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# Constraining a class of $B-L$ extended models from vacuum stability and perturbativity

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The precise knowledge of the Standard Model (SM) Higgs boson and top-quark masses and couplings are crucial to understand the physics beyond it. An SM-like Higgs boson having a mass in the range of 123–127 GeV squeezes the parameters for physics beyond the Standard Model. In recent the LHC era many TeV-scale neutrino mass models have earned much attention as they pose many interesting phenomenological aspects. We have contemplated $B-L$ extended models which are theoretically well motivated and phenomenologically interesting, and they successfully explain neutrino mass generation. In this article we analyze the detailed structures of the scalar potentials for such models. We compute the criteria which guarantee that the vacuum is bounded from below in all directions. In addition, perturbativity (triviality) bounds are also necessitated. Incorporating all such effects, we constrain the parameters of such models by performing their renormalization-group evolutions.

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## I. INTRODUCTION

The recent announcements from both ATLAS [1] and CMS [2] have revealed the existence of a new boson having a mass in the range 123–127 GeV. The data so far indicates a close resemblance to one having some of the measured properties of the Standard Model (SM) Higgs. However, it has yet to be confirmed whether this boson is the SM Higgs or a beyond the Standard Model artifact. This long awaited quest will only be examined more vigorously in the near future with the help of more data.

If the newly discovered particle is indeed the SM Higgs boson then its mass can carry a signature of new physics which embeds SM at low energy. The Higgs mass can be recast solely in terms of the Higgs quartic coupling, $\lambda_h$. The stability of the electroweak (EW) vacuum demands a positive $\lambda_h$. Now if the SM is the only existing theory in nature then this condition, $\lambda_h > 0$,[1] must be maintained at each scale of its evolution up to the Planck scale ($M_{\mathrm{Pl}}$). The evolution of $\lambda_h$ with the renormalization (mass) scale limits two boundary values—one at the EW scale for which we have $\lambda_h(M_{\mathrm{Pl}}) = \pi$, and one at the Planck scale for which we have 0—from the demands of perturbativity of the coupling (triviality) and the stability of the vacuum (vacuum stability), respectively. It has been noted in Refs. [3–5] that the SM electroweak vacuum is not stable up to the Planck scale for most of the SM parameters (top-quark mass, Higgs mass and strong coupling $\alpha_s$). Thus it indicates that some new physics might be there before the SM vacuum attains instability. Thus the physics beyond Standard Model is expected to take care of the stability of the vacuum of the full scalar potential along with the electroweak ones. In brief, the present range of the SM-like Higgs mass entertains the presence of new physics solely from the vacuum stability point of view.

Apart from this, we already have hints of new physics beyond the Standard Model from the neutrino sector. Many experimental observations, like neutrino oscillations, confirm that neutrinos have tiny nonzero masses which cannot be accommodated naturally within the SM. Thus we must have physics beyond the Standard Model to explain this feature. Among the neutrino mass generation procedures the seesaw mechanism [6–15] is very popular. In usual (natural) seesaw models light neutrino masses are $\sim m_D^2/M$ where the Dirac-type mass $m_D \sim 100$ GeV and $M$ is the Majorana mass of a heavy fermion which gets integrated out during the process. The mass of this heavy fermion, $M$, determines the scale of the seesaw models which needs to be very high ($\sim 10^{11}$ GeV) to avoid any fine-tuning in $m_D$. As the natural scale of the seesaw is very high these models suffer from a lack of testability. But it is also possible to construct low-scale ($\sim$TeV) models by either importing some new fields [16] or incorporating higher-dimensional operators [17–21]. These models not only generate the correct order of neutrino masses and mixing, but are also phenomenologically interesting as the scale of these theories are well within the reach of present experiments like the LHC. These models are extended by some extra gauge symmetry and/or new particles. The presence of these new fields might affect the evolution of the SM couplings—like the gauge, Higgs quartic, and top Yukawa couplings—if they couple to the SM particles. Hence it is necessary to examine the status of the SM vacuum once these new

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[1]This is the necessary—but not sufficient—condition to confirm the sole existence of the SM up to the Planck scale.

 

physics models come into play. Thus by using knowledge of the SM parameters and from the demand of vacuum stability[2] the new parameters involved in the theory might be severely constrained. In the literature the stability of the vacua was discussed in several scenarios considering beyond Standard Models. These models are extended by the extra gauge symmetry and/or the addition of new particles. Quantum corrections of the quartic couplings depend on the spin of the particles belonging to a particular model. The fermion-loop contributions contain a relative minus sign compared to the bosonic fields. Thus the Yukawa couplings tend to spoil the stability unlike the gauge and other scalar self-couplings. Vacuum stability in different variants of seesaw models has been adjudged in Refs. [22–28] which has a richer particle spectrum compared to the SM. In the context of gauge extensions, the vacuum stability for the alternative left-right symmetric model has been discussed in Ref. [29].

In a theory involving multiple scalar fields the structure of the potential is complicated. The vacuum stability criteria depend on some combinations of the scalar quartic couplings. Moreover, the perturbativity (triviality) bounds also play crucial roles in finding a consistent parameter space compatible with the choice of new physics scales. Nontachyonic scalar masses are guaranteed with these constraints. It has been noted that some of the quartic couplings can be recast in terms of the heavy scalar masses and thus can be constrained from a phenomenological point of view. On the contrary, few of them do not have that much impact on scalar masses; rather, they determine the splitting among the narrowly spaced massive scalar modes. Our present collider experiments are still not sensitive enough to address these fine splittings, and thus the quartic couplings are beyond the reach of any experimental verification. But these couplings can be constrained through vacuum stability and perturbativity (triviality) depending on the choice of the scale of new physics.

In this paper we concentrate on the $U(1)_{B-L}$ extended models which are classified into two categories: SM $\otimes$ $U(1)_{B-L}$ or the left-right (LR) symmetry. We have adopted two variants of the LR-symmetric models containing (i) two SU(2) triplet scalars $\Delta_{L(R)}$, and (ii) two SU(2) doublet scalars, $H_{L(R)}$. In Sec. II we introduce the basic structures of these models. Then we include the renormalization group evolutions of all the necessary couplings and show how the vacuum stability and perturbativity (triviality) bounds constrain the parameter space of each model in Sec. III. We have analyze the structure of the potentials in detail and compute the criteria for vacuum stability using the formalism shown in Ref. [30]. All vacuum stability conditions corresponding to different models are listed in Appendix B.

[2]In this paper we are considering stability up to the Planck scale. We are not considering the metastability which does not require the vacuum to be bounded from below. If the decay lifetime of a vacuum is larger than the lifetime of the Universe then that vacuum is metastable. But as our procedure concerns only the boundedness of the scalar potential it fails to pin down the existence of the metastable vacuum.

## II. MODELS

The Standard Model symmetry group is expressed as $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. It has been noted in Ref. [31] that an extra U(1) gauge symmetry along with the SM can provide solutions to some of the unaddressed issues in the Standard Model. These extra Abelian symmetry groups can, in general, originate from different high-scale grand unified theories (GUTs), like SO(10) and E(6). These larger groups contain $U(1)_{B-L}$ as a part of the intermediate gauge symmetries. In nonsupersymmetric GUT models the $U(1)_{B-L}$ breaking scale can be lowered to a few TeV[3] [32], which is consistent with unification pictures. In our present study we concentrate on TeV-scale $U(1)_{B-L}$ extended models where neutrino mass generation can be explained. However, any high-scale roots of these models are not considered and are kept for future work.

### A. $U(1)_{B-L}$

The gauge group under consideration is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$. This minimal model contains an extra complex singlet scalar field $S$ and this extra $B-L$ symmetry is broken once it acquires a vacuum expectation value (VEV) [33–35]. Thus the VEV determines the symmetry-breaking scale of this symmetry and also the mass of the extra neutral gauge boson $Z_{B-L}$. For the purpose of our study we will focus only on the relevant part of the Lagrangian, namely the scalar kinetic and potential terms and the lepton Yukawa couplings. The scalar kinetic term is

$$\mathcal{L}_s = (D^\mu \Phi)^\dagger (D_\mu \Phi) + (D^\mu S)^\dagger (D_\mu S) - V(\Phi, S). \quad (1)$$

Here the potential $V(\Phi, S)$ is given as

$$V(\Phi, S) = m^2 \Phi^\dagger \Phi + \mu^2 |S|^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 |S|^4 + \lambda_3 \Phi^\dagger \Phi |S|^2, \quad (2)$$

where $\Phi$ and $S$ are the complex scalar doublet and singlet fields, respectively. After gauging away the extra modes and acquiring the VEVs these fields are redefined as

$$\Phi \equiv \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + \phi) \end{pmatrix}, \qquad S \equiv \frac{1}{\sqrt{2}}(v_{B-L} + s), \quad (3)$$

where the EW-symmetry-breaking VEV $v$ and the $B-L$-breaking VEV $v_{B-L}$ are real and positive.

[3]This is also true for supersymmetric GUT models; see Ref. [32].

We also find the scalar mass matrix in the following form:

$$\mathcal{M} = \begin{pmatrix} \lambda_1 v^2 & \frac{\lambda_3 v_{B-L} v}{2} \\ \frac{\lambda_3 v_{B-L} v}{2} & \lambda_2 v_{B-L}^2 \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}. \quad (4)$$

After diagonalizing this mass matrix we construct two physical scalar states: a light $h$ and a heavy $H$, having masses $M_h$ and $M_H$, respectively,

$$M_{H,h}^2 = \frac{1}{2}\left[\mathcal{M}_{11} + \mathcal{M}_{22} \pm \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}\right]. \quad (5)$$

The scalar mixing angle $\alpha$ can be expressed as

$$\tan(2\alpha) = \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11} - \mathcal{M}_{22}} = \frac{\lambda_3 v v_{B-L}}{\lambda_1 v^2 - \lambda_2 v_{B-L}^2}. \quad (6)$$

Using Eqs. (5) and (6) the quartic coupling constants $\lambda_1$, $\lambda_2$, and $\lambda_3$ can be recast in the following forms:

$$\lambda_1 = \frac{1}{4v^2}\{(M_H^2 + M_h^2) - \cos 2\alpha(M_H^2 - M_h^2)\},$$
$$\lambda_2 = \frac{1}{4v_{B-L}^2}\{(M_H^2 + M_h^2) + \cos 2\alpha(M_H^2 - M_h^2)\},$$
$$\lambda_3 = \frac{1}{2vv_{B-L}}\{\sin 2\alpha(M_H^2 - M_h^2)\}. \quad (7)$$

It can be noted from the last equation in Eq. (7) that we would get a duplicate set of solutions with inverted signs for both $\alpha$ and $\lambda_3$. Hence one choice of positive $\alpha$ suffices as presented at Sec. III A.

