Branching ratios and direct CP asymmetries in $D \to PV$ decays

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We study the two-body hadronic $D \to PV$ decays, where P (V) denotes a pseudoscalar (vector) meson, in the factorization-assisted topological-amplitude approach proposed in our previous work. This approach is based on the factorization of short-distance and long-distance dynamics into Wilson coefficients and hadronic matrix elements of four-fermion operators, respectively, with the latter being parametrized in terms of several nonperturbative quantities. We further take into account the ρ-ω mixing effect, which improves the global fit to the branching ratios involving the ρ^0 and ω mesons. Combining short-distance dynamics associated with penguin operators and the hadronic parameters determined from the global fit to branching ratios, we predict direct CP asymmetries. In particular, the direct CP asymmetries in the $D^0 \to K^0 \bar{K}^{*0}$, $\bar{K}^0 K^{*0}$, $D^+ \to \pi^+ \rho^0$, and $D_s^+ \to K^+ \omega$, $K^+ \phi$ decays are found to be of $\mathcal{O}(10^{-3})$, which can be observed at the LHCb and future Belle II experiment. We also predict the CP asymmetry observables of some neutral D meson decays.

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I. INTRODUCTION

Recent measurements of direct CP asymmetries in two-body hadronic D meson decays have stimulated great theoretical efforts on their study. The difference between the direct CP asymmetries of the $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays, $\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = [-0.82 \pm 0.21(\mathrm{stat}) \pm 0.11(\mathrm{syst})]\%$, was observed by LHCb [1] and confirmed by other collaborations. For example, the CDF and Belle measurement gave $\Delta A_{CP} = [-0.62 \pm 0.21(\mathrm{stat}) \pm 0.10(\mathrm{syst})]\%$ [2] and $\Delta A_{CP} = [-0.87 \pm 0.41(\mathrm{stat}) \pm 0.06(\mathrm{syst})]\%$ [3], respectively. The quantity ΔA_{CP} is expected to be much smaller in the Standard Model (SM) because the responsible penguin contributions are suppressed by both the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the Wilson coefficients [4,5], $A_{CP} \sim (|V_{cb}^* V_{ub}|/|V_{cs}^* V_{us}|)$ $(\alpha_s/\pi) \sim 10^{-4}$. The dramatic deviation of the data from the expectation has been investigated in the SM and in new physics models by employing different approaches.

To predict direct CP asymmetries, a reliable evaluation of the penguin contributions to two-body hadronic D meson decays is necessary. In Refs. [6,7] the tree amplitudes were determined by fitting the topology parametrization to measured branching ratios, while the penguin amplitudes were calculated in the QCD-improved factorization [8,9]. It has been noticed that the penguin amplitudes derived from the QCD-improved factorization lead to a tiny ΔA_{CP} of order 10^{-5} [7]. Allowing the penguin amplitudes to be of the same order as the tree ones discretionally, ΔA_{CP} reaches $-0.13\% \sim O(10^{-3})$ [7]. In another work [10] also based on the topology parametrization, the penguin contribution via an internal b quark was identified as the major source of CP violation, since it cannot be related to the tree amplitudes. This penguin contribution, including its strong phase, was constrained by the LHCb data and then adopted to predict direct CP asymmetries of other decay modes. Therefore, it is difficult to tell whether the large $\Delta A_{CP} \sim O(10^{-2})$ arise from new physics [11–18], if one follows the approaches in the literature.

To estimate the penguin contribution precisely, we have proposed a theoretical framework for two-body hadronic D meson decays, named as the factorization-assisted topological-amplitude (FAT) approach [19], which combines the conventional naive factorization hypothesis and topological-amplitude parametrization. It is based on the factorization of short-distance (long-distance) dynamics into Wilson coefficients (hadronic matrix elements of four-fermion operators, i.e., topological amplitudes). Because of the small charm quark mass just above 1 GeV, a perturbation theory for the hadronic matrix elements may not be reliable. The idea is to identify as complete as possible

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the important sources of nonperturbative dynamics in the hadronic matrix elements and parametrize them in the framework of the factorization hypothesis. Fitting our parametrization to abundant data of D meson decay branching ratios, all the nonperturbative parameters can be determined. Once the nonperturbative parameters have been determined, the replacement of the Wilson coefficients works for estimating the penguin contributions. For those penguin amplitudes, which cannot be related to tree amplitudes through the above replacement, we have shown that they are either factorizable or suppressed by the helicity conservation. If they are factorizable, such as the scalar penguin annihilation contribution, data from other processes can be used for their determination. We are then able to predict the direct CP asymmetries in D meson decays without ambiguity.

The FAT approach has been applied to the study of the $D \to PP$ decays [19], where P represents a pseudoscalar meson. It has been shown that our framework greatly improves the global fit to the measured $D \to PP$ branching ratios. In particular, we have obtained $\Delta A_{CP} = -1.00 \times 10^{-3}$, which discriminates the opposite postulations on large (small) direct CP asymmetries in singly Cabibbo-suppressed D meson decays [20] ([21]). After the publication of our work, the LHCb collaboration updated the data [22],

$$\Delta A_{CP} = [-0.34 \pm 0.15(\mathrm{stat}) \pm 0.10(\mathrm{syst})]\%, \qquad (1)$$

where the central value is lower than the previous one. Two sources of D meson production have been employed by the LHCb in the measurements of ΔA_{CP}: the $D^{*+} \to D^0\pi^+$ channel with the flavor of the neutral D meson being determined by the emitted pion and semileptonic b-hadron decays where the flavor of the neutral D meson is tagged by the accompanying charged lepton. The former, from more data collected in the fall of 2011, led to Eq. (1) with lower statistical uncertainty. The latter from almost one-third of the samples of the D^* analysis gave [23]

$$\Delta A_{CP} = [+0.49 \pm 0.30(\mathrm{stat}) \pm 0.14(\mathrm{syst})]\%. \qquad (2)$$

The sign flip of the central value indicates that the direct CP asymmetry in the $D^0 \to K^+K^-$, $\pi^+\pi^-$ decays may be small, so it could fluctuate into negative or positive values.

In this paper we shall extend the FAT approach to the $D \to PV$ decays with V denoting a vector meson. Their data of branching ratios are also abundant enough for fixing nonperturbative parameters, and their direct CP asymmetries are of great phenomenological importance and interest. Compared to Ref. [19], we further take into account the ρ-ω mixing effect, which improves the global fit to the branching ratios involving the ρ^0 and ω mesons. It will be shown that the measured branching ratios of the D_s^+ decays into $K^+\bar{K}^{*0}$, $\bar{K}^0K^{*+}$, $\pi^+\rho^0$, and $\pi^+\omega$, which could not be accommodated simultaneously in the diagrammatic approach [24], are explained. This overall improvement between the predictions and the data is attributed to the SU(3) symmetry breaking effects included in our topological-amplitude parametrization. Besides, the direct CP asymmetries in the $D^0 \to K^0\bar{K}^{*0}$, $\bar{K}^0K^{*0}$, $D^+ \to \pi^+\rho^0$, and $D_s^+ \to K^+\omega$, $K^+\phi$ modes reach 10^{-3}, which can be observed at the LHCb or future Belle II. We also calculate the CP asymmetry observables of some neutral D meson decays. Our predictions presented in this work would help analyze CP asymmetries in three-body D meson decays. For example, the result for the $D^0 \to \pi^0\rho^0$ mode is relevant to the $D^0 \to \pi^+\pi^-\pi^0$ channel.

In Sec. II we construct our parametrization of the tree contributions to the $D \to PV$ branching ratios in the FAT approach. In Sec. III the penguin contributions from the operators $O_{3\text{–}6}$, from $O_{1,2}$ through the quark loops, and from the magnetic penguin O_{8g} are formulated. The direct CP asymmetries in the $D \to PV$ decays are then predicted. Section IV is the conclusion. We discuss the scalar penguin contributions in Appendix A and the ρ-ω mixing in Appendix B.

II. BRANCHING RATIOS

In the FAT approach the hadronic matrix elements of the four-fermion operators, including the emission, W-annihilation, and W-exchange amplitudes, are parametrized into the magnitudes χ's and the strong phases ϕ's. An important ingredient is the Glauber strong phase factor [25] associated with a pion in the nonfactorizable annihilation amplitudes, which might originate from the unique role of the pion as a Nambu—Goldstone boson and a quark-antiquark bound state simultaneously. The Glauber phase modifies the relative angle and the interference between the annihilation and emission amplitudes involving pions. The predicted $D^0 \to \pi^+\pi^-$ ($D^0 \to K^+K^-$) branching ratio is then reduced (enhanced), and the the long-standing puzzle related to these branching ratios [24,26] is resolved. In this work, we only consider the tree contributions to the branching ratios and neglect the penguin ones which are suppressed by Wilson coefficients and CKM matrix elements.

