = \text{SU}(3)_i \times \text{SU}(2)_i \times \text{U}(1)_i$ . The first two generations and the third generation of matter transform under  $G_1$  and  $G_2$ , respectively. The two Higgs doublets  $H_u$  and  $H_d$  are in  $G_2$ . The other fields include SUSY-breaking messengers and the Higgs fields which break  $G_1 \times G_2$  into the SM. For convenience, we will use field representations under  $\text{SU}(5)$  to illustrate their representations under  $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ .

The GMSB mechanism is employed. Two sets of messenger fields  $T_1$  ( $\bar{T}_1$ ) and  $T_2$  ( $\bar{T}_2$ ) are introduced. They transform nontrivially under  $G_1$  and  $G_2$ , respectively. Without losing generality, we will focus on the quark/squark sector. At the scale of SUSY breaking, squarks have the following masses:

$$m_{\tilde{Q}_{1,2}}^2 \sim \left( \frac{g_1^2 F}{16\pi^2 M} \right)^2 \quad \text{and} \quad m_{\tilde{Q}_3}^2 \simeq \left( \frac{g_2^2 F}{16\pi^2 M} \right)^2. \quad (1)$$

$M$  stands for the messenger scale and  $\sqrt{F}$  is the measure of SUSY breaking.  $g_1$  and  $g_2$  represent coupling constants for  $G_1$  and  $G_2$ , respectively. Therefore, we can realize the effective SUSY sparticle spectrum by requiring  $g_1$  to be much larger than  $g_2$ . Note that Eq. (1) for  $m_{\tilde{Q}_{1,2}}^2$  is not exact, because  $g_1$  is too large.

A pair of Higgs fields  $\Phi$  and  $\bar{\Phi}$  charged under  $G_1 \times G_2$  as  $\mathbf{5} \times \bar{\mathbf{5}}$  and  $\bar{\mathbf{5}} \times \mathbf{5}$  is introduced.  $\Phi$  and  $\bar{\Phi}$  have a mass  $M_\Phi$ , and they obtain vacuum expectation values (VEVs) as  $\langle \Phi \rangle = \langle \bar{\Phi} \rangle = VI_2 \times I_3$ , where  $V \sim M_\Phi$ , and  $I_2$  and  $I_3$  are the unit matrix in the subspace of  $\text{SU}(2)_1 \times \text{SU}(2)_2$  and  $\text{SU}(3)_1 \times \text{SU}(3)_2$ , respectively. As a result,  $G_1 \times G_2$  break diagonally to the SM gauge group  $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ . Below the scale of  $M_\Phi$ , the effective theory of this model looks like effective SUSY with the following relations among gauge coupling constants:

$$\frac{1}{g_s^2} = \frac{1}{g_{s1}^2} + \frac{1}{g_{s2}^2}, \quad \frac{1}{g^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}, \quad \frac{1}{g'^2} = \frac{1}{g_1'^2} + \frac{1}{g_2'^2}. \quad (2)$$

Because of the relation  $g_1^2 \gg g_2^2$ , the SM gauge couplings are almost fully determined by those in  $G_2$ . More details about the breaking of  $G_1 \times G_2$  can be found in Ref. [8].

While this model in many aspects is similar to that of Ref. [8], new features come in because of the separation of the three generations.

Before  $G_1 \times G_2$  breaking, the three generations are put in different gauge sectors, so there are no marginal operators giving Yukawa couplings between the first two generations of matter and the third generation. However, higher-dimensional operators such as  $H_d Q_3 \Phi_{d\bar{a}} \bar{d}_{1(2)}$  are allowed by  $G_1 \times G_2$  symmetry. Here  $\Phi_{d\bar{a}}$  is the component of the Higgs  $\Phi$  with quantum numbers  $(3, 1, -\frac{1}{3}) * (\bar{3}, 1, \frac{1}{3})$  under  $\text{SU}(3)_1 \times \text{SU}(2)_1 \times \text{U}(1)_1 \times \text{SU}(3)_2 \times \text{SU}(2)_2 \times \text{U}(1)_2$ . These kinds of operators can be produced by integrating out appropriate heavy fields, just like the Froggatt-Nielsen mechanism [17]. As a result, they are suppressed by  $M$  (mass scale for the heavy fields that have been integrated out),  $\frac{1}{M} H_d Q_3 \Phi_{d\bar{a}} \bar{d}_{1(2)}$ . When  $G_1 \times G_2$  is spontaneously broken, i.e.,  $\Phi_{d\bar{a}}$  gets a VEV, there will be terms like  $\frac{V}{M} H_d Q_3 \bar{d}_{1(2)}$  that lead to Yukawa interactions between first/second generation fermions and third generation fermions. Taking  $\frac{V}{M}$  to be a small quantity  $\sim 0.1 - 0.01$ , the mass hierarchy between the third generation and the first two is obvious. Roughly speaking, this paves the way to obtain the realistic fermion mass pattern, mixing, and  $CP$  violation. We can say that the hierarchy among the three generations of fermions and that among the three generations of sfermions are closely connected to each other in this model.

