Light-front quark model consistent with Drell-Yan-West duality and quark counting rules

Thomas Gutsche,[1] Valery E. Lyubovitskij,[1,2] Ivan Schmidt,[3] and Alfredo Vega[4,5]

[1]*Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

[2]*Department of Physics, Tomsk State University, 634050 Tomsk, Russia*

[3]*Departamento de Física y Centro Científico Tecnológico de Valparaíso (CCTVal), Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

[4]*Instituto de Física y Astronomía, Universidad de Valparaíso, Avenida Gran Bretaña 1111, Valparaíso, Chile*

[5]*Centro de Astrofísica de Valparaíso, Avenida Gran Bretaña 1111, Valparaíso, Chile*

(Received 4 June 2013; published 26 March 2014)

From the matching of soft-wall AdS/QCD and light-front QCD for electromagnetic form factors of hadrons with arbitrary twist, we derive an effective light-front wave function (LFWF). The scale at which such matching is performed is set as an initial scale. Together with the LFWF, we also obtain expressions for parton distribution functions (PDFs) and generalized parton distributions (GPDs) at this scale. Doing the hard evolution of PDFs and GPDs, we show that these quantities obey model-independent constraints imposed, e.g., by the Drell-Yan-West duality at large x and large scales, while the hadronic form factors at large momenta are in full agreement with quark counting rules. We finally propose a phenomenological light-front wave function for hadrons with arbitrary twist dimension (mesons, baryons, and multiquark states). It explicitly depends on the scale. The wave function evolves from the expression at the initial scale predicted by soft-wall AdS/QCD to high scales, where it gives the correct scaling behavior of PDFs, GPDs, and form factors for both pions and nucleons. For other hadronic states, the proposed wave function produces form factors consistent with quark counting rules and gives predictions for parton distributions. As an application, we build a light-front quark model for nucleons based on the proposed wave function.

DOI: 10.1103/PhysRevD.89.054033 PACS numbers: 12.38.Aw, 12.38.Lg, 13.40.Gp, 14.20.Dh

I. INTRODUCTION

The main objective of this paper is to derive a phenomenological light-front wave function (LFWF) for hadrons with arbitrary twist dimension (mesons, baryons, and multiquark states). At an initial scale, this LFWF is constrained by the soft-wall AdS/QCD model, and at higher scales it gives the correct scaling behavior of parton distribution functions (PDFs) and form factors, both for pions and nucleons. The explicit form of the wave function at large scales is extracted from the hard evolution of PDFs and generalized parton distributions (GPDs). The proposed wave function produces form factors consistent with quark counting rules and also gives predictions for the corresponding parton distributions. As an application, we construct a light-front (LF) quark model for nucleons based on the proposed wave function. This model is by construction consistent with the Drell-Yan-West (DYW) relation [1] between the large-Q^2 behavior of nucleon electromagnetic form factors and the large-x behavior of the structure functions (see, also, Ref. [2] for an extension to inelastic scattering) and with quark counting rules [3]. Based on the findings of Refs. [1–3] one can, e.g., relate the behavior of the quark distribution function in the nucleon $q_v(x) \sim (1-x)^p$ at $x \to 1$ to the scaling of the proton Dirac form factor $F_1^p(Q^2) \sim 1/(Q^2)^{(p+1)/2}$ at large Q^2 (the parameter p is related to the number of valence constituents N in the hadron; hence, for $N = 3$, we have $p = 3$). In Refs. [4,5], the large-x scaling of pion and nucleon PDFs and GPDs has been obtained in the framework of perturbative QCD. In particular, the pion PDF behaves as $(1-x)^2$ for $x \to 1$, while the nucleon spin nonflip and spin-flip PDFs behave at large x as $(1-x)^3$ and $(1-x)^5$, respectively. The importance of these scaling laws and their role in the description of hadron structure has been stressed and studied in detail in the literature, see, e.g., Refs. [4–7].

In this paper, we show that these important features of nucleon structure can be also fulfilled in the framework of a simple light-front quark model. It is based on a phenomenological wave function with a specific dependence on the transverse momentum $\mathbf{k}_\perp$ and the light-cone variable x. We also derive a phenomenological LFWF for hadrons with arbitrary twist dimension τ, which gives the required scaling for the corresponding form factors and also provides predictions for unknown PDFs and GPDs.

II. LIGHT-FRONT WAVE FUNCTION FOR HADRON WITH ARBITRARY TWIST

In this section, we propose a phenomenological LFWF $\psi_\tau(x, \mathbf{k}_\perp, \mu)$ for hadrons with arbitrary twist dimension τ without referring to the flavor structure and with scale dependence μ. The main idea is that at some initial scale, these functions are derived from the matching of the electromagnetic form factors of hadrons with arbitrary spin in the soft-wall AdS/QCD approach and in light-front QCD. Then we propose that these wave functions have a scale dependence, which is also encoded in the corresponding PDFs and GPDs. Next, doing the hard evolution of PDFs and GPDs, we find the evolution of the LFWFs, which fulfill the following constraints: (1) at large scales $\mu \to \infty$ and for large $x \to 1$, this function must produce the scaling of PDFs as $(1-x)^\tau$; (2) at large Q^2, the form factor scales as $1/(Q^2)^{\tau-1}$.

In this paper, we consider two examples: $\tau = 2$ (leading twist contribution to the pion wave function) and $\tau = 3$ (leading twist contribution to the nucleon wave function). We extract the LFWF at the initial scale μ_0 from the matching of the following expressions for the hadronic form factors: from one side in LF QCD we use the Drell-Yan-West formula [1]

$$F_\tau(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_\tau^\dagger(x, \mathbf{k}'_\perp)\psi_\tau(x, \mathbf{k}_\perp), \quad (1)$$

where $\psi(x, \mathbf{k}_\perp) \equiv \psi(x, \mathbf{k}_\perp; \mu_0)$, $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$, and $Q^2 = \mathbf{q}_\perp^2$ and from soft-wall AdS/QCD [8–13] we have

$$F_\tau(Q^2) = \int_0^\infty dz V(Q, z)\varphi_\tau^2(z) = \frac{\Gamma(\frac{Q^2}{4\kappa^2} + 1)\Gamma(\tau)}{\Gamma(\frac{Q^2}{4\kappa^2} + \tau)}. \quad (2)$$

In the last expression,

$$\phi_\tau(z) = \sqrt{\frac{2}{\Gamma(\tau-1)}}\kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2} \quad (3)$$

is the hadronic wave function with twist τ depending on the scale (holographic) variable z [8], which is dual to the corresponding bulk profile confined in the quadratic dilaton potential. We also introduce

$$V(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right), \quad (4)$$

which is the bulk-to-boundary propagator dual to the electromagnetic field. Note that the hadronic factors predicted in soft-wall AdS/QCD [8–13] are given by an analytical expression and also possess the correct powerlike scaling at large Q^2 with

$$F_\tau(Q^2) \sim \frac{1}{(Q^2)^{\tau-1}}. \quad (5)$$

This AdS/QCD result for the hadronic form factor is in leading order $\mathcal{O}(\alpha_s^0)$ in the strong coupling α_s expansion. As a result of the matching of Eqs. (1) and (2), we derive the following expression for the LFWF at the initial scale μ_0:

$$\psi_\tau(x, \mathbf{k}_\perp) = N_\tau \frac{4\pi}{\kappa} \sqrt{\log(1/x)}(1-x)^{\frac{\tau-4}{2}} \times \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right], \quad (6)$$

where

$$N_\tau = \sqrt{\tau - 1}. \quad (7)$$

Note that the derived LFWF is not symmetric under the exchange $x \to 1 - x$, since it was extracted from matching the matrix element of the bare electromagnetic current between the dressed LFWF in LF QCD and the matrix element of the dressed electromagnetic current between hadronic wave functions in AdS/QCD. In addition, the AdS/QCD prediction for the PDF is specified by large values of $x \to 1$. The same observation was made before in Ref. [14] for the case of the Callan-Gross relation between nucleon structure functions F_1 and F_2. It was found that $F_2 = 2F_1$ [14], which means that $x \to 1$.