A. Parametrization of tree amplitudes

In this subsection we parametrize the tree contributions which dominate the $D \to PV$ branching ratios. The relevant effective weak Hamiltonian is given by

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} V_{\mathrm{CKM}}[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], \qquad (3)$$

where G_F is the Fermi coupling constant, V_{CKM} represents the product of the corresponding CKM matrix elements, and $C_{1,2}$ are the Wilson coefficients. The current-current operators are defined by

$$O_1 = (\bar{u}_\alpha q_{2\beta})_{V-A}(\bar{q}_{1\beta}c_\alpha)_{V-A},$$
$$O_2 = (\bar{u}_\alpha q_{2\alpha})_{V-A}(\bar{q}_{1\beta}c_\beta)_{V-A}, \tag{4}$$

with $q_{1,2}$ being the d or s quark, α, β being the color indices, and $(\bar{q}q')_{V-A}$ representing $\bar{q}\gamma_\mu(1-\gamma_5)q'$. The relevant eight topological diagrams are displayed in Fig. 1, where $T_{P(V)}$ represents the color-favored tree amplitude with the $D \to P(V)$ transition, $C_{P(V)}$ represents the color-suppressed tree amplitude with the $D \to P(V)$ transition, $E_{P(V)}$ represents the W-exchange amplitude with the pseudoscalar (vector) meson containing the antiquark from the weak vertex, and $A_{P(V)}$ represents the W-annihilation amplitude with the pseudoscalar (vector) meson containing the a ntiquark from the weak vertex.

For the emission type, we ignore the nonfactorizable contributions to the color-favored amplitudes because the factorizable ones dominate. The amplitudes T_P and C_P are formulated as [19]

$$T_P(C_P) = \frac{G_F}{\sqrt{2}} V_{\rm CKM} a_1(\mu)(a_2^P(\mu)) f_V m_V F_1^{DP}(m_V^2) 2(\varepsilon_V \cdot p_D), \tag{5}$$

where f_V (m_V, ε_V) is the decay constant (mass, polarization vector) of the vector meson, F_1^{DP} is the $D \to P$ transition form factor, and p_D is the D meson momentum. The amplitudes T_V and C_V are formulated as

$$T_V(C_V) = \frac{G_F}{\sqrt{2}} V_{\rm CKM} a_1(\mu)(a_2^V(\mu)) f_P m_V A_0^{DV}(m_P^2) 2(\varepsilon_V \cdot p_D), \tag{6}$$

where f_P is the decay constant of the pseudoscalar meson and A_0^{DV} is the $D \to V$ transition form factor. The

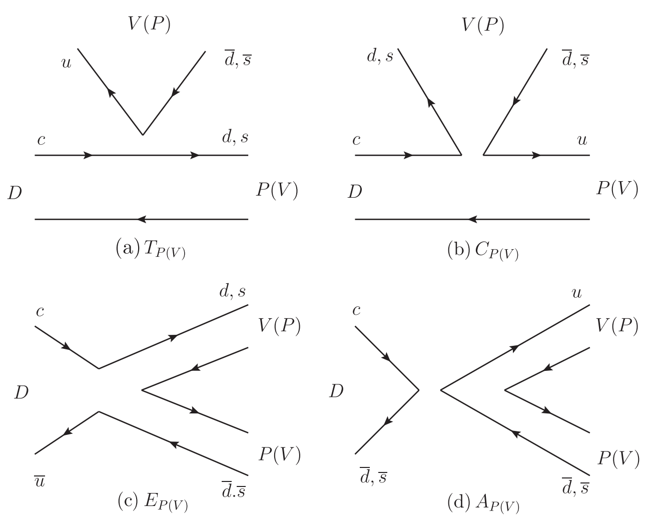

FIG. 1. Eight topological diagrams contributing to the $D \to PV$ decays with (a) the color-favored tree amplitude $T_{P(V)}$, (b) the color-suppressed tree amplitude $C_{P(V)}$, (c) the W-exchange amplitude $E_{P(V)}$, and (d) the W-annihilation amplitude $A_{P(V)}$.

associated scale-dependent Wilson coefficients a_1 and $a_2^{P,V}$ are given by

$$a_1(\mu) = C_2(\mu) + \frac{C_1(\mu)}{N_C},$$
$$a_2^{P(V)}(\mu) = C_1(\mu) + C_2(\mu)\left(\frac{1}{N_C} + \chi^C_{P(V)} e^{i\phi^C_{P(V)}}\right), \tag{7}$$

with N_C being the number of colors. The parameters $\chi^C_{P,V}$ and $\phi^C_{P,V}$ describe the magnitudes and the strong phases of the nonfactorizable contributions in the color-suppressed amplitudes, since final-state interaction (FSI) and resonance effects cannot be neglected in D meson decays. We set the scale of the Wilson coefficients to the energy release in individual decay modes as suggested by the perturbative QCD (PQCD) approach [27]: it depends on masses of final states and on the scale Λ that characterizes the soft degrees of freedom in the D meson [19],

$$\mu = \sqrt{\Lambda m_D (1 - r^2_{V(P)})}, \tag{8}$$

$r_{V(P)} = m_{V(P)}/m_D$ being the mass ratio of the vector (pseudoscalar) meson emitted from the weak vertex over the D meson. The evolution of the Wilson coefficients for c quark decays can be found in Ref. [19].

Because the factorizable contributions to the annihilation-type amplitudes are down by helicity suppression [28], only the nonfactorizable contributions are considered. The W-exchange and W-annihilation amplitudes are parametrized as

$$E_{P,V} = \frac{G_F}{\sqrt{2}} V_{\rm CKM} C_2(\mu) \chi^E_{q(s)} e^{i\phi^E_{q(s)}} f_D m_D \frac{f_P f_V}{f_\pi f_\rho}(\varepsilon_V \cdot p_D), \tag{9}$$

$$A_{P,V} = \frac{G_F}{\sqrt{2}} V_{\rm CKM} C_1(\mu) \chi^A_{q(s)} e^{i\phi^A_{q(s)}} f_D m_D \frac{f_P f_V}{f_\pi f_\rho}(\varepsilon_V \cdot p_D), \tag{10}$$

where f_D, f_π, and f_ρ are the decay constants of the D meson, π meson, and ρ meson, respectively. The parameters $\chi^{E,A}_{q,s}$ and $\phi^{E,A}_{q,s}$ characterize the strengths and the strong phases of the corresponding amplitudes, with the subscripts q and s differentiating the strongly produced light-quark (u or d) and strange-quark pair. The ratios over f_π and f_ρ in Eqs. (9) and (10) take into account the $SU(3)$ breaking effects from the decay constants. As in the emission-type amplitudes, the scale of the Wilson coefficients,

$$\mu = \sqrt{\Lambda m_D (1 - r_P^2)(1 - r_V^2)}, \tag{11}$$

also depends on the initial- and final-state masses.

As shown above, we have followed the parametrization for the $D \to PP$ decays [19] by considering the nonfactorizable amplitudes χ_q and χ_s in this work. Note that

χ_P and χ_V were adopted in Ref. [24], which describe the nonfactorizable contributions with the spectator antiquark going into the P and V mesons, respectively. However, as χ_P and χ_V appear together in some $D \to PV$ modes, such as $D^+ \to \pi^+\omega$, their difference reflects the isospin symmetry breaking, which ought to be tiny. Certainly, they do not always appear together. For example, only χ_P appears in the $D^0 \to \pi^+\rho^-$ decay. Viewing that χ_P and χ_V may violate the isospin symmetry, we prefer χ_q and χ_s, whose difference reflects the SU(3) symmetry breaking that could be significant. It turns out that the parametrization with χ_q and χ_s has a lower χ^2 in the global fit than the parametrization with χ_P and χ_V does. That is, the SU(3) symmetry breaking is more crucial than the isospin symmetry breaking in D-meson decays.

It was proposed in Ref. [29] that a kind of soft gluons, named the Glauber gluons, exist in two-body heavy meson decays, which may lead to additional strong phases in the nonfactorizable amplitudes. The multiple Fock states of a pion have been proposed to reconcile its simultaneous roles as a $q\bar{q}$ bound state and a Nambu—Goldstone boson [30]. It was then speculated that the Glauber effect becomes significant due to the huge soft cloud formed by higher Fock states of a pion [29]. According to Ref. [19], we multiply a phase factor $\exp(iS_\pi)$ to the nonfactorizable annihilation-type amplitudes, as a pion is involved in the final state, while leaving the emission-type amplitudes unchanged, in which the factorizable contributions usually dominate. In summary, our parametrization of the $D \to PV$ decays is composed of 14 global free parameters: the soft scale Λ; the magnitudes of the nonfactorizable amplitudes, $\chi^C_{P,V}$ and $\chi^{E,A}_{q,s}$; the strong phases of the nonfactorizable amplitudes, $\phi^C_{P,V}$ and $\phi^{E,A}_{q,s}$; and the Glauber phase, S_π. Compared to the $D \to PP$ analysis [19], there are only two more free parameters.

B. Numerical analysis

The partial decay width of a $D \to PV$ mode is expressed as

$$\Gamma(D \to PV) = \frac{|\vec{p}_V|}{8\pi m_D^2}\sum_{\text{pol}}|\mathcal{A}|^2, \tag{12}$$

or equivalently as

$$\Gamma(D \to PV) = \frac{|\vec{p}_V|^3}{8\pi m_V^2}|\tilde{\mathcal{A}}|^2, \tag{13}$$

which are related to each other via $\mathcal{A} = \tilde{\mathcal{A}}(\varepsilon \cdot p_D)$ and the summation over the polarization states of the vector boson, $\sum_{\text{pol}}|\varepsilon \cdot p_D|^2 = (m_D^2/m_V^2)|\vec{p}_V|^2$. Note that only the longitudinal polarization state of the vector meson contributes to the $D \to PV$ decays. We perform the global fits based on the above two formulas and find the same solutions. This is in contrast to the observation in Ref. [24], where different solutions were obtained from Eqs. (12) and (13). For the decay constants of the pseudoscalar and vector mesons and the $D \to P$ transition form factors F_1^{DP} in Eq. (5), we take the same values as in Ref. [31]. The $D \to V$ transition form factors $A_0^{DV}(q^2)$'s have been calculated with poor precision: their values at $q^2 = 0$ range from about 0.6 to 0.8 [32] and are chosen as in Table I. Our fits include all the channels with measured branching ratios except $D_s^+ \to \eta'\rho^+$, i.e., 33 experimental data of branching ratios in total. The $D_s^+ \to \eta'\rho^+$ mode is excluded for the following reason. It is the only η'-involved decay with a measured branching ratio, and the input of the $D_s^+ \to \eta'$ transition form factor is uncertain, whose variation easily changes the fit to this mode. Therefore, its data, with less satisfactory quality, does not constrain the relevant parameters effectively.