This approach has been discussed in Ref. [14]. There, a vectorlike  $\mathbf{5}$  representation was introduced as the mediator that is integrated out. Similarly, in  $G_2$ , we introduce a full vectorlike generation  $(L, \bar{d})$  and  $(Q, \bar{u}, \bar{e})$  as the representations  $\bar{\mathbf{5}}$  and  $\mathbf{10}$ , respectively. The masses of these vectorlike fields are taken to be of the same order,  $M_\Psi$ .

In the minimal supersymmetric Standard Model, to obtain a 126 GeV Higgs, we either need multi-TeV top squarks which lead to severe fine-tuning, or we must turn to the large- $A_t$  scenario, in which case sub-TeV top squarks are enough. In conventional GMSB, the contribution of  $A_t$  is negligible and the fine-tuning induced by top squarks that are too heavy seems to be unavoidable. To preserve naturalness, we will produce a large- $A_t$  term by extending conventional GMSB.

We choose the messenger  $T_2$  ( $\bar{T}_2$ ) to be the representation  $\mathbf{10}$  ( $\bar{\mathbf{10}}$ ). It was found that [18] a large- $A_t$  term can be produced without a large Higgs mass. This is due to the direct interaction between the Higgs and the messenger in the superpotential,

$$y H_u T_2^O T_2^{\bar{u}}, \quad (3)$$

where  $T_2^O$  and  $T_2^{\bar{u}}$  are components of  $T_2$  that have the same gauge quantum numbers as  $Q_3$  and  $\bar{u}_3$ , respectively. It should be pointed out that this term is the only one of direct interaction between messengers and ordinary matter that employs messenger parity. The one-loop contribution to

the soft term  $H_u \tilde{Q}_3 \tilde{u}_3$  is extracted from the wave-function renormalization for the superfield  $H_u$  [19],

$$A_t \sim -\frac{y^2 y_t F}{16\pi^2 M}. \quad (4)$$

In Ref. [20], the same method was used to produce a large- $A_t$  term except that there it was  $Q_3$  and  $\bar{u}_3$  instead of  $H_u$  that interacted directly with the messenger. More issues about this method of producing a large- $A_t$  term can be found in Refs. [18,21].

### III. STRONG UNIFICATION

We will consider the unification of the gauge coupling constants. Because the SM gauge couplings are almost fully determined by those of  $G_2$ , what we will really care about above the  $G_1 \times G_2$  breaking scale, is the  $G_2$  gauge coupling constants. So far quite a few new fields have been introduced. The particle content and the mass spectrum are summarized in the following with an emphasis on those that are charged under  $G_2$ . There is only one chiral generation in  $G_2$ . The two Higgs doublets are in  $G_2$ . The bifundamental Higgs  $\Phi$  ( $\bar{\Phi}$ ) is charged under  $SU(5)_1 \times SU(5)_2$  as  $\mathbf{5} \times \bar{\mathbf{5}}$  ( $\bar{\mathbf{5}} \times \mathbf{5}$ ) with a mass  $M_\Phi$ . The messenger  $T_2$  ( $\bar{T}_2$ ) is charged under  $G_2$  as the representation  $\mathbf{10}$  ( $\bar{\mathbf{10}}$ ) with a mass  $M$ . Besides, there is an extra vectorlike generation charged under  $G_2$  as the representation  $\bar{\mathbf{5}} + \mathbf{10}$  ( $\mathbf{5} + \bar{\mathbf{10}}$ ) with a mass  $M_\Psi$ . For simplicity and definiteness, we will identify  $M_\Phi$  and  $M_\Psi$  with  $M$  in the following analysis.

