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Higher spin cosmology

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We construct cosmological solutions of higher spin gravity in $2+1$ dimensional de Sitter space. We show that a consistent thermodynamics can be obtained for their horizons by demanding appropriate holonomy conditions. This is equivalent to demanding the integrability of the Euclidean boundary conformal field theory partition function, and it reduces to Gibbons-Hawking thermodynamics in the spin-2 case. By using the prescription of Maldacena, we relate the thermodynamics of these solutions to those of higher spin black holes in AdS_3.

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I. INTRODUCTION

In $2+1$ dimensions, pure gravity has no (perturbative) dynamics because curvature is completely rigid. But despite the lack of any gravitational attraction, gravity in $2+1$ dimensions is nontrivial—black hole solutions were discovered by Banados, Teitelboim and Zanelli (BTZ) [1] as quotients of AdS_3 [2]. This fact makes $2+1$ D gravity an excellent theoretical laboratory for testing a variety of nonperturbative issues in quantum gravity, without the added complications of curvature dynamics which play a huge role in higher dimensions. However, effort in this direction did not begin in earnest until the work of Witten [3] (see also [4]). He demonstrated that $2+1$ D gravity can be recast as a Chern-Simons gauge theory, with the gauge group $SL(2,R)\times SL(2,R)$ when the cosmological constant Λ is <0, and the gauge group $SL(2,C)$ when Λ is >0.

The negative cosmological constant case drew a lot of attention, partly because that was the context in which the above-mentioned BTZ black holes were discovered, but also because of the earlier work of Brown and Henneaux [5] who showed that the asymptotic symmetry algebra of AdS_3 gravity is a Virasoro algebra. In fact, this latter result is now widely recognized as a precursor to the celebrated AdS-CFT duality [6], where a fully *quantum* theory of gravity in AdS_{d+1} is conjectured to have an equivalent description in terms of a conformal field theory supported on the boundary of AdS_{d+1}. However, there was no such happy ending in the case of a positive cosmological constant, namely dS_3. Although the counterpart to the BTZ black hole quotients were constructed in [7] (also see [8]), the fact that de Sitter is a cosmological spacetime with a spacelike boundary [9] has made the development of a consistent dS/CFT proposal much more confusing. Various interesting attempts were made in [10–12], but there seems to be a fundamental difficulty in realizing de Sitter space in *any* kind of unitary quantum set up as a stable vacuum [13–18].

On an entirely different theme, theories of interacting gauge fields with an infinite tower of higher spins ($s\geq 2$) have been studied as a toy version of a full string theory[1] by Fradkin and Vasiliev [20–25], building on the early work of Fronsdal [26]. Higher spin theories in three dimensions, as demonstrated in [27], are considerably simpler than theories in higher dimensions due to the absence of any local propagating degrees of freedom. In addition, it is possible to truncate the infinite tower of higher spin fields to spin, $s\leq N$. The complicated nonlinear interactions of the higher spin fields can be reformulated in terms of an $SL(N,R)\times SL(N,R)$ Chern-Simons gauge theory (for the AdS_3 case) or an $SL(N,C)$ Chern-Simons (for dS_3). Therefore $2+1$ dimensional higher spin theories are a generalization of Chern-Simons gravity—one gets back to the spin-2 pure gravity theory when one sets $N=2$.

The aim of this paper is to construct cosmological solutions in higher spin dS_3 gravity. We work specifically with the case where the rank of the gauge group, $N=3$. The solutions we construct are the higher spin generalizations of dS_3 quotients such as Kerr-dS_3 and quotient cosmology [7,8,28] and should be thought of as the de Sitter counterparts of the spin-3 charged AdS_3 black hole solutions of [29,30]. It has been shown recently by two of us that big-bang–type singularities contained in quotient cosmologies in the purely $SL(2)$ sector of this higher spin theory can be removed by performing a spin-3 gauge transformation [28]. But the problem of constructing spin-3 charged cosmologies was left open. In this paper, we fill this gap and discuss the thermodynamics of their cosmological horizons.

The plan of the paper is the following. In Sec. II, we recap the formulation of a spin-3 field coupled to gravity in

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[1] Since the latter has infinite dimensional gauge invariance, see the work by Sundborg [19].