TABLE I. Values of $D \to V$ transition form factors $A_0^{DV}(0)$.

$D \to \rho$	$D \to K^*$	$D \to \omega$	$D_s \to K^*$	$D_s \to \phi$
0.76	0.73	0.70	0.76	0.78

The global fit leads to the nonperturbative parameters

$$\begin{aligned}
&\Lambda = 0.44\ \text{GeV}, \quad S_\pi = -0.96,\\
&\chi_P^C = -0.40, \quad \phi_P^C = -0.53, \quad \chi_V^C = -0.53,\\
&\phi_V^C = -0.25, \quad \chi_q^E = 0.25, \quad \phi_q^E = 1.73,\\
&\chi_q^A = 0.11, \quad \phi_q^A = -0.35, \quad \chi_s^E = 0.29,\\
&\phi_s^E = 3.11, \quad \chi_s^A = 0.10, \quad \phi_s^A = 1.60,
\end{aligned} \tag{14}$$

with the fitted $\chi^2 = 2.8$ per degree of freedom. Since the weak phases associated with the tree contributions are tiny, roughly the same branching ratios will be obtained, if the strong phases in Eq. (14) flip the sign. We select the above outcomes to keep the strong phases of the emission-type amplitudes in consistence with those in Ref. [19]. The value of Λ is in the correct order of magnitude for characterizing the soft degrees of freedom in the D meson and close to that derived from the $D \to PP$ fit [19]. The Glauber phase S_π is not very different from what was obtained in the $D \to PP$ analysis [19] and is consistent with the value extracted from the data for the direct CP asymmetries in the $B \to \pi K$ decays [29].

The branching ratios of the Cabibbo-favored, singly Cabibbo-suppressed, and doubly Cabibbo-suppressed $D \to PV$ decays corresponding to the parameters in Eq. (14) are listed in Tables II, III, and IV, respectively. Our results are also compared with the experimental data [33] and with those from other theoretical approaches, such as the fit based on the diagrammatic approach [24], the calculations including the FSI effects of nearby resonances [34], and the combination of the generalized factorization and the pole model [31]. Our results in the column Br(FAT) basically agree with the data. Note that the branching ratios $\text{Br}(D_s^+ \to \eta'\rho^+) = (12.5 \pm 2.2)\%$ given by the Particle Data Group [33] was from an old measurement [35]. It

TABLE II. Branching ratios for the Cabibbo-favored $D \to PV$ decays in units of percentage. Our results without (FAT) and with the ρ-ω mixing (FAT[mix]) are compared to the experimental data [33], the fitted results from the diagrammatic approach [24], the results including the FSI effects [34], and the calculations from the combination of the generalized factorization and the pole model [31]. The involved amplitudes of the decays are also shown, with those outside the parentheses being dominant.

Modes	Amplitudes	Br(FSI)	Br(diagrammatic)	Br(pole)	Br(FAT)	Br(FAT[mix])	Br(exp)
$D^0 \to \pi^+ K^{*-}$	T_V, (E_P)	4.69	5.91 ± 0.70	3.1 ± 1.0	6.21	6.09	$5.44^{+0.70}_{-0.53}$
$D^0 \to \pi^0 \bar{K}^{*0}$	C_P, (E_P)	3.49	2.82 ± 0.34	2.9 ± 1.0	3.42	3.25	3.44 ± 0.35
$D^0 \to \bar{K}^0 \rho^0$	C_V, (E_V)	0.88	1.54 ± 1.15	1.7 ± 0.7	1.31	1.17	$1.26^{+0.14}_{-0.16}$
$D^0 \to \bar{K}^0 \omega$	C_V, (E_V)	2.16	2.26 ± 1.38	2.5 ± 0.7	2.26	2.22	2.22 ± 0.12
$D^0 \to \bar{K}^0 \phi$	E_P	0.90	0.868 ± 0.139	0.8 ± 0.2	0.800	0.800	0.834 ± 0.074
$D^0 \to K^- \rho^+$	T_P, (E_V)	11.19	10.8 ± 2.2	8.8 ± 2.2	9.6	9.6	10.8 ± 0.7
$D^0 \to \eta \bar{K}^{*0}$	C_P, (E_P, E_V)	0.51	0.96 ± 0.32	0.7 ± 0.2	0.55	0.57	0.96 ± 0.30
$D^0 \to \eta' \bar{K}^{*0}$	C_P, (E_P, E_V)	0.005	0.012 ± 0.003	0.016 ± 0.005	0.018	0.018	< 0.11
$D^+ \to \pi^+ \bar{K}^{*0}$	T_V, C_P	0.64	1.83 ± 0.49	1.4 ± 1.3	1.70	1.70	1.51 ± 0.16
$D^+ \to \bar{K}^0 \rho^+$	T_P, C_V	11.77	9.2 ± 6.7	15.1 ± 3.8	6.4	6.0	9.6 ± 2.0
$D_s^+ \to \pi^+ \rho^0$	A_P, A_V	0.080		0.4 ± 0.4	0	0.004	0.020 ± 0.012
$D_s^+ \to \pi^+ \omega$	A_P, A_V	0.0		0	0.30	0.26	0.25 ± 0.07
$D_s^+ \to \pi^+ \phi$	T_V	2.89	4.38 ± 0.35	4.3 ± 0.6	3.4	3.4	4.5 ± 0.4
$D_s^+ \to \pi^0 \rho^+$	A_P, A_V	0.080		0.4 ± 0.4	0	0	
$D_s^+ \to K^+ \bar{K}^{*0}$	C_P, (A_V)	3.86		4.2 ± 1.7	4.08	4.07	3.95 ± 0.2
$D_s^+ \to \bar{K}^0 K^{*+}$	C_V, (A_P)	3.37		1.0 ± 0.6	2.5	3.1	5.4 ± 1.2
$D_s^+ \to \eta \rho^+$	T_P, (A_P, A_V)	9.49		8.3 ± 1.3	8.2	8.8	8.9 ± 0.8
$D_s^+ \to \eta' \rho^+$	T_P, (A_P, A_V)	2.61		3.0 ± 0.5	1.7	1.6	5.6 ± 1.1[a]

[a]data from Ref. [36]

was then questioned for exceeding the inclusive η' fraction $(11.7 \pm 1.8)\%$ [33], which includes all η' involved modes. This controversy was resolved by the recent CLEOc measurement with the branching fraction $\mathrm{Br}(D_s^+ \to \eta' \rho^+) = (5.6 \pm 1.1)\%$ [36], which is closer to our prediction.

It was noticed in Refs. [31,34] that the prediction for the $D_s^+ \to \pi^+ \rho^0$ branching ratio is much larger than the data, while the $D_s^+ \to \pi^+ \omega$ branching ratio, predicted to be zero, is sizable in experiments. The inconsistence observed in Refs. [31,34] was explained via the topological amplitudes of these two modes:

$$\mathcal{A}(D_s^+ \to \pi^+ \rho^0) = \frac{1}{\sqrt{2}}(A_P - A_V), \tag{15}$$

$$\mathcal{A}(D_s^+ \to \pi^+ \omega) = \frac{1}{\sqrt{2}}(A_P + A_V). \tag{16}$$

The factorizable W-annihilation contributions A_P^f and A_V^f obey $A_P^f = -A_V^f$, which holds in the pole-dominant model [31] because of the antisymmetric space wave function of the two P-wave final states and can also be derived in the PQCD approach [37]. Then the two contributions are constructive in the $\pi^+ \rho^0$ mode and destructive in the $\pi^+ \omega$ mode, contrary to the implication of the data. In our approach only the nonfactorizable contributions are considered due to the helicity suppression of the factorizable ones as shown in Eq. (10), such that the relation $A_P = A_V$ leads to the vanishing $D_s^+ \to \pi^+ \rho^0$ branching ratio [see the value in the column Br(FAT) of Table II]. The difference between our prediction and those in Refs. [31,34] for the $D_s^+ \to \pi^0 \rho^+$ branching ratio can be understood in the same way.

In the diagrammatic approach [24], where the global fit was performed only for the Cabibbo-favored modes with the flavor SU(3) symmetry, it is impossible to find a reasonable solution to the $D_s^+ \to \pi^+ \rho^0$, $\pi^+ \omega$, $K^+ \bar{K}^{*0}$, and $\bar{K}^0 K^{*+}$ data simultaneously. Besides, the fit in Ref. [31] indicated that the $D_s^+ \to \bar{K}^0 K^{*+}$ branching ratio is much lower than the $D_s^+ \to K^+ \bar{K}^{*0}$ one. This is also the case observed in the naive factorization, because of the form factor relation $F_1^{DK} \approx A_0^{DK^*}$, but with the decay constant $f_K < f_{K^*}$. Note that the $D_s^+ \to \pi^+ \rho^0$ and $\pi^+ \omega$ decays involve the W-annihilation amplitudes with strongly produced light-quark pairs, while the $D_s^+ \to K^+ \bar{K}^{*0}$ and $\bar{K}^0 K^{*+}$ decays involve strongly produced strange-quark pairs. Since we have included the significant SU(3) breaking effects from the nonfactorizable contributions in Eq. (10), better agreement between the predictions and the data for the above modes has been attained in our global fit.