Below the  $G_1 \times G_2$  breaking scale  $M$ , this model is just that of the minimal SUSY SM, so the SM gauge coupling running can be calculated in the usual way. At the scale  $M$ , the SM gauge couplings are identified with those in  $G_2$ . Above the scale  $M$ , since there are so many complete representations of  $SU(5)_2$ , while the unification energy scale does not change, gauge couplings in  $G_2$  grow so fast that they may come across their Landau poles as they evolve to the unification scale. The situation where gauge couplings reach their common Landau pole is called strong unification [8–10]. Using strong unification to predict the gauge couplings at the EW scale seems unreasonable because of the strong coupling domain, where the perturbative method is not reliable. However, as shown in Ref. [10], the ratios of gauge couplings in  $G_2$  will reach their infra-fixed points at the scale  $M$ . Thus, we can determine the SM gauge couplings at the EW scale, with the ratios of the gauge couplings in  $G_2$  at the scale  $M$  as a boundary condition where the perturbative calculation already works.

The boundary conditions for the gauge couplings of  $G_2$  at the scale  $M$  satisfy the following relation [10]:

$$\alpha'_2(M) b'_2 = \alpha_2(M) b_2 = \alpha_{s2}(M) b_{s2}, \quad (5)$$

where  $\alpha'_2(M) = \frac{g_2^2(M)}{4\pi}$ , and so forth.  $b'_2 = \frac{73}{5}$ ,  $b_2 = 9$ , and  $b_{s2} = 5$  are one-loop beta functions above the scale  $M$  for

$g'_2$ ,  $g_2$ , and  $g_{s2}$ , respectively. In the following, we will first calculate the prediction for  $\alpha_s(M_Z)$  in a simply way to illustrate the usage of strong unification, and then take into consideration two-loop contributions and low-scale threshold effects induced by the sparticles' mass splitting among different generations.

In the first case,  $\alpha'(M)$  and  $\alpha(M)$  can be determined by  $\alpha'(M_Z)$  and  $\alpha(M_Z)$  through the following equations:

$$\alpha'^{-1}(M) = \alpha'^{-1}(M_Z) - \frac{b'}{2\pi} \ln \frac{M}{M_Z}, \quad (6)$$

$$\alpha^{-1}(M) = \alpha^{-1}(M_Z) - \frac{b}{2\pi} \ln \frac{M}{M_Z}. \quad (7)$$

With Eq. (5), we can get  $M \sim 10^8$  GeV and  $\alpha_s^{-1}(M) = 15.17$ . Finally,  $\alpha_s(M_Z)$  is calculated to be 0.119 as follows:

$$\alpha_s^{-1}(M_Z) = \alpha_s^{-1}(M) + \frac{b_s}{2\pi} \ln \frac{M}{M_Z}. \quad (8)$$

After the inclusion of low-scale threshold effects and dominated two-loop contributions, Eqs. (6)–(8) will be replaced by the following equations:

$$\begin{aligned} \alpha'^{-1}(M) = \alpha'^{-1}(M_Z) &- \frac{\tilde{b}'}{2\pi} \ln \frac{m_{\tilde{Q}_3}}{M_Z} - \frac{\tilde{b}'}{2\pi} \ln \frac{m_{\tilde{Q}_{1,2}}}{m_{\tilde{Q}_3}} \\ &- \frac{b'}{2\pi} \ln \frac{M}{m_{\tilde{Q}_{1,2}}} - \frac{1}{4\pi} \frac{b_{11}}{b'} \ln \frac{\alpha'(M)}{\alpha'(M_Z)} \\ &- \frac{1}{4\pi} \frac{b_{12}}{b} \ln \frac{\alpha(M)}{\alpha(M_Z)} - \frac{1}{4\pi} \frac{b_{13}}{b_s} \ln \frac{\alpha_s(M)}{\alpha_s(M_Z)}, \quad (9) \end{aligned}$$