$2+1$-dimensions as a Chern-Simons gauge theory with a noncompact gauge group, $SL(3,C)$. We fix all notations and conventions for the map between the second order variables (metric and spin-3 field) and the gauge connection here. We also review and discuss the variational principle for asymptotically de Sitter–like connections in the gauge theory formulation. In Sec. III, we first review pure gravity i.e. $SL(2,C)$ sector solutions, namely the Kerr de Sitter universe and the quotient cosmology. We do this both in the metric and in the gauge theory setup. Then we construct higher spin extensions of these geometries by modifying their gauge connection and adding spin-3 charges in a manner consistent with the triviality of gauge connection holonomies along contractible cycles. These solutions are shown to contain cosmological horizons and, in the case of quotient cosmology, higher spin big bang/ big crunch–like causal singularities. In the final section, Sec. IV, these holonomy conditions are shown to be necessary for the consistency of thermodynamics associated with cosmological horizons. These consistency conditions turn out to be identical to demanding integrability of a "boundary CFT partition function." Using the prescription of Maldacena [12], we relate thermodynamics of our solutions to those of higher spin AdS_3 black holes. Our formulation gives the same results as the Gibbons-Hawking results when we restrict to the spin-2 case and work in the metric language.

II. $SL(3,C)$ CHERN-SIMONS FORMULATION OF HIGHER SPIN dS_3 GRAVITY

Here we quickly review the basics of the $SL(3,C)$ CS gauge theory generalizing Witten's construction [3] as presented in detail in [28,31] (see also [32] for a discussion on higher spins in dS_3). One simply defines the higher spin (up to spin-3) theory, i.e. an interacting theory of gravity and a spin-3 field, by the action [27]

$$I_{CS}[A] = \frac{k}{4\pi d_R}\int_M \mathrm{Tr}\left(AdA + \frac{2}{3}A^3\right) - \frac{k}{4\pi d_R}\int_M \mathrm{Tr}\left(\bar{A}d\bar{A} + \frac{2}{3}\bar{A}^3\right). \quad (1)$$

The constant $d_R = -2\mathrm{Tr}(T_0T_0)$ is a characteristic of the representation size. Here the gauge field A is a complex $SL(3)$ matrix-valued one-form. In the basis of $SL(3)$ matrices, $\{T_a, T_{ab}; a, b = 0,1,2\}$ as listed in [33], we can expand the gauge field as

$$A = \left(\omega_\mu^a + \frac{i}{l}e_\mu^a\right)T_a dx^\mu + \left(\omega_\mu^{ab} + \frac{i}{l}e_\mu^{ab}\right)T_{ab}dx^\mu = \left(\omega_\mu + \frac{i}{l}e_\mu\right)dx^\mu,$$

$$\omega_\mu = \omega_\mu^a T_a + \omega_\mu^{ab}T_{ab}, \quad e_\mu = \omega_\mu^a T_a + \omega_\mu^{ab}T_{ab}.$$

Then, the more familiar metric and spin-3 fields can be extracted from the (imaginary parts of the basis coefficients) of the gauge field [33]:

$$g_{\mu\nu} = \frac{1}{2!}\mathrm{Tr}(e_\mu e_\nu), \qquad \phi_{\mu\nu\lambda} = \frac{1}{3!}\mathrm{Tr}(e_{(\mu}e_\nu e_{\lambda)}), \quad (2)$$

while the three-dimensional Newton's constant (in units of the dS radius, l) is given by the Chern-Simons level number,

$$\frac{G_3}{l} = \frac{1}{4ik}. \quad (3)$$

We work in the prevalent general relativity convention where $8G_3 = 1$. Since the gauge group $SL(3,C)$ is noncompact, the Chern-Simons level number is *not* quantized.

Now let us consider the variation of the action (1). Generically a variation has a bulk (volume) piece proportional to the equation of motion and boundary pieces supported on temporal and spatial boundaries,

$$\delta I = \int d^3x(E.O.M) + \int d^2x\pi_\mu\delta A^\mu|_{t_i}^{t_f} + \int dtdx^j\pi_\mu^j\delta A^\mu|_{x_{i,\min}}^{x_{i,\max}}. \quad (4)$$

To have a good variational principle one has to ensure that these boundary pieces vanish (on shell) by prescribing initial and final conditions and spatial boundary conditions. If the prescribed conditions do not lead to a vanishing contribution for the boundary pieces of the variation, then one has to add supplementary boundary terms to the action to cancel these. One crucial point to be noted here, in contrast to the AdS case, is that the action (1) already defines a good variational principle without any supplementary boundary terms. This is because asymptotically de Sitter spaces have *closed* spatial sections and the only boundary contributions are from future infinity ($t_f \to \infty$) and at some time coordinate in the past ($t_i = \mathrm{const}$). As the variational principle is usually defined with vanishing variations at the initial and final times,

$$\delta A|_{t_i,t_f} = 0, \quad (5)$$

these boundary pieces vanish. However, we shall *not* demand that the future data are fixed (i.e. $\delta A|_{t_f\to\infty} \neq 0$), and we look to set up a variational principle by demanding instead that the conjugate momentum vanishes,