It is seen that some ω-involved branching ratios have been overestimated compared to the corresponding ρ^0-involved ones. For example, the predicted $D^0 \to \pi^0 \omega$ branching ratio in the column Br(FAT) of Table III exceeds the upper bound from the experimental value, $\mathrm{Br}(D^0 \to \pi^0 \omega) < 0.26 \times 10^{-3}$, while the $D^0 \to \pi^0 \rho^0$ one is slightly lower. The predicted $D^+ \to \pi^+ \rho^0$ and $D_s^+ \to K^+ \rho^0$ branching ratios are also lower than the data, while the $D^+ \to \pi^+ \omega$ and $D_s^+ \to K^+ \omega$ ones may be overestimated.

TABLE III. Same as Table II for the singly Cabibbo-suppressed $D \to PV$ decays in units of 10^{-3}.

Modes	Amplitudes	Br(FSI)	Br(diagrammatic)	Br(pole)	Br(FAT)	Br(FAT[mix])	Br(exp)
$D^0 \to \pi^+\rho^-$	$T_V,\ (E_P)$	6.5	3.92 ± 0.46	3.5 ± 0.6	4.74	4.66	4.96 ± 0.24
$D^0 \to \pi^0\rho^0$	$C_P,\ C_V,\ (E_P, E_V)$	1.7	2.96 ± 0.98	1.4 ± 0.6	3.55	3.83	3.72 ± 0.22
$D^0 \to \pi^0\omega$	$C_P,\ C_V,\ (E_P, E_V)$	0.08	0.10 ± 0.18	0.08 ± 0.02	0.85	0.18	< 0.26
$D^0 \to \pi^0\phi$	C_P	1.1	1.22 ± 0.08	1.0 ± 0.3	1.11	1.11	1.31 ± 0.10
$D^0 \to \pi^-\rho^+$	$T_P,\ (E_V)$	8.2	8.34 ± 1.69	10.2 ± 1.5	10.2	10.0	9.8 ± 0.4
$D^0 \to K^+K^{*-}$	$T_V,\ (E_P)$	2.8	1.99 ± 0.24	1.6 ± 0.3	1.72	1.73	1.56 ± 0.12
$D^0 \to K^0\bar{K}^{*0}$	$E_P,\ E_V$	0.99	0.29 ± 0.22	0.16 ± 0.05	1.1	1.1	< 1
$D^0 \to \bar{K}^0K^{*0}$	$E_P,\ E_V$	0.99	0.29 ± 0.22	0.16 ± 0.05	1.1	1.1	< 0.56
$D^0 \to K^-K^{*+}$	$T_P,\ (E_V)$	4.5	4.25 ± 0.86	4.7 ± 0.8	4.37	4.37	4.38 ± 0.21
$D^0 \to \eta\rho^0$	$C_P,\ C_V,\ (E_P, E_V)$	0.24	1.11 ± 0.86	0.05 ± 0.01	0.54	0.45	
$D^0 \to \eta\omega$	$C_P,\ C_V,\ (E_P, E_V)$	1.9	3.08 ± 1.42	1.2 ± 0.3	2.4	2.0	
$D^0 \to \eta\phi$	$C_P,\ (E_P, E_V)$	0.57	0.31 ± 0.10	0.23 ± 0.06	0.19	0.18	0.14 ± 0.05
$D^0 \to \eta'\rho^0$	$C_P,\ C_V,\ (E_P, E_V)$	0.10	0.14 ± 0.02	0.08 ± 0.02	0.21	0.27	
$D^0 \to \eta'\omega$	$C_P,\ C_V,\ (E_P, E_V)$	0.001	0.07 ± 0.02	0.0001 ± 0.0001	0.04	0.02	
$D^+ \to \pi^+\rho^0$	$T_V,\ C_P,\ (A_P, A_V)$	1.7		0.8 ± 0.7	0.42	0.58	0.81 ± 0.15
$D^+ \to \pi^+\omega$	$T_V,\ C_P,\ (A_P, A_V)$	0.35		0.3 ± 0.3	0.95	0.80	< 0.34
$D^+ \to \pi^+\phi$	C_P	5.9	6.21 ± 0.43	5.1 ± 1.4	5.65	5.65	$5.42^{+0.22}_{-0.24}$
$D^+ \to \pi^0\rho^+$	$T_P,\ C_V,\ (A_P, A_V)$	3.7		3.5 ± 1.6	2.7	2.5	
$D^+ \to K^+\overline{K^{*0}}$	$T_V,\ (A_V)$	2.5		4.1 ± 1.0	3.61	3.60	$3.675^{+0.14}_{-0.21}$
$D^+ \to \bar{K}^0K^{*+}$	$T_P,\ (A_P)$	1.70		12.4 ± 2.4	11	11	32 ± 14
$D^+ \to \eta\rho^+$	$T_P,\ C_V,\ (A_P, A_V)$	0.002		0.4 ± 0.4	0.7	2.2	< 15
$D^+ \to \eta'\rho^+$	$T_P,\ C_V,\ (A_P, A_V)$	1.3		0.8 ± 0.1	0.7	0.8	
$D_s^+ \to \pi^+K^{*0}$	$T_V,\ (A_V)$	3.3		1.5 ± 0.7	2.52	2.35	2.25 ± 0.39
$D_s^+ \to \pi^0K^{*+}$	$C_V,\ (A_V)$	0.29		0.1 ± 0.1	0.8	1.0	
$D_s^+ \to K^+\rho^0$	$C_P,\ (A_P)$	2.4		1.0 ± 0.6	1.9	2.5	2.7 ± 0.5
$D_s^+ \to K^+\omega$	$C_P,\ (A_P)$	0.72		1.8 ± 0.7	0.6	0.07	< 2.4
$D_s^+ \to K^+\phi$	$T_V,\ C_P,\ (A_V)$	0.15		0.3 ± 0.3	0.166	0.166	0.184 ± 0.045
$D_s^+ \to K^0\rho^+$	$T_P,\ (A_P)$	19.5		7.5 ± 2.1	9.1	9.6	
$D_s^+ \to \eta K^{*+}$	$T_P,\ C_V,\ (A_P, A_V)$	0.24		1.0 ± 0.4	0.2	0.2	
$D_s^+ \to \eta' K^{*+}$	$T_P,\ C_V,\ (A_P, A_V)$	0.24		0.6 ± 0.2	0.2	0.2	

The above observation implies that the inclusion of the ρ-ω mixing effect may improve the consistency between the predictions and the data for these decays. We notice the similar pattern in the $D \to PP$ analysis [19]: for the $D \to \pi^0 K$ decays, whose branching ratios are larger, our predictions are consistent with the data. For the $D^0 \to \pi^0\pi^0$ decay, whose branching ratio is smaller, it was underestimated. Taking into account the π-η-η' mixing, for which the mixing matrix element M_{12} is negative [38], the predicted $D^0 \to \pi^0\pi^0$ branching ratio can be increased and match the data better. We point out that M_{12} is negative in the ρ-ω mixing [38], so its effect on the ω-involved modes is opposite and in the desired tendency. It is intriguing that both the $D \to PP$ and $D \to PV$ decays exhibit the meson mixing mechanism.

Motivated by the above argument, we include the ρ-ω mixing defined by

$$|\rho^0\rangle = |\rho_I^0\rangle - \varepsilon|\omega_I\rangle,$$
$$|\omega\rangle = \varepsilon|\rho_I^0\rangle + |\omega_I\rangle, \tag{17}$$

up to $\mathcal{O}(\varepsilon^2)$ corrections, where $|\rho_I^0\rangle$ and $|\omega_I\rangle$ denote the isospin eigenstates. The decay constants of the $|\rho_I^0\rangle$ and $|\omega_I\rangle$ states are related to those of the physical states in Appendix B, through the evaluation of the $V^0 \to e^+e^-$ decay width. Choosing the mixing angle $\varepsilon = 0.12$, which is reasonable viewing the large uncertainty of this parameter [39], we obtain another set of nonperturbative parameters,

$$\Lambda = 0.44\ \mathrm{GeV}, \qquad S_\pi = -0.85,$$
$$\chi_P^C = -0.40, \qquad \phi_P^C = -0.53, \qquad \chi_V^C = -0.63,$$
$$\phi_V^C = -0.42, \qquad \chi_q^E = 0.26, \qquad \phi_q^E = 1.74,$$
$$\chi_q^A = 0.17, \qquad \phi_q^A = -0.77, \qquad \chi_s^E = 0.29,$$
$$\phi_s^E = 3.10, \qquad \chi_s^A = 0.10, \qquad \phi_s^A = 1.61, \tag{18}$$

with the fitted $\chi^2 = 2.3$ per degree of freedom. The corresponding $D \to PV$ branching ratios are listed in the column Br(FAT[mix]) of Tables II, III, and IV. After including the mixing, the predicted $D^0 \to \pi^0\omega$ branching ratio is reduced to 0.18×10^{-3} mainly due to the lower ω meson decay constant and is below the observed upper bound. The branching ratios of most other ω-involved modes are also decreased considerably. On the contrary, the branching ratios of most ρ^0-involved modes are enhanced,

TABLE IV. Same as Table II for the doubly Cabibbo-suppressed $D \to PV$ decays in units of 10^{-4}, except with the absence of Br(FSI).

