Baryon octet and decuplet phenomenology in a three-flavor extended linear sigma model

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We present an effective model, which is an extension of the usual linear sigma model, that contains a low energy multiplet for every hadronic particle type. These multiplets are a scalar nonet, a pseudoscalar nonet, a vector nonet, an axial-vector nonet, a baryon octet and a baryon decuplet. Tree-level baryon masses and possible two-body decuplet decays are calculated. The baryon masses are generated through spontaneous symmetry breaking. The calculated quantities are used to determine the model parameters through a multiparametric minimalization process, which compares the calculated physical quantities with their experimental values. We found that the calculated quantities are in good agreement with the experimental data.

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I. INTRODUCTION

The phase diagram of QCD, the theory of strong interaction, is a heavily studied field both theoretically (see e.g. [1–10] and references therein) and experimentally (see e.g. [11–15] and references therein). Our aim is to develop a model for that problem, which also reproduces the vacuum phenomenology.

QCD can be solved perturbatively only at very high energies. Although it is possible to solve QCD nonperturbatively on the lattice, that is computationally demanding and not very well suited for instance for scattering problems, or for high densities. We are therefore left with effective theories. The underlying principle in the construction of such theories is that they share the same global symmetries as QCD [16].

For massless quarks (which is a very good approximation for u and d and less good for s quarks) the global symmetry of QCD is $U(N_f)_R \times U(N_f)_L \equiv U(N_f)_V \times U(N_f)_A$, the so-called chiral symmetry (R stands for right-handed, L for left-handed quark flavors, and N_f denotes the number of massless quark flavors). The $U(1)_A$ part of the symmetry is broken by topological charges [17], while in the vacuum $SU(3)_A$ is spontaneously broken [18] due to the existence of quark-antiquark condensates.

There are different ways in which the chiral symmetry of QCD can be realized. In the QCD Lagrangian, the symmetry is linearly realized, while in the vacuum and at low energies, the symmetry is nonlinearly realized. Linear realizations of chiral symmetry have the property that states are doubled. In nonlinear realizations [19], there can be states without associated chiral partners. Around the phase transition the chiral partners are degenerate, so none of them is negligible. Therefore, in order to investigate the mechanism of chiral symmetry restoration, which is one of our final aims, effective theories with linearly realized chiral symmetry [20] are most appropriate.

The last version of our model [21] contained the scalar, pseudoscalar, vector, and axial-vector nonets of mesons. That model described the vacuum phenomenology of mesons very well. In this paper we include the nucleon-octet and the Delta decuplet to extend the vacuum phenomenology for baryons as well. Other investigations concerning baryon phenomenology can be found for instance in [22–31].

Our paper is organized as follows. In Sec. II we describe our model with some of the details taken from [21] relegated to Appendix A. Section III is dedicated to calculation of tree-level baryon masses and decay widths, while Sec. IV contains our results of the fitting procedure. We conclude in Sec. V.

II. THE MODEL

The model construction is based on the idea of inclusion of the lowest lying multiplets for every hadronic particle type, where we assume that mesons are $q\bar{q}$ and baryons are qqq states. This means that for mesons we included a scalar, a pseudoscalar, a vector and an axial-vector nonet, while for baryons an octet and a decuplet. Accordingly, our Lagrangian consists of a mesonic and a baryonic part, the latter also includes the baryon-meson interaction terms, $\mathcal{L} = \mathcal{L}_{\mathrm{meson}} + \mathcal{L}_{\mathrm{baryon}}$, from which we already constructed and analyzed the meson part in [21], and it is presented briefly in Appendix A.

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The Lagrangian of the baryon sector is constructed as follows. In addition to the kinetic terms of the octet and decuplet baryons we included such interaction terms with the lowest possible dimension, that either describe decuplet decays into one octet baryon and one (pseudo)scalar—which are the physically relevant two-body decays of the decuplet—or such baryon-(pseudo)scalar interactions that generate octet/decuplet mass terms via spontaneous symmetry breaking. The lowest possible dimension for the decuplet decay terms is four containing one vector-spinor, one spinor, and one (pseudo)scalar field, and together with the kinetic terms are taken from the leading order expansion of the nonlinear sigma model [19] (for more details see Appendix B). In case of the baryon-(pseudo)scalar interaction terms (that can produce octet/decuplet mass terms) the lowest possible dimension is five, containing two spinor and two (pseudo)scalar fields. Correspondingly, we included every possible $SU(3)_V$ invariant [32], which can be constructed with the given number of fields (see e.g. Appendix C of [33]).