$$\pi_\mu|_{t_f\to\infty} \to 0. \quad (6)$$

Such a variational principle will be made to appear natural in Sec. IV, where the close parallel between de Sitter and anti-de Sitter cases is brought out. This will often restrict us to a subclass of solutions which are specified by their future falloff behaviors (which close under gauge transformations),

$$\lim_{t\to\infty} A_\mu \sim t^{\alpha_\mu} \tag{7}$$

for some real *bounded* exponent α_μ. This is the analogue of non-normalizable falloffs in AdS. These falloff behaviors are fixed by conducting the asymptotic (future or past) symmetry analysis in a manner closely parallel to the AdS_3 counterpart [33,34], as was done in [31]. By demanding that the asymptotic symmetries of this larger theory still contain the Virasoro algebras already present in the $SL(2,C)$ case, it was found that the suitable falloff behaviors at future infinity for the $SL(3,C)$ gauge connections are

$$A_{\bar{w}} = 0, \qquad A_\rho = b^{-1}\partial_\rho b, \qquad A - A_{\mathrm{dS}_3} \xrightarrow{\tau\to\infty} \mathcal{O}(1). \tag{8}$$

Here b is a gauge transformation $\in SL(3,C)$. However, as we shall see in the next section, in order to construct gauge field configurations with nonvanishing higher spin charges, one has to violate the asymptotic falloffs (8), and hence one has to supplement the action (1) with boundary terms. Again this is parallel to the situation for higher spin AdS black hole solutions [29] for which the boundary counterterms were worked out in [35,36].

III. HIGHER SPIN DE SITTER COSMOLOGIES

We are interested in constructing solutions of the $SL(3,C)$ gauge theory describing spacetimes of positive cosmological constant which have nonzero spin-3 charges in addition to the spin-2 charges, i.e. energy and angular momentum. Since these are higher spin extensions of the pure gravity solutions or $SL(2)$ sector, let us first review the solutions of the $SL(2)$ sector obtained by taking quotients of pure three-dimensional de Sitter space [8].

A. Kerr-dS_3 universe

The first class of $SL(2)$ quotients of pure de Sitter space is the so-called Kerr-de Sitter universe (KdS_3). This is very similar to de Sitter space itself, in the sense that these solutions have two regions bounded by cosmological horizons and have future and past infinite regions outside the cosmological horizons. However, the topology of the past and future infinities of KdS_3 is that of a cylinder, $S^1\times R$, in contrast to the de Sitter space, for which they have topology of a sphere, S^2.

In static Schwarzschild-like coordinates, the KdS_3 metric [7,8] reads

$$ds^2 = -N^2(r)dt^2 + N^{-2}(r)dr^2 + r^2(d\phi + N_\phi dt)^2,$$
$$N^2(r) = M - \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \quad N_\phi = -\frac{J}{2r^2}. \tag{9}$$

Introducing the outer and inner radii,

$$r_\pm^2 = Ml^2\left(\sqrt{1+(J/Ml)^2} \pm 1\right)/2, \tag{10}$$

one can rewrite Eq. (9) as

$$ds^2 = -\frac{(r^2+r_-^2)(r_+^2-r^2)}{r^2l^2}dt^2 + \frac{r^2l^2}{(r^2+r_-^2)(r_+^2-r^2)}dr^2 + r^2\left(d\phi + \frac{r_+r_-}{r^2}\frac{dt}{l}\right)^2, \quad r < r_+ \tag{11}$$

and we note that this geometry has a horizon at $r = r_+$. This metric can be analytically continued across the outside, i.e. for $r > r_+$:

$$ds^2 = -\frac{r^2l^2}{(r^2+r_-^2)(r^2-r_+^2)}dr^2 + \frac{(r^2+r_-^2)(r^2-r_+^2)}{r^2l^2}dt^2 + r^2\left(d\phi + \frac{r_+r_-}{r^2}\frac{dt}{l}\right)^2. \tag{12}$$

In this region r is timelike while t is spacelike.