The idea to extract LFWFs by matching to AdS/QCD has originally been suggested in Ref. [8], considering the pion electromagnetic form factor in two approaches—AdS/QCD and light-front QCD. In a series of papers [8,9,15–18], this problem was further discussed in detail. Therefore, in the case of twist $\tau = 2$, the LFWF coincides with the effective LFWF previously extracted from AdS/QCD [17]:

$$\psi_{\tau=2}(x, \mathbf{k}_\perp) = \frac{4\pi}{\kappa}\frac{\sqrt{\log(1/x)}}{1-x} \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right]. \quad (8)$$

We stress again that the extraction of the LFWF from soft-wall AdS/QCD is done using the expressions for the electromagnetic hadron form factors in both approaches (LF QCD and AdS/QCD) at the initial scale, i.e., when hard and soft evolutions are neglected. The PDFs $q_\tau(x)$ and GPDs $H_\tau(x, Q^2)$ in terms of the LFWFs at the initial scale are defined as

$$q_\tau(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |\psi_\tau(x, \mathbf{k}_\perp)|^2,$$
$$H_\tau(x, Q^2) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_\tau^\dagger(x, \mathbf{k}'_\perp)\psi_\tau(x, \mathbf{k}_\perp). \quad (9)$$

A straightforward calculation gives

$$q_\tau(x) = (\tau - 1)(1-x)^{\tau-2} \quad (10)$$

and

$$H_\tau(x,Q^2) = q_\tau(x)\exp\left[-\frac{Q^2}{4\kappa^2}\log(1/x)\right]. \quad (11)$$

Next we extend our LFWFs to an arbitrary scale as

$$\psi_\tau(x,\mathbf{k}_\perp,\mu) = N_\tau(\mu)\frac{4\pi}{\kappa}\sqrt{\log(1/x)} \times x^{a_1(\tau,\mu)}(1-x)^{b_1(\tau,\mu)} \times (1+c_1(\tau,\mu)\sqrt{x}+c_2(\tau,\mu)x)^{1/2} \times \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{x^{a_2(\tau,\mu)}(1-x)^{b_2(\tau,\mu)}}\right]. \quad (12)$$

Here, x is the light-cone variable, $\mathbf{k}_\perp$ is the transverse momentum, κ is the scale parameter, and $N_\tau(\mu)$ is the normalization constant

$$\frac{1}{N_\tau^2(\mu)} = B(1+a(\tau,\mu),1+b(\tau,\mu)) + c_1(\tau,\mu)B\left(\frac{3}{2}+a(\tau,\mu),1+b(\tau,\mu)\right) + c_2(\tau,\mu)B(2+a(\tau,\mu),1+b(\tau,\mu)), \quad (13)$$

where $B(a,b)$ is the beta function, $a = 2a_1 + a_2$, and $b = 2b_1 + b_2$. The parameters $a_i(\tau,\mu)$, $b_i(\tau,\mu)$, and $c_i(\tau,\mu)$ depend on twist τ and the scale μ. We assume that the scale dependence of the LFWF follows from the scale dependence of the PDFs and GPDs in their evolution.

In this case, the expressions for the PDFs and GPDs read as

$$q_\tau(x,\mu) = N_\tau^2(\mu)x^{a(\tau,\mu)}(1-x)^{b(\tau,\mu)} \times (1+c_1(\tau,\mu)\sqrt{x}+c_2(\tau,\mu)x) \quad (14)$$

and

$$H_\tau(x,Q^2,\mu) = q_\tau(x,\mu) \times \exp\left[-\frac{Q^2}{4\kappa^2}\frac{\log(1/x)}{x^{a_2(\tau,\mu)}(1-x)^{b_2(\tau,\mu)-2}}\right]. \quad (15)$$

The combinations of the scale-dependent parameters $a = 2a_1 + a_2$ and $b = 2b_1 + b_2$ encode the hard-gluon evolution of the PDFs, the parameters a_2 and b_2—the evolution of the Q^2-dependent part of the GPDs. In particular, regarding the evolution of the GPDs, we will follow Ref. [19] and keep the value of Q^2 fixed. Hence, for each value of Q^2, there exists a corresponding set of parameters a_2 and b_2, which run from the initial scale μ_0 to any desired hard resolution scale. Notice that in Ref. [19], the evolution of the Q^2-dependent part of GPDs was encoded in the scale-dependent transverse parameter κ. In our case, this parameter is universal, i.e., scale independent. One should stress that the parameters $a(\mu,\tau)$, $b(\mu,\tau)$, and $c_i(\mu,\tau)$ control specific regions of the light-cone variable. The parameter a is responsible for the Regge-like behavior of the PDF at small $x \to 0$, the parameter b provides the quark counting rules at large $x \to 1$, and the parameters c_1 and c_2 affect intermediate values of x. Such a parametrization of the PDFs at the initial scale was proposed in Ref. [20] and presently used in the global fits of PDFs to data (see, e.g., the review in Ref. [21]). At the initial scale, the parameters a_i, b_i, and c_i are fixed as

$$a_1(\tau,\mu_0) = a_2(\tau,\mu_0) = c_1(\tau,\mu_0) = c_2(\tau,\mu_0) = 0, \quad b_1(\tau,\mu_0) = \frac{\tau-4}{2}, \quad b_2(\tau,\mu_0) = 2. \quad (16)$$

We perform the evolution of the PDFs and GPDs following Ref. [22] (see also Ref. [23]) and explicitly determine the parameters $a_i(\tau,\mu)$, $b_i(\tau,\mu)$, and $c_i(\tau,\mu)$.

Now we perform the evolution of the PDFs and calculate explicitly the parameters $a_i(\tau,\mu)$ and $b_i(\tau,\mu)$. Due to the matching to soft-wall AdS/QCD, the evolved PDFs coincide with the PDF evaluated in the holographic approach [8,9,15–18]. The evolution of the hadron PDF characterizing the twist dimension τ in the soft region $\mu < \mu_0$ is given by an expression with a universal running on the scale (i.e., the same for any twist). It leads

$$q_\tau(x,\mu) = \int^{\mu^2}\frac{d^2\mathbf{k}_\perp}{16\pi^3}|\psi_\tau(x,\mathbf{k}_\perp)|^2 = q_\tau(x)f(x), \quad (17)$$

where

$$f(x) = 1 - \exp\left[-\frac{\mu^2}{\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right] \quad (18)$$

is a function encoding the soft evolution of the hadronic PDF universal for any twist. In Fig. 1, we display the two-dimensional plot of the x dependence of the function $f(x)$ at $\mu = \kappa$. One can see that it is very close to 1. In Fig. 2, we display the three-dimensional plot for the quantity $xq_\pi(x,\mu)$ corresponding to the pion ($\tau = 2$).