Due to the presence of an extra $U(1)_{B-L}$ gauge theory the SM gauge kinetic terms are modified by

$$\mathcal{L}_{B-L}^{KE} = -\frac{1}{4}F'^{\mu\nu}F'_{\mu\nu}, \quad (8)$$

where,

$$F'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu. \quad (9)$$

The covariant derivative for $\mathrm{SU}(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$ sector in this model is modified as

$$D_\mu \equiv \partial_\mu + ig_2 T^a W_\mu^a + ig_1 Y B_\mu + i(\tilde{g}Y + g_{B-L}Y_{B-L})B'_\mu. \quad (10)$$

The SM gauge bosons $B_\mu$ and $W_\mu^3$ will mix with the new gauge boson $B'_\mu$ to create two massive physical fields $Z$ and $Z_{B-L}$ and one massless photon field $A$. Assuming there is no kinetic mixing at tree level, i.e., $\tilde{g} = 0$ at the EW scale, the physical gauge-boson masses are given as

$$M_Z^2 = \frac{1}{4}(g_1^2 + g_2^2)v^2, \quad (11)$$

$$M_{Z_{B-L}}^2 = 4g_{B-L}^2 v_{B-L}^2. \quad (12)$$

Along with the Standard Model particles, three right-handed neutrinos ($\nu_R$) are introduced.[4] The relevant term of the Lagrangian of the Yukawa interactions can be written as

$$-\mathcal{L}_Y = y_{ij}^l \overline{l_{iL}}\,\tilde{\Phi}\,\nu_{jR} + y_{ij}^h \overline{(\nu_R)_i^c}\nu_{jR} S + \text{H.c.}, \quad (13)$$

where $\tilde{\Phi} = i\sigma_2\Phi^*$ with $\sigma_2$ being the Pauli matrix. The second term of the above equation is the Majorana mass term. Note from Eq. (13) that the conservation of $B-L$ charge requires that the singlet scalar field, $S$, must have $Q_{B-L} = -2$. When the SM Higgs and singlet scalar $S$ acquire VEVs the neutrino mass matrix takes the form

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix}, \quad (14)$$

where $m_D = y^l \frac{v}{\sqrt{2}}$ and $m_R = \sqrt{2}y^h v_{B-L}$. The light ($m_{\nu_l}$) and heavy ($m_{\nu_h}$) neutrino masses are

$$m_{\nu_l} = -m_D^T m_R^{-1} m_D, \quad (15)$$

$$m_{\nu_h} = m_R. \quad (16)$$

In this model the heavy neutrino mass $m_R$ is also generated through the Yukawa terms unlike the gauge-invariant Majorana mass term in type-I seesaw models. It can be noted that with $m_R \sim \mathcal{O}$ (TeV), $y^l$ needs to be very small to generate light neutrino masses $\sim\mathcal{O}$ (eV). But $y^h$ can be large $\sim\mathcal{O}(1)$ as $v_{B-L}$ is around the TeV scale. Thus successful light neutrino mass generation does not constrain $y^h$. But as the heavy neutrino is also coupled to the SM-like Higgs, $y^h$ affects the vacuum stability of the scalar potential in this model and gets constrained. The gauge coupling $g_{B-L}$ and the VEV of the $B-L$-breaking scale are also free parameters. In the following section we show how these parameters are constrained from the vacuum stability of the scalar potential and also from the perturbativity (triviality) of the couplings.

### B. Left-right symmetry

The full LR-symmetric gauge group is written as $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$. The $\mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$ is broken to $\mathrm{U}(1)_Y$ at a scale higher than the EW-symmetry-breaking one. Thus the hypercharge generator is a linear combination of the $\mathrm{SU}(2)_R$ and $\mathrm{U}(1)_{B-L}$ generators. In this model, the hypercharge, $Y$, can be

[4] One right-handed neutrino ($Q_{B-L} = -1$) for each generation is required for the sake of gauge anomaly cancellation.

reconstructed from the $SU(2)_R$ and $U(1)_{B-L}$ quantum numbers as

$$Y = T_{3R} + (B-L)/2, \tag{17}$$

with $T_{3R}$ being the third component of $SU(2)_R$ isospin.

Here we briefly present two variants of minimal left-right symmetric models:

(i) The scalar sector consists of a bidoublet ($\Phi$), one left-handed triplet ($\Delta_L$), and one right-handed triplet ($\Delta_R$) [36–39].

(ii) The scalar sector consists of a bidoublet ($\Phi$), one left-handed doublet ($H_L$), and one right-handed doublet ($H_R$) [40–42].

### *1. LR model with triplet scalars*

The most generic scalar potential of this model with bidoublet and triplet scalars ($\Phi, \Delta_{L,R}$) is given in Appendix 2. The explicit structures of the scalars can be presented in the following form:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \quad \Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix}.$$

These fields transform under $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge groups in the following manner:

$$\Phi \equiv (2,2,0), \quad \Delta_R \equiv (1,3,2), \quad \Delta_L \equiv (3,1,2). \tag{18}$$

Once neutral components of these scalars acquire vacuum expectation values, they can be written in the following form:

$$\langle\Phi\rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\theta} \end{pmatrix}, \quad \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix},$$
$$\langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \tag{19}$$

where, for simplicity we have chosen $v_2 = 0$ without loss of generality. With these structures for the vacuum expectation values, symmetry breaking occurs in two stages. The symmetry group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ breaks down to $SU(2)_L \otimes U(1)_Y$ by $v_R$ at the high scale. Consequently, the vacuum expectation value $v_1$ of the bidoublet breaks $SU(2)_L \otimes U(1)_Y$ to $U(1)_{EM}$. So the total number of Goldstone bosons will be six. Now the Higgs sector has 20 degrees of freedom (eight real fields for the bidoublet and six each for the triplet fields). Hence, the remaining 14 fields will be massive scalars and they are as follows:

(1) Two doubly charged scalars ($H_1^{\pm\pm}, H_2^{\pm\pm}$).
(2) Two singly charged scalars ($H_1^{\pm}, H_2^{\pm}$).
(3) Four neutral $CP$-even scalars ($H_0^0, H_1^0, H_2^0, H_3^0$).
(4) Two neutral ($CP$-odd) pseudoscalars ($A_0^0, A_1^0$).

Since we already mentioned that the scale $v_R$ is much higher than the VEV of electroweak breaking $v_1$, the scalar masses can be expressed in leading-order terms[5] [43,44],

$$\begin{aligned} M_{H_0^0}^2 &\simeq 2\lambda_1 v_1^2, \\ M_{H_1^0}^2 &\simeq \frac{1}{2}\lambda_{12} v_R^2, \\ M_{H_2^0}^2 \simeq M_{A_1^0}^2 \simeq M_{H_2^\pm}^2 &\simeq 2\lambda_5 v_R^2, \\ M_{H_3^0}^2 \simeq M_{A_2^0}^2 \simeq M_{H_1^\pm}^2 \simeq M_{H_1^{\pm\pm}}^2 &\simeq \frac{1}{2}(\lambda_7 - 2\lambda_5) v_R^2, \\ M_{H_2^{\pm\pm}}^2 &\simeq 2\lambda_6 v_R^2. \end{aligned} \tag{20}$$

$M_{H_0^0}$ is the Standard Model Higgs boson and is denoted as $M_h$ from here onwards. For simplicity and to reduce the number of free parameters, we consider degenerate heavy scalars at the $v_R$ scale, i.e., $M_{H_1^0} = M_{H_2^0} = M_{H_3^0} = M_{H_2^{\pm\pm}} = M_H$. It is important to note that the remaining quartic couplings only contribute in the scalar masses as subleading terms and they are proportional to the $v_1^2$ at the electroweak symmetry-breaking (EWSB) scale. Hence, $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_8$, $\lambda_9$, $\lambda_{10}$, and $\lambda_{11}$ induce only the relative mass splittings among these heavy scalars which are almost phenomenologically unaccessible at present experiments.

The kinetic term of the scalar part can be written as

$$\begin{aligned} \mathcal{L}_{\rm kin} = &\, {\rm Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] + {\rm Tr}[(D_\mu\Delta_L)^\dagger(D^\mu\Delta_L)] \\ &+ {\rm Tr}[(D_\mu\Delta_R)^\dagger(D^\mu\Delta_R)], \end{aligned} \tag{21}$$

where,

$$\begin{aligned} D_\mu\Phi &= \partial_\mu\Phi - ig_{2L}T^a W^a_{L\mu}\Phi + ig_{2R}\Phi T^a W^a_{R\mu}, \\ D_\mu\Delta_{(L/R)} &= \partial_\mu\Delta_{(L/R)} - ig_{(2L/2R)}[T^a W^a_{(L/R)\mu}, \Delta_{(L/R)}] \\ &\quad - ig_{B-L}B_\mu\Delta_{(L/R)}. \end{aligned} \tag{22}$$

We choose the gauge couplings $g_{2L}$ and $g_{2R}$ for the $SU(2)_L$ and $SU(2)_R$ gauge groups, respectively, to be same for the sake of minimality of the model in terms of the number of parameters. After spontaneous breaking of the LR and EW symmetries, two charged $W^\pm_{L/R}$ and two neutral $Z_{L/R}$ gauge bosons become massive, while the photon $A$ remains massless,

[5]These leading-order terms match exactly with the masses of the heavy scalars at the scale $v_R$, i.e., before the EWSB. After the EWSB, some correction terms are generated which are proportional to $v_1^2$. But as $v_R \gg v_1$, the splitting among the masses of these heavy scalars are negligible compared to their relative masses. It is important to note that this "$\simeq$" will be replaced by "$=$" in Eq. (20) when these masses are given at the $v_R$ scale.

$$M^2_{W^\pm_L} = \frac{1}{4} g_2^2 v_1^2, \qquad M^2_{W^\pm_R} = \frac{1}{4} g_2^2 (v_1^2 + 2v_R^2),$$
$$M^2_{Z_{L,R}} = \frac{1}{4}[(g_2^2 v_1^2 + 2v_R^2(g_2^2 + g_{B-L}^2)) \mp \sqrt{\{g_2^2 v_1^2 + 2v_R^2(g_2^2 + g_{B-L}^2)\}^2 - 4g^2(g_2^2 + 2g_{B-L}^2) v_1^2 v_R^2}]. \tag{23}$$

Under the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ quarks and leptons are doublets,

$$L_{i(L/R)} = \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_{(L/R)}, \qquad Q_{i(L/R)} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_{(L/R)}. \tag{24}$$

The most general lepton Yukawa Lagrangian can be written as,

$$-\mathcal{L}_Y = [\bar{L_L}(y^l \Phi + \tilde{y}^l \tilde{\Phi}) L_R + h.c] + y_L^h \overline{L_R^c} \tilde{\Delta}_L L_L + y_R^h \bar{L_L^c} \tilde{\Delta}_R L_R, \tag{25}$$

where, $\tilde{\Phi} = i\sigma_2 \Phi^*$ and $\tilde{\Delta}_{L/R} = i\sigma_2 \Delta_{L/R}$. Here we have considered that the Yukawa matrices are diagonal.[6] The neutral fermion masses are generated once the $\Phi$ and $\Delta$ acquire VEVs. The neutral fermion mass matrix is given as

$$M_\nu = \begin{pmatrix} m_\nu^{II} & m_D \\ m_D^T & m_R \end{pmatrix}, \qquad m_D = \frac{1}{\sqrt{2}} y^l v_1,$$
$$m_R = \sqrt{2} y^h v_R, \qquad m_\nu^{II} = \sqrt{2} y^h v_L, \tag{26}$$

where, $y_L^h = y_R^h = y^h$ because of left-right symmetry. Thus the light neutrino mass

$$m_{\nu_l} = m_\nu^{II} - m_D^T m_R^{-1} m_D, \tag{27}$$

is generated through type-II (first term) and type-I (second term) seesaw mechanisms.

As the VEV of the left-handed triplet scalar is constrained by $\rho$ parameter of the SM it cannot be larger than $\sim\mathcal{O}$(few GeV). Thus it is indeed possible to generate light neutrino masses ~eV with $v_L \sim$ eV while the neutrino Yukawa coupling can be $\sim\mathcal{O}(1)$. In our further analysis we consider $v_L = 0$, and thus the type-II seesaw mechanism is absent here. The heavy neutrino mass $m_R$ is also generated through the Yukawa terms and is proportional to $v_R$. It can be noted that with $m_R \sim \mathcal{O}$ (TeV), the Dirac term $m_D$ needs to be very small to generate light neutrino masses $\sim\mathcal{O}$ (eV). But $y^h$ can be as large as $\sim\mathcal{O}(1)$ even when $v_R$ is around the TeV scale. Thus successful light neutrino mass generation is still possible while keeping $y^h$ as large as $\sim\mathcal{O}(1)$. But $y^h$ affects the vacuum stability of the scalar potential in this model as the heavy neutrino is also coupled to the SM-like Higgs. In the following section we show how these parameters are constrained due to vacuum stability and perturbativity (triviality).

It has been noted that the minimal left-right symmetric model is constrained by flavor-changing neutral currents (FCNCs) [46–49]. The model we have worked with contains the bidoublet whose one of the VEV is zero. Thus there is no FCNC problem in this model. There are also constraints from neutral kaon mixing, i.e., the kaon mass difference. Our choice of the $v_R$ scale and the masses for the heavy neutral scalars takes care of those bounds. As the VEVs and the Yukawa couplings in our scenario are real there is neither a source of nor spontaneous or explicit $CP$ violation. But since we have considered the Yukawa matrices to be diagonal we will boil down to the trivial, i.e., identity Cabibbo-Kobayashi-Maskawa and Pontecorvo-Maki-Nakagawa-Sakata matrices. To fit all the masses and mixings we need to go for the nonminimal extension of this model and that certainly modify the set of renormalization group evolutions (RGEs) that we have used here.