To make contact with the gauge theory we write down the $SL(2,C)$ connections for the two regions. Introducing $\mathcal{N}^2(r) \equiv \frac{(r^2+r_-^2)(r^2-r_+^2)}{r^2l^2} = -N^2(r)$, the gauge field expressions are

$$A^0 = N(r)\left(d\phi + i\frac{dt}{l}\right), \qquad A^1 = \frac{lN_\phi - i}{N(r)}\frac{dr}{l},$$
$$A^2 = \left(rN_\phi + i\frac{r}{l}\right)\left(d\phi + i\frac{dt}{l}\right); \quad r < r_+,$$
$$A^0 = -\frac{lN_\phi - i}{\mathcal{N}(r)}\frac{dr}{l}, \qquad A^1 = \mathcal{N}(r)\left(d\phi + i\frac{dt}{l}\right),$$
$$A^2 = \left(rN_\phi + i\frac{r}{l}\right)\left(d\phi + i\frac{dt}{l}\right); \quad r > r_+. \tag{13}$$

In the exterior region, $r > r_+^2$, one can make the transformation to Fefferman-Graham–like coordinates (ρ, w, $\bar{w}$) defined by

$$\rho = \ln\left(\frac{\sqrt{r^2-r_+^2} + \sqrt{r^2+r_-^2}}{2l}\right),$$
$$w = \phi + it/l, \qquad \bar{w} = \phi - it/l \tag{14}$$

and obtain the form of the metric,

$$ds^2 = -l^2d\rho^2 + \frac{1}{2}(Ldw^2 + \bar{L}d\bar{w}^2) + \left(l^2e^{2\rho} + \frac{L\bar{L}}{4}e^{-2\rho}\right)dwd\bar{w}, \tag{15}$$

where the zero modes L, $\bar{L}$ are defined by

$$L+\bar{L}=Ml, \qquad L-\bar{L}=iJ. \tag{16}$$

Note that ρ here is a time coordinate (see e.g. [37]).

This coordinate system is better suited than the Schwarzschild one for conducting the asymptotic symmetry analysis of dS_3 and its identification with Euclidean Virasoro algebra and its charges [8]. As we have worked out in our previous paper [28], the corresponding $SL(2,C)$ gauge field is

$$A=iT_0d\rho+\left[\left(e^{\rho}-\frac{L}{2l}e^{-\rho}\right)T_1 +i\left(e^{\rho}+\frac{L}{2l}e^{-\rho}\right)T_2\right]dw. \tag{17}$$

One can obtain the above Kerr-dS_3 connection from a primitive connection a given by

$$a=\left[\left(1-\frac{L}{2l}\right)T_1+i\left(1+\frac{L}{2l}\right)T_2\right]dw, \tag{18}$$

free of any ρ dependence, by performing a single-valued gauge transformation on a:

$$A=\mathcal{B}^{-1}a\mathcal{B}+\mathcal{B}^{-1}d\mathcal{B}, \tag{19}$$

for

$$\mathcal{B}=\exp\,(i\rho T_0)=\exp(\rho L_0) \tag{20}$$

(because $\mathcal{B}$, being a sole function of ρ, is single valued in the ϕ direction).

B. Quotient cosmology

One can also construct de Sitter quotients containing (spinning) big bang/big crunch singularities [8] (also reviewed in [38]). These quotients are locally given by the same exterior Kerr-de Sitter metric (12). But since t and r switch their roles and become spacelike and timelike, respectively, we are better off switching their roles in the metric itself,

$$ds^2=-\frac{t^2l^2}{(t^2+r_-^2)(t^2-r_+^2)}dt^2+\frac{(t^2+r_-^2)(t^2-r_+^2)}{t^2l^2}dr^2 + t^2\left(d\phi+\frac{r_+r_-}{t^2}dr\right)^2. \tag{21}$$

The quotient cosmology arises when we compactify r into a circle. With r and ϕ both being periodic, the future and past infinities of this quotient cosmology have the topology of a torus, $S^1\times S^1$, as opposed to $R\times S^1$ for the case of the Kerr-de Sitter universe. Also with a periodic r, this metric cannot be extended to $-r_+<t<r_+$, where $g_{rr}<0$ and one has closed timelike curves. Removing this region then leaves us with a big bang (big crunch)–like solution for $t>r_+$ ($t<r_+$), with the r-ϕ torus degenerating to a circle [8]. This is an example of a causal structure singularity [2], and these are the analogues of higher dimensional curvature singularities in $2+1$ dimensions. These singularities were shown to be removable via a higher spin gauge transformation when we embedded this metric into a spin-3 $SL(3)$ theory in [28].

Since the quotient cosmology is metrically identical to the exterior regions of the Kerr-de Sitter universe, the Fefferman-Graham gauge metric expression (15) and the gauge connection expressions (17), (18), (20) also carry over with the coordinate changes,

$$\rho=\ln\left(\frac{\sqrt{t^2-r_+^2}+\sqrt{t^2+r_-^2}}{2l}\right), \qquad w=\phi+ir/l, \qquad \bar{w}=\phi-ir/l. \tag{22}$$

C. The higher spin cosmological gauge fields

In the $SL(3)$ theory, the general primitive connection that satisfies asymptotic (future) de Sitter falloff conditions is

$$a'=\left[\left(1-\frac{L}{2l}\right)T_1+i\left(1+\frac{L}{2l}\right)T_2+\frac{W}{8l}W_{-2}\right]dw. \tag{23}$$

L and W can be functions of z, but we will consider the constant case in analogy with [29]. Explicit forms for the generators can be found in [28].