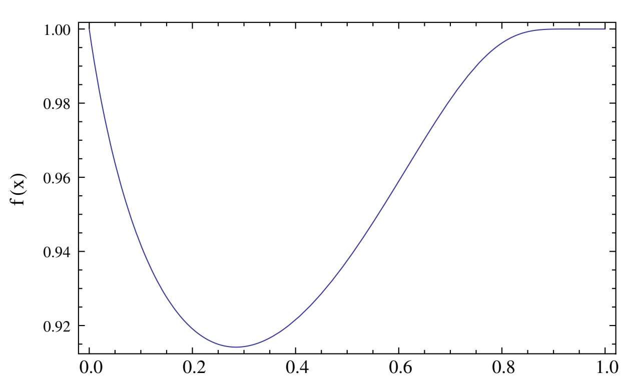

FIG. 1 (color online). Soft evolution of the hadron PDF. Dependence of the factor $f(x)$ on x at $\mu/\kappa = 1$.

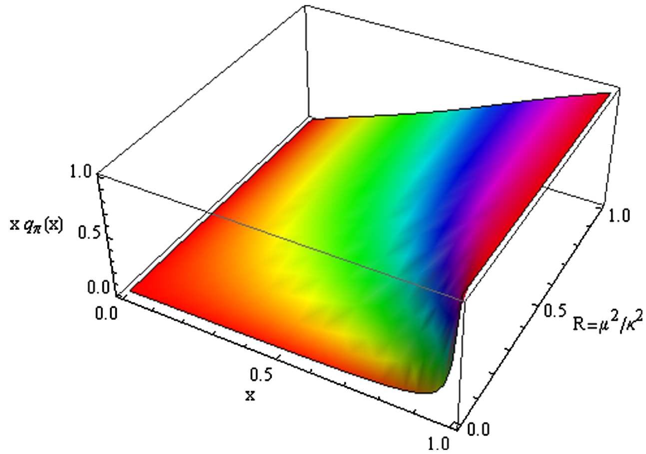

FIG. 2 (color online). Soft evolution of the pion PDF $xq_\pi(x,\mu)$ for $R = \mu^2/\kappa^2$ running from 0.01 to 1.

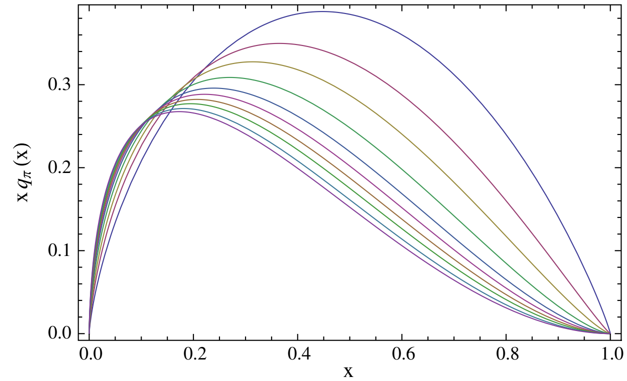

FIG. 3 (color online). Hard evolution of the pion PDF $q_\pi(x,\mu)$ for $\mu = 1, 2, 4, 10, 25, 50, 100, 200, 500$, and 1000 GeV. An increase of the scale leads to lowering of the maximum of the curves.

Next we consider the hard evolution of the hadronic wave functions, or the PDFs and GPDs. First, we consider the evolution of the PDFs. In the QCD evolution of the PDFs, in the leading order in the strong coupling constant α_s, we follow the ideas of Ref. [22]. We choose for the initial scale $\mu_0 = 313$ MeV, which in Ref. [22] was fixed for the evolution of the pion PDF. For the other hadrons, we use the same initial scale $\mu_0 = 313$ MeV. The master formula for the evolution of the Mellin moments of the PDF is given by [22]

$$\frac{\int_0^1 dx x^n q(x,\mu)}{\int_0^1 dx x^n q(x,\mu_0)} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n^{(0)}/2\beta_0}, \tag{19}$$

where $q_\tau(x,\mu_0) = (\tau-1)(1-x)^{\tau-2}$ and

$$\int_0^1 dx x^n q(x,\mu_0) = (\tau-1)B(n+1,\tau-1). \tag{20}$$

One can see that for the pion ($\tau = 2$), the PDF at the initial scale is $q_{\tau=2}(x,\mu_0) = 1$, which is in full agreement with the result of Ref. [22].

The coefficients

$$\gamma_n^{(0)} = -2C_F\left(3 + \frac{2}{(n+1)(n+2)} - 4\sum_{k=1}^{n+1}\frac{1}{k}\right) \tag{21}$$

are the anomalous dimensions, $C_F = 4/3$, $\beta_0 = 9$, and

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda_{\rm QCD}^2)} \tag{22}$$

is the running strong coupling constant with $\Lambda_{\rm QCD} = 226$ MeV.

Following Ref. [22] in evolving the pion PDF to $\mu = 4$ GeV, we are able reproduce the analysis of the Drell-Yan data from the E615 experiment [24]. Evolving the pion PDF to very large scales $\mu \to \infty$, we reproduce the power scaling $(1-x)^2$ for large $x \to 1$. In particular, the use of the coefficients c_1 and c_2 are not necessary; they can be put to zero in the case of the pion. The combinations $2a_1(\mu) + a_2(\mu)$ and $2b_1(\mu) + b_2(\mu)$ run from 0 to -0.5 and from 0 to 2, respectively. In Fig. 3, we present the evolution of the pion PDF from the scale $\mu = 1$ GeV to $\mu = 1000$ GeV. In Fig. 4, we compare our results, which exactly coincide with the prediction of Ref. [22], to the ones of the E615 experiment [24] for $\mu = 4$ GeV. At this scale, the theoretical result for the pionic PDF is very well approximated by the form

$$q_\pi(x,\mu = 4\ \text{GeV}) = 1.108x^{-0.381}(1-x)^{1.323}. \tag{23}$$

Note that this result corresponds to the leading order and close to the next-to-leading order analysis of the E615 experiment [24]. In Ref. [25], it was shown that inclusion of the next-to-leading logarithmic threshold resummation

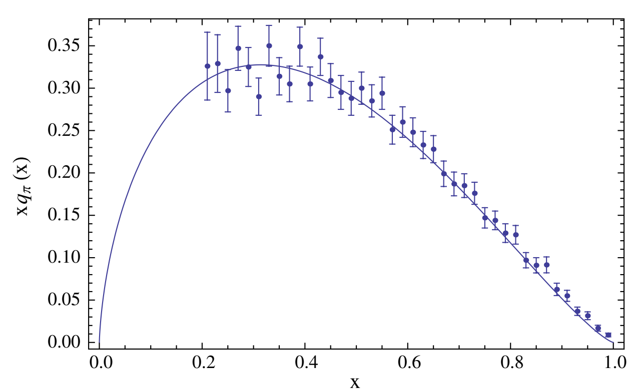

FIG. 4 (color online). Comparison of the evolved pion PDF at the scale $\mu = 4$ GeV in Ref. [22] and in our approach to the analysis of the E615 experiment [24].

effects leads to a considerably softer pion PDF $q_\pi(x, \mu = 4\ \mathrm{GeV}) \sim (1-x)^{2.34}$ at large x.