### 2. LR model with doublet scalars

In this case the scalar sector consists of a bidoublet ($\Phi$), one left-handed doublet ($H_L$), and one right-handed doublet ($H_R$). The scalar potential is depicted in Appendix A 3. In terms of the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group these fields can be written as

$$\Phi \equiv (2, 2, 0), \qquad H_L \equiv (2, 1, 1), \quad \text{and} \quad H_R \equiv (1, 2, 1). \tag{28}$$

The structure of $H_{L/R}$ is written as

$$H_{L/R} = \begin{pmatrix} h^+_{L/R} \\ h^0_{L/R} \end{pmatrix}. \tag{29}$$

The neutral components of $\Phi$ and $H_{L/R}$ acquire the vacuum expectation values

$$\langle\Phi\rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\theta} \end{pmatrix}, \qquad \langle H_L\rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix},$$
$$\langle H_R\rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix}. \tag{30}$$

[6]There exist two different discrete symmetries which can relate left- and right-handed fields [45]. Yukawa matrices are diagonal as we have considered the parity operation as defined in Ref. [43] to relate $L$ and $R$ fields.

As before, we put $v_2 = 0$. The scalar sector consists of 16 real scalar fields out of which six will be Goldstone bosons. Finally we will have four $CP$-even scalars: two $CP$-odd scalars and two charged scalars. Among the $CP$-even scalars one is the Standard Model Higgs boson with mass $M_h$ and the other three are taken as degenerate heavy scalars having mass $M_H$. The parameters in the Higgs potential can be recast in terms of the masses of the neutral and charged scalars. The details about the scalar sector have been discussed in Ref. [50]. The gauge sector is similar to the previous case, i.e., the LR model with triplet scalars.

In the limit $v_R \gg v_1$ and assuming all the heavy scalars are degenerate, we have

$$f_1 = (M_H/v_R)^2 = \kappa_1 = -\kappa_2, \tag{31}$$

whereas the minimization of the potential requires

$$\frac{v_1^2}{v_R^2} = \frac{f_1 - 2\beta_1}{4\lambda_1}.$$

The structure of the covariant derivative in this model is very similar to that for the triplet scenario [see Eq. (22)],

$$\begin{aligned} D_\mu \Phi &= \partial_\mu - i g_{2L} T^a W^a_{L\mu} \Phi + i g_{2R} \Phi T^a W^a_{R\mu}, \\ D_\mu H_{(L/R)} &= \partial_\mu H_{(L/R)} - i g_{(2L/2R)} T^a W^a_{(L/R)\mu} H_{(L/R)} \\ &\quad - i g_{B-L} B_\mu H_{(L/R)}. \end{aligned} \tag{32}$$

Following the previous convention we also set $g_{2L} = g_{2R} = g_2$. After spontaneous breaking of the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry, two charged $W^{\pm}_{L/R}$ and two neutral $Z_{L/R}$ gauge bosons become massive, while photon the $A$ remains massless,

$$\begin{aligned} M^2_{W^\pm_L} &= \frac{1}{4} g_2^2 v_1^2, \quad M^2_{W^\pm_R} = \frac{1}{4} g_2^2 (v_1^2 + v_R^2), \\ M^2_{Z_{L,R}} &= \frac{1}{8} [(2 g_2^2 v_1^2 + v_R^2 (g_2^2 + g_{B-L}^2)) \\ &\quad \mp \sqrt{4 g_2^4 v_1^4 + (g_2^2 + g_{B-L}^2) v_R^4 - 4 g_2^2 g_{B-L}^2 v_1^2 v_R^2}]. \end{aligned} \tag{33}$$

In the left-right symmetric model with a doublet scalar the leptonic part of the Yukawa interaction can be written as

$$-\mathcal{L} = \bar{L}_L (y_1 \Phi + y_2 \tilde{\Phi}) L_R + \text{H.c.}, \tag{34}$$

where the $SU(2)_L \otimes SU(2)_R$ quantum numbers of $L_L$ and $L_R$ are (2,1) and (1,2), respectively. So from this Lagrangian the Dirac mass term for the neutrinos can be written as

$$m_D = y_1 v_1. \tag{35}$$

Here, it is not possible to write the renormalizable Majorana mass term for the light and heavy neutrinos. But we can add nonrenormalizable effective terms as

$$\mathcal{L}_{\text{eff}} = \frac{\eta_L}{M} L_L L_L H_L H_L + \frac{\eta_R}{M} L_R L_R H_R H_R, \tag{36}$$

where, $M$ is some very high scale and the $\eta$'s are dimensionless parameters that denote the strength of these nonrenormalizable couplings. Once $H_R$ acquires a VEV the right-handed neutrino mass is generated as

$$m_R \simeq \frac{\eta_R v_R^2}{M}.$$

Here we consider that $\langle H_L \rangle = v_L = 0$, and thus this effective term does not contribute to the light neutrino mass. The neutrino mass matrix in the $(\nu_l, \nu_h)$ basis reads as

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix}, \tag{37}$$

and the light neutrino mass can be written as

$$m_{\nu_l} = -m_D^T m_R^{-1} m_D, \tag{38}$$

which is a variant of the type-I seesaw mechanism.

In the left-right symmetric model associated with two doublet scalars, neutrino masses cannot be generated through a type-II seesaw mechanism due to the lack of a left-handed triplet scalar.[7] Thus the type-I seesaw mechanism is the natural choice in this case. But the right-handed neutrino masses are generated through an effective operator suppressed by a heavy scale. This may provide a possible explanation how the right-handed neutrinos can be lowered to the TeV scale. Here, the correct order of light neutrino masses are generated if the Dirac-type neutrino Yukawa coupling needs to be very small, unless one considers the special textures for the Dirac Yukawa couplings. Then vacuum stability is automatically satisfied as these Dirac Yukawa couplings are much smaller. Thus here only the quartic couplings get constrained through the vacuum stability and perturbativity (triviality) of the couplings. Within a framework very similar to this it is indeed possible to generate light neutrino masses of the correct order without lowering the Yukawa coupling as the light neutrino masses are independent of $v_R$ but are suppressed by some high scale [51,52]. In that case the vacuum stability constraints cannot be avoided and play the most crucial role in constraining the Yukawa couplings and other parameters.

[7] Although, through an effective operator the Majorana mass term for light neutrinos can be generated; see Eq. (36). But this contribution is absent here as we have set $v_L = 0$.

## III. VACUUM STABILITY

The presence of new physics introduces exotic non-SM particles in the theory and if they couple to the SM fields then the RGEs of the Higgs quartic coupling ($\lambda_h$) will be modified. Moreover, additional quartic interactions of extra scalar fields should also be introduced. Extended gauge interactions from the larger gauge groups as well as Yukawa interactions would contribute to these evolution equations. Now the question arises of whether or not the vacuum is stable in the presence of the new physics. In particular, when we have narrowed down a preferred range of the Higgs mass between 123–127 GeV, the new physics could be constrained by the vacuum stability criteria. To adjudge the stability of these models we have considered the one-loop RGEs of all the required parameters. In passing we would like to mention that the allowed parameter space in our analysis is the minimal set which will be extended once one includes the higher-order renormalization group (RG) effects. The RGEs for the SM and each of the $B-L$ models which are used in our calculation are given in an Appendix. Since we are dealing with the TeV-scale models, all the SM RGEs will be modified once the new physics effects are switched on. Thus from the EW scale to the TeV scale (specific values are dictated in plots) the RGEs will be SM-like and from the TeV scale to the Planck scale they will be the modified ones, and during the process proper matching conditions are incorporated at the TeV scale.

### A. $U(1)_{B-L}$ model

It is clear from the structure of the potential as shown in Eq. (2) for the $U(1)_{B-L}$ model, that the vacuum stability conditions are different from those for the SM due to the presence of an extra singlet scalar. If all the quartic couplings are positive, the potential will be trivially bounded from below, i.e., the vacuum is stable and these stability conditions read simply as $\lambda_{1,2,3} > 0$. But it is indeed possible to allow $\lambda_3$ to be negative and still have the vacuum be stable. Thus vacuum stability conditions beyond the trivial ones allow for a larger parameter space and need to be accommodated in these conditions. We find the nontrivial vacuum stability criteria using the proposal dictated in Ref. [30], which are shown in Appendix B 1,

$$4\lambda_1\lambda_2 - \lambda_3^2 > 0, \quad \lambda_1 > 0, \qquad \lambda_2 > 0. \tag{39}$$

Together with these we have also incorporated perturbativity constraints on quartic couplings by demanding an upper limit, i.e., $|\lambda_i| < 1 (i = 1, 2, 3)$.

Noting from Eqs. (5) and (6) that the physical Higgs field is an admixture of two scalar fields $\phi$ and $s$, in our study the scalar mixing angle $\alpha$ is considered to be a free parameter instead of the quartic couplings $\lambda_i (i = 1, 2, 3)$. This model consists of two different scales in the theory: the EW scale and the $B-L$ symmetry-breaking scale. Thus two RGEs are invoked for the analysis. As we have two Abelian couplings in this model, there might be mixing between them [53,54]. To simplify the situation, and of course without hampering any other conclusions, we impose no mixing between the $Z_{B-L}$ and $Z$ gauge bosons at the tree level. This follows from the condition $\tilde{g}(Q_{\rm EW}) = 0$ as already discussed above [Eq. (11)]. As a consequence the $B-L$-breaking VEV $v_{B-L}$ relates to the new $Z_{B-L}$ boson mass given in Eq. (12). For demonstration, we have picked the perturbative value of this additional gauge coupling at the breaking scale as, $g_{B-L} = 0.1$. For simplicity we further assume that heavy neutrinos are degenerate and fixed at $m_{\nu_h}^{1,2,3} \equiv m_{\nu_h} \simeq 200$ GeV, which are within the allowed values. We have used the central value of the light Higgs mass ($M_h$) at 125 GeV, the top-quark mass at 173.2 GeV and the strong coupling constant $\alpha_s$ at 0.1184. Thus the remaining free parameters in our study are $M_H$, $\alpha$ and $v_{B-L}$. We have explored the correlated constraints on these parameters from vacuum stability.

The set of RGEs of different couplings that we have used in our analysis are encoded in Appendix C 2 [35]. The parameter space consistent with vacuum stability in the

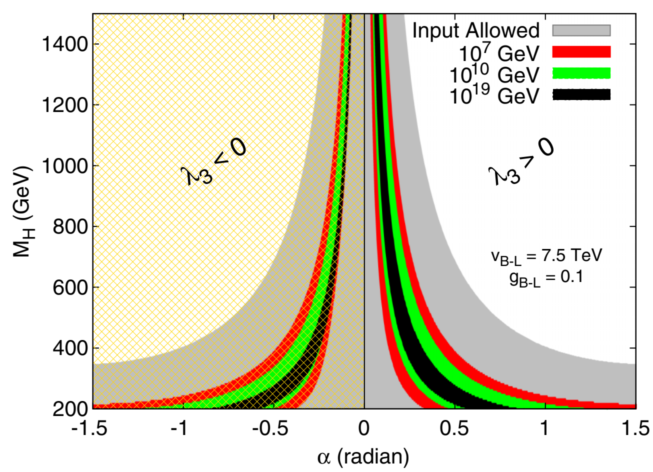


FIG. 1 (color online). The allowed parameter space in the heavy Higgs mass ($M_H$) and scalar mixing angle ($\alpha$) plane consistent with vacuum stability and perturbativity bounds are shown. The gray region is the domain of allowed input parameters. The red (dark gray), green (light gray), and black sub-parameter spaces show the domain of $M_H$ and $\alpha$ for which this $B-L$ theory is valid up to $10^7$, $10^{10}$ and $10^{19}$ GeV, respectively and expectedly the region in both sides squeeze for demanding higher scale of validity of the theory. The Majorana neutrino mass is fixed at 200 GeV and the $B-L$-breaking VEV ($v_{B-L}$) is set at 7.5 TeV. The $U(1)_{B-L}$ gauge coupling is taken to be 0.1 which implies $M_{Z_{B-L}} = 1.5$ TeV. The shaded region satisfies $\lambda_3 < 0$ [as well as $\alpha < 0$ from Eq. (7)]. Thus the nontrivial vacuum stability conditions are satisfied in this region. These conditions are more stringent than the trivial one applied in the positive-$\alpha$ region. Although the pattern of the allowed parameter space is very similar for both the positive and negative $\alpha$ regions, the $\alpha > 0$ region covers a larger parameter space.