A. Lagrangian

The baryonic part of the Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\text{baryon}} &= \mathrm{Tr}[\bar{B}(i\not{D} - M_{(8)})B] - \mathrm{Tr}\{\bar{\Delta}_\mu \cdot [(i\not{D} - M_{(10)})g^{\mu\nu} - i(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + \gamma^\mu(i\not{D} + M_{(10)})\gamma^\nu]\Delta_\nu\} \\ &+ C\mathrm{Tr}\left[\bar{\Delta}^\mu \cdot \left(-\frac{1}{f}(\partial_\mu - ieA_\mu^e[T_3,\Phi]) - \frac{1}{f}[\Phi, V_\mu] + A_\mu\right)B\right] + \text{H.c.} - \xi_1 \mathrm{Tr}(\bar{B}B)\mathrm{Tr}(\Phi^\dagger\Phi) \\ &- \xi_2\mathrm{Tr}(\bar{B}\{\{\Phi,\Phi^\dagger\},B\}) - \xi_3\mathrm{Tr}(\bar{B}[\{\Phi,\Phi^\dagger\},B]) - \xi_4(\mathrm{Tr}(\bar{B}\Phi)\mathrm{Tr}(\Phi^\dagger B) + \mathrm{Tr}(\bar{B}\Phi^\dagger)\mathrm{Tr}(\Phi B)) \\ &- \xi_5\mathrm{Tr}(\bar{B}\{[\Phi,\Phi^\dagger],B\}) - \xi_6\mathrm{Tr}(\bar{B}[[\Phi,\Phi^\dagger],B]) - \xi_7(\mathrm{Tr}(\bar{B}\Phi)\mathrm{Tr}(\Phi^\dagger B) - \mathrm{Tr}(\bar{B}\Phi^\dagger)\mathrm{Tr}(\Phi B)) \\ &- \xi_8(\mathrm{Tr}(\bar{B}\Phi B\Phi^\dagger) - \mathrm{Tr}(\bar{B}\Phi^\dagger B\Phi)) + \chi_1\mathrm{Tr}(\bar{\Delta}\cdot\Delta)\mathrm{Tr}(\Phi^\dagger\Phi) + \chi_2\mathrm{Tr}((\bar{\Delta}\cdot\Delta)\{\Phi,\Phi^\dagger\}) \\ &+ \chi_3\mathrm{Tr}((\bar{\Delta}\cdot\Phi)(\Phi^\dagger\cdot\Delta) + (\bar{\Delta}\cdot\Phi^\dagger)(\Phi\cdot\Delta)) + \chi_4\mathrm{Tr}((\bar{\Delta}\cdot\Delta)[\Phi,\Phi^\dagger]), \qquad (1)\end{aligned}$$

where $B, \Delta_\mu, \Phi, V_\mu, A_\mu, A_\mu^e$ stands for the baryon octet, the baryon decuplet, the scalar-pseudoscalar meson octet, the vector-meson octet, the axial-vector meson octet and the electromagnetic field, respectively. $M_{(8)}$, $M_{(10)}$ are the bare baryon octet and decuplet masses, f is related to the pion decay constant, while T_a denotes the $SU(3)$ generators, [,] and $\{,\}$ denote commutator and anticommutator, respectively. Moreover, the covariant derivatives are defined as

$$\begin{aligned}D_\mu B &= \partial_\mu B + i[B, V_\mu] + \frac{1}{f}\{[A_\mu,\Phi],B\}, \\ D_\mu \Delta_\nu^{ijk} &= \partial\Delta_\nu^{ijk} + \left(\frac{1}{f}[A_\mu,\Phi]^i_l - iV_{\mu l}{}^i\right)\Delta_\nu^{ljk} \\ &+ \left(\frac{1}{f}[A_\mu,\Phi]^j_l - iV_{\mu l}{}^j\right)\Delta_\nu^{ilk} \\ &+ \left(\frac{1}{f}[A_\mu,\Phi]^k_l - iV_{\mu l}{}^k\right)\Delta_\nu^{ijl},\end{aligned}$$

and we used the following dot notation:

$$\begin{aligned}(\bar{\Delta}\cdot\Delta)^m_k &\equiv \bar{\Delta}_{ijk}\Delta^{ijm}, \\ (\bar{\Delta}\cdot\Phi)^m_k &\equiv \bar{\Delta}_{ijk}\Phi^i_l\epsilon^{jlm}, \\ (\Phi\cdot\Delta)^m_k &\equiv \Delta^{ijm}\Phi^l_i\epsilon_{jlm}. \qquad (2)\end{aligned}$$

The explicit forms of the baryon multiplets are as follows:

$$\begin{aligned}B &= \sqrt{2}\sum_{i=1}^{8} B_a T_a \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{2}}\Lambda^0 \end{pmatrix}, \qquad (3)\end{aligned}$$

$$\begin{aligned}\Delta_\mu^{111} &= \Delta_\mu^{++}, \qquad \Delta_\mu^{112} = \frac{1}{\sqrt{3}}\Delta_\mu^+, \\ \Delta_\mu^{122} &= \frac{1}{\sqrt{3}}\Delta_\mu^0, \qquad \Delta_\mu^{222} = \Delta_\mu^-, \qquad (4)\end{aligned}$$

$$\begin{aligned}\Delta_\mu^{113} &= \frac{1}{\sqrt{3}}\Sigma_\mu^{\star +}, \\ \Delta_\mu^{123} &= \frac{1}{\sqrt{6}}\Sigma_\mu^{\star 0}, \\ \Delta_\mu^{223} &= \frac{1}{\sqrt{3}}\Sigma_\mu^{\star -}, \qquad (5)\end{aligned}$$

$$\begin{aligned}\Delta_\mu^{133} &= \frac{1}{\sqrt{3}}\Xi_\mu^{\star 0}, \\ \Delta_\mu^{233} &= \frac{1}{\sqrt{3}}\Xi_\mu^{\star -}, \qquad (6)\end{aligned}$$

$$\Delta_\mu^{333} = \Omega_\mu^-, \qquad (7)$$

while the explicit form of the scalar-pseudoscalar octet is

$$\begin{aligned}\Phi &\equiv \Phi_S + \Phi_P \\ &= \sum_{i=0}^{8}(S_i + iP_i)T_i \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_N+a_0^0)+i(\eta_N+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N-a_0^0)+i(\eta_N-\pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix},\end{aligned} \tag{8}$$

and the remaining two multiplets can be found in Appendix A.