In Figs. 5–7, we plot the evolution of the valence u quark PDF in the nucleon and perform a comparison to the global fit of Martin, Stirling, Thorne, and Watt at leading order (MSTWLO) [21]. For x values in the range $0.2 \le x \le 1$, we get qualitative agreement with the valence u quark PDF extracted from data analysis [21].

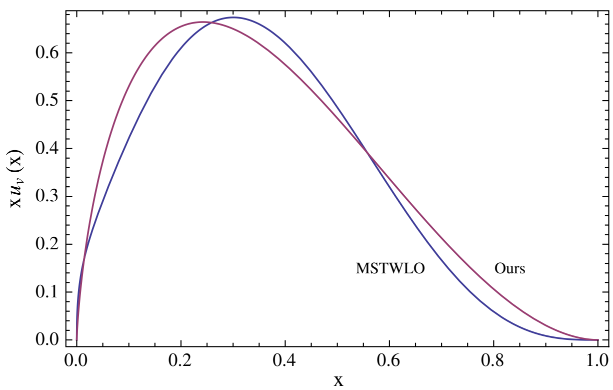

FIG. 5 (color online). PDF $xu_v(x,\mu)$ at $\mu = 1$ GeV.

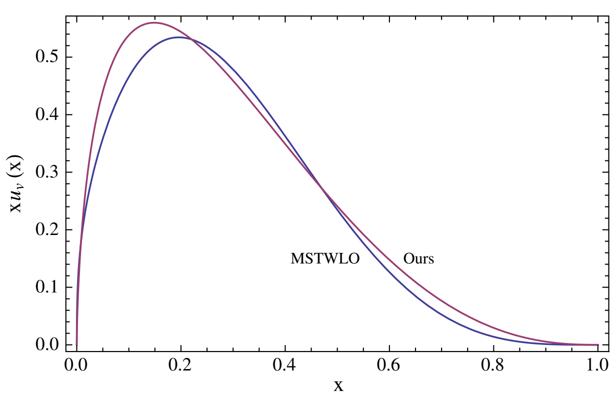

FIG. 6 (color online). PDF $xu_v(x,\mu)$ at $\mu = 10$ GeV.

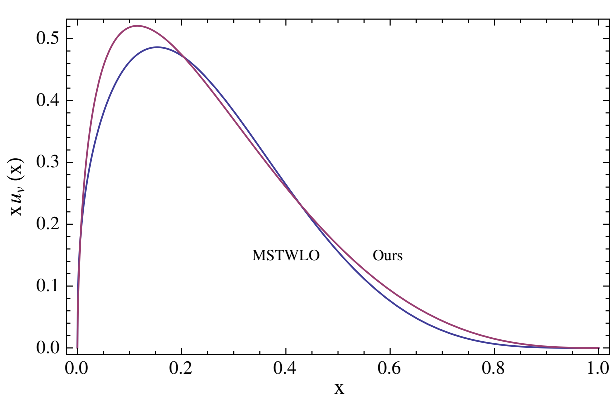

FIG. 7 (color online). PDF $xu_v(x,\mu)$ at $\mu = 100$ GeV.

Finally, at large scales μ and for $x \to 1$, the hadronic PDFs scale as $q_\tau(x,\mu) \sim (1-x)^\tau$. In particular, for the nucleon spin-nonflip PDF, we get $\sim(1-x)^3$, which is in agreement with the DYW duality. In the case of the pion, our result $\sim(1-x)^2$ agrees with the prediction of Refs. [4,5]. Therefore, the scaling of the PDFs is universal at large scales $\mu \to \infty$ and for large $x \to 1$, and it is specific to the partonic structure of a hadron (i.e., it depends on τ).

Next we present the evolution of the GPDs at fixed Q^2. Since our GPDs have a simple form at the initial scale—they are the product of the PDF at the initial scale and the factor $x^{Q^2/(4\kappa^2)}$—we apply for the evolution of the GDPs the formula for the Mellin moments of the PDFs by changing x^n to $x^{n+Q^2/(4\kappa^2)}$. The results for the evolution of the pion GPD and the valence u quark GPD in the nucleon at $Q^2 = 10\ \mathrm{GeV}^2$ are shown in Figs. 8 and 9. The scale parameter κ is fixed at 350 MeV.

One can easily show that the hadronic form factors are independent of scale. In the soft region $\mu < \mu_0$, we have

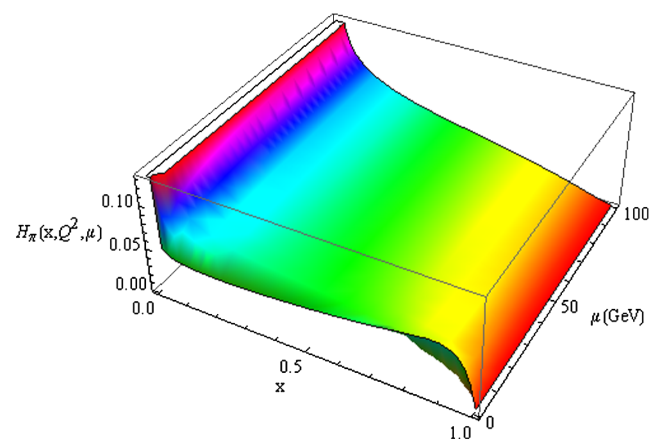

FIG. 8 (color online). $H_\pi(x, Q^2, \mu)$ at $Q^2 = 10\ \mathrm{GeV}^2$ and $\mu = 1$–100 GeV.

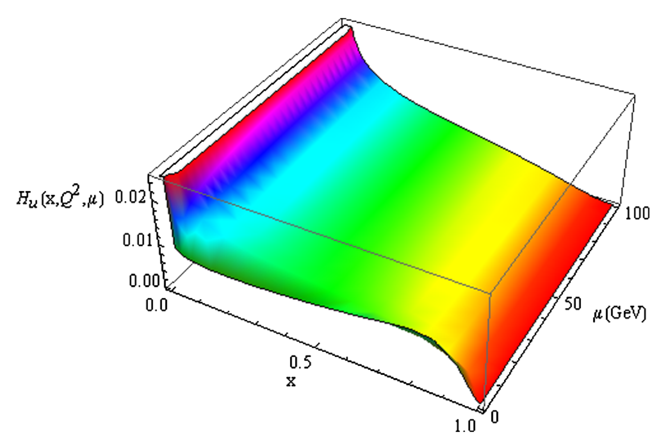

FIG. 9 (color online). $H_u(x, Q^2, \mu)$ at $Q^2 = 10\ \mathrm{GeV}^2$ and $\mu = 1$–100 GeV.