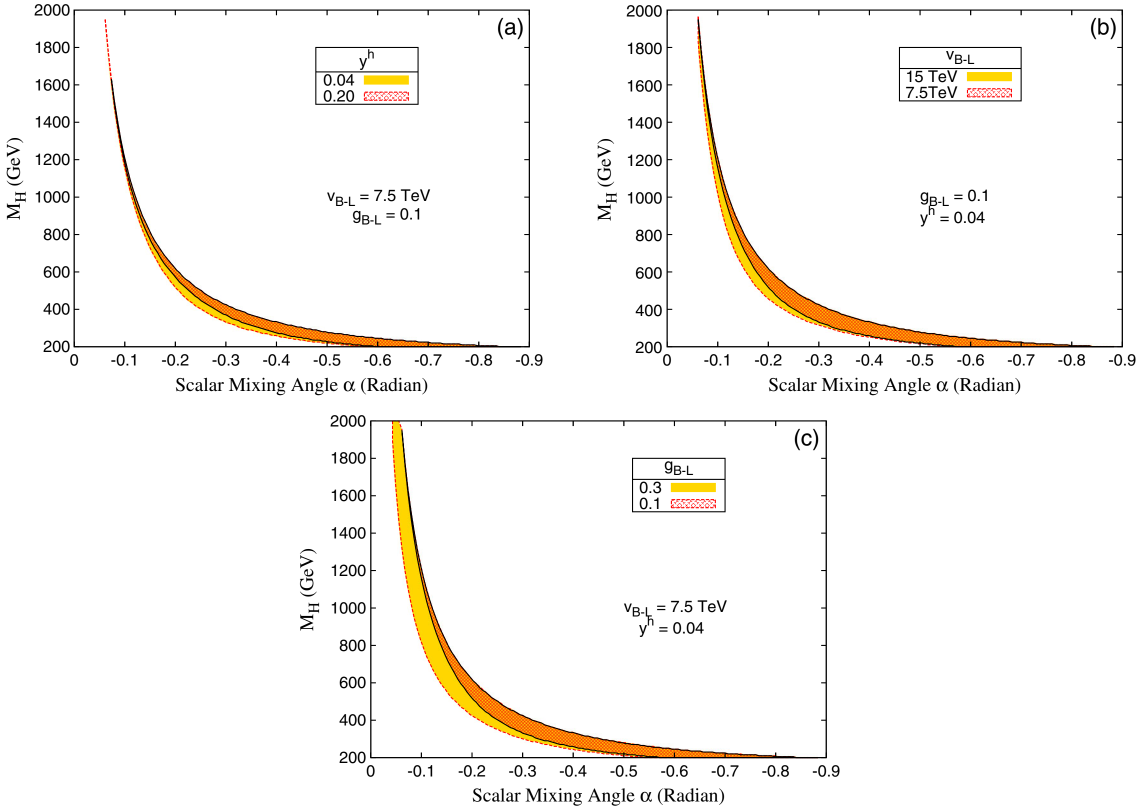


FIG. 2 (color online). Allowed parameter space in the $M_H - \alpha$ plane, with $\alpha$ varying between $[0, -\pi/2]$, consistent with vacuum stability and perturbativity (triviality) bounds up to the Planck scale. (a): The Majorana neutrino Yukawa coupling $y^h$ is varied keeping $v_{B-L}$ and $g_{B-L}$ fixed. (b): Two different sets of the $B-L$-breaking VEV, $v_{B-L}$ are chosen keeping $g_{B-L}$ and $y^h$ fixed. (c): In this plot $g_{B-L}$ varies while $v_{B-L}$ and $y^h$ are kept constant. In our analysis any value of $g_{B-L}$ for $v_{B-L} = 7.5$ TeV that is greater than 0.34 is disallowed as the coupling becomes nonperturbative before the Planck scale. Corresponding regions for positive $\alpha$ are not shown here, as they remain unaffected and are the same as those given in the blue strip in Fig. 1 owing to the trivial conditions.

heavy Higgs mass ($M_H$) and scalar mixing angle ($\alpha$) plane is depicted in Fig. 1. All the couplings are perturbative throughout their evolutions. The gray region is the domain of allowed input parameters. The red, green, and black sub-parameter spaces show the domain of $M_H$ and $\alpha$ for which this $B-L$ theory is valid up to $10^7$, $10^{10}$ and $10^{19}$ GeV, respectively. In this figure, for a particular heavy scalar mass each part of this allowed domain is restricted at some minimum (maximum) value of $\alpha$ due to the vacuum stability (perturbativity) of the quartic couplings. The Majorana neutrino mass is fixed at 200 GeV and the $B-L$-breaking VEV ($v_{B-L}$) is set at 7.5 TeV. The $U(1)_{B-L}$ gauge coupling is taken to be 0.1 which implies $M_{Z_{B-L}} = 1.5$ TeV consistent with present experimental bounds [55]. The yellow shaded region contains the set of allowed parameters for $\lambda_3 < 0$ [as well as $\alpha < 0$ from Eq. (7)]. Though the pattern of the allowed parameter space in the positive-$\lambda_3$ region is very similar, it is not exactly symmetric. The outer boundaries above each color in Fig. 1 match exactly for both the positive- and negative-$\alpha$ region. This is not surprising because the outer boundary is determined by the perturbativity of the couplings and thus is not affected by the vacuum stability conditions which are different for different signs of $\lambda_3$. However, the lower boundaries are the outcome of the need to satisfy the criteria of vacuum stability. Allowed parameters in the yellow shaded region (which represents $\lambda_3 < 0$) in Fig. 1 are reflected by the nontrivial vacuum stability condition in Eq. (39), which sequentially plays a role in determining the lower boundaries in the allowed parameters. Thus expectedly in the positive-$\alpha$ region the allowed parameter space is larger than that for negative $\alpha$. Also, note that $\alpha = 0$ leads to the decoupling limit when the heavy scalar will not affect the vacuum stability. The parameter space has also shrunk as the validity of the model must be closer to the Planck scale as can be inferred from Fig. 1.

To study the dependence of different parameters as shown in Fig. 1, we plot the allowed parameter space in the $M_H - \alpha$ plane which remains consistent with vacuum stability and where all the couplings are perturbative up to the Planck Scale. In Fig. 2(a) the Majorana neutrino Yukawa coupling $y^h$ is varied while keeping $v_{B-L}$ and $g_{B-L}$ fixed. As $y^h$ increases, the allowed parameter space is shrunk since the Yukawa coupling affects the quartic couplings negatively in their RG evolutions. Thus larger Yukawa couplings spoil the vacuum stability. Figure 2(b) shows the dependence on the $B-L$-breaking VEV for fixed $g_{B-L}$ and $y^h$. $v_{B-L}$ determines the scale of new physics beyond the Standard Model, i.e., from where the RGEs are being modified due to the presence of new particles. The larger $v_{B-L}$ implies that new set of RGEs come into play later. In the $B-L$ extended model $\lambda_3$ is inversely proportional to $v_{B-L}$ at the EW scale [see Eq. (7)]. Thus for the same set of values of $M_H$ and $\alpha$, $\lambda_3$ is smaller for larger $v_{B-L}$ at 15 TeV. The RGE of $\lambda_3$ is such that for our choice of parameters it grows with the mass scale. Thus there is a possibility of generating a large $\lambda_3$ such that vacuum stability and perturbativity conditions are not validated at some higher scale. This plot therefore shows that it is possible to have a larger allowed parameter space for larger values of $v_{B-L}$. Finally in Fig. 2(c), $g_{B-L}$ varies while $v_{B-L}$ and $Y_\nu$ are kept constant. As the larger values of the gauge couplings affect the RGEs of the quartic couplings positively, the vacuum stability is improved. Thus with a larger value of gauge coupling a larger parameter space is allowed. But the U(1) couplings increases with the mass scale. Hence the couplings with much larger values at the low scale might be nonperturbative at the high scale. In our analysis, when $v_{B-L}$ is at 7.5 TeV, any value of $g_{B-L}$ greater than 0.34 is disallowed as the coupling becomes nonperturbative before the Planck scale.

### B. Left-right symmetry

#### *1. LR model with triplet scalars*

In this model the scalar potential for the left-right symmetric model with a triplet scalar as shown in Appendix A 2 contains many quartic couplings. To find the condition of vacuum stability we have considered all two-field, three-field and four-field directions and found their stability criteria. Detailed field directions corresponding to the potential together with calculated stability conditions are listed in Appendix B 2. Finally, the effective nontrivial vacuum stability conditions which are necessary and sufficient are

$$\lambda_1 > 0, \qquad \lambda_5 > 0, \qquad \lambda_5 + \lambda_6 > 0,$$
$$\lambda_5 + 2\lambda_6 > 0, \qquad \lambda_{12} - 2\sqrt{\lambda_1 \lambda_5} < 0. \tag{40}$$

Along with the above conditions, we find an additional condition $\lambda_{12} > 0$ from Eq. (20).

The renormalization group evolutions that we have considered in our analysis are depicted in Appendix C 3 [46]. In Fig. 3 we show the constraints on the universal quartic coupling $\lambda_u(\equiv\lambda_2, \lambda_3, \lambda_4, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11})$ for the LR model with triplet scalars in the low-$v_R$ region. The yellow shaded region is disallowed from low-energy data ($M_{W_R} > 3.5$ TeV) [56–59] and the green shaded region is excluded from direct searches at the LHC ($M_{W_R} > 2.5$ TeV) [60–63]. These limits can be extracted

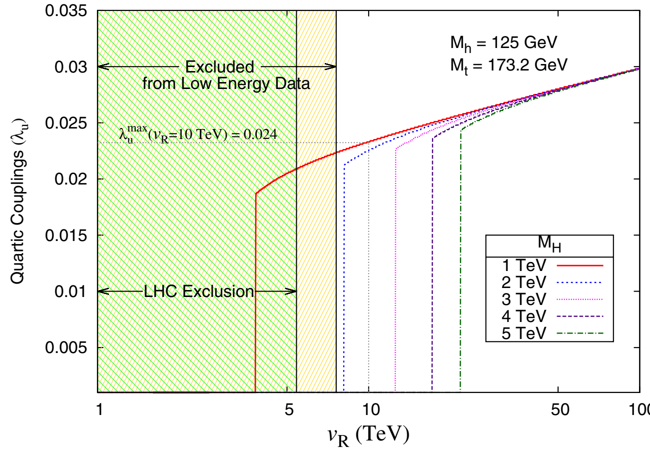


FIG. 3 (color online). Constraints on the universal quartic coupling $\lambda_u(\equiv\lambda_2, \lambda_3, \lambda_4, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11})$ for the LR model with triplet scalars in the low-$v_R$ region. The yellow shaded region is disallowed from low-energy data ($M_{W_R} > 3.5$ TeV) and the green shaded region is excluded from direct searches at the LHC ($M_{W_R} > 2.5$ TeV).

using Eq. (33). In our analysis we also set the Majorana Yukawa, $y^h$ at 0.25. We note that, for any particular heavy scalar mass ($M_H$), the universal quartic coupling $\lambda_u$ is disallowed above the corresponding line shown in the figure. For example, as seen from the plot, the maximum allowed value of the universal quartic coupling is 0.024 if one considers an LR-breaking scale at 10 TeV and a heavy scalar mass at 1 TeV. The allowed maximum quartic coupling is lowered for a heavier scalar which can be understood from vacuum stability and perturbativity.