We can transform from the primitive connection a' to A', the fully ρ-dependent form, by applying the transformation (20),

$$A'=iT_0d\rho+\left[\left(e^{\rho}-\frac{L}{2l}e^{-\rho}\right)T_1 +i\left(e^{\rho}+\frac{L}{2l}e^{-\rho}\right)T_2+\frac{W}{8l}e^{-2\rho}W_{-2}\right]dw, \tag{24}$$

which we call the Fefferman-Graham gauge because it manifests the proper $\rho\to\infty$ falloff behaviors, Eq. (8), as derived in [31,33].

As the trace of W_{-2} with any $SL(3)$ generator is zero, we find that the metric obtained from $A',\bar{A}'$ is the same as (15). But the spin-3 field now attains a nonzero value. These nonvanishing components of spin-3 fields are given by

$$\begin{aligned}\varphi_{www}&=-\frac{i}{8}l^2W,\\ \varphi_{ww\bar{w}}&=-\frac{i}{24}l\bar{L}We^{-2\rho}+\frac{i}{24}l^2\bar{W},\\ \varphi_{w\bar{w}\bar{w}}&=-\frac{i}{96}\bar{L}^2We^{-4\rho}+\frac{i}{24}l\bar{L}\,\bar{W}\,e^{-2\rho},\\ \varphi_{\bar{w}\bar{w}\bar{w}}&=\frac{i}{32}\bar{L}^2\bar{W}e^{-4\rho}.\end{aligned} \tag{25}$$

In order to construct metrics (cosmologies) with *non-vanishing* spin-3 charges (which will necessarily violate the asymptotically dS falloff), we propose the following ansatz for the primitive connection corresponding to a general spin-3 cosmology,

$$a' = \left[\left(1-\frac{L}{2l}\right)T_1 + i\left(1+\frac{L}{2l}\right)T_2 + \frac{W}{8l}W_{-2}\right]dw + \mu[W_2 + w_0 W_0 + w_{-2}W_{-2} + t(T_1 - iT_2)]d\bar{w}, \tag{26}$$

where μ, w_0, w_{-2} and t are constants. The motivation for this comes from the fact that under a suitable set of analytical continuations of the charges and sign of the cosmological constant (which will be elaborated in the following sections), de Sitter higher spin cosmologies turn into the Euclidean sections of AdS higher spin black hole solutions of [29] [much like in the case of pure gravity or the $SL(2)$ sector, Kerr-dS_3 solutions continue on to Euclidean BTZ black holes].

Now, the connection (26) is an off-shell object, and it contains too many independent parameters. Restricting on shell, we find that the connection has to be of the form

$$a' = \left[\left(1-\frac{L}{2l}\right)T_1 + i\left(1+\frac{L}{2l}\right)T_2 + \frac{W}{8l}W_{-2}\right]dw + \mu\left[W_2 - \frac{L}{2l}W_0 + \frac{L^2}{16l^2}W_{-2} + \frac{W}{l}(T_1 - iT_2)\right]d\bar{w}. \tag{27}$$

Now, although the connection is on shell, it is still arbitrary in the sense that one does *not* know whether such solutions make a regular or singular contribution to the Hartle-Hawking wave function (or, equivalently, when continued to Euclidean AdS, the corresponding Gibbons-Hawking partition function, Z_{ECFT}). Just as in the second-order or metric formulation of gravity, this is guaranteed by demanding the regularity of the Euclidean section of the metric, in the case of the first-order or connection formulation, it is fixed by demanding *triviality* of the gauge connection A along contractible circle(s). The nontrivial topology of the connection is captured by the holonomy matrix or the Wilson loop operator along any contractible circle $\mathcal{C}$,

$$\mathrm{Hol}_{\mathcal{C}}(A) \equiv \mathcal{B}^{-1}\exp\left[\oint_{\mathcal{C}} dx^\mu a_\mu\right]\mathcal{B} = e^{H_{\mathcal{C}}}. \tag{28}$$