$$F_\tau(Q^2)|_{\mu<\mu_0} = \int_0^1 dx \int^{\mu^2} \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_\tau^\dagger(x,\mathbf{k}'_\perp)\psi_\tau(x,\mathbf{k}_\perp) = F_\tau(Q^2)\int_0^1 dx q_\tau(x,\mu) = F_\tau(Q^2), \quad (24)$$

where

$$\int_0^1 dx q_\tau(x,\mu) = 1. \quad (25)$$

Therefore, it coincides with the initial scale expression (1). For the hard scales $\mu > \mu_0$, the same result follows from the master formula for the $n = 0$ Mellin moment of the GPD

$$\int_0^1 dx H_\tau(x,Q^2,\mu>\mu_0) \equiv \int_0^1 dx H_\tau(x,Q^2,\mu_0). \quad (26)$$

In the limit of large scales $\mu \gg \mu_0$, and for large $x \to 1$ the form factor behaves as

$$F_\tau^\infty(Q^2) = \int_0^1 dx H_\tau^\infty(x,Q^2),$$
$$H_\tau^\infty(x,Q^2) = (\tau+1)(1-x)^\tau \times \exp\left[-\frac{Q^2}{4\kappa^2}\log(1/x)(1-x)^{\frac{2}{\tau-1}}\right]. \quad (27)$$

The Q^2 dependence of the GPD generalizes the so-called modified Regge ansatz [6] for the nucleon helicity nonflip GPD with

$$H_{\tau=3}(x,Q^2) \sim (1-x)^3 e^{-\frac{Q^2}{\Lambda^2}\log(1/x)(1-x)} \quad (28)$$

for the specific choice $\tau = 3$ to the case of hadrons with arbitrary twist dimension τ (Λ is the scale parameter). E.g., for the pion ($\tau = 2$), we get

$$H_{\tau=2}(x,Q^2) \sim (1-x)^2 e^{-\frac{Q^2}{\Lambda^2}\log(1/x)(1-x)^2}. \quad (29)$$

Now we write the pion form factor using formula (2) for $\tau = 2$ with

$$F_\pi(Q^2) \equiv F_{\tau=2}(Q^2) = \frac{1}{1+Q^2/(4\kappa^2)}. \quad (30)$$

Note, as soon as the hadronic form factor is independent of scale, it is convenient to calculate it at the initial scale, where it is given by the analytical expression (2). In Fig. 10, we plot the pion form factor multiplied by Q^2 and compare it to the data. It is also important to present results for the other two fundamental properties of the pion—pion decay constant F_π and the electromagnetic radius $\langle r_\pi^2\rangle$. In our approach, these quantities are given by the analytical expressions

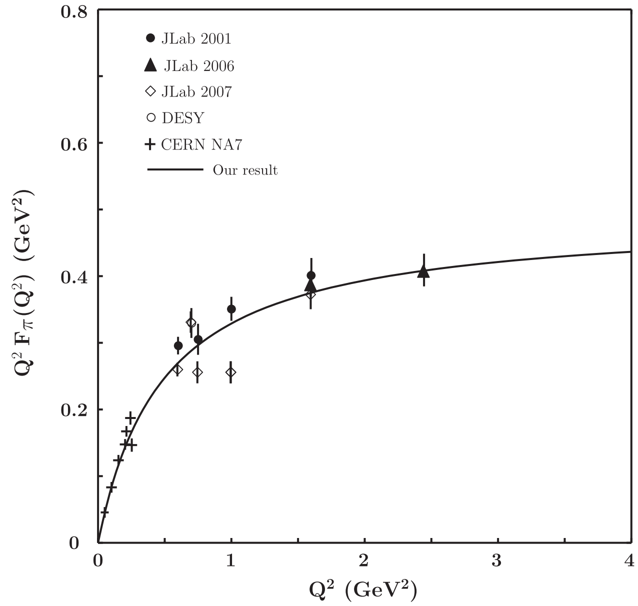

FIG. 10. Pion form factor $Q^2F_\pi(Q^2)$ (data taken from Refs. [27–32]).

$$F_\pi = 2\sqrt{N_c}\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\psi_{\tau=2}(x,\mathbf{k}_\perp) = \kappa\sqrt{\frac{N_c}{\pi}}\left(1-\frac{1}{\sqrt{2}}\right) \quad (31)$$

and

$$\langle r_\pi^2\rangle = -6\frac{dF_{\tau=2}(Q^2)}{dQ^2}\bigg|_{Q^2=0} = \frac{6}{4\kappa^2}. \quad (32)$$

For $\kappa = 350$ MeV, we get the results $F_\pi = 100$ MeV and $\langle r_\pi^2\rangle = 0.477$ fm^2, which compare well to the data $F_\pi^{\rm exp} = 92.4$ MeV and $\langle r_\pi^2\rangle_{\rm exp} = 0.452$ fm^2 [26]. Nucleon form factors will be discussed in the next section.

III. LIGHT-FRONT QUARK-DIQUARK MODEL FOR THE NUCLEON

In this section, we consider the application of the phenomenological LFWF in setting up a light-front quark-diquark model for the nucleon. First we collect the well-known decompositions [33] of the nucleon Dirac and Pauli form factors $F_{1,2}^N$ ($N = p, n$) in terms the distributions of valence quarks in nucleons $F_{1,2}^q$ ($q = u, d$) those defining GPDs ($\mathcal{H}^q$ and $\mathcal{E}^q$) [34] of valence quarks

$$F_i^{p(n)}(Q^2) = \frac{2}{3}F_i^{u(d)}(Q^2) - \frac{1}{3}F_i^{d(u)}(Q^2),$$
$$F_1^q(Q^2) = \int_0^1 dx\mathcal{H}^q(x,Q^2,\mu),$$
$$F_2^q(Q^2) = \int_0^1 dx\mathcal{E}^q(x,Q^2,\mu). \quad (33)$$

At $Q^2 = 0$, the GPDs are related to the quark densities—valence $q_v(x,\mu)$ and magnetic $\mathcal{E}_q(x,\mu)$—as

$$\mathcal{H}^q(x,0,\mu) = q_v(x,\mu), \quad \mathcal{E}^q(x,0,\mu) = \mathcal{E}^q(x,\mu), \tag{34}$$

which are normalized as

$$n_q = F_1^q(0) = \int_0^1 dx q_v(x,\mu),$$
$$\kappa_q = F_2^q(0) = \int_0^1 dx \mathcal{E}^q(x,\mu). \tag{35}$$

The number of u or d valence quarks in the proton is denoted by n_q, and κ_q is the quark anomalous magnetic moment.

Next we recall the definitions of the nucleon Sachs form factors $G_{E/M}(Q^2)$ and the electromagnetic $\langle r^2_{E/M}\rangle^N$ radii in terms of the Dirac and Pauli form factors

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2} F_2^N(Q^2),$$
$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2),$$
$$\langle r_E^2\rangle^N = -6 \frac{dG_E^N(Q^2)}{dQ^2}\bigg|_{Q^2=0},$$
$$\langle r_M^2\rangle^N = -\frac{6}{G_M^N(0)} \frac{dG_M^N(Q^2)}{dQ^2}\bigg|_{Q^2=0}, \tag{36}$$

where $G_M^N(0) \equiv \mu_N$ is the nucleon magnetic moment.