In Fig. 4 we check the compatibility for the stable vacuum in the left-right symmetric breaking scale $v_R$ and heavy scalar $M_H$ allowed region in the LR model with a triplet scalar. Each color represents a particular set of light Higgs masses ($M_h$) and top masses ($M_t$) in their respective plots. In Fig. 4(a) the Higgs mass is fixed at 125 GeV and the top-quark mass varies from 170 GeV to 175 GeV whereas, in Fig. 4(b) the top-quark mass is fixed at 173.2 GeV and the Higgs mass varies from 122 GeV to 127 GeV. The upper-left region (shaded with light blue) above the line $M_H = v_R$ is disallowed since quartic couplings are nonperturbative in this domain. The blank (white) strip is also ruled out as the values of the couplings in this region are such that they become nonperturbative before reaching the Planck scale. In the lower-right region (shaded with light pink) the quartic coupling related to the heavy Higgs mass becomes extremely small [$\leq \mathcal{O}(10^{-7})$]. We choose the universal quartic coupling $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_8$, $\lambda_9$, $\lambda_{10}$, $\lambda_{11} = \lambda_u$ to be fixed at 0.03. This choice of $\lambda_u$ allows only $v_R \geq 100$ TeV which can be inferred from Fig. 3. The insets in both panels show the higher $v_R$ scale where color patches terminate, representing the very scale where in fact the Standard Model breaks down for a particular Higgs mass or top-quark mass at one loop.

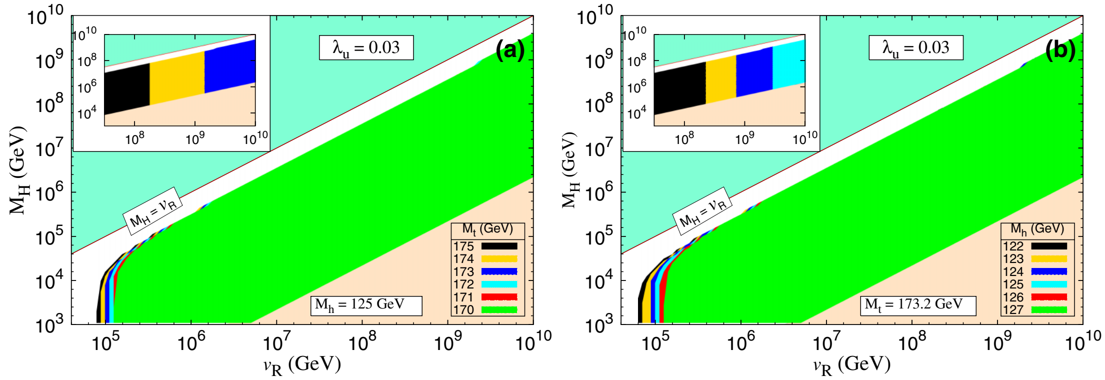


FIG. 4 (color online). Compatibility for the stable vacuum in the $v_R$ and heavy scalar $M_H$ allowed region in the LR model with a triplet scalar. Each color (or shade) represents a particular set of light Higgs masses ($M_h$) and top masses ($M_t$) in their respective plots. In (a) the Higgs mass is fixed at 125 GeV for different top-quark masses whereas, in (b) the top-quark mass is fixed at 173.2 GeV and the Higgs mass is allowed to vary. The upper-left region (shaded with light blue/gray) above the line $M_H = v_R$ is disallowed since quartic couplings are nonperturbative in this domain. In the lower-right region (shaded with light pink/gray) the quartic coupling related to the heavy scalar mass becomes extremely small [$\leq \mathcal{O}(10^{-7})$]. We fix the universal quartic coupling $\lambda_u$ at 0.03. The insets in both panels show the higher $v_R$ scale where color/shade patches terminate, representing the very scale where in fact the Standard Model breaks down for a particular Higgs mass or top-quark mass at one loop. As expected, from left to right the top quark mass decreases in (a), whereas the Higgs mass increses in (b).

### *2. LR model with doublet scalars*

Using a similar technique as that used in the previous section we depict all the multiple-field directions of the potential and the corresponding stability criteria in Appendix B 3. We find the nontrivial vacuum stability conditions which read as

$$\lambda_1 > 0, \qquad 2\beta_1 + f_1 > 0, \qquad 2\beta_1 - f_1 > 0. \tag{41}$$

We have also noted the required RGEs for our analysis in Appendix 4 [64]. In Fig. 5 we constrain the universal quartic coupling $\lambda_u(\equiv\lambda_2, -\lambda_3)$ for the LR model with

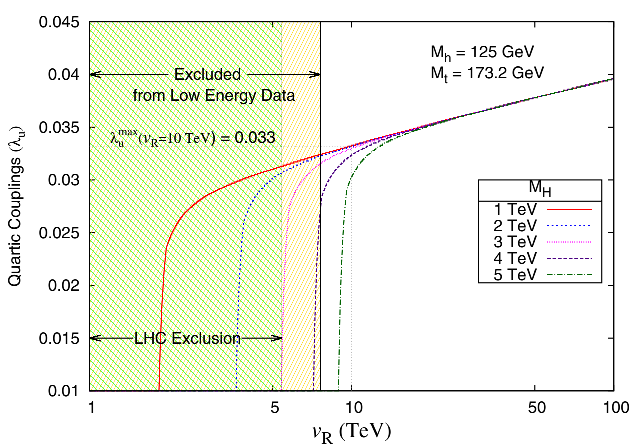


FIG. 5 (color online). Constraints on the universal quartic coupling $\lambda_u(\equiv\lambda_2, -\lambda_3)$ for the LR model with doublet scalars in the low-$v_R$ region for different sets of heavy scalar masses $M_H$. The yellow shaded region is disallowed from low-energy data ($M_{W_R} > 3.5$ TeV) and the green shaded region is excluded from direct searches at the LHC ($M_{W_R} > 2.5$ TeV).

doublet scalars in the low-$v_R$ region for different sets of heavy scalar masses $M_H$. Similar to the previous case, the yellow shaded region in the plot is disallowed from low-energy data ($M_{W_R} > 3.5$ TeV) and the green shaded region is excluded from direct searches at the LHC ($M_{W_R} > 2.5$ TeV).

As we noticed in Fig. 5, for any particular heavy scalar mass ($M_H$), the universal quartic coupling $\lambda_u$ is disallowed above the corresponding line. For example, as seen from the plot, the maximum allowed value of the universal quartic coupling is 0.033 if one considers an LR-breaking scale at 10 TeV and a heavy scalar mass at 1 TeV. As before, the allowed maximum quartic coupling is lowered for a heavier scalar.

In Fig. 6 we check the compatibility for the stable vacuum in the $v_R$ and heavy scalar $M_H$ allowed region in the LR model with doublet scalars. Each color represents a particular set of light Higgs masses ($M_h$) and top masses ($M_t$) in their respective plots. In Fig. 6(a) the Higgs mass is fixed at 125 GeV and the top-quark mass varies from 170 GeV to 175 GeV whereas, in Fig. 6(b) the top-quark mass is fixed at 173.2 GeV and the Higgs mass varies from 122 GeV to 127 GeV. The upper-left region (shaded with light blue) above the line $M_H = v_R$ is disallowed since the quartic couplings are nonperturbative at the low scale itself in this domain. The blank (white) strip is also ruled out as the values of the couplings in this region are such that they become nonperturbative before reaching the Planck scale. In the lower-right region (shaded with light pink) the quartic coupling related to a heavy Higgs mass becomes extremely small [$\leq \mathcal{O}(10^{-7})$]. We choose the universal quartic coupling $\lambda_1 = -\lambda_2 = \lambda_u$ to be fixed at 0.04. Here, the choice of $\lambda_u$ allows only $v_R \geq 100$ TeV. The insets in

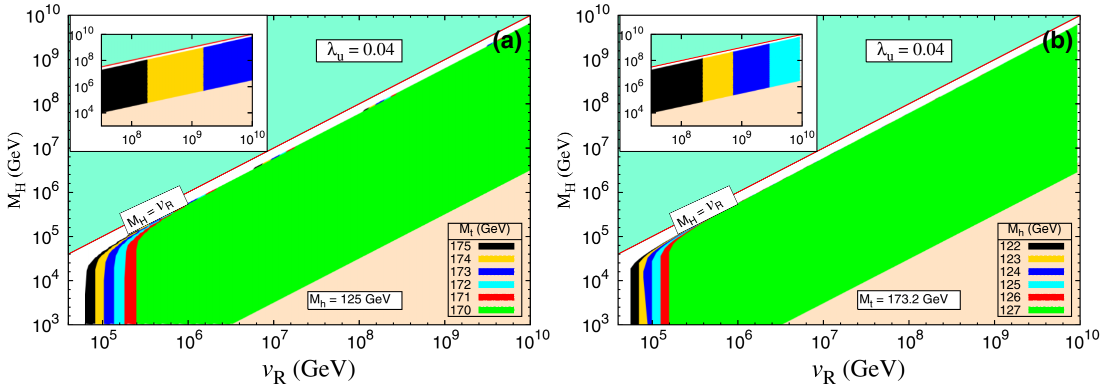


FIG. 6 (color online). Compatibility for stable vacuum in $v_R$ and heavy Higgs $M_H$ allowed region in LR model with doublet scalars. Each color (or shade) represents a particular set of light Higgs mass ($M_h$) and top mass ($M_t$) in respective plot. In figure (a) Higgs mass is fixed at 125 GeV and top quark mass is varying, where as, in figure (b) top quark mass is fixed at 173.2 GeV and Higgs mass is varying. Upper-left region (shaded with light blue/gray) above the line $M_H = v_R$ is disallowed since quartic couplings are non-perturbative at the low scale itself in this domain. Lower-right region (shaded with light pink/gray) quartic coupling related with heavy Higgs mass becomes extremely small ($\leq \mathcal{O}(10^{-7})$). We choose universal quartic coupling $\lambda_u$ fixed at 0.04. Inset to both figures shows the higher $v_R$ scale where color/shade patches terminate, representing the very scale where in fact Standard Model breaks down for a particular Higgs mass or top quark mass at one loop. As expected, from left to right the top quark mass decreases in (a), whereas the Higgs mass increases in (b).

both panels show the higher $v_R$ scale where color patches terminate, representing the very scale where in fact the Standard Model breaks down for a particular Higgs mass or top-quark mass at one loop.

## IV. CONCLUSIONS

We have noted that one needs to study the scalar potential to understand the structure of the vacuum and its compatibility with successful spontaneous symmetry breaking. In addition, the perturbativity (triviality) of the couplings also plays a crucial role. We have analyzed the structure of the scalar potentials of $B-L$ extended models—namely, SM $\otimes$ $U(1)_{B-L}$ and the left-right symmetry—with different scalar representations. We have computed the criteria for the potential to be bounded from below, i.e., the conditions for vacuum stability. We also performed the renormalization group evolutions of the parameters (couplings) of these models at the one-loop level with proper matching conditions. We have shown how the phenomenologically inaccessible couplings can be constrained for different choices of scales of new physics. They in turn also affect the RGEs of the other couplings. We have noted that the new physics effects must be switched on before the SM vacuum faces the instability. This helps the vacuum stability of the full scalar potential and achieves a consistent spontaneous symmetry breaking. We have analyzed these aspects by varying the Higgs and top-quark mass over their allowed ranges. In summary, it is meaningful to mention that more precise knowledge of the SM parameters—like the Higgs mass, the top-quark mass and the strong coupling—will constrain the parameters (couplings, masses, scales) of new physics and might direct us towards the correct theory for beyond the Standard Model physics. In principle one can study the left-right symmetric models including the radiative correction in the scalar potential and use the Coleman-Weinberg mechanism, e.g., Ref. [50] has considered this scenario and calculated the flat directions using the one-loop effective potential. This will certainly change the correlations among the parameters of the scalar potential leading to the stable vacuum. While submitting our paper Ref. [65] appeared, where the vacuum stability for SM $\otimes$ $U(1)_{B-L}$ was discussed. The viewpoints of our analysis are quite different from that work.