An important point here is that in the physical scalar sector of low energy QCD beside the scalar $q\bar{q}$ octet included in our model there are other states like glueballs and tetraquarks having similar or even lower mass than the $q\bar{q}$ states, which in principle can mix with their corresponding $q\bar{q}$ partner. However the scalar tetraquarks, like $f_0(500)$, have a much smaller mass than the diquark state with the same quantum number, thus we expect that their mixings are small. The glueball f_0—which should have mass around 1.5 GeV—probably has a considerable mixing with the f_0 states considered here (see the discussion in [21]), which should be investigated, but the properties of the scalar sector are not included in the fitting procedure and are beyond the scope of this paper. On the other hand, the scalars have no direct influence on the properties of the baryons considered here, because none of these baryons has large partial decay widths into scalars and baryons [34], which means either their couplings are weak or the considered scalar mass is too large. In both cases the contribution of the scalars to the self-energies of the baryons is small. Thus at first glance a more precise description of the scalar sector is not essential in this discussion.

It is worth noting that the pseudoscalar (P_a), axial-vector (A_a^μ) and baryon octet (B_a) fields are not physically observable in their current form, since for example P_1 is not observable, only the combination $(P_1 - iP_2)/\sqrt{2}$, which is π^+. Thus for later calculation it is worth transforming the above fields into physically observable forms,[1] as already shown in their matrix form. This can be done with the following 8×8 (in case of the baryon octet) and 9×9 (in case of the meson nonets) transformations as

$$\begin{aligned} Q^{(8)} &= \mathrm{diag}\left(\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}, 1, \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}, 1\right), \\ Q^{(9)} &= \mathrm{diag}\left(1, \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}, 1, \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}, 1\right), \end{aligned} \tag{9}$$

with which the fields can be written as

$$\begin{aligned} P_A &= Q_{Aa}^{(9)}P_a = (P_0, \pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \bar{K}^0, P_8), \\ A_A^\mu &= Q_{Aa}^{(9)}A_a^\mu = (A_0, a_1^+, a_1^-, a_1^0, K_1^+, K_1^-, K_1^0, \bar{K}_1{}^0, A_8)^\mu, \\ B_A &= Q_{Aa}^{(8)}B_a = (\Sigma^+, \Sigma^-, \Sigma^0, p, -\Xi^-, n, \Xi^0, \Lambda^8). \end{aligned} \tag{10}$$

As one can see from the Lagrangian, the baryonic sector has 16—yet unknown—parameters: two bare masses $M_{(8)}$ and $M_{(10)}$; eight octet baryon-(pseudo)scalar couplings $\xi_1, \ldots, \xi_8$; four decuplet baryon-(pseudo)scalar couplings $\chi_1, \ldots, \chi_4$; one Δ-decay constant C, and the parameter f. However, as one shall see in the next chapter not all the 16 parameters are independent or some of them do not even appear in the formulas of the physical quantities considered here.

III. BARYON MASSES AND DECUPLET DECAYS

After the Lagrangian is fixed, as a usual procedure we require nonzero vacuum expectation values for certain scalar fields, namely for the nonstrange σ_N and strange σ_S scalar fields,[2] which corresponds to the isospin symmetric case (see e.g. [35]). The vacuum expectation values will be denoted by

$$\phi_N \equiv \langle\sigma_N\rangle, \qquad \phi_S \equiv \langle\sigma_S\rangle. \tag{11}$$

Then one should shift the σ_N, σ_S scalar fields by their expectation values in the Lagrangian in order to get the tree-level masses and decay widths around the true vacuum,

$$\sigma_N \to \sigma_N + \phi_N, \qquad \sigma_S \to \sigma_S + \phi_S. \tag{12}$$

It is easy to see that the terms proportional to ξ_5, ξ_6, χ_4 and ξ_7, ξ_8 do not contribute to the masses. In the case of the first three terms it is due to the fact that $[T_0, T_8] = 0$, while in the case of the second two terms it is because the scalar octet is Hermitian ($\Phi_S = \Phi_S^\dagger$). Moreover, in the expression of the baryon octet masses ξ_1 and $M_{(8)}$, while in case of the

[1]In the 0–8 sector of the (pseudo)scalars, where there is particle mixing, another orthogonal transformation is needed in order to transform them into physically observable particles.

[2]We use the so-called nonstrange-strange basis defined in Eq. (A5).

decuplet baryon masses χ_1 and $M_{(10)}$ always appear in the same combination, thus without loss of generality we can set $\xi_1 = \chi_1 = 0$. Although, in scattering processes both ξ_1 and χ_1 would be needed, these processes are not considered here.