Modes	Amplitudes	Br(diagrammatic)	Br(pole)	Br(FAT)	Br(FAT[mix])	Br(exp)
$D^0 \to \pi^0 K^{*0}$	C_P, (E_V)	0.54 ± 0.18	0.8 ± 0.3	1.0	0.9	
$D^0 \to \pi^- K^{*+}$	T_P, (E_V)	3.59 ± 0.72	2.7 ± 0.6	4.82	4.72	3.39 ± 1.41
$D^0 \to K^+ \rho^-$	T_V, (E_P)	1.45 ± 0.17	0.9 ± 0.3	1.4	1.5	
$D^0 \to K^0 \rho^0$	C_V, (E_P)	0.91 ± 0.51	0.5 ± 0.2	0.4	0.3	
$D^0 \to K^0 \omega$	C_V, (E_P)	0.58 ± 0.40	0.7 ± 0.2	0.6	0.6	
$D^0 \to K^0 \phi$	E_V	0.06 ± 0.05	0.20 ± 0.06	0.2	0.2	
$D^0 \to \eta K^{*0}$	C_P, (E_P, E_V)	0.33	0.08	0.2	0.2	
$D^0 \to \eta' K^{*0}$	C_P, (E_P, E_V)	0.0040 ± 0.0006	0.004 ± 0.001	0.005	0.005	
$D^+ \to \pi^+ K^{*0}$	C_P, (A_V)		2.2 ± 0.9	3.33	3.33	3.75 ± 0.60
$D^+ \to \pi^0 K^{*+}$	T_P, (A_V)		4.0 ± 0.9	4.0	3.9	
$D^+ \to K^+ \rho^0$	T_V, (A_P)		0.5 ± 0.4	1.9	2.4	2.0 ± 0.5
$D^+ \to K^+ \omega$	T_V, (A_P)		1.8 ± 0.5	0.9	0.7	
$D^+ \to K^+ \phi$	A_V		0.2 ± 0.2	0.01	0.02	
$D^+ \to K^0 \rho^+$	C_V, (A_P)		0.5 ± 0.4	2.3	3.3	
$D^+ \to \eta K^{*+}$	T_P, (A_P, A_V)		1.4 ± 0.2	1.0	1.0	
$D^+ \to \eta' K^{*+}$	T_P, (A_P, A_V)		0.020 ± 0.007	0.01	0.01	
$D_s^+ \to K^+ K^{*0}$	T_V, C_P	0.20 ± 0.05	0.2 ± 0.2	0.23	0.23	0.90 ± 0.53
$D_s^+ \to K^0 K^{*+}$	T_P, C_V	1.17 ± 0.86	2.3 ± 0.6	1.2	1.1	

since the ρ meson decay constant is increased. However, some of them are lowered, such as the branching ratios of $D^0 \to \bar{K}^0 \rho^0$ and $D^0 \to \eta \rho^0$. The mixing effect has only a minor correction to the ρ^0 meson decay constant, which is overcome by the changes of the parameters in Eq. (18). Note that the $D^+ \to \pi^+ \omega$ branching ratio around 8.0×10^{-4} is still higher than the experimental upper bound 3.4×10^{-4} even after including the ρ-ω mixing. With the very limited number of free parameters in our global fit, this outcome is acceptable.

To improve the global fit, we can include the nonfactorizable contributions to the amplitude T or the factorizable contributions to the amplitudes E and A, both of which are expected to be small and have been neglected. With four more free parameters introduced in each case, the χ^2 is reduced from 44.4 to 36.6 and 36.3, respectively. The additional contributions turn out to be tiny and change the results for the branching ratios by only about 7%. The original parameters remain almost the same, implying that the additionally introduced parameters are indeed less important. Improvement can also be achieved by tuning the inputs of the form factors and the mixing angle, but it will not be pursued in this paper.

III. DIRECT *CP* ASYMMETRIES

In this section we predict the direct CP asymmetries of the $D \to PV$ decays, which are defined by

$$A_{CP} = \frac{\Gamma(D \to PV) - \Gamma(\bar{D} \to \bar{P}\,\bar{V})}{\Gamma(D \to PV) + \Gamma(\bar{D} \to \bar{P}\,\bar{V})} \tag{19}$$

by estimating the penguin contributions in the FAT approach. The quark-loop and magnetic penguin contributions are included and absorbed into the Wilson coefficients of the penguin operators. It has been found that the strong phases from the quark loops and from the scalar penguin annihilation dominate the direct CP asymmetries [19].

A. Parametrization of penguin amplitudes

The effective weak Hamiltonian for the penguin contributions is written as

$$\Delta H_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} \left[\sum_{i=3}^{6} C_i(\mu) O_i(\mu) + C_{8g}(\mu) O_{8g}(\mu) \right], \tag{20}$$

where the QCD-penguin and chromomagnetic-penguin operators are defined by

$$\begin{aligned}
O_3 &= \Sigma_q (\bar{u}_\alpha c_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V-A}, \\
O_4 &= \Sigma_q (\bar{u}_\alpha c_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V-A}, \\
O_5 &= \Sigma_q (\bar{u}_\alpha c_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V+A}, \\
O_6 &= \Sigma_q (\bar{u}_\alpha c_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V+A}, \\
O_{8g} &= \frac{g_s}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu} (1+\gamma_5) T^a G^{a\mu\nu} c,
\end{aligned} \tag{21}$$

with m_c being the charm quark mass, T^a being a color matrix, and $G^{a\mu\nu}$ being the gluon field tensor. The eight topological penguin diagrams for the $D \to PV$ decays are displayed in Fig. 2, in which the color-favored penguin amplitude $PT_{P(V)}$, the color-suppressed penguin amplitude $PC_{P(V)}$, the gluon-annihilation penguin amplitude $PE_{P(V)}$, and the gluon-exchange penguin amplitude $PA_{P(V)}$ correspond to the tree amplitudes $T_{P(V)}$, $C_{P(V)}$, $E_{P(V)}$, and $A_{P(V)}$, respectively.

The contributions from the $(V-A)(V-A)$ operators $O_{3,4}$ can be simply obtained by substituting the associated Wilson

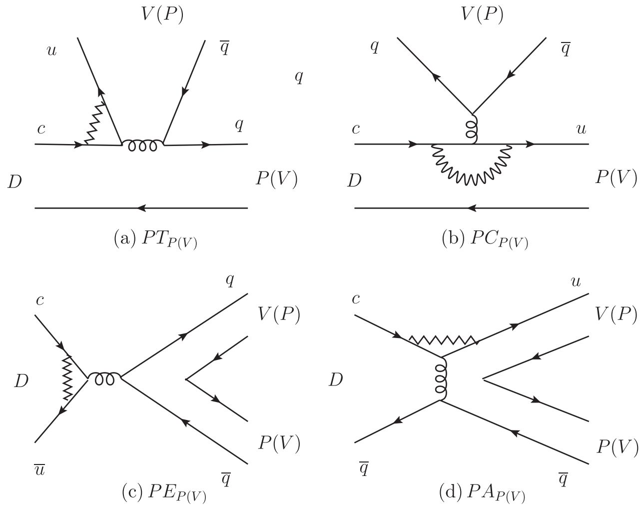

FIG. 2. Topological penguin diagrams contributing to the $D \to PV$ decays with (a) the color-favored penguin amplitude $PT_{P(V)}$, (b) the color-suppressed penguin amplitude $PC_{P(V)}$, (c) the gluon-annihilation penguin amplitude $PE_{P(V)}$, and (d) the gluon-exchange penguin amplitude $PA_{P(V)}$.

coefficients and CKM matrix elements in the tree amplitudes, while the contributions from the $(V-A)$ $(V+A)$ operators $O_{5,6}$ need to be treated separately. The nonfactorizable contributions to the color-favored penguin amplitudes are ignored as in the color-favored tree amplitudes. Since a vector meson cannot be generated from the scalar or pseudoscalar operator, PT_P does not receive contributions from O_5 or O_6. The penguin amplitude PT_V is expressed as

$$\begin{aligned} PT_V &= -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} [a_4(\mu) \langle V|(\bar{q}c)_{V-A}|D\rangle \langle P|(\bar{u}q)_{V-A}|0\rangle \\ &\quad - 2a_6(\mu) \langle V|(\bar{q}c)_{S-P}|D\rangle \langle P|(\bar{u}q)_{S+P}|0\rangle] \\ &= -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} [a_4(\mu) - r_X a_6(\mu)] \\ &\quad \times f_P m_V A_0^{DV}(m_P^2) 2(\varepsilon \cdot p_D), \end{aligned} \tag{22}$$

with the chiral factor $r_X = 2m_0^P/m_c$ and the Wilson coefficients $a_4 = C_4 + C_3/N_c$ and $a_6 = C_6 + C_5/N_c$.