The light-front representation [35,36] for the Dirac and Pauli quark form factors is

$$F_1^q(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Big[\psi_{+q}^{+*}(x,\mathbf{k}'_\perp)\psi_{+q}^{+}(x,\mathbf{k}_\perp) + \psi_{-q}^{+*}(x,\mathbf{k}'_\perp)\psi_{-q}^{+}(x,\mathbf{k}_\perp)\Big],$$
$$F_2^q(Q^2) = -\frac{2M_N}{q^1 - iq^2} \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \times \Big[\psi_{+q}^{+*}(x,\mathbf{k}'_\perp)\psi_{+q}^{-}(x,\mathbf{k}_\perp) + \psi_{-q}^{+*}(x,\mathbf{k}'_\perp)\psi_{-q}^{-}(x,\mathbf{k}_\perp)\Big]. \tag{37}$$

Here, M_N is the nucleon mass, and $\psi_{\lambda_q q}^{\lambda_N}(x,\mathbf{k}_\perp)$ are the LFWFs at the initial scale μ_0 with specific helicities for the nucleon $\lambda_N = \pm$ and for the struck quark $\lambda_q = \pm$, where plus and minus correspond to $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. We work in the frame with $q = (0,0,\mathbf{q}_\perp)$, and, therefore, the Euclidean momentum squared is $Q^2 = \mathbf{q}_\perp^2$.

In the quark-scalar diquark model, the generic ansatz for the massless LFWFs at the initial scale μ_0 reads

$$\psi_{+q}^{+}(x,\mathbf{k}_\perp) = \phi_q^{(1)}(x,\mathbf{k}_\perp),$$
$$\psi_{-q}^{+}(x,\mathbf{k}_\perp) = -\frac{k^1 + ik^2}{xM_N}\phi_q^{(2)}(x,\mathbf{k}_\perp),$$
$$\psi_{+q}^{-}(x,\mathbf{k}_\perp) = \frac{k^1 - ik^2}{xM_N}\phi_q^{(2)}(x,\mathbf{k}_\perp),$$
$$\psi_{-q}^{-}(x,\mathbf{k}_\perp) = \phi_q^{(1)}(x,\mathbf{k}_\perp), \tag{38}$$

where $\varphi_q^{(1)}(x,\mathbf{k}_\perp)$ and $\phi_q^{(2)}(x,\mathbf{k}_\perp)$ are wave functions, which are generalizations of the proposed twist-three LFWF by inclusion of the parameters $a_q^{(i)}$ and $b_q^{(i)}$,

$$\varphi_q^{(i)}(x,\mathbf{k}_\perp) = N_q^{(i)} \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}} x^{a_q^{(i)}}(1-x)^{b_q^{(i)}} \times \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right], \tag{39}$$

where $N_q^{(i)}$ are the normalization constants fixed by conditions (35). The parameters $a_q^{(i)}$ and $b_q^{(i)}$ are fitted to optimize the description of the nucleon electromagnetic properties, magnetic moments, charge radii, and form factors as

$$a_u^{(1)} = 0.285, \qquad a_d^{(1)} = 0.7,$$
$$b_u^{(1)} = 0.050, \qquad b_d^{(1)} = 1, \tag{40}$$

$$a_u^{(2)} = 0.244, \qquad a_d^{(2)} = 0.445,$$
$$b_u^{(2)} = -0.109, \qquad b_d^{(2)} = 0.336. \tag{41}$$

The scale parameter $\kappa = 350$ MeV remains the same as in the preceding calculations. For this set of parameters, the normalization constants $N_q^{(i)}$ are fixed as

$$N_u^{(1)} = 2.918, \qquad N_d^{(1)} = 5.653,$$
$$N_u^{(2)} = 1.459, \qquad N_d^{(2)} = 1.413. \tag{42}$$

TABLE I. Electromagnetic properties of nucleons.

Quantity	Our results	Data [26]
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	−1.913	−1.913
r_E^p (fm)	0.789	0.8768 ± 0.0069
$\langle r_E^2\rangle^n$ (fm^2)	−0.108	−0.1161 ± 0.0022
r_M^p (fm)	0.757	0.777 ± 0.013 ± 0.010
r_M^n (fm)	0.773	$0.862^{+0.009}_{0.008}$

Substituting our ansatz for the LFWF in Eq. (37) and integrating over $\mathbf{k}_\perp$, we get the following expressions for the quark form factors:

$$F_1^q(Q^2) = n_q \frac{I_1^q(Q^2)}{I_1^q(0)}, \qquad F_2^q(Q^2) = \kappa_q \frac{I_2^q(Q^2)}{I_2^q(0)}, \quad (43)$$

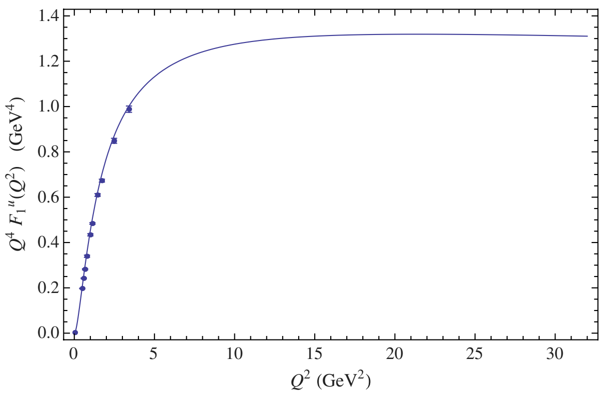

FIG. 11 (color online). Dirac u quark form factor multiplied by Q^4.

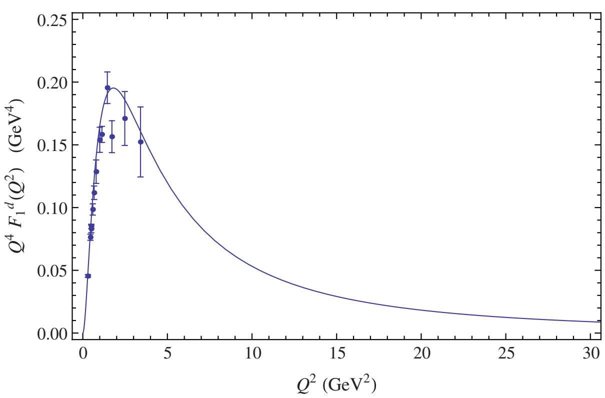

FIG. 12 (color online). Dirac d quark form factor multiplied by Q^4.

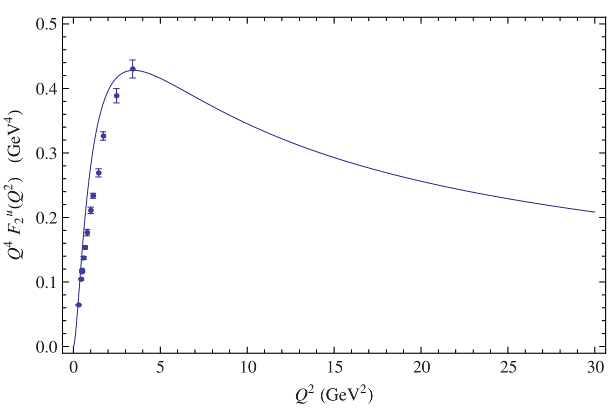

FIG. 13 (color online). Pauli u quark form factor multiplied by Q^4.