After some straightforward calculation the terms quadratic in the fields B and Δ_μ can be determined, and consequently the three-level baryon masses are found to be

$$m_p = m_n = M_{(8)} + \frac{1}{2}\xi_2(\Phi_N^2 + 2\Phi_S^2) + \frac{1}{2}\xi_3(\Phi_N^2 - 2\Phi_S^2), \tag{13}$$

$$m_\Xi = M_{(8)} + \frac{1}{2}\xi_2(\Phi_N^2 + 2\Phi_S^2) - \frac{1}{2}\xi_3(\Phi_N^2 - 2\Phi_S^2), \tag{14}$$

$$m_\Sigma = M_{(8)} + \xi_2\Phi_N^2, \tag{15}$$

$$m_\Lambda = M_{(8)} + \frac{1}{3}\xi_2(\Phi_N^2 + 4\Phi_S^2) + \frac{1}{3}\xi_4\left(\Phi_N - \sqrt{2}\Phi_S\right)^2, \tag{16}$$

$$m_\Delta = M_{(10)} + \frac{1}{2}\chi_2\Phi_N^2, \tag{17}$$

$$m_{\Sigma^\star} = M_{(10)} + \frac{1}{3}\chi_2(\Phi_N^2 + \Phi_S^2) + \frac{1}{6}\chi_3\left(\Phi_N - \sqrt{2}\Phi_S\right)^2, \tag{18}$$

$$m_{\Xi^\star} = M_{(10)} + \frac{1}{6}\chi_2(\Phi_N^2 + 4\Phi_S^2) + \frac{1}{6}\chi_3\left(\Phi_N - \sqrt{2}\Phi_S\right)^2, \tag{19}$$

$$m_\Omega = M_{(10)} + \chi_2\Phi_S^2. \tag{20}$$

A. Decay widths

According to the PDG [34], one can consider four physically allowed two-body decays of the decuplet baryons. These are the following:

$$\Delta \to p\pi, \qquad \Sigma^\star \to \Lambda\pi, \qquad \Xi^\star \to \Xi\pi, \qquad \Sigma^\star \to \Sigma\pi. \tag{21}$$

After applying the field shifts Eq. (12) in the C-term of the Lagrangian Eq. (1) the corresponding interaction part is given by

$$\mathcal{L}_{\Delta\to BP} = -\frac{C}{f} G_{ijk}^{ab}\bar{\Delta}_{ijk}^{\mu}(\partial_\mu P_a)B_b + C\cdot G_{ijk}^{ab}\bar{\Delta}_{ijk}^{\mu}A_{a\mu}B_b, \tag{22}$$

where the G_{ijk}^{ab} coupling constant reads as

$$G_{ijk}^{ab} \equiv \frac{\sqrt{2}}{4}\epsilon_{ilm}(\lambda^a)_{jl}(\lambda^b)_{km}. \tag{23}$$

Looking at Eq. (22) one could ask why the second term present, since it does not contain pseudoscalars, however all of the decays in Eq. (21) does. Actually, due to a mixing between the (axial)vectors and the (pseudo)scalars in the meson sector a redefinition of certain (axial)vector fields is needed (see Appendix A for details), which will bring in the (pseudo)scalars into the second term [see Eq. (A8)].

Using the Eq. (9) transformations and the field redefinitions Eq. (A8) in Eq. (22) the resulting Lagrangian is

$$\begin{aligned}\mathcal{L}_{\Delta\to BP} = {}& \frac{G}{\sqrt{2}}\Delta_\mu^{--}(\partial^\mu\pi^+)p - \frac{G}{2}\Sigma_\mu^{\star -}(\partial^\mu\pi^+)\Lambda^0 \\ & - \frac{G}{2\sqrt{3}}\Sigma_\mu^{\star -}[(\partial^\mu\pi^+)\Sigma^0 + (\partial^\mu\pi^0)\Sigma^+] \\ & + \frac{G}{2\sqrt{3}}\Xi_\mu^{\star -}[(\partial^\mu\pi^+)\Xi^- + (\partial^\mu\pi^0)\Xi^0],\end{aligned} \tag{24}$$

with

$$G = CZ_\pi\left(g_1 w_{a_1} - \frac{i}{f}\right),$$

where Z_π and w_{a_1} are defined in Appendix A. Moreover, terms including the same decaying particle with different charges are not written out, since they would result in the same decay widths. According to Eq. (C7) the decay width can be calculated as

$$\Gamma_{\Delta\to PB} = I_{\Delta\to PB}\frac{k^3}{12 m_\Delta}(m_B + E_B)|G_{(\Delta\to PB)}|^2, \tag{25}$$

where k and E_B are given by Eqs. (C8) and (C9), while the isospin factor $I_{\Delta\to PB}$ is one for $\Delta^{++}\to p\pi^+$ and $\Sigma^{\star +}\to\Lambda\pi^+$, two for $\Sigma^{\star +}\to\Sigma^{0}_{+}\pi^{+}_{0}$, since there are two channels $\Sigma^+\pi^0$ and $\Sigma^0\pi^+$, and three for $\Xi^{\star 0}\to\Xi^{0}_{-}\pi^{0}_{+}$, where there is one charged $\Xi^-\pi^+$ and one neutral channel $\Xi^0\pi^0$. Accordingly, for the decays of Eq. (21), the decay widths are given by

$$\Gamma_{\Delta\to\pi p} = \frac{k_p^3}{24 m_\Delta}(m_p + E_p)G^2,$$

$$\Gamma_{\Sigma^\star\to\pi\Lambda} = \frac{k_\Lambda^3}{48 m_{\Sigma^\star}}(m_\Lambda + E_\Lambda)G^2,$$