$$\begin{aligned} \alpha^{-1}(M) = \alpha^{-1}(M_Z) &- \frac{\tilde{b}}{2\pi} \ln \frac{m_{\tilde{Q}_3}}{M_Z} - \frac{\tilde{b}}{2\pi} \ln \frac{m_{\tilde{Q}_{1,2}}}{m_{\tilde{Q}_3}} \\ &- \frac{b}{2\pi} \ln \frac{M}{m_{\tilde{Q}_{1,2}}} - \frac{1}{4\pi} \frac{b_{21}}{b'} \ln \frac{\alpha'(M)}{\alpha'(M_Z)} \\ &- \frac{1}{4\pi} \frac{b_{22}}{b} \ln \frac{\alpha(M)}{\alpha(M_Z)} - \frac{1}{4\pi} \frac{b_{23}}{b_s} \ln \frac{\alpha_s(M)}{\alpha_s(M_Z)}, \quad (10) \end{aligned}$$

$$\begin{aligned} \alpha_s^{-1}(M) = \alpha_s^{-1}(M_Z) &- \frac{\tilde{b}_s}{2\pi} \ln \frac{m_{\tilde{Q}_3}}{M_Z} - \frac{\tilde{b}_s}{2\pi} \ln \frac{m_{\tilde{Q}_{1,2}}}{m_{\tilde{Q}_3}} \\ &- \frac{b_s}{2\pi} \ln \frac{M}{m_{\tilde{Q}_{1,2}}} - \frac{1}{4\pi} \frac{b_{31}}{b'} \ln \frac{\alpha'(M)}{\alpha'(M_Z)} \\ &- \frac{1}{4\pi} \frac{b_{32}}{b} \ln \frac{\alpha(M)}{\alpha(M_Z)} - \frac{1}{4\pi} \frac{b_{33}}{b_s} \ln \frac{\alpha_s(M)}{\alpha_s(M_Z)}, \quad (11) \end{aligned}$$

with

$$\begin{aligned}
\tilde{b}' &= \frac{41}{10}, & \tilde{b}' &= \frac{79}{15}, & b' &= \frac{33}{5}, & b_{11} &= \frac{199}{25}, & b_{12} &= \frac{27}{5}, & b_{13} &= \frac{88}{5}, \\
\tilde{b} &= -\frac{19}{6}, & \tilde{b} &= -\frac{1}{3}, & b &= 1, & b_{21} &= \frac{9}{5}, & b_{22} &= 25, & b_{23} &= 24, \\
\tilde{b}_s &= -7, & \tilde{b}_s &= -\frac{13}{3}, & b_s &= -3, & b_{31} &= \frac{11}{5}, & b_{32} &= 9, & b_{33} &= 14.
\end{aligned} \tag{12}$$

It is found that  $\alpha_s(M_Z) \sim 0.117$  and  $M \sim 10^7$  GeV, when  $m_{\tilde{Q}_3}$  and  $m_{\tilde{Q}_{1,2}}$  take the typical values 1 and 10 TeV, respectively. This value is more closer to world average value  $0.1184 \pm 0.0007$  [22] than the value 0.122 obtained with the conventional unification boundary condition instead of the strong unification boundary condition.

In the above discussion, we have taken the limit as  $\frac{g_1^2}{g_2^2}$  goes to infinity so that the SM gauge coupling can be identified with that in  $G_2$  at the Higgsing scale. There will be a several percent uncertainty in this identification if we take into consideration the fact that  $\frac{g_1^2}{g_2^2}$  is finite. This uncertainty will affect the prediction for  $\alpha_s(M_Z)$  substantially. For example, with a typical value  $20 \sim 40$  for  $\frac{g_1^2}{g_2^2}$ ,  $\alpha_s(M_Z)$  will have an uncertainty of about 0.005. However, if three gauge couplings in  $G_1$  sector has the same ratio with the counterparts in  $G_2$  sector,

$$\frac{g'_1}{g_2} = \frac{g_1}{g_2} = \frac{g_{s1}}{g_{s2}} \tag{13}$$

this uncertainty will disappear. This is because the boundary condition Eq. (5) just depends on the ratio of gauge coupling constants.