A remark is in order. The penguin operators, with a sum over light quark flavors, form a U-spin singlet, but the tree operators do not. It is then expected from symmetry considerations that the penguin matrix elements may have magnitudes and strong phases different from those of the tree ones. Take the $D_s^+ \to \pi^+ K^{*0}$ channel as an example. The $\bar{s}s$ quark pair in $O_{3,4}$ can also contribute to this decay through final-state rescattering $\bar{s}s \to \bar{d}d$, that then introduces an additional source of strong phases and differentiates the $O_{3,4}$ amplitudes from the $O_{1,2}$ amplitudes. However, our formalism relies on the factorization of short-distance and long-distance dynamics, so the weak vertex is regarded as a hard vertex. The s and $\bar{s}$ quarks emitted from the penguin operator fly back to back, and the chance for them to have rescattering is small; namely, final-state rescattering is regarded as a subleading effect. We then have specific quark flavors for the external lines of the decay, to which the tree operators $O_1 = (\bar{u}_\alpha d_\beta)_{V-A}(\bar{d}_\beta c_\alpha)_{V-A}$ and $O_2 = (\bar{u}_\alpha d_\alpha)_{V-A}(\bar{d}_\beta c_\beta)_{V-A}$, and only the $\bar{d}d$ components of the penguin operators, $O_3 = (\bar{u}_\alpha c_\alpha)_{V-A}(\bar{d}_\beta d_\beta)_{V-A}$ and $O_4 = (\bar{u}_\alpha c_\beta)_{V-A}(\bar{d}_\beta d_\alpha)_{V-A}$, contribute (not considering $O_{5,6}$ here). The above $O_{1(2)}$ and $O_{3(4)}$ are identical according to the Fiertz identity, and they lead to the same hadronic matrix elements.

The factorizable contributions to $PC_{P,V}$ from $O_{5,6}$ are easily derived with the relations between the $(V+A)$ and $(V-A)$ currents used, $\langle V|(\bar{q}_1 q_2)_{V+A}|0\rangle = \langle V|(\bar{q}_1 q_2)_{V-A}|0\rangle$ and $\langle P|(\bar{q}_1 q_2)_{V+A}|0\rangle = -\langle P|(\bar{q}_1 q_2)_{V-A}|0\rangle$. The nonfactorizable contributions from O_4 and O_6 are related to the tree contributions in the following way. Since the hadronic matrix element of the $(V-A)$ $(V-A)$ operator has been parametrized as the product of $(1/N_c + \chi^C e^{i\phi^C})$, the decay constant, and the form factor as shown in Eqs. (5)–(7), the nonfactorizable contributions from O_2 and O_4 carry the same strong phase ϕ^C. It has been confirmed by PQCD analytical formulas for two-body hadronic D meson decays [40] that the nonfactorizable contributions from O_4 and O_6 to PC_V are identical without including the Wilson coefficients $C_4(\mu)$ and $C_6(\mu)$. Relative to PC_V, an additional negative sign is added to the contribution from O_6 to PC_P. We then arrive at the parametrization of the color-suppressed penguin amplitudes

$$\begin{aligned} PC_P &= -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} [a_3^P(\mu) + a_5^P(\mu)] \\ &\quad \times f_V m_V F_1^{DP}(m_V^2) 2(\varepsilon \cdot p_D), \\ PC_V &= -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} [a_3^V(\mu) - a_5^V(\mu)] \\ &\quad \times f_P m_V A_0^{DV}(m_P^2) 2(\varepsilon \cdot p_D), \end{aligned} \tag{23}$$

with the Wilson coefficients

$$\begin{aligned} a_3^{P(V)}(\mu) &= C_3(\mu) + C_4(\mu)\left(\frac{1}{N_c} + \chi_{P(V)}^C e^{i\phi_{P(V)}^C}\right), \\ a_5^{P(V)}(\mu) &= C_5(\mu) + C_6(\mu)\left(\frac{1}{N_c} - \chi_{P(V)}^C e^{i\phi_{P(V)}^C}\right). \end{aligned} \tag{24}$$

All the factorizable contributions to the annihilation-type penguin diagrams are neglected because of the helicity suppression, except those to the diagrams $PA_{P,V}$ from $O_{5,6}$. They are expressed as

$$\begin{aligned} PA_{P(V)}^f &= -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} a_6(\mu) \langle VP(PV)|(\bar{u}c)_{V-A}(\bar{q}q)_{V+A}|D\rangle \\ &= -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} a_6(\mu)(-2) \langle VP(PV)| \\ &\quad \times (\bar{u}q)_{S+P}|0\rangle \langle 0|(\bar{q}c)_{S-P}|D\rangle, \end{aligned} \tag{25}$$

after the Fierz transformation and the factorization hypothesis are applied. In the pole resonance model, Eq. (25) becomes

$$\begin{aligned}
PA_P^f &= -\frac{G_F}{\sqrt{2}}V_{cb}^* V_{ub} a_6(\mu)(-2)\langle VP|H_s|P^*\rangle \frac{1}{m_D^2 - m_{P^*}^2}\langle P^*|(\bar{u}q)_{S+P}|0\rangle\langle 0|(\bar{q}c)_{S-P}|D\rangle \\
&= 2\frac{G_F}{\sqrt{2}}V_{cb}^* V_{ub} a_6(\mu)(2g_{PPV}\, p_D\cdot\epsilon_V)\frac{1}{m_D^2 - m_{P^*}^2}(f_{P^*}m_{P^*}^0)\left(f_D\frac{m_D^2}{m_c}\right), \\
PA_V^f &= -\frac{G_F}{\sqrt{2}}V_{cb}^* V_{ub} a_6(\mu)(-2)\langle PV|H_s|P^*\rangle \frac{1}{m_D^2 - m_{P^*}^2}\langle P^*|(\bar{u}q)_{S+P}|0\rangle\langle 0|(\bar{q}c)_{S-P}|D\rangle \\
&= 2\frac{G_F}{\sqrt{2}}V_{cb}^* V_{ub} a_6(\mu)(-2g_{PPV}\, p_D\cdot\epsilon_V)\frac{1}{m_D^2 - m_{P^*}^2}(f_{P^*}m_{P^*}^0)\left(f_D\frac{m_D^2}{m_c}\right),
\end{aligned} \tag{26}$$

where P^* represents the pole resonant pseudoscalar meson and H_s is the corresponding strong Hamiltonian. The corresponding effective coupling constants g_{PPV}'s are obtained from $\rho \to \pi\pi$, $K^*(892)^0 \to \pi^+K^-$, and $\phi \to K^+K^-$, as handled in Ref. [31]. We set g_{PPV} to be $g_q = 4.2$, if none of the three strongly coupled mesons contains s quarks, to be $g_s = 4.6$ if two of them contain s quarks, and to be $g_{ss} = 4.5$ if all of them contain s quarks.

The PQCD approach [40] suggests that the nonfactorizable contributions to $PA_{P,V}$ from O_5 almost vanish, leading to the parametrization

$$\begin{aligned}
PA_P^{nf} &\propto C_3(\mu)\chi_{q(s)}^A e^{i\phi_{q(s)}^A}, \\
PA_V^{nf} &\propto C_3(\mu)\chi_{q(s)}^A e^{i\phi_{q(s)}^A}.
\end{aligned} \tag{27}$$

The sum of Eqs. (26) and (27) completes the parametrization of the amplitudes $PA_{P,V}$, which turn out to carry strong phases different from those of the tree amplitudes $A_{P,V}$ in Eq. (10). For the nonfactorizable contributions to the amplitudes $PE_{P,V}$, the PQCD approach [40] suggests that the dominant pieces from O_4 and O_6 are formulated in the same way as

$$\begin{aligned}
PE_P &\propto [C_4(\mu) - C_6(\mu)]\chi_{q(s)}^E e^{i\phi_{q(s)}^E}, \\
PE_V &\propto [C_4(\mu) - C_6(\mu)]\chi_{q(s)}^E e^{i\phi_{q(s)}^E}.
\end{aligned} \tag{28}$$

The quark-loop contributions from the tree operators can be absorbed into the Wilson coefficients as [8]

$$\begin{aligned}
C_{3,5}(\mu) &\to C_{3,5} - \frac{\alpha_s(\mu)}{8\pi N_c}\sum_{q=d,s}\frac{\lambda_q}{\lambda_b}C^q(\mu,\langle l^2\rangle), \\
C_{4,6}(\mu) &\to C_{4,6} + \frac{\alpha_s(\mu)}{8\pi}\sum_{q=d,s}\frac{\lambda_q}{\lambda_b}C^q(\mu,\langle l^2\rangle),
\end{aligned} \tag{29}$$

where $\langle l^2\rangle$ is the averaged invariant mass squared of the virtual gluon emitted from the quark loop; λ_q is defined as $V_{cq}^* V_{uq}$ for the quark $q = d$, s or b; and the function C^q is given by

$$C^q(\mu,\langle l^2\rangle) = \left[-\frac{2}{3} - 4\int_0^1 dx x(1-x)\ln\frac{m_q^2 - x(1-x)\langle l^2\rangle}{\mu^2}\right] \times C_2(\mu), \tag{30}$$

with the quark mass m_q. We set the value of $\langle l^2\rangle$ to be $(P_P/2 + P_V/2)^2 = m_D^2/4$ by assuming that each spectator of a light meson is likely to carry half of the meson momentum. We have checked that our predictions for direct *CP* asymmetries stayed stable as $\langle l^2\rangle$ ranges from $m_D^2/25$ to m_D^2. The chromomagnetic-penguin contribution can be further absorbed into the Wilson coefficients, leading to [8]

$$\begin{aligned}
C_{3,5}(\mu) &\to C_{3,5} - \frac{\alpha_s(\mu)}{8\pi N_c}\sum_{q=d,s}\frac{\lambda_q}{\lambda_b}C^q(\mu,\langle l^2\rangle) \\
&\quad + \frac{1}{N_c}\frac{\alpha_s(\mu)}{4\pi}\frac{m_c^2}{\langle l^2\rangle}[C_{8g}(\mu) + C_5(\mu)], \\
C_{4,6}(\mu) &\to C_{4,6} + \frac{\alpha_s(\mu)}{8\pi}\sum_{q=d,s}\frac{\lambda_q}{\lambda_b}C^q(\mu,\langle l^2\rangle) \\
&\quad - \frac{\alpha_s(\mu)}{4\pi}\frac{m_c^2}{\langle l^2\rangle}[C_{8g}(\mu) + C_5(\mu)].
\end{aligned} \tag{31}$$