## ACKNOWLEDGMENTS

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## APPENDIX A: SCALAR POTENTIAL FOR DIFFERENT MODELS

### 1. $U(1)_{B-L}$ model

$$V(\Phi, S) = m^2\Phi^\dagger\Phi + \mu^2|S|^2 + \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2|S|^4 + \lambda_3\Phi^\dagger\Phi|S|^2. \quad \text{(A1)}$$

### 2. LR model with triplet scalars

The most general form of the scalar potential can be written as in Ref. [64],

$$
\begin{aligned}
&V_{\mathrm{LRT}}(\Phi,\Delta_L,\Delta_R)\\
&\quad= -\mu_1^2\{\mathrm{Tr}[\Phi^\dagger\Phi]\} - \mu_2^2\{\mathrm{Tr}[\tilde{\Phi}\Phi^\dagger] + \mathrm{Tr}[\tilde{\Phi}^\dagger\Phi]\} - \mu_3^2\{\mathrm{Tr}[\Delta_L^\dagger\Delta_L] + \mathrm{Tr}[\Delta_R^\dagger\Delta_R]\} + \lambda_1\{(\mathrm{Tr}[\Phi^\dagger\Phi])^2\}\\
&\qquad + \lambda_2\{(\mathrm{Tr}[\tilde{\Phi}\Phi^\dagger])^2 + (\mathrm{Tr}[\tilde{\Phi}^\dagger\Phi])^2\} + \lambda_3\{\mathrm{Tr}[\tilde{\Phi}\Phi^\dagger]\mathrm{Tr}[\tilde{\Phi}^\dagger\Phi]\} + \lambda_4\{\mathrm{Tr}[\Phi^\dagger\Phi](\mathrm{Tr}[\tilde{\Phi}\Phi^\dagger] + \mathrm{Tr}[\tilde{\Phi}^\dagger\Phi])\}\\
&\qquad + \lambda_5\{(\mathrm{Tr}[\Delta_L\Delta_L^\dagger])^2 + (\Delta_R\Delta_R^\dagger)^2\} + \lambda_6\{\mathrm{Tr}[\Delta_L\Delta_L]\mathrm{Tr}[\Delta_L^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R]\mathrm{Tr}[\Delta_R^\dagger\Delta_R^\dagger]\} + \lambda_7\{\mathrm{Tr}[\Delta_L\Delta_L^\dagger]\mathrm{Tr}[\Delta_R\Delta_R^\dagger]\}\\
&\qquad + \lambda_8[\Delta_L\Delta_L^\dagger]\{\mathrm{Tr}[\Delta_L\Delta_L^\dagger]\mathrm{Tr}[\Delta_R\Delta_R^\dagger]\} + \lambda_9\{\mathrm{Tr}[\Phi^\dagger\Phi](\mathrm{Tr}[\Delta_L\Delta_L^\dagger] + \mathrm{Tr}[\Delta_R\Delta_R^\dagger])\} + (\lambda_{10} + i\lambda_{11})\{\mathrm{Tr}[\Phi\tilde{\Phi}^\dagger]\mathrm{Tr}[\Delta_R\Delta_R^\dagger]\\
&\qquad + \mathrm{Tr}[\Phi^\dagger\tilde{\Phi}]\mathrm{Tr}[\Delta_L\Delta_L^\dagger]\} + (\lambda_{10} - i\lambda_{11})\{\mathrm{Tr}[\Phi^\dagger\tilde{\Phi}]\mathrm{Tr}[\Delta_R\Delta_R^\dagger] + \mathrm{Tr}[\tilde{\Phi}^\dagger\Phi]\mathrm{Tr}[\Delta_L\Delta_L^\dagger]\} + \lambda_{12}\{\mathrm{Tr}[\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger]\\
&\qquad + \mathrm{Tr}[\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger]\} + \lambda_{13}\{\mathrm{Tr}[\Phi\Delta_R\Phi^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\Phi^\dagger\Delta_L\Phi\Delta_R^\dagger]\} + \lambda_{14}\{\mathrm{Tr}[\tilde{\Phi}\Delta_R\Phi^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\tilde{\Phi}^\dagger\Delta_L\Phi\Delta_R^\dagger]\}\\
&\qquad + \lambda_{15}\{\mathrm{Tr}[\Phi\Delta_R\tilde{\Phi}^\dagger\Delta_L^\dagger] + \mathrm{Tr}[\Phi^\dagger\Delta_L\tilde{\Phi}\Delta_R^\dagger]\},
\end{aligned}
$$

where all the coupling constants are real.

### 3. LR model with doublet scalars

The scalar potential for the LR model with doublet scalars can be written as

$$
\begin{aligned}
V_{\mathrm{LRD}}(\Phi,H_L,H_R) &= 4\lambda_1(\mathrm{Tr}[\Phi^\dagger\Phi])^2 + 4\lambda_2(\mathrm{Tr}[\Phi^\dagger\tilde{\Phi}] + \mathrm{Tr}[\Phi\tilde{\Phi}^\dagger])^2 + 4\lambda_3(\mathrm{Tr}[\Phi^\dagger\tilde{\Phi}] - \mathrm{Tr}[\Phi\tilde{\Phi}^\dagger])^2\\
&\quad + \frac{\kappa_1}{2}(H_L^\dagger H_L + H_R^\dagger H_R)^2 + \frac{\kappa_2}{2}(H_L^\dagger H_L - H_R^\dagger H_R)^2 + \beta_1(\mathrm{Tr}[\Phi^\dagger\tilde{\Phi}] + \mathrm{Tr}[\Phi\tilde{\Phi}^\dagger])(H_L^\dagger H_L + H_R^\dagger H_R)\\
&\quad + f_1(H_L^\dagger(\tilde{\Phi}\tilde{\Phi}^\dagger - \Phi\Phi^\dagger)H_L - H_R^\dagger(\Phi^\dagger\Phi - \tilde{\Phi}^\dagger\tilde{\Phi})H_R).
\end{aligned}
$$

## APPENDIX B: CALCULATION OF NONTRIVIAL VACUUM STABILITY CONDITIONS

Here we have gathered the structure of the scalar potential in the two-, three-, and four-field directions. We have calculated vacuum stability conditions from these field directions while keeping in mind that the conditions should cover most of the parameter space spanned by the quartic couplings.

### 1. $U(1)_{B-L}$ model

For the $U(1)_{B-L}$ model the potential has a simple structure and the stability conditions can be calculated easily. The quartic potential has the form

$$\lambda_1|\Phi|^4 + \lambda_2|S|^4 + \lambda_3|\Phi|^2|S|^2,$$

and we can easily write this potential as

$$\left(\sqrt{\lambda_1}|\Phi|^2 + \frac{\lambda_3}{2\sqrt{\lambda_1}}|S|^2\right)^2 + \left(\lambda_2 - \frac{\lambda_3^2}{4\lambda_1}\right)|S|^4.$$

Clearly the above equation is positive definite if

$$\lambda_1 > 0, \qquad \lambda_2 > 0, \quad 4\lambda_1\lambda_2 - \lambda_3^2 > 0.$$

These are the nontrivial vacuum stability conditions with $\lambda_3 < 0$. The trivial boundary conditions are when all the $\lambda_{1,2,3} > 0$.

### 2. LR model with triplet scalars

The absence of any tachyonic pseudoscalar modes imposes the condition $\lambda_{12} > 0$.

#### *a. Two-field directions and stability conditions*

$$
\begin{aligned}
{}^{2F}V_1(\phi_1^0,\phi_1^+) &= \lambda_1(\phi_1^{02} + \phi_1^{+2})^2,\\
{}^{2F}V_2(\phi_1^0,\delta^0) &= \lambda_5\delta^{04} + \lambda_1\phi_1^{04},\\
{}^{2F}V_3(\phi_1^0,\delta^+) &= (\lambda_5+\lambda_6)\delta^{+4} + \lambda_1\phi_1^{04}\\
&\quad + \frac{1}{2}(\lambda_{12} + 2\lambda_9)\delta^{+2}\phi_1^{02},\\
{}^{2F}V_4(\phi_1^0,\delta^{++}) &= \lambda_5\delta^{++4} + \lambda_1\phi_1^{04} + \lambda_{12}\delta^{++2}\phi_1^{02},\\
{}^{2F}V_5(\phi_1^+,\delta^0) &= \lambda_5\delta^{04} + \lambda_1\phi_1^{+4} + \lambda_{12}\delta^{02}\phi_1^{+2},\\
{}^{2F}V_6(\phi_1^+,\delta^+) &= (\lambda_5+\lambda_6)\delta^{+4} + \lambda_1\phi_1^{+4}\\
&\quad + \frac{1}{2}(\lambda_{12} + 2\lambda_9)\delta^{+2}\phi_1^{+2},\\
{}^{2F}V_7(\phi_1^+,\delta^{++}) &= \lambda_5\delta^{++4} + \lambda_1\phi_1^{04},\\
{}^{2F}V_8(\delta^0,\delta^+) &= \lambda_5(\delta^{02} + \delta^{+2})^2 + \lambda_6\delta^{+4},\\
{}^{2F}V_9(\delta^0,\delta^{++}) &= \lambda_5(\delta^{02} + \delta^{++2})^2 + 4\lambda_6\delta^{+2}\delta^{02},\\
{}^{2F}V_{10}(\delta^+,\delta^{++}) &= \lambda_5(\delta^{+2} + \delta^{++2})^2 + \lambda_6\delta^{+4}.
\end{aligned}
$$

*Stability conditions*

$$
\begin{aligned}
{}^{2F}V_1 &\to \lambda_1 > 0,\\
{}^{2F}V_2, {}^{2F}V_4, {}^{2F}V_5, {}^{2F}V_7 &\to \lambda_1 > 0; \quad \lambda_5 > 0,\\
{}^{2F}V_3, {}^{2F}V_6 &\to \lambda_1 > 0; \quad \lambda_5 + \lambda_6 > 0,\\
{}^{2F}V_8, {}^{2F}V_9, {}^{2F}V_{10} &\to \lambda_5 > 0; \quad \lambda_5 + \lambda_6 > 0,
\end{aligned}
$$

*b. Three-field directions and stability conditions*

$$
\begin{aligned}
{}^{3F}V_1(\phi_1^0,\phi_1^+,\delta^0) &= \lambda_1(\phi_1^{02}+\phi_1^{+2})^2+\lambda_5\delta^{02}+\lambda_{12}\delta^{02}\phi_1^{02},\\
{}^{3F}V_2(\phi_1^0,\phi_1^+,\delta^+) &= \lambda_1(\phi_1^{02}+\phi_1^{+2})^2+(\lambda_5+\lambda_6)\delta^{+4}+\frac{1}{2}(\lambda_{12}+2\lambda_9)(\phi_1^{02}+\phi_1^{+2})\delta^{+2},\\
{}^{3F}V_3(\phi_1^0,\phi_1^+,\delta^{++}) &= \lambda_1(\phi_1^{02}+\phi_1^{+2})^2+\lambda_5\delta^{++4}+\lambda_{12}\phi_1^{02}\delta^{++2},\\
{}^{3F}V_4(\phi_1^0,\delta^0,\delta^+) &= \lambda_1\phi_1^{04}+\lambda_5(\delta^{02}+\delta^{+2})^2+\lambda_6\delta^{+4}+\frac{1}{2}(\lambda_{12}+2\lambda_9)\phi_1^{02}\delta^{+2},\\
{}^{3F}V_5(\phi_1^0,\delta^0,\delta^{++}) &= \lambda_5(\delta^{02}+\delta^{++2})^2+\lambda_1\phi_1^{04}+4\lambda_6{\delta_0}^2\delta^{++2}+\lambda_{12}\delta^{++2}\phi_1^{02}+2\lambda_9\delta^0\delta^{++}\phi_1^{02},\\
{}^{3F}V_6(\phi_1^0,\delta^+,\delta^{++}) &= \lambda_1\phi_1^{04}+\lambda_5(\delta^{++2}+\delta^{+2})^2+\lambda_6\delta^{+4}+\frac{1}{2}\lambda_{12}\phi_1^{02}(2\delta^{++2}+\delta^{+2})+\lambda_9\delta^{+2}\phi_1^{02},\\
{}^{3F}V_7(\phi_1^+,\delta^0,\delta^+) &= \lambda_1\phi_1^{+4}+\lambda_5(\delta^{02}+\delta^{+2})^2+\lambda_6\delta^{+4}+\frac{1}{2}\lambda_{12}\phi_1^{+2}(2\delta^{02}+\delta^{+2})+\lambda_9\delta^{+2}\phi_1^{+2},\\
{}^{3F}V_8(\phi_1^+,\delta^0,\delta^{++}) &= \lambda_5(\delta^{02}+\delta^{++2})^2+\lambda_1\phi_1^{+4}+4\lambda_6{\delta_0}^2\delta^{++2}+\lambda_{12}\delta^{02}\phi_1^{+2}+2\lambda_9\delta^0\delta^{++}\phi_1^{+2},\\
{}^{3F}V_9(\phi_1^+,\delta^+,\delta^{++}) &= \lambda_1\phi_1^{+4}+\lambda_5(\delta^{+2}+\delta^{++2})^2+\lambda_6\delta^{+4}+\frac{1}{2}(\lambda_{12}+2\lambda_9)\phi_1^{+2}\delta^{+2},\\
{}^{3F}V_{10}(\delta^0,\delta^+,\delta^{++}) &= \lambda_5(\delta^{02}+\delta^{+2}+\delta^{++2})^2+\lambda_6(\delta^{+2}+2\delta^0\delta^{++})^2.
\end{aligned}
$$