#### IV. SUMMARY AND DISCUSSION

In summary, we have presented a model of realizing effective SUSY. Two sets of the SM-like gauge group  $G_1 \times G_2 = \text{SU}(3)_1 \times \text{SU}(2)_1 \times \text{U}(1)_1 \times \text{SU}(3)_2 \times \text{SU}(2)_2 \times \text{U}(1)_2$  have been introduced which break diagonally to the SM gauge group at the energy scale  $M \sim 10^7$  GeV. The gauge couplings in  $G_1$  have been assumed to be much larger than those in  $G_2$ . GMSB has been adopted. The first two generations (third generation) are charged only under  $G_1$  ( $G_2$ ). The effective SUSY spectrum has been obtained naturally. The fine-tuning for a 126 GeV Higgs is greatly reduced. With all the fields necessary and their masses fixed, we predicted  $\alpha_s(M_Z)$  in the scenario of strong unification.

Compared to our previous works [7,8], in addition to the effective SUSY spectrum, the following new features have arisen.

- (i) An extra vectorlike generation charged under  $G_2$  has been introduced as a mediator, so as to reproduce realistic Yukawa couplings between the first two

generations and the third generation, i.e., the suitable fermion mass hierarchy and the Cabibbo-Kobayashi-Maskawa mixing matrix.

- (ii) The fine-tuning for an 126 GeV Higgs is greatly reduced by a large- $A_t$  term produced by a direct Higgs-messenger interaction, because the messenger for  $G_2$  has been specified to be a **10** representation under SU(5), which is absent in conventional GMSB.

The following three main aspects clarify the differences between our model and that of Ref. [14].

- (a) In Ref. [14], the gauge couplings in  $G_1$  and  $G_2$  were comparable. Only a messenger for  $G_1$  was introduced, and the third generation particles could feel SUSY breaking only after the breaking of  $G_1 \times G_2$ , so that  $m_{\tilde{Q}_3}$  was suppressed by an additional factor  $\frac{V}{M}$  in comparison with  $m_{\tilde{Q}_{1,2}}$ .
- (b) There was no need to produce a large- $A_t$  term in Ref. [14]. Due to the comparability of the gauge couplings in  $G_1$  and  $G_2$ , and the low scale of  $M_\Phi \sim 10^4$  GeV, a nondecoupling  $D$ -term contribution to the Higgs mass could be significant.
- (c) In Ref. [14], the unification of the gauge couplings was “weak” as compared to strong unification.

We conclude with our final remarks. First, it is worth pointing out that despite the term strong “unification,”  $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$  does not necessarily unify into a larger simple group, so that there can be no proton decay at all, and thus there is no so-called doublet-triplet splitting problem. Because  $g_2 \ll g_1$ , the  $G_2$  unification scale is the same as that of the traditional grand unified theory (GUT), namely about  $3 \times 10^{16}$  GeV. Second, the gauge couplings in  $G_1$  are expected to be a realization of the GUT. We have not studied this much because, on the one hand, it does not affect our physical results, and on the other hand, the couplings are too strong to use the perturbation method. Third, the LHC has set a lower bound on the gluino mass  $m_{\tilde{g}} > 1$  TeV [2], and this bound would also apply to  $m_{\tilde{Q}_3}$  in traditional GMSB. This is not the case for this model, because  $G_1 \times G_2$  breaking also contributes an additional mass to the gluino  $\sim \frac{g_2^2 V^2}{M}$  from the mixing with the fermionic component of  $\Phi$ . This contribution is expected to be larger than the purely soft mass  $\frac{g_2^2 F}{16\pi^2 M}$  [8]. Besides, the Higgs-mediated SUSY-breaking contribution reduces  $m_{\tilde{Q}_3}$ . In a word, this model allows for an interesting mass pattern,  $m_{\tilde{Q}_{1,2}} \gg m_{\tilde{g}} \gg m_{\tilde{Q}_3}$ . Finally, in this work

we have assumed that the first two generations are in the same gauge group. It is imaginable that we can introduce one more version of the SM group to further split these two generations. Namely, we may expect a model of  $[SU(3) \times SU(2) \times U(1)]^3$ , which first breaks into  $G_1 \times G_2$  at some higher energy scale.

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