B. Penguin-induced *CP* violation

We list the predicted direct *CP* asymmetries in the $D \to PV$ decays without and with various corrections (QCD-penguins, chromomagnetic penguins, quark loops, pole resonances, and $\rho - \omega$ mixing) in Table V. For the D^0 decays, the direct *CP* asymmetries cannot be measured directly in experiments owing to the D^0-$\bar{D}^0$ mixing. However, we can obtain the time-integrated *CP* asymmetries by adding the contributions from the indirect *CP* asymmetries to the direct ones, as done in Ref. [1]. It can be found from Table V that the $D^0 \to K^0\bar{K}^{*0}$ and $D^0 \to \bar{K}^0 K^{*0}$ modes do not receive contributions from the quark loops or the chromomagnetic penguins, since these two contributions to the Wilson coefficients $C_4(\mu)$ and $C_6(\mu)$ cancel exactly with each other in the amplitudes $PE_{P,V}$. We can also find that the direct *CP* asymmetries of the ω-involved modes change considerably with the ρ-ω mixing effect. Similarly to the case of branching ratios, the mixing

TABLE V. Direct CP asymmetries for the $D \to PV$ decays in units of 10^{-3}. The results excluding and including various corrections (QCD-penguins, chromomagnetic penguins, quark loops, pole resonances, and ρ-ω mixing) one by one are listed. The relevant amplitudes of the decays are also shown, with those outside the parentheses being dominant.

Modes	Amplitudes	A_{CP} (tree)	A_{CP} (+penguin)	A_{CP} (+cm,ql)	A_{CP} (+pole)	A_{CP}(mixing)
$D^0 \to \pi^+\rho^-$	PT, PA, (PE)	0	−0.03	0.02	−0.02	−0.03
$D^0 \to \pi^0\rho^0$	PT, PC, (PE, PA)	0	−0.01	−0.02	−0.02	−0.03
$D^0 \to \pi^0\omega$	PT, PC, (PE, PA)	0	0.0002	0.04	0.04	0.02
$D^0 \to \pi^0\phi$	PC	0	−0.0002	−0.0002	−0.0002	−0.0002
$D^0 \to \pi^-\rho^+$	PT, PA, (PE)	0	0.01	−0.04	−0.01	−0.01
$D^0 \to K^+K^{*-}$	PT, PA, (PE)	0	0.05	−0.03	−0.01	−0.01
$D^0 \to K^0\bar{K}^{*0}$	PE	−0.7	−0.7	−0.7	−0.7	−0.7
$D^0 \to \bar{K}^0K^{*0}$	PE	−0.7	−0.7	−0.7	−0.7	−0.7
$D^0 \to K^-K^{*+}$	PT, PA, (PE)	0	−0.04	0.03	0	0
$D^0 \to \eta\rho^0$	PT, PC, (PE, PA)	0.8	0.8	0.8	0.8	1.0
$D^0 \to \eta\omega$	PT, PC, (PE, PA)	−0.2	−0.1	−0.2	−0.2	−0.1
$D^0 \to \eta\phi$	PC, (PE)	0	0.003	0.003	0.003	0.003
$D^0 \to \eta'\rho^0$	PT, PC, (PE, PA)	−0.5	−0.3	−0.2	−0.2	−0.1
$D^0 \to \eta'\omega$	PT, PC, (PE, PA)	1.8	1.2	1.2	1.2	2.2
$D^+ \to \pi^+\rho^0$	PT, PC, PA	0	−0.5	−0.5	0.7	0.5
$D^+ \to \pi^+\omega$	PT, PC, (PA)	0	0.06	0.03	0.03	−0.05
$D^+ \to \pi^+\phi$	PC	0	−0.0001	−0.0001	−0.0001	−0.0001
$D^+ \to \pi^0\rho^+$	PT, PC, PA	0	0.03	−0.2	0.2	0.2
$D^+ \to K^+\bar{K}^{*0}$	PT, PA	0.1	0.5	0.1	0.2	0.2
$D^+ \to \bar{K}^0K^{*+}$	PT, PA	0.08	0.07	0.15	0.04	0.04
$D^+ \to \eta\rho^+$	PT, PC, (PA)	−0.7	−0.7	−0.7	−0.7	−0.6
$D^+ \to \eta'\rho^+$	PT, PC, (PA)	0.2	0.1	0.3	0.3	0.5
$D_s^+ \to \pi^+K^{*0}$	PT, PA	0.2	0.2	0.2	−0.2	−0.1
$D_s^+ \to \pi^0K^{*+}$	PT, PC, PA	0.2	0.2	0.3	−0.3	−0.2
$D_s^+ \to K^+\rho^0$	PT, PC, PA	−0.01	−0.05	−0.1	0.3	0.3
$D_s^+ \to K^+\omega$	PT, PC, PA	0.03	0.09	0.2	−0.6	−2.3
$D_s^+ \to K^+\phi$	PT, PC, PA	0	0.4	0.3	−0.8	−0.8
$D_s^+ \to K^0\rho^+$	PT, PA	0.04	0.03	−0.02	0.2	0.3
$D_s^+ \to \eta K^{*+}$	PT, PC, PA	0.5	0.4	0.8	−0.3	1.1
$D_s^+ \to \eta' K^{*+}$	PT, PC, PA	−0.1	−0.1	−0.2	−0.4	−0.5

effect lowers the ω meson decay constant, which has considerable influence on both the tree and penguin amplitudes of the ω-involved modes.

The $D \to PP$ analysis has indicated that the direct CP asymmetries of the $D^0 \to \pi^+\pi^-$ and K^+K^- decays reach $\mathcal{O}(10^{-4})$ [19]. It seems that the direct CP asymmetries of the corresponding $D \to PV$ decays, such as $D^0 \to \pi^+\rho^-$, $\pi^-\rho^+$, K^+K^{*-}, and K^-K^{*+}, should be of the same order. However, tiny values for these four modes are predicted as shown in Table V. We investigate the $D^0 \to \pi^+\rho^-$ decay specifically, whose direct CP asymmetry receives contributions mainly from the penguin amplitudes PT_V and PA_V (for which the nonfactorizable contributions are negligible). According to Eqs. (22) and (26), PT_V and PA_V carry nearly the same magnitude and phase, but with an opposite sign between them (the corresponding two amplitudes in $D^0 \to \pi^+\pi^-$ have the same sign). Therefore, it is numerically coincident that they cancel each other, $PA_V + PT_V \approx 0$. The small direct CP asymmetries in these decays are then understood.

The direct CP asymmetries in several modes, including $D^0 \to K^0\bar{K}^{*0}$, $\bar{K}^0K^{*0}$, $\eta\rho^0$, $\eta'\omega$, $D^+ \to \pi^+\rho^0$, $\eta\rho^+$, and $D_s^+ \to K^+\omega$, $K^+\phi$, ηK^{*+}, reach $\mathcal{O}(10^{-3})$ as shown by Table V, which are expected to be observed at the LHCb or Belle II in the future. In particular, the detecting efficiency of the final states in the $D^+ \to \pi^+\rho^0$ and $D_s^+ \to K^+\omega, K^+\phi$ decays is high. The direct CP asymmetry in the $D^+ \to \pi^+\phi$ mode has been recently measured by the LHCb, and the datum $(-0.04 \pm 0.14 \pm 0.13)\%$ [41] is consistent with zero as predicted in the FAT approach.

The contributions from new physics to electroweak interactions can be easily absorbed into the Wilson coefficients in the FAT approach. Given a new-physics model, we can calculate how the Wilson coefficients are modified in order to match the observed direct CP asymmetries and then use the new Wilson coefficients to predict direct CP asymmetries in other modes. For example, if a new-physics model has a considerable impact only on the chromomagnetic penguin operator O_{8g}, which is allowed by the constraints from the D^0-$\bar{D}^0$ mixing [42], we extract $C_{8g} \approx 11$ from the first measurement of ΔA_{CP} by LHCb [1]. Then we predict the direct CP asymmetries in the $D \to PV$ modes and find that two of them are hopefully measured: about 1% for $D^+ \to \pi^+\rho^0$ and about −1% for $D_s^+ \to K^+\phi$.

TABLE VI. *CP* asymmetry observables of some neutral D meson decays.

Modes	C_f	S_f	$S_{\bar{f}}$	D_f	$D_{\bar{f}}$
$D^0 \to \pi^+\rho^-$	−0.4	−0.1	−0.1	0.9	0.9
$D^0 \to \pi^-\rho^+$	0.4	0.1	0.1	0.9	0.9
$D^0 \to K^+K^{*-}$	−0.4	−0.2	−0.2	0.9	0.9
$D^0 \to K^-K^{*+}$	0.4	0.2	0.2	0.9	0.9

For those decays for which the tree amplitudes do not contribute to the *CP* asymmetries, their *CP* asymmetries are proportional to the penguin amplitudes [34]. In other words, they are simply proportional to the QCD-penguin Wilson coefficients $C_{3\text{–}6}(\mu)$. In some new physics models, these coefficients are synchronously varied and will become about 1 order larger in order to accommodate the measured ΔA_{CP}. As a consequence, the direct *CP* asymmetries of most modes listed in Table V will be enhanced by 1 order of magnitude. Specifically, the direct *CP* asymmetry of the $D^+ \to \pi^+\rho^0$ decay can reach 1% level.