$$\Gamma_{\Xi^\star\to\pi\Xi} = \frac{k_\Xi^3}{48 m_{\Xi^\star}}(m_\Xi + E_\Xi)G^2,$$

$$\Gamma_{\Sigma^\star\to\pi\Sigma} = \frac{k_\Sigma^3}{72 m_{\Sigma^\star}}(m_\Sigma + E_\Sigma)G^2, \tag{26}$$

with

$$G^2 = C^2 Z_\pi^2\left(g_1^2 w_{a_1}^2 + \frac{1}{f^2}\right).$$

IV. χ^2 FIT AND RESULTS

In the fitting procedure we used a χ^2 minimalization method to determine the parameters of the baryon Lagrangian as we did for the meson Lagrangian in [21] from where we took the parameters of the mesonic sector. Our aim was to find a parameter set with which the calculated values of the observables deviate from their experimental values only within a given error. Since isospin breaking is neglected, our calculation is at tree level and our model is an effective model of QCD, not the QCD itself, we do not expect that it reproduces all the observables perfectly. Accordingly, we artificially set the errors to 5% for the masses and to 10% for the decay width, since they have a larger uncertainty.

In the baryon Lagrangian there are eight unknown parameters, namely, $M_{(8)}$, ξ_2, ξ_3 and ξ_4 are describing the octet masses, $M_{(10)}$, χ_2 and χ_3 the decuplet masses, while $G \equiv CZ_\pi\sqrt{g_1^2 w_{a_1}^2 + 1/f^2}$ the decay widths. In order to determine these parameters we define the χ^2 as

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i}\right]^2, \tag{27}$$

where $x_1, \ldots, x_N$ are the unknown parameters, the M observables $Q_i(x_1, \ldots, x_N)$ are calculated from the model, while Q_i^{exp} are taken from the PDG [34] with the chosen error δQ_i as discussed above. For the multiparametric minimalization of $\chi^2(x_1, \ldots, x_N)$ the MINUIT [36] code was used. In this particular case we have eight parameters to fit for the 12 observables. The resulting parameters are given in Table I along with their theoretical errors, which characterize how sensitive quantities are to the change of the given variable. For instance the large error of χ_3 in Table I means that χ_3 should be changed by 4387.18 in order to change χ^2 by one. The values of the observables along with their experimental values and errors can be found in Table II. It is important to note that all the parameters appeared already in the meson sector are fixed during the fit and their values are presented in Table III. It can be seen from Table II that the octet masses can be described perfectly, which is not so surprising, since we have four parameters to fit for four quantity and all the equations are linear in the parameters. It is more interesting that the decuplet masses are given with almost the same precision as the octet masses, even if we have only three independent parameters in this sector to fit. Finally, for the decuplet decays we have only one parameter for four physical

TABLE I. Baryon parameters and their theoretical errors.

Parameter	Value
$M_{(8)}$ [GeV]	1.92 ± 0.05
ξ_2 [GeV^{-1}]	-27.01 ± 1.57
ξ_3 [GeV^{-1}]	79.35 ± 16.70
ξ_4 [GeV^{-1}]	139.33 ± 1063.42
$M_{(10)}$ [GeV]	-1.27 ± 0.03
χ_2 [GeV^{-1}]	184.42 ± 2.13
χ_3 [GeV^{-1}]	213.00 ± 4387.18
G [GeV^{-1}]	9.88 ± 2.16

TABLE II. Calculated and experimental values of the observables along with their theoretical and experimental errors.

Observable	Fit [MeV]	Experiment [MeV]
m_p	939.0 ± 59.6	939.0 ± 47.0
m_Λ	1116.0 ± 67.0	1116.0 ± 55.8
m_Σ	1193.0 ± 69.3	1193.0 ± 59.7
m_Ξ	1318.0 ± 75.3	1318.0 ± 65.9
m_Δ	1231.9 ± 58.5	1232.0 ± 61.6
$m_{\Sigma^\star}$	1385.5 ± 50.6	1385.0 ± 69.3
$m_{\Xi^\star}$	1532.3 ± 51.1	1533.0 ± 76.7
m_Ω	1672.3 ± 78.3	1672.0 ± 83.6
$\Gamma_{\Delta\to p\pi}$	72.4 ± 3.5	110.0 ± 11.0
$\Gamma_{\Sigma^\star\to\Lambda\pi}$	29.1 ± 1.4	32.0 ± 3.2
$\Gamma_{\Sigma^\star\to\Sigma\pi}$	3.9 ± 0.2	4.3 ± 0.4
$\Gamma_{\Xi^\star\to\Xi\pi}$	12.0 ± 0.6	9.5 ± 1.0

TABLE III. Meson parameters and their errors.