*Stability conditions*

$$
\begin{aligned}
{}^{3F}V_1,{}^{3F}V_3 &\to \lambda_1>0;\quad \lambda_5>0,\\
{}^{3F}V_2 &\to \lambda_1>0;\quad \lambda_5+\lambda_6>0,\\
{}^{3F}V_4,{}^{3F}V_6,{}^{3F}V_7,{}^{3F}V_9 &\to \lambda_1>0;\quad \lambda_5>0;\quad \lambda_5+\lambda_6>0,\\
{}^{3F}V_5,{}^{3F}V_8 &\to \lambda_1>0;\quad \lambda_5>0;\quad \lambda_5+2\lambda_6>0,\\
{}^{3F}V_{10} &\to \lambda_5>0;\quad \lambda_5+\lambda_6>0;\quad \lambda_5+2\lambda_6>0.
\end{aligned}
$$

*c. Four-field directions and stability conditions*

$$
\begin{aligned}
{}^{4F}V_1(\phi_1^0,\phi_1^+,\delta^0,\delta^+) &= \lambda_5(\delta^{02}+\delta^{+2})^2+\lambda_6\delta^{+4}+\lambda_1(\phi_1^{02}+\phi_1^{+2})^2\\
&\quad+\frac{1}{2}\lambda_{12}(2\delta^{02}\phi_1^{+2}+2\sqrt{2}\phi_1^0\phi_1^+\delta^0\delta^++\delta^{+2}(\phi_1^{02}+\phi_1^{+2}))+\lambda_9\delta^{+2}(\phi_1^{02}+\phi_1^{+2}),\\
{}^{4F}V_2(\phi_1^0,\phi_1^+,\delta^0,\delta^{++}) &= \lambda_5(\delta^{02}+\delta^{++2})^2+4\lambda_6\delta^{02}\delta^{++2}+\lambda_1(\phi_1^{02}+\phi_1^{+2})^2+\lambda_{12}(\delta^{++2}\phi_1^{02}+\delta^{02}\phi_1^{+2})\\
&\quad+2\lambda_9\delta^0\delta^{++}(\phi_1^{02}+\phi_1^{+2}),\\
{}^{4F}V_3(\phi_1^0,\phi_1^+,\delta^+,\delta^{++}) &= \lambda_5(\delta^{+2}+\delta^{++2})^2+\lambda_6\delta^{+4}+\lambda_1(\phi_1^{02}+\phi_1^{+2})^2+\frac{1}{2}\lambda_{12}(2\delta^{++2}\phi_1^{02}-2\sqrt{2}\phi_1^0\phi_1^+\delta^+\delta^{++}\\
&\quad+\delta^{+2}(\phi_1^{02}+\phi_1^{+2}))+\lambda_9\delta^{+2}(\phi_1^{02}+\phi_1^{+2}),\\
{}^{4F}V_4(\phi_1^0,\delta^0,\delta^+,\delta^{++}) &= \lambda_5(\delta^{02}+\delta^{+2}+\delta^{++2})^2+\lambda_6(\delta^{+2}+2\delta^0\delta^{++})^2+\lambda_1\phi_1^{04}+\frac{1}{2}\lambda_{12}\phi_1^{02}(2\delta^{02}+\delta^{+2})\\
&\quad+\lambda_9\phi_1^{02}(\delta^{+2}+2\delta^0\delta^{++}),\\
{}^{4F}V_5(\phi_1^+,\delta^0,\delta^+,\delta^{++}) &= \lambda_5(\delta^{02}+\delta^{+2}+\delta^{++2})^2+\lambda_6(\delta^{+2}+2\delta^0\delta^{++})^2+\lambda_1\phi_1^{+4}\\
&\quad+\frac{1}{2}\lambda_{12}\phi_1^{+2}(2\delta^{02}+\delta^{+2})+\lambda_9\phi_1^{+2}(\delta^{+2}+2\delta^0\delta^{++}).
\end{aligned}
$$

*Stability conditions*

$$\begin{aligned}
{}^{4F}V_1 &\to \lambda_1 > 0; \quad \lambda_5 > 0; \quad \lambda_5 + 2\lambda_6 > 0,\\
{}^{4F}V_2 &\to \lambda_1 > 0; \quad \lambda_5 > 0; \quad \lambda_5 + \lambda_6 > 0,\\
{}^{4F}V_3 &\to \lambda_1 > 0; \quad \lambda_5 > 0; \quad \lambda_5 + \lambda_6 > 0; \quad \lambda_{12} - 2\sqrt{2\lambda_1\lambda_5} < 0,\\
{}^{4F}V_4, {}^{4F}V_5 &\to \lambda_1 > 0; \quad \lambda_5 > 0; \quad \lambda_5 + \lambda_6 > 0; \quad \lambda_5 + 2\lambda_6 > 0,
\end{aligned}$$

### 3. LR model with doublet scalars

#### *a. Two-field directions and stability conditions*

$$\begin{aligned}
{}^{2F}V_1(\phi_1^0, \phi_1^+) &= \lambda_1(\phi_1^{02} + \phi_1^{+2})^2,\\
{}^{2F}V_2(\phi_1^+, h_R^+) &= \lambda_1\phi_1^{+4} + \frac{2\beta_1 + f_1}{2} h_R^{+2}\phi_1^{+2},\\
{}^{2F}V_3(\phi_1^0, h_R^+) &= \lambda_1\phi_1^{04} + \frac{2\beta_1 - f_1}{2} h_R^{+2}\phi_1^{02},\\
{}^{2F}V_4(\phi_1^+, h_R^0) &= \lambda_1\phi_1^{+4} + \frac{2\beta_1 - f_1}{2} h_R^{0\,2}\phi_1^{+2},\\
{}^{2F}V_5(\phi_1^0, h_R^0) &= \lambda_1\phi_1^{04} + \frac{2\beta_1 + f_1}{2} h_R^{0\,2}\phi_1^{02}.
\end{aligned}$$

*Stability conditions*

$${}^{2F}V_1 \to \lambda_1 > 0, \quad {}^{2F}V_2, {}^{2F}V_5 \to \lambda_1 > 0; \quad 2\beta_1 + f_1 > 0, {}^{2F}V_3, \quad {}^{2F}V_4 \to \lambda_1 > 0; \quad 2\beta_1 - f_1 > 0,$$

#### *b. Three-field directions and stability conditions*

$$\begin{aligned}
{}^{3F}V_1(\phi_1^0, \phi_1^+, h_R^0) &= \lambda_1(\phi_1^{02} + \phi_1^{+2})^2 + h_R^{0\,2}\left(\beta_1(\phi_1^{02} + \phi_1^{+2}) + \frac{1}{2} f_1(\phi_1^{02} - \phi_1^{+2})\right),\\
{}^{3F}V_2(\phi_1^0, \phi_1^+, h_R^+) &= \lambda_1(\phi_1^{02} + \phi_1^{+2})^2 + h_R^{+2}\left(\beta_1(\phi_1^{02} + \phi_1^{+2}) + \frac{1}{2} f_1(\phi_1^{+2} - \phi_1^{02})\right),\\
{}^{3F}V_3(\phi_1^0, h_R^0, h_R^+) &= \frac{1}{2}\phi_1^{02}(f_1(h_R^{0\,2} - h_R^{+2}) + 2\beta_1(h_R^{0\,2} + h_R^{+2}) + 2\lambda_1\phi_1^{02}),\\
{}^{3F}V_3(\phi_1^+, h_R^0, h_R^+) &= \frac{1}{2}\phi_1^{+2}(f_1(h_R^{+2} - h_R^{0\,2}) + 2\beta_1(h_R^{0\,2} + h_R^{+2}) + 2\lambda_1\phi_1^{+2}).
\end{aligned}$$

*Stability conditions*

$${}^{3F}V_1, {}^{3F}V_2, {}^{3F}V_3, {}^{3F}V_4 \to \lambda_1 > 0; \quad 2\beta_1 + f_1 > 0; \quad 2\beta_1 - f_1 > 0.$$

#### *c. Four-field directions and stability conditions*

$$\begin{aligned}
{}^{4F}V_1(\phi_1^0, \phi_1^+, h_R^0, h_R^+) = \frac{1}{2}&(f_1(h_R^+(\phi_1^0 - \phi_1^+) + h_R^0(\phi_1^0 + \phi_1^+))(h_R^0(\phi_1^0 - \phi_1^+) - h_R^+(\phi_1^0 + \phi_1^+))\\
&+ 2(\phi_1^{02} + \phi_1^{+2})((h_R^{0\,2} + h_R^{+2})\beta_1 + \lambda_1(\phi_1^{02} + \phi_1^{+2}))).
\end{aligned}$$

*Stability conditions*

$${}^{4F}V_1 \to \lambda_1 > 0; \quad 2\beta_1 + f_1 > 0; \quad 2\beta_1 - f_1 > 0.$$

## APPENDIX C: RENORMALIZATION GROUP EVOLUTION EQUATIONS

### 1. Standard Model RGEs

For the Standard Model we have used the renormalization group evolution equations from Ref. [66] with matching conditions for the top Yukawa and Higgs quartic couplings at their pole masses.

### 2. $U(1)_{B-L}$ model

#### *a. Gauge RG equations*

The renormalization group equations for $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ gauge couplings $g_3$ and $g_2$ are

$$16\pi^2 \frac{d}{dt} g_3 = g_3^3\left[-1+\frac{4}{3}n_g\right] = \frac{g_3^3}{16\pi^2}[-7],$$
$$16\pi^2 \frac{d}{dt} g_2 = g_2^3\left[-\frac{22}{3}+\frac{4}{3}n_g+\frac{1}{6}\right] = \frac{g_2^3}{16\pi^2}\left[-\frac{19}{6}\right],$$

where $n_g$ is the number of generations.

The renormalization group equations for the Abelian gauge couplings $g_1, g_{B-L}$ and $\tilde{g}$ are

$$16\pi^2 \frac{d}{dt} g_1 = \left[\frac{41}{6} g_1^3\right],$$
$$16\pi^2 \frac{d}{dt} g_{B-L} = \left[12 g_{B-L}^3 + \frac{32}{3} g_{B-L}\tilde{g} + \frac{41}{6} g_{B-L}\tilde{g}^2\right],$$
$$16\pi^2 \frac{d}{dt} \tilde{g} = \left[\frac{41}{6}\tilde{g}(\tilde{g}^2+2g_1^2) + \frac{32}{3} g_{B-L}(\tilde{g}^2+g_1^2) + 12 g_{B-L}^2\tilde{g}\right],$$

#### *b. Fermion RG equations*

The RG evolution equation for the top-quark Yukawa coupling $Y_t$ is

$$16\pi^2 \frac{d}{dt} Y_t = Y_t\left[\frac{9}{2}Y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 - \frac{17}{12}\tilde{g}^2 - \frac{2}{3}g_{B-L}^2 - \frac{5}{3}\tilde{g}g_{B-L}\right].$$

In the case of right-handed neutrinos the RGEs we are considering degenerate the right-handed neutrino Yukawa coupling and we are in a basis where these couplings are diagonal; then, we have

$$16\pi^2 \frac{d}{dt} y_i^h = y_i^h[4(y_i^h)^2 + 2\mathrm{Tr}[(y^h)^2] - 6g_{B-L}^2].$$

#### *c. Scalar RG equations*

The RGEs for the scalar couplings $\lambda_1, \lambda_2$ and $\lambda_3$ are

$$16\pi^2 \frac{d}{dt}\lambda_1 = \left[24\lambda_1^2 + \lambda_3^2 - 6Y_t^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2g_1^2 + \frac{3}{4}g_2^2\tilde{g}^2 + \frac{3}{4}g_1^2\tilde{g}^2 + \frac{3}{8}\tilde{g}^4 + 12\lambda_1Y_t^2 - 9\lambda_1g_2^2 - 3\lambda_1g_1^2 - 3\lambda_1\tilde{g}^2\right],$$
$$8\pi^2 \frac{d}{dt}\lambda_2 = \left[10\lambda_2^2 + \lambda_3^2 - \frac{1}{2}\mathrm{Tr}[(y^h)^4] + 48g_{B-L}^4 + 4\lambda_2\mathrm{Tr}[(y^h)^2] - 24\lambda_2g_{B-L}^2\right],$$
$$8\pi^2 \frac{d}{dt}\lambda_3 = \lambda_3\left[6\lambda_1 + 4\lambda_2 + 2\lambda_3 + 3Y_t^2 - \frac{3}{4}(3g_2^2 - g_1^2 - \tilde{g}^2) + 2\mathrm{Tr}[(y^h)^2] - 12g_{B-L}^2\right] + 6\tilde{g}^2g_{B-L}^2.$$

### 3. LR model with triplet scalars

#### *a. Gauge RG equations*

$$16\pi^2 \frac{d}{dt} g_3 = g_3^3(-7),\; 16\pi^2 \frac{d}{dt} g_2 = g_2^3\left(-\frac{15}{6}\right),\; 16\pi^2 \frac{d}{dt} g_{B-L} = g_{B-L}^3\left(\frac{28}{9}\right).$$

Note that in our case $g_{2L} = g_{2R} = g_2$.