where $I_i^q(Q^2)$ are the structure integrals given by

$$I_1^q(Q^2) = \int_0^1 dx x^{2a_q^{(1)}} (1-x)^{1+2b_q^{(1)}} R_q(x, Q^2), \times \exp\left[-\frac{Q^2}{4\kappa^2}\log(1/x)\right], \quad (44)$$

$$I_2^q(Q^2) = 2\int_0^1 dx x^{2a_q^{(1)}-1} (1-x)^{2+2b_q^{(1)}} \sigma_q(x) \times \exp\left[-\frac{Q^2}{4\kappa^2}\log(1/x)\right], \quad (45)$$

with

$$R_q(x, Q^2) = 1 + \sigma_q^2(x)\frac{(1-x)^2}{x^2} \times \frac{\kappa^2}{M_N^2 \log(1/x)}\left(1 - \frac{Q^2}{4\kappa^2}\log(1/x)\right),$$

$$\sigma_q(x) = \frac{N_q^{(2)}}{N_q^{(1)}} x^{a_q^{(2)} - a_q^{(1)}} (1-x)^{b_q^{(2)} - b_q^{(1)}}. \quad (46)$$

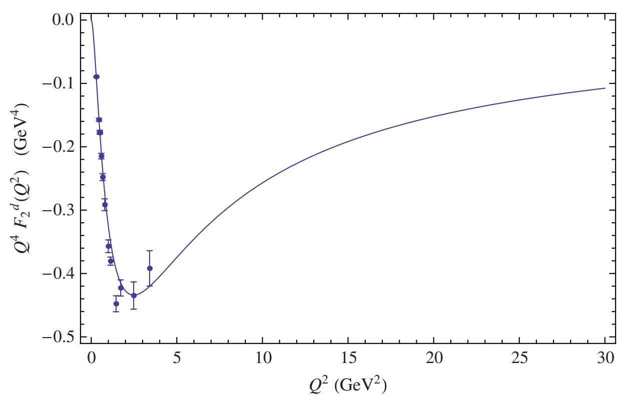

FIG. 14 (color online). Pauli d quark form factor multiplied by Q^4.

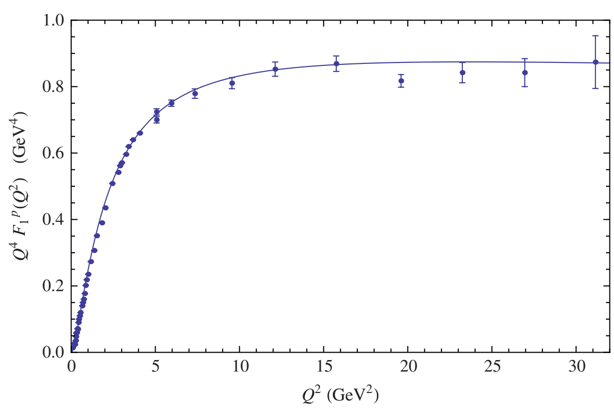

FIG. 15 (color online). Dirac proton form factor multiplied by Q^4.

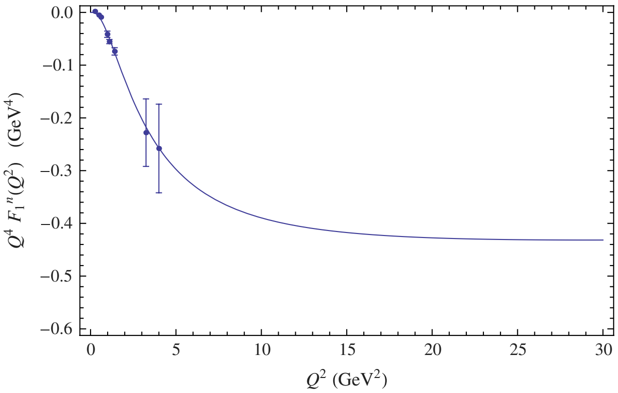

FIG. 16 (color online). Dirac neutron form factor multiplied by Q^4.

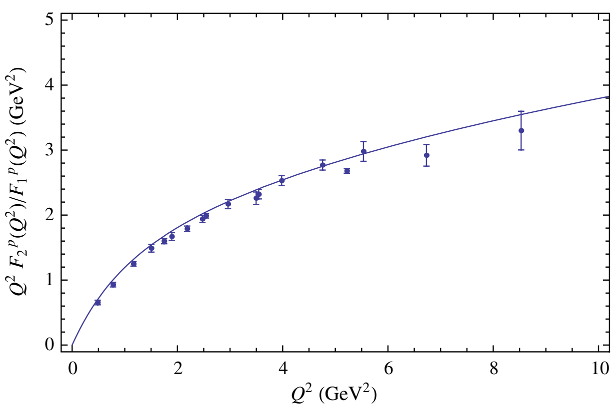

FIG. 17 (color online). Ratio $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$.

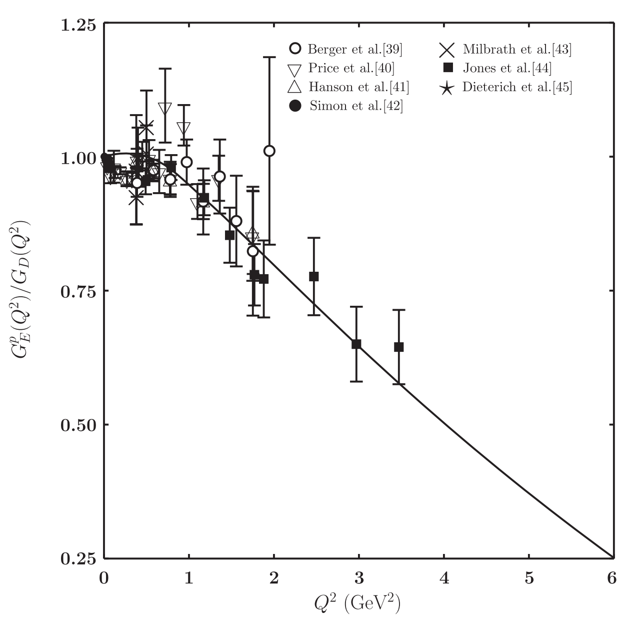

FIG. 18. Ratio $G_E^p(Q^2)/G_D(Q^2)$.

Finally, we discuss our numerical results. The results of the fit to magnetic moments in terms of the nuclear magneton (n.m.) and electromagnetic radii are shown in Table I in comparison with the data [26]. In Figs. 11–22, we present the plots of the Dirac and Pauli form factors of (u and d) quarks and nucleons. The data in Figs. 11–14 and Figs. 15–17 are taken from Ref. [37] and Ref. [38],

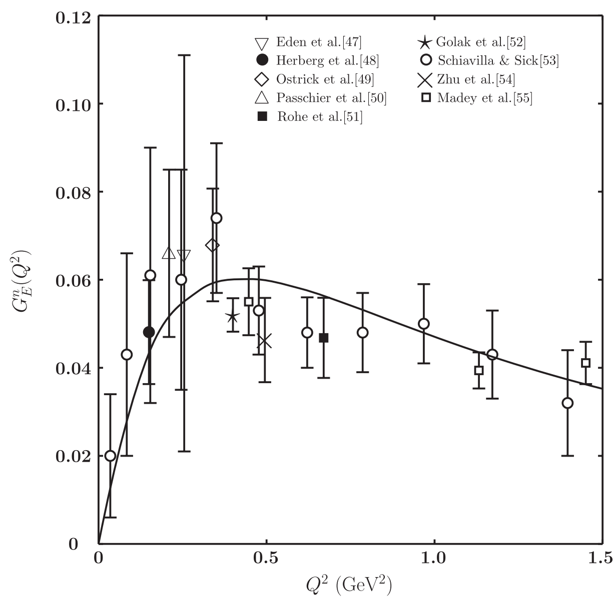

FIG. 19. Charge neutron form factor $G_E^n(Q^2)$.