Parameter	Value
C_1 [GeV^2]	-0.9183 ± 0.0006
C_2 [GeV^2]	0.4135 ± 0.0147
c_1 [GeV^{-2}]	450.5420 ± 7.0339
δ_S [GeV^2]	0.1511 ± 0.0038
g_1	5.8433 ± 0.0176
g_2	3.0250 ± 0.2329
ϕ_N [GeV]	0.1646 ± 0.0001
ϕ_S [GeV]	0.1262 ± 0.0001
h_2	9.8796 ± 0.6627
h_3	4.8667 ± 0.0864
λ_2	68.2972 ± 0.0435

observables and as expected the tree-level expressions, which differ from each other only in their kinematic parts, cannot give back all the experimental values with a very good precision. The unnatural values of $M_{(8)}$ and $M_{(10)}$ do not concern us, since with appropriately chosen values of ξ_1 and χ_1 we can achieve any values for $M_{(8)}$ and $M_{(10)}$.

V. CONCLUSIONS

We have presented a possible baryon octet and decuplet extension to our previous meson model [21]. We included interaction terms, such as $\Delta - B - P$ suggested by the lowest order chiral perturbation theory, other interaction terms like the $B - B - \Phi - \Phi$ kind of terms was introduced in order to generate baryon masses. In the last case we included every possible $SU(3)_V$ invariant.

From the constructed Lagrangian we calculated the tree-level masses and physically relevant decuplet decay widths and we found that in general they are in good agreement with the experimental data taken from the PDG [34].

As a continuation other interaction terms which contain derivatives could also be introduced (see e.g. [26]), which are important if one would like to investigate scattering processes as well. Another interesting direction is to move on to finite temperature and/or densities with these fields included in our model. However, this task seems not an easy one. For instance it is not obvious how one can switch from the baryon octet and decuplet degrees of freedom, which are the appropriate degrees of freedom at low temperature and densities, to the constituent quarks, which are better candidates for degrees of freedom as one approaches the phase transition region.

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APPENDIX A: MESON LAGRANGIAN

The meson Lagrangian is basically the same, as in [21] with the exception that the dilaton field is completely neglected and it reads as

$$\begin{aligned}\mathcal{L}_{\text{meson}} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2)\\ &+ \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2+R_\mu^2)\right] + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] + c_1(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\}\\ &+ \mathrm{Tr}\{R_{\mu\nu}[R^\mu,R^\nu]\}) + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2+R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2]\\ &+ 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) + g_3[\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\mathrm{Tr}(L_\mu L^\mu L_\nu L^\nu)\\ &+ \mathrm{Tr}(R_\mu R^\mu R_\nu R^\nu)] + g_5\mathrm{Tr}(L_\mu L^\mu)\mathrm{Tr}(R_\nu R^\nu) + g_6[\mathrm{Tr}(L_\mu L^\mu)\mathrm{Tr}(L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu)\mathrm{Tr}(R_\nu R^\nu)],\end{aligned} \tag{A1}$$

where

$$\begin{aligned}D^\mu\Phi &\equiv \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA^{e\mu}[T_3,\Phi],\\ L^{\mu\nu} &\equiv \partial^\mu L^\nu - ieA^{e\mu}[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^{e\nu}[T_3, L^\mu]\},\\ R^{\mu\nu} &\equiv \partial^\mu R^\nu - ieA^{e\mu}[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^{e\nu}[T_3, R^\mu]\}.\end{aligned}$$

The quantities Φ, R^μ, and L^μ represent the scalar-pseudoscalar, the left- and right-handed vector nonets:

$$\Phi = \sum_{i=0}^{8}(S_i + iP_i)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star+} + iK^+\\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N-\pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0\\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S\end{pmatrix}, \tag{A2}$$

$$L^\mu = \sum_{i=0}^{8}(V_i^\mu + A_i^\mu)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} + \frac{f_{1N}+a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{\star +} + K_1^+\\ \rho^- + a_1^- & \frac{\omega_N-\rho^0}{\sqrt{2}} + \frac{f_{1N}-a_1^0}{\sqrt{2}} & K^{\star 0} + K_1^0\\ K^{\star -} + K_1^- & \bar{K}^{\star 0} + \bar{K}_1^0 & \omega_S + f_{1S}\end{pmatrix}^\mu, \tag{A3}$$

$$R^\mu = \sum_{i=0}^{8}(V_i^\mu - A_i^\mu)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} - \frac{f_{1N}+a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{\star +} - K_1^+\\ \rho^- - a_1^- & \frac{\omega_N-\rho^0}{\sqrt{2}} - \frac{f_{1N}-a_1^0}{\sqrt{2}} & K^{\star 0} - K_1^0\\ K^{\star -} - K_1^- & \bar{K}^{\star 0} - \bar{K}_1^0 & \omega_S - f_{1S}\end{pmatrix}^\mu, \tag{A4}$$

where the assignment to physical particles is also shown, except in the 0–8 sector, where there is particle mixing [9,21,37] and the physical fields are given by certain orthogonal transformation from the nonphysical fields. Here, $T_i (i = 0, \ldots, 8)$ denote the generators of $U(3)$, while S_i represents the scalar, P_i the pseudoscalar, V_i^μ the vector, and A_i^μ the axial-vector meson fields, and $A^{e\mu}$ is the electromagnetic field. It should be noted that here and throughout the article we use the so-called nonstrange-strange basis in the (0–8) sector, defined as

$$\begin{aligned} \varphi_N &= \frac{1}{\sqrt{3}}(\sqrt{2}\varphi_0 + \varphi_8), \\ \varphi_S &= \frac{1}{\sqrt{3}}(\varphi_0 - \sqrt{2}\varphi_8), \\ \varphi_i &\in (S_i, P_i, V_i^\mu, A_i^\mu), \end{aligned} \tag{A5}$$