The triviality of the connection is ensured when this holonomy matrix is equal to identity. Equivalently, this means that the matrix $H_{\mathcal{C}}$ has eigenvalues $(0, -2\pi i, 2\pi i)$. In the case of Kerr-dS_3 and its spin-3 generalizations, one has a contractible thermo-angular circle,

$$(t, \phi) \sim (t + i\beta, \phi + i\beta\Omega). \tag{29}$$

Equivalently, if one defines $\tau = \frac{\beta}{2\pi}(1 - i\Omega l)$, this thermo-angular circle can be reexpressed as $(w, \bar{w}) \sim (w + 2\pi\tau/l, \bar{w} + 2\pi\bar{\tau}/l)$. The associated holonomy is

$$\begin{aligned}\mathrm{Hol}(a) &= \mathcal{B}^{-1}\exp\left[\int_0^{i\beta} dt a_t + \int_0^{i\beta\Omega} d\phi a_\phi\right]\mathcal{B} \\ &= \mathcal{B}^{-1}\exp\left[\int_0^{i\beta} dt i(a_w - a_{\bar{w}})/l + \int_0^{i\beta\Omega} d\phi(a_w + a_{\bar{w}})\right]\mathcal{B} \\ &= \mathcal{B}^{-1}\exp\left[-(2\pi\tau a_w - 2\pi\bar{\tau}a_{\bar{w}})/l\right]\mathcal{B} \\ &= e^{W(a)}.\end{aligned} \tag{30}$$

This means that the matrix $W(a)$ should have eigenvalues $(0, -2\pi i, 2\pi i)$[2]. These two (complex) conditions entirely fix the charges L, W in terms of the potentials β, μ.

For generic gauge connections it is nontrivial to compute the holonomy matrix exactly. Since all we need are its eigenvalues, we are perfectly fine to work with the matrix $\exp(\tilde{w}(a))$, $\tilde{w}(a) \equiv -(2\pi\tau a_w - 2\pi\bar{\tau}a_{\bar{w}})$ instead, since it is related to the holonomy matrix $\exp(W(a))$ by a single-valued gauge transformation (similarity transformation) $\mathcal{B}$ and hence has the same eigenvalue spectrum. Demanding that the eigenvalues of the $\tilde{w}_z$ be $(0, -2\pi i, 2\pi i)$ implies

$$\det(\tilde{w}(a)) = 0, \qquad \mathrm{Tr}[\tilde{w}(a)^2] = -8\pi^2, \qquad \mathrm{Tr}[\tilde{w}(a)] = 0, \tag{31}$$

which translate to the following relations determining the charges L, W in terms of the potentials τ, μ,

$$\begin{aligned}&27l^2\tau^3 W - 36l\tau^2\alpha L^2 - 54l\tau\alpha^2 LW - 54l\alpha^3 W^2 - 8\alpha^3 L^3 = 0 \\ &1 - \frac{2\tau^2 L}{l^3} - \frac{6\tau\alpha W}{l^3} + \frac{4}{3}\frac{\alpha^2 L^2}{\tau^2 l^2} = 0,\end{aligned} \tag{32}$$

where $\alpha = \mu\bar{\tau}$.

[2] Of course, one could consider the eigenvalues to be integer multiples of $\pm 2\pi i$, in general, to get trivial holonomy. This ambiguity is directly tied to the ambiguity in identifying the period of the thermal circle with the inverse temperature, which in turn is ultimately tied to fixing the asymptotics of the geometry [39]. The choice $(0, -2\pi i, 2\pi i)$ can be found in Sec. 5.4 of [29]. It is easy to see from (30) that once we fix a period τ, scaling the holonomy eigenvalues by N can only be accomplished by scaling the primitive connection a in (27). But this results in a metric (and connection A) that will violate the standard Brown-Henneaux falloffs (and their higher spin generalizations). This is again ultimately tied to the fact that the asymptotic falloffs are defined after fixing the asymptotic coordinates; the Killing vectors are normalized at infinity. Note that the norm of the Killing vector that turns null at the horizon is crucial for determining the surface gravity or temperature [39].

Since we are already familiar with the exact solution for purely spin-2 charges, i.e. mass and angular momentum, we can now obtain a solution to the charges in the presence of spin-3 potentials in a perturbation series in the spin-3 chemical potential, μ:

$$W = \sum_{i=1}^{\infty} a_i \mu^i \qquad L = \frac{l}{2\tau^2} + \sum_{j=1}^{\infty} b_j \mu^j. \tag{33}$$

Substituting this into Eq. (32) and solving both equations to quadratic order μ, we get the following perturbative solution for L and W,

$$\frac{L}{l} = \frac{l^2}{2\tau^2} - \frac{5\alpha^2 l^4}{6\tau^6} + \cdots,$$
$$\frac{W}{l} = \frac{\alpha l^4}{3\tau^5} - \frac{20\alpha^3 l^6}{27\tau^9} + \cdots. \tag{34}$$