As shown at the end of Sec. II, the neglected contributions, such as the nonfactorizable T and the factorizable E and A, lead to small corrections to the branching ratios. The corrections from the corresponding penguin contributions, parametrized in a similar way, then modify the predicted direct *CP* asymmetries. Their effects can be used to estimate the uncertainties for predictions in our approach, which are found to be about 17%. Besides, the signs of the predicted direct *CP* asymmetries never flip. This level of precision should be acceptable, considering the tremendous difficulty to analyze D meson decays theoretically.

Finally, the *CP* asymmetry observables of some neutral D meson decays with the final states f, which follow the definitions in Ref. [43], are listed in Table VI. The ρ-ω mixing has a negligible influence on these observables. The other neutral D meson decays are not considered, since their time evolution effect is tiny.

IV. SUMMARY

In this paper we have analyzed the branching ratios and direct *CP* asymmetries of the $D \to PV$ decays in the FAT approach, which was proposed in Ref. [19]. Briefly speaking, we have improved the topology parametrization by taking into account mode-dependent QCD dynamics, for instance, the evolution of the Wilson coefficients with the energy release in individual modes, flavor SU(3) symmetry breaking effects, and strong phases from FSI and from the Glauber gluons in nonfactorizable annihilation-type amplitudes. The ρ-ω mixing effect has been included, which improves the global fit to the branching ratios involving the ρ^0 and ω mesons. The puzzle from the $D_s^+ \to \pi^+\rho^0$, $\pi^+\omega$ branching ratios observed in the previous studies has been also resolved. Combining the short-distance dynamics associated with the penguin operators and the hadronic parameters determined from the global fit to the measured branching ratios, we have predicted the direct *CP* asymmetries in the $D \to PV$ decays. The parametrization of some nonfactorizable contributions from the operator O_6 was guided by the PQCD analysis for two-body hadronic D meson decays. Fortunately, these contributions do not dominate our predictions for the direct *CP* asymmetries in most of the $D \to PV$ modes. It was found that the direct *CP* asymmetries in the $D^0 \to K^0\bar{K}^{*0}$, $\bar{K}^0K^{*0}$, $D^+ \to \pi^+\rho^0$, and $D_s^+ \to K^+\omega$, $K^+\phi$ decays reach $\mathcal{O}(10^{-3})$, which may be observed at the LHCb or Belle II. The *CP* asymmetry observables of some neutral D meson decays have also been calculated. Many of our predictions can be confronted with future data.

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APPENDIX A: STRONG MATRIX ELEMENTS

In this appendix we determine the relative sign between the hadronic matrix elements $\langle PV|H_s|P^*\rangle$ (with the pseudoscalar meson P being emitted) and $\langle VP|H_s|P^*\rangle$ (with the vector meson V being emitted) in Eq. (26). Since the strong vertex $H_s \propto iV_\mu(P_1\partial^\mu P_2 - P_2\partial^\mu P_1)$ is antisymmetric under the exchange of the mesons P_1 and P_2, we need to differentiate P_1 and P_2 to avoid a wrong sign. It is achieved by comparing an emission amplitude in the pole resonance model to that in the naive factorization method. We use the emission amplitude for the decay $D^0 \to \rho^+\pi^-$ to fix the sign of $\langle VP|H_s|P^*\rangle$ and use that of the decay $D^0 \to \pi^+\rho^-$ to fix the sign of $\langle PV|H_s|P^*\rangle$. We consider the following decay amplitudes:

$$\begin{aligned}
\langle\rho^+\pi^-|H_{\rm eff}|D^0\rangle &= \langle\rho^+|(\bar{u}d)_{V-A}|0\rangle\langle 0|(\bar{d}c)_{V-A}|D^{*+}\rangle \\
&\quad\times\frac{1}{m_\rho^2 - m_{D^*}^2}\langle D^{*+}\pi^-|H_s|D^0\rangle \\
&= f_\rho m_\rho f_{D^*} m_{D^*}\frac{1}{m_{D^*}^2 - m_\rho^2}\langle D^{*+}\pi^-|H_s|D^0\rangle, \\
\langle\pi^+\rho^-|H_{\rm eff}|D^0\rangle &= \langle\pi^+|(\bar{u}d)_{V-A}|0\rangle\langle 0|(\bar{d}c)_{V-A}|D^+\rangle \\
&\quad\times\frac{1}{m_\pi^2 - m_B^2}\langle D^+\rho^-|H_s|D^0\rangle \\
&= -f_\pi f_D m_\pi^2\frac{1}{m_D^2 - m_\pi^2}\langle D^+\rho^-|H_s|D^0\rangle.
\end{aligned} \tag{A1}$$

For the emission amplitudes to get positive values as in the naive factorization, $\langle D^{*+}\pi^-|H_s|D^0\rangle$ should be positive and

$\langle D^+\rho^-|H_s|D^0\rangle$ should be negative. Therefore, we have the strong matrix elements

$$\langle VP|H_s|P^*\rangle = 2g_{PPV}p_D\cdot\epsilon_V,$$
$$\langle PV|H_s|P^*\rangle = -2g_{PPV}p_D\cdot\epsilon_V. \tag{A2}$$

APPENDIX B: ρ-ω MIXING

In this appendix we formulate the ρ-ω mixing and its effect on the decay constants of the ρ^0 and ω mesons. As elaborated in Sec. II, this mixing plays an important role in the evaluation of the branching ratio and the direct CP asymmetry of a decay mode involving ρ^0 or ω. The isospin eigenstates introduced in Eq. (17) are written as

$$|\rho_I^0\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle - |\bar{d}d\rangle),$$
$$|\omega_I\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle). \tag{B1}$$

The decay constants f_ρ^0 and f_ω^0 of the isospin eigenstates are defined via

$$\langle 0|\frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)|\rho^0\rangle = f_\rho^0 m_\rho \varepsilon_\mu^\rho,$$
$$\langle 0|\frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)|\omega\rangle = f_\omega^0 m_\omega \varepsilon_\mu^\omega, \tag{B2}$$

where $m_{\rho,\omega}$ and $\varepsilon^{\rho,\omega}$ are the the physical masses and polarization vectors, respectively.

We can obtain the decay constant of a light neutral vector meson V^0 through the $V^0\to e^+e^-$ decay width, which occurs through the electromagnetic current

$$j_\mu^{\rm em} = Q_u\bar{u}\gamma_\mu u + Q_d\bar{d}\gamma_\mu d$$
$$= \frac{1}{3\sqrt{2}}j_\mu^{I=0} + \frac{1}{\sqrt{2}}j_\mu^{I=1}, \tag{B3}$$

with the quark charges $Q_{u,d}$ and the isospin currents $j_\mu^{I=0,1} = (\bar{u}\gamma_\mu u \pm \bar{d}\gamma_\mu d)/\sqrt{2}$. The $V^0\to e^+e^-$ amplitude is proportional to the matrix element $\langle 0|j_\mu^{\rm em}|V^0\rangle$, for which we have, from Eq. (17),

$$\langle 0|j_\mu^{\rm em}|\rho^0\rangle = \left(\frac{1}{\sqrt{2}}f_\rho^0 m_\rho - \frac{\epsilon}{3\sqrt{2}}f_\omega^0 m_\omega\right)\varepsilon_\mu^\rho \equiv \mathcal{T}_\rho \varepsilon_\mu^\rho,$$
$$\langle 0|j_\mu^{\rm em}|\omega\rangle = \left(\frac{1}{3\sqrt{2}}f_\omega^0 m_\omega + \frac{\epsilon}{\sqrt{2}}f_\rho^0 m_\rho\right)\varepsilon_\mu^\omega \equiv \mathcal{T}_\omega \varepsilon_\mu^\omega. \tag{B4}$$

The decay width is then expressed as

$$\Gamma(V^0\to e^+e^-) = \frac{4\pi}{3}\frac{\alpha^2}{m_V^3}|\mathcal{T}_V|^2, \tag{B5}$$

with the fine structure constant α. The decay constant of the physical $\rho^0(\omega)$ meson, $f_{\rho(\omega)}$, can be read off the experimental measurements of $\Gamma_{\rho^0(\omega)}$. Therefore, the physical decay constants are related to those for the isospin eigenstates via

$$\left|\frac{1}{\sqrt{2}}f_\rho m_\rho\right|^2 = \left|\frac{1}{\sqrt{2}}f_\rho^0 m_\rho - \frac{\epsilon}{3\sqrt{2}}f_\omega^0 m_\omega\right|^2,$$
$$\left|\frac{1}{3\sqrt{2}}f_\omega m_\omega\right|^2 = \left|\frac{1}{3\sqrt{2}}f_\omega^0 m_\omega + \frac{\epsilon}{\sqrt{2}}f_\rho^0 m_\rho\right|^2. \tag{B6}$$

Then, we obtain the decay constants for the isospin eigenstates,

$$f_\rho^0 = f_\rho + \frac{\epsilon}{3}\frac{m_\omega}{m_\rho}f_\omega,$$
$$f_\omega^0 = f_\omega - 3\epsilon\frac{m_\rho}{m_\omega}f_\rho, \tag{B7}$$

where the higher-order terms in ϵ have been neglected.

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