which is more suitable for our calculations. Moreover, H and Δ are constant external fields defined as

$$H = H_0 T_0 + H_8 T_8 = \begin{pmatrix} \frac{h_{0N}}{2} & 0 & 0 \\ 0 & \frac{h_{0N}}{2} & 0 \\ 0 & 0 & \frac{h_{0S}}{\sqrt{2}} \end{pmatrix}, \tag{A6}$$

$$\Delta = \Delta_0 T_0 + \Delta_8 T_8 = \begin{pmatrix} \frac{\tilde{\delta}_N}{2} & 0 & 0 \\ 0 & \frac{\tilde{\delta}_N}{2} & 0 \\ 0 & 0 & \frac{\tilde{\delta}_S}{\sqrt{2}} \end{pmatrix} \equiv \begin{pmatrix} \delta_N & 0 & 0 \\ 0 & \delta_N & 0 \\ 0 & 0 & \delta_S \end{pmatrix}. \tag{A7}$$

Shifting the fields σ_N and σ_S with their nonzero expectation values ϕ_N and ϕ_S [Eq. (11)], the quadratic terms of the Lagrangian, from which the tree-level meson masses originate, can be determined. The quadratic terms contain, beside the mixing in the $N - S$ (or 0–8) sector of the scalar and pseudoscalar octet, vector-scalar and axial-vector-pseudoscalar mixing terms as well. The later can be resolved by redefinition of certain (axial)vector fields (for details see [21]). In our case, only one such field enters in the calculations, namely the a_1^μ axial-vector meson, which should be redefined as

$$a_1^{\mu\pm,0} \longrightarrow a_1^{\mu\pm,0} + Z_\pi w_{a_1} \partial^\mu \pi^{\pm,0}, \tag{A8}$$

where

$$Z_\pi = \frac{m_{a_1}}{\sqrt{m_{a_1}^2 - g_1^2 \phi_N^2}}, \tag{A9}$$

$$w_{a_1} = \frac{g_1 \phi_N}{m_{a_1}^2}, \tag{A10}$$

and the a_1^μ axial-vector mass is given by

$$m_{a_1}^2 = m_1^2 + \frac{1}{2}(2g_1^2 + h_1 + h_2 - h_3)\phi_N^2 + \frac{h_1}{2}\phi_S^2 + 2\delta_N. \tag{A11}$$

Since in all decuplet decays [Eq. (21)] a pion is formed, we also need the explicit expression of the pion mass, which is

$$m_\pi^2 = Z_\pi^2 \left[m_0^2 + \left(\lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^2 + \lambda_1 \phi_S^2 \right]. \tag{A12}$$

The parameters of the meson Lagrangian are determined through the comparison of the calculated tree-level expressions of the spectrum and decay widths [21] with their experimental value taken from [34]. Some of the parameters only appear in certain combinations in every calculated quantities, namely,

$$\begin{aligned} C_1 &= m_0^2 + \lambda_1(\phi_N^2 + \phi_S^2) \quad \text{and} \\ C_2 &= m_1^2 + \frac{h_1}{2}(\phi_N^2 + \phi_S^2) \end{aligned} \tag{A13}$$

are such combinations. Moreover without the loss of generality we can set $\delta_N = 0$, while all the other meson parameters, taken from [21], are given in Table III.

APPENDIX B: ON THE CONSTRUCTION OF THE LAGRANGIAN

The leading order chiral Lagrangian containing baryon octet, baryon decuplet and pseudoscalar octet fields is (see e.g. [33])

$$\begin{aligned} \mathcal{L}_{\text{chiral}}^{(1)} = \mathrm{Tr}[\bar{B}(iD - M_{(8)})B] &- \mathrm{Tr}\{\bar{\Delta}_\mu \cdot [(iD - M_{(10)})g^{\mu\nu} \\ &- i(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + \gamma^\mu(iD + M_{(10)})\gamma^\nu]\Delta_\nu\} \\ &+ F\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5[iU_\mu, B]) + D\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5\{iU_\mu, B\}) \\ &+ C\{\mathrm{Tr}[(\bar{\Delta}_\mu \cdot iU^\mu)B] + \mathrm{H.c.}\} \\ &+ H\mathrm{Tr}[(\bar{\Delta}^\mu \cdot \gamma_\nu\gamma_5\Delta_\mu)iU^\nu], \end{aligned} \tag{B1}$$

where

$$D_\mu B = \partial_\mu B + \Gamma_\mu B + B\Gamma_\mu^+, \tag{B2}$$

$$D_\mu \Delta_\nu^{ijk} = \partial_\mu \Delta_\nu^{ijk} + (\Gamma_\mu)_l^i \Delta_\nu^{ljk} + (\Gamma_\mu)_l^j \Delta_\nu^{ilk} + (\Gamma_\mu)_l^k \Delta_\nu^{ijl}, \tag{B3}$$

$$\text{with}\quad \Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}(u^\dagger L_\mu u + u R_\mu u^\dagger);$$
$$U_\mu = -\frac{1}{2}u(\nabla_\mu U)^\dagger u,$$
$$\text{with}\quad \nabla_\mu U = \partial_\mu U + i(UR_\mu - L_\mu U), \tag{B4}$$