These solutions satisfy the "integrability conditions" (as can be checked order by order),

$$\frac{\partial L}{\partial \alpha} = \frac{\partial W}{\partial \tau}, \tag{35}$$

pointing out the existence of a bulk (Euclidean) action I,

$$\delta I \sim \delta\tau L + \delta\alpha W, \tag{36}$$

with L and W being functions of τ, α. The basic reason why Eq. (35) arises is because we are demanding that there be an underlying partition function description for the system (the exponential of the action being the semiclassical partition function). The integrability condition is the statement that the double derivatives of the partition function (with respect to α and τ) commute. A closely related discussion can be found in Sec. (5.2) of [29]. The precise form of the action functional requires taking care of various subtleties (see [42]). We will make use of their results when we make comparisons with the AdS case.

For the (higher spin) AdS case, the integrability conditions were understood [29] to be integrability conditions of a *boundary* CFT partition function Z_{CFT} dual to the higher spin AdS bulk theory (as expected from AdS/CFT). The on-shell bulk action $I^{\text{on-shell}}$ is the saddle-point contribution to Z_{CFT}, corresponding to the classical higher spin black hole configuration. Similarly, it will be shown in Sec. IV that the integrability conditions Eqs. (32), (34) for the case of (higher spin) de Sitter connections apply to a putative dual *Euclidean* CFT partition function $Z_{\mathrm{CFT}*}$. It will also be shown that the two partition functions (Z_{CFT}, $Z_{\mathrm{CFT}*}$) are related by a suitable "Wick rotation" of the Cherns-Simons level number (cosmological constant) and gauge theory charges (mass, spin, spin-3 charges).

Finally, we apply the radial gauge transformation (20) to obtain full radial dependence,

$$A' = iT_0 d\rho + \left[\left(e^{\rho} - e^{-\rho}\frac{L}{2l}\right)T_1 + i\left(e^{\rho} + e^{-\rho}\frac{L}{2l}\right)T_2 + e^{-2\rho}\frac{W}{8l}W_{-2}\right]dw$$
$$+ \mu\left[e^{2\rho}W_2 - \frac{L}{2l}W_0 + e^{-2\rho}\frac{L^2}{16l^2}W_{-2} + e^{-\rho}\frac{W}{l}(T_1 - iT_2)\right]d\bar{w},$$
$$\bar{A}' = -iT_0 d\rho + \left[\left(e^{\rho} - e^{-\rho}\frac{\bar{L}}{2l}\right)T_1 - i\left(e^{\rho} + e^{-\rho}\frac{\bar{L}}{2l}\right)T_2 + e^{-2\rho}\frac{\bar{W}}{8l}W_{-2}\right]d\bar{w}$$
$$+ \bar{\mu}\left[e^{2\rho}W_2 - \frac{L}{2l}W_0 + e^{-2\rho}\frac{L^2}{16l^2}W_{-2} + e^{-\rho}\frac{\bar{W}}{l}(T_1 + iT_2)\right]dw. \tag{37}$$

The corresponding metric expression is

$$ds^2 = -l^2 d\rho^2 + \left(\frac{lL}{2} + \frac{lW\bar{\mu}}{2} - \frac{1}{2}e^{-2\rho}L\bar{W}\bar{\mu} - \frac{\bar{L}^2\bar{\mu}^2}{3}\right)dw^2 + \left(\frac{l\bar{L}}{2} + \frac{l\bar{W}\mu}{2} - \frac{1}{2}e^{-2\rho}\bar{L}W\mu - \frac{L^2\mu^2}{3}\right)d\bar{w}^2$$
$$\times\left(\frac{1}{2}e^{2\rho}l^2 - \frac{3\ lW\mu}{4} - \frac{3l\bar{W}\bar{\mu}}{4} + \frac{L^2\mu\bar{\mu}}{8} + \frac{L\bar{L}\mu\bar{\mu}}{12} + \frac{\bar{L}^2\mu\bar{\mu}}{8} + \frac{1}{8}e^{-2\rho}(L\bar{L} + 4W\bar{W}\mu\bar{\mu})\right)dwd\bar{w}, \tag{38}$$

while the expressions for the nonvanishing spin-3 field components are

$$\psi_{\rho\rho w} = \frac{1}{18} i l^2 \bar{L}\,\bar{\mu}, \qquad \psi_{\rho\rho\bar{w}} = -\frac{1}{18} i l^2 L\mu,$$