#### *b. Fermion RG equations*

$$16\pi^2 \frac{d}{dt} Y_t = \left[8Y_t^3 - Y_t\left(\frac{2}{3}g_1^2 - \frac{9}{2}g_2^2 - 8g_3^2\right)\right],$$
$$16\pi^2 \frac{d}{dt} Y_i^M = \left[2Y_i^M\left(-\frac{3}{4}g_1^2 - \frac{9}{4}g_2^2\right) + 2Y_i^M\mathrm{Tr}[(Y^M)^2] + 6(Y_i^M)^3\right].$$

### *c. Scalar RG equations*

To write down the scalar RG equations, we classified 15 scalar couplings into three categories depending on how they coupled with scalar fields.

(i) *Coefficients with* $\Phi^4$

$$16\pi^2\frac{d}{dt}\lambda_1 = 32\lambda_1^2 + \frac{5}{3}\lambda_{12}^2 + \frac{1}{2}\lambda_{13}^2 + 2\lambda_{14}^2 + 64\lambda_2^2 + 16\lambda_1\lambda_3 + 16\lambda_3^2 + 48\lambda_4^2 + 6\lambda_{12}\lambda_9 + 6\lambda_9^2 + 12\lambda_1 Y_t^2 - 6Y_t^4 - 18\lambda_1 g_2^2 + 3g_2^4,$$

$$16\pi^2\frac{d}{dt}\lambda_2 = 6(\lambda_{10}^2 - \lambda 11^2) + \frac{3}{2}\lambda_{14}\lambda_{15} + 24\lambda_1\lambda_2 + 48\lambda_2\lambda_3 + 12\lambda_4^2 + 12\lambda_2 Y_t^2 - 18\lambda_2 g_2^2,$$

$$16\pi^2\frac{d}{dt}\lambda_3 = 12(\lambda_{10}^2 + \lambda_{11}^2) - (\lambda_{12}^2 - \lambda_{13}^2) - \frac{1}{2}(\lambda_{14}^2 + \lambda_{15}^2) + 128\lambda_2^2 + 24\lambda_1\lambda_3 + 16\lambda_3^2 + 24\lambda_4^2 + 12\lambda_3 Y_t^2 + 3Y_t^4 - 18\lambda_3 g_2^2 + \frac{3}{2}g_2^2,$$

$$16\pi^2\frac{d}{dt}\lambda_4 = 48\lambda_4(\lambda_1 + 2\lambda_2 + \lambda_3) + 6\lambda_{10}(2\lambda_9 + \lambda_{12}) + \frac{3}{2}\lambda_{13}(\lambda_{14} + \lambda_{15}) + 12\lambda_4 Y_t^2 - 18\lambda_4 g_2^2.$$

(ii) *Coefficients with* $\Delta^4$

$$16\pi^2\frac{d}{dt}\lambda_5 = 28\lambda_5^2 + 16\lambda_6(\lambda_5 + \lambda_6) + 16(\lambda_{10}^2 + \lambda_{11}^2) + 2\lambda_{12}^2 + 3\lambda_7^2 + 4\lambda_9(\lambda_9 + \lambda_{12}) + 2\lambda_5 Y_t^2 - 16Y_t^4 - 12\lambda_5 g_{B-L}^2 + 6g_{B-L}^4 + 12g_{B-L}^2 g_2^2 - 24\lambda_5 g_2^2 + 9g_2^4,$$

$$16\pi^2\frac{d}{dt}\lambda_6 = 12\lambda_6(\lambda_6 + 2\lambda_5 - g_{B-L}^2 - 2g_2^2) + 12\lambda_8^2 - \lambda_{12}^2 + 8Y_t^4 + 8\lambda_6 Y_t^2 - 12g_{B-L}^2 g_2^2 + 3g_2^4,$$

$$16\pi^2\frac{d}{dt}\lambda_7 = 4\lambda_7^2 + 16\lambda_7(2\lambda_5 + \lambda_6) + 32(\lambda_{10}^2 - \lambda_{11}^2) + 2(\lambda_{12}^2 + \lambda_{13}^2) + 4(\lambda_{14}^2 + \lambda_{15}^2) + 32\lambda_8^2 + 8\lambda_{12}\lambda_9 + \lambda_9^2 + 8\lambda_7 Y_t^2 - 12\lambda_7(g_{B-L}^2 + g_2^2) + 12g_{B-L}^4,$$

$$16\pi^2\frac{d}{dt}\lambda_8 = \lambda_{13}^2 + 4\lambda_{14}\lambda_{15} + 8\lambda_8(\lambda_5 + 5\lambda_6 + \lambda_7 + Y_t^2) - 12\lambda_8(2g_{B-L}^2 + g_2^2).$$

(iii) *Coefficients with* $\Phi^2\Delta^2$

$$16\pi^2\frac{d}{dt}\lambda_9 = \lambda_9(20\lambda_1 + 8\lambda_3 + 16\lambda_5 + 8\lambda_6 + 6\lambda_7 + 4\lambda_9 + 6Y_t^2 + 4\mathrm{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2) + 6g_2^4 + 16(\lambda_{10}^2 + \lambda_{11}^2) + \lambda_{12}(8\lambda_1 + \lambda_{12}) + 3\lambda_{13}^2 + 12\lambda_{14}^2 + 8\lambda_{12}\lambda_3 + 48\lambda_{10}\lambda_4 + \lambda_{12}(6\lambda_5 + 8\lambda_6 + 3\lambda_7),$$

$$16\pi^2\frac{d}{dt}\lambda_{10} = \lambda_{10}(4\lambda_1 + 4\lambda_{12} + 48\lambda_2 + 16\lambda_3 + 16\lambda_4 + 16\lambda_5 + 8\lambda_6 + 6\lambda_7 + 8\lambda_9 + 6Y_t^2 + 4\mathrm{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2) - 3\lambda_{13}(\lambda_{14} + \lambda_{15}) + 12\lambda_4\lambda_9,$$

$$16\pi^2\frac{d}{dt}\lambda_{11} = \lambda_{11}(4\lambda_1 + 4\lambda_{12} - 48\lambda_2 + 16\lambda_3 + 16\lambda_5 + 8\lambda_6 - 6\lambda_7 + 8\lambda_9 + 6Y_t^2 + 4\mathrm{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2),$$

$$16\pi^2\frac{d}{dt}\lambda_{12} = \lambda_{12}(4\lambda_1 + 4\lambda_{12} - 8\lambda_3 + 4\lambda_5 - 8\lambda_6 + 8\lambda_9 + 6Y_t^2 4\mathrm{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2) - 12(\lambda_{14}^2 - \lambda_{15}^2),$$

$$16\pi^2\frac{d}{dt}\lambda_{13} = \lambda_{13}(4\lambda_1 + 4\lambda_{12} + 8\lambda_3 + 2\lambda_7 + 8\lambda_8 + 8\lambda_9 + 3Y_t^2 + \mathrm{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2) + (8\lambda_4 + 16\lambda_{10})(\lambda_{14} + \lambda_{15}),$$

$$16\pi^2\frac{d}{dt}\lambda_{14} = \lambda_{14}(4\lambda_1 - 4\lambda_{12} + 2\lambda_7 + 8\lambda_9 + 6Y_t^2 + 4\mathrm{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2) + 4\lambda_{13}(\lambda_4 + 2\lambda_{10}) + 8\lambda_{15}(2\lambda_2 + \lambda_8),$$

$$16\pi^2\frac{d}{dt}\lambda_{15} = \lambda_{15}(4\lambda_1 + 12\lambda_{12} + 2\lambda_7 + 8\lambda_9 + 6Y_t^2 + 4\mathrm{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2) + 4\lambda_{13}(\lambda_4 + 4\lambda_{10}) + 8\lambda_{14}(2\lambda_2 + \lambda_8).$$

### 4. LR model with doublet scalars

#### *a. Gauge RG equations*

$$16\pi^2\frac{d}{dt}g_3 = g_3^3(-7), \quad 16\pi^2\frac{d}{dt}g_2 = g_2^3\left(-\frac{17}{6}\right), \quad 16\pi^2\frac{d}{dt}g_{B-L}^2 = g_{B-L}^3(3).$$

Note that in our case $g_{2L} = g_{2R} = g_2$.

#### *b. Fermion RG equations*

$$64\pi^2\frac{d}{dt}Y_t = \left(-\frac{2}{9}g_{B-L}^2 - 9g_2^2 - 32g_3^2\right)Y_t + 7Y_t^3.$$

#### *c. Scalar RG equations*

(i) *Coefficients with* $\Phi^4$

$$128\pi^2\frac{d}{dt}\lambda_1 = \lambda_1(-72g_2^2 + 256(\lambda_1+\lambda_2-\lambda_3) + 24Y_t^2) + 1024(\lambda_1^2+\lambda_2^2) + 32\beta_1^2 + 8f_1^2 + 9g_2^4 - 12Y - Y_t^4,$$

$$512\pi^2\frac{d}{dt}\lambda_2 = \lambda_2(-288g_2^2 + 768\lambda_1 + 3072\lambda_2 + 1024\lambda_3 + 96Y_t^2) - 8f_1^2 + 3g_2^4 - 3Y_t^4,$$

$$256\pi^2\frac{d}{dt}\lambda_3 = \lambda_3(-144g_2^2 - 384\lambda_1 - 512\lambda_2 - 1536\lambda_3 + 48Y_t^2) + 4f_1^2 - 3g_2^4 - 3Y_t^4.$$

(ii) *Coefficients with* $H_{L/R}^4$

$$512\pi^2\frac{d}{dt}\kappa_1 = \kappa_1(-96g_{B-L}^2 - 144g_2^2 + 576\kappa_1 + 384\kappa_2) + 192\kappa_2^2 + 256\beta_1^2 + 128f_1^2 + 24g_{B-l}^4 + 12g_{B-L}^2g_2^2 + 9g_2^4,$$

$$512\pi^2\frac{d}{dt}\kappa_2 = \kappa_2(-96g_{B-L}^2 - 144g_2^2 + 512\kappa_1 + 384\kappa_2) + 128f_1^2 + 12g_{B-L}^2g_2^2 + 9g_2^4.$$

(iii) *Coefficeients with* $\Phi^2H_{L/R}^2$

$$256\pi^2\frac{d}{dt}\beta_1 = -4\beta_1[-8\beta_1 + 6g_{B-L}^2 + 27g_2^2 - 2(20\kappa_1 + 4\kappa_2 + 40\lambda_1 + 32\lambda_2 - 32\lambda_3 + 3Y_t^2)] + 24f_1^2 + 9g_2^4,$$

$$256\pi^2\frac{d}{dt}f_1 = f_1(16\beta_1 - 6g_{B-L}^2 - 27g_2^2 + 8(\kappa_1+\kappa_2) + 16(\lambda_1 - 4\lambda_2) + 64\lambda_3 + 6Y_t^2).$$


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