and it should be noted that the convention for the left- ($L_\mu \equiv V_\mu + A_\mu$) and right-handed ($R_\mu \equiv V_\mu - A_\mu$) fields is just the opposite as in [33]. Moreover the U and u fields are defined as

$$U = e^{i2\tilde{\Phi}/f}, \qquad u = e^{i\tilde{\Phi}/f}. \tag{B5}$$

Here U is an $SU(3)$ matrix, which parametrizes the $\tilde{\Phi}$ pseudoscalar octet nonlinearly according to the Callan-Coleman-Wess-Zumino prescription [38], while the f constant with energy dimension one is related to the pion decay constant. In order to get the relevant terms from Eq. (B1) it should be expanded in $\tilde{\Phi}$. Expanding Eqs. (B2)–(B4) in $\tilde{\Phi}$ results in

$$D_\mu B = \partial_\mu B + i[B, V_\mu] + \frac{1}{f}\{[A_\mu, \tilde{\Phi}], B\} + \mathcal{O}(\tilde{\Phi}^2), \tag{B6}$$

$$\begin{aligned} D_\mu \Delta_\nu^{ijk} = \partial_\mu \Delta_\nu^{ijk} &+ \left(\frac{1}{f}[A_\mu, \tilde{\Phi}]^i_l - iV^i_{\mu l}\right)\Delta_\nu^{ljk} \\ &+ \left(\frac{1}{f}[A_\mu, \tilde{\Phi}]^j_l - iV^j_{\mu l}\right)\Delta_\nu^{ilk} \\ &+ \left(\frac{1}{f}[A_\mu, \tilde{\Phi}]^k_l - iV^k_{\mu l}\right)\Delta_\nu^{ijl} + \mathcal{O}(\tilde{\Phi}^2), \end{aligned} \tag{B7}$$

$$U_\mu = \frac{i}{f}\partial_\mu \tilde{\Phi} - \frac{1}{f}[\tilde{\Phi}, V_\mu] - iA_\mu + \mathcal{O}(\tilde{\Phi}^2), \tag{B8}$$

which should be substituted into Eq. (B1) and replace $\tilde{\Phi}$ by Φ to get the first three terms of the baryon Lagrangian Eq. (1).

More details about the chiral Lagrangian and its expansion up to different orders can be found in [39–41].

APPENDIX C: TWO-BODY, TREE-LEVEL DECAY WIDTH OF DECUPLETS

As can be found in any standard textbook (see e.g. [42]), the tree-level two-body decay width can be written as

$$\Gamma_{A\to BC} = I\frac{k}{8\pi m_A^2}|\mathcal{M}_{A\to BC}|^2, \tag{C1}$$

where A is the decaying particle, B and C are the resulting particles, $k \equiv k_C = k_B$ is the magnitude of the momentum of the resulting particles in the rest frame of A, $\mathcal{M}_{A\to BC}$ is the matrix element and I is the isospin factor, which shows how many independent decay channels we have (see later). In our case A is a vector-spinor, B is a pseudoscalar and C is a spinor field. According to Eq. (24), the structure of the interaction Lagrangian is $G_{(A\to BC)}A_\mu(\partial^\mu B)C$, from which the matrix element can be written as

$$i\mathcal{M}_{A\to BC} = G_{(A\to BC)} \cdot u^A_\mu(k_A, s) \cdot ik^\mu_B \cdot \bar{u}^C(k_C, s'). \tag{C2}$$

Taking the average for the incoming and sum for the outgoing polarizations the absolute square of the matrix element is given by

$$|\mathcal{M}_{A\to BC}|^2 = |G_{(A\to BC)}|^2 \mathrm{Tr}\Bigg\{\underbrace{\frac{1}{4}\sum_{s=-3/2}^{3/2} u^A_\mu(k_A, s)\bar{u}^A_\nu(k_A, s)}_{-(\not{k}_A+m_A)P^A_{\mu\nu}} \underbrace{\sum_{s'=-1/2}^{1/2} u^C(k_C, s')\bar{u}^C(k_C, s')}_{\not{k}_C+m_C}\Bigg\}k^\mu_B k^\nu_B, \tag{C3}$$

where the P projector is defined as [43]

$$P^A_{\mu\nu} = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3m_A^2}k^A_\mu k^A_\nu + \frac{1}{3m_A}(k^A_\mu\gamma_\nu - k^A_\nu\gamma_\mu). \tag{C4}$$

After some straightforward calculation the matrix element can be written as

$$|\mathcal{M}_{A\to BC}|^2 = \frac{2}{3}|G_{(A\to BC)}|^2 k^2 m_A(m_C + E_C), \tag{C5}$$

$$\text{with,}\qquad E_c = \frac{m_A^2 + m_C^2 - m_B^2}{2m_A}. \tag{C6}$$

Consequently, the decay width reads

$$\Gamma_{A\to BC} = I\frac{k^3}{12m_A}(m_C + E_C)|G_{(A\to BC)}|^2, \tag{C7}$$

$$\text{where } k = \sqrt{\frac{(m_A^2 - m_B^2 - m_C^2)^2 - 4m_B^2 m_C^2}{4m_A^2}}, \tag{C8}$$

$$\text{and}\quad E_c = \frac{m_A^2 + m_C^2 - m_B^2}{2m_A}. \tag{C9}$$

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