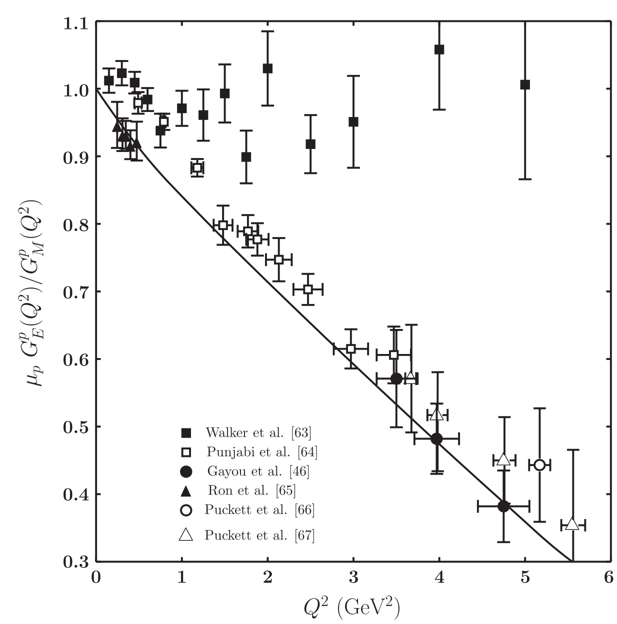

FIG. 20. Ratio $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$.

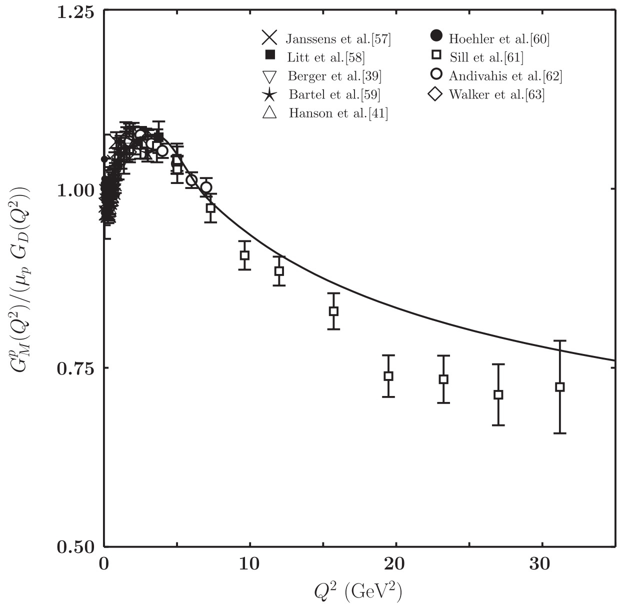

FIG. 21. Ratio $G_M^p(Q^2)/(\mu_p G_D(Q^2))$.

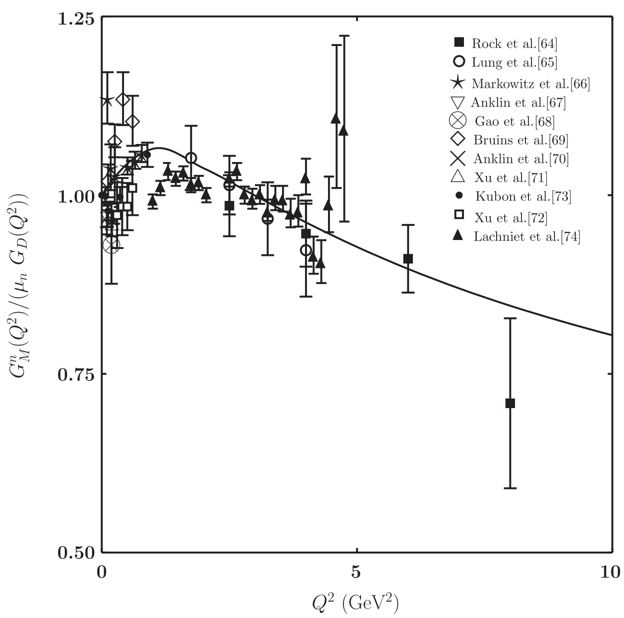

FIG. 22. Ratio $G_M^n(Q^2)/(\mu_n G_D(Q^2))$.

respectively. In Figs. 18–23, we also show the results for the Sachs nucleon form factors and compare them with the dipole formula $G_D(Q^2) = 1/(1 + Q^2/0.71\ \mathrm{GeV}^2)^2$ and with the data [39–78]. We have overall good agreement with the data.

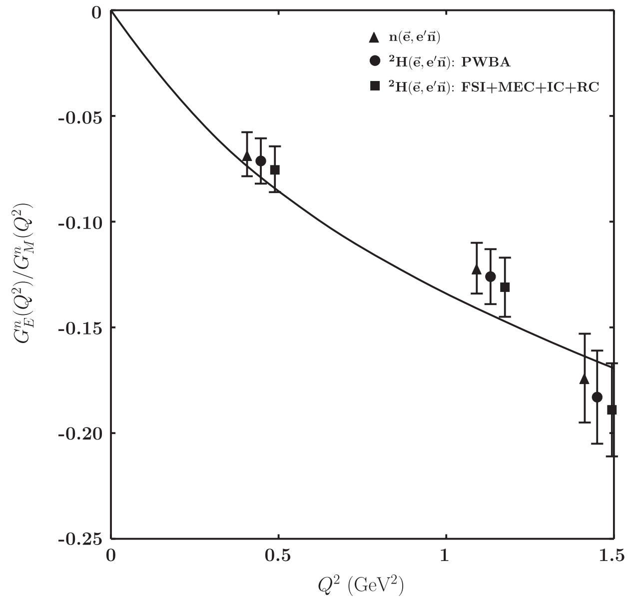

FIG. 23. Ratio $G_E^n(Q^2)/G_M^n(Q^2)$. Experimental data are taken from Ref. [56].

IV. CONCLUSION

In conclusion, we want to summarize the main result of our paper. We demonstrated how to construct a light-front quark model (or wave function) consistent with model-independent scaling laws—the DYW duality [1] and quark counting rules [3]—and which is explicitly dependent on the scale resolution. The proposed LFWF can also be generalized to sea quarks, antiquarks and gluons because it explicitly depends on twist. Such an extension work is planned to be done.

ACKNOWLEDGMENTS

The authors thank Stan Brodsky, Guy de Téramond, Wojciech Broniowski, and Werner Vogelsang for useful discussions. This work was supported by the DFG under Contract No. LY 114/2-1, by FONDECYT (Chile) under Grant No. 1100287, and by CONICYT (Chile) under Grant No. 7912010025. V. E. L. would like to thank the Departamento de Física y Centro Científico Tecnológico de Valparaíso (CCTVal), Universidad Técnica Federico Santa María, Valparaíso, Chile for warm hospitality and the Tomsk Polytechnic University, Russia for the invitation to give lectures on gauge/gravity duality and for warm hospitality.

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