$$\psi_{www} = -\frac{1}{8} i l^2 W + \frac{1}{16} i l L^2 \bar{\mu} + \frac{1}{24} i l L\bar{L}\,\bar{\mu} + \frac{1}{16} i l \bar{L}^2 \bar{\mu} - \frac{1}{12} i l \bar{L} W \bar{\mu}^2 + \frac{1}{27} i \bar{L}^3 \bar{\mu}^3$$
$$+ \frac{1}{4} i e^{-2\rho}\left(l W \bar{W}\,\bar{\mu} - \frac{1}{6} L\bar{L}\,\bar{W}\,\bar{\mu} - \frac{1}{2}\bar{L}^2 \bar{W}\bar{\mu}^2\right) - \frac{1}{8} i e^{-4\rho} W\bar{W}^2\bar{\mu}^2 + \frac{i e^{-4\rho}\bar{L}^2\bar{W}^2\bar{\mu}^3}{16 l},$$

$$\psi_{\bar{w}\bar{w}\bar{w}} = -\frac{1}{4} i e^{4\rho} l^3 \mu + \frac{1}{2} i e^{2\rho} l^2 W \mu^2 - \frac{1}{24} i l L\bar{L}\mu + \frac{1}{12} i l L\bar{W}\mu^2 - \frac{1}{27} i L^3\mu^3 - \frac{1}{4} i l W^2 \mu + \frac{1}{24} i e^{-2\rho} L\bar{L} W \mu^2$$
$$+ \frac{1}{32} i e^{-4\rho}\left(\bar{L}^2 \bar{W} - \frac{L^2\bar{L}^2\mu}{2l}\right),$$

$$\psi_{ww\bar{w}} = \frac{1}{12} i e^{2\rho}\left(l^2 L\bar{\mu} + \frac{1}{3} i l^2 \bar{L}\,\bar{\mu}\right) + \frac{1}{24} i l^2 \bar{W} - \frac{1}{18} i l L^2 \mu - \frac{1}{18} i l L W \mu\bar{\mu} - \frac{1}{72} i L^2 \bar{L}\mu\bar{\mu}^2 - \frac{1}{108} i L\bar{L}^2\mu\bar{\mu}^2 - \frac{1}{72} i \bar{L}^3 \mu\bar{\mu}^2$$
$$+ \frac{i e^{-2\rho}}{12}\left(\frac{1}{12} L\bar{L}^2\bar{\mu} + \frac{1}{4}\bar{L}^3\bar{\mu} - l\bar{W}^2\bar{\mu} - \frac{1}{2} l\bar{L} W + \frac{2}{3} L^2 \bar{W}\mu\bar{\mu} + \frac{1}{36}\bar{L} W\bar{W}\mu\bar{\mu}^2\right)$$
$$- \frac{i e^{-4\rho}}{24}\left(\frac{\bar{L}^3\bar{W}\bar{\mu}^2}{2l} - \bar{L} W\bar{W}\,\bar{\mu} - \bar{W}^3\bar{\mu}^2 + \frac{L^2\bar{W}^2\mu\bar{\mu}^2}{2l}\right),$$

$$\psi_{ww\bar{w}} = \frac{1}{12} i e^{4\rho} l^3 \bar{\mu} - i e^{2\rho} l^2 \left(\frac{1}{9} L\mu + \frac{1}{6} W\mu\bar{\mu}\right)$$
$$+ \frac{1}{12} i \left(l L W \mu^2 + \frac{1}{6} l \bar{L}^2 \bar{\mu} - \frac{1}{3} l \bar{L}\,\bar{W}\mu\bar{\mu} + \frac{1}{6} L^3\mu^2\bar{\mu} + \frac{1}{9} L^2\bar{L}\mu^2\bar{\mu} + \frac{1}{6} L\bar{L}^2\mu^2\bar{\mu} + l W^2\mu^2\bar{\mu}\right)$$
$$+ \frac{1}{12} i e^{-2\rho}\left(l\bar{L}\,\bar{W} - \frac{1}{3} L^2\bar{L}\mu - \frac{1}{6}\bar{L}^2 W\mu\bar{\mu} - \frac{1}{3} L W\bar{W}\mu^2\bar{\mu}\right) - \frac{1}{24} i e^{-4\rho}\left(\bar{L}^2 W - \frac{\bar{L}^4\bar{\mu}}{8l} + \bar{L}\bar{W}^2\bar{\mu} - \frac{L^2\bar{L}\,\bar{W}\mu\bar{\mu}}{2l}\right). \tag{39}$$

D. Schwarzschild gauge

The Fefferman-Graham (FG) gauge expressions only cover part of the spacetime outside the horizon. In this section, we describe the solution of the gauge connection in Schwarzschild gauge. For simplicity, we will consider the purely nonrotating case from now on,

$$L = \bar{L}, \quad \bar{W} = -W \quad \text{and} \quad \bar{\mu} = -\mu. \tag{40}$$

The metric in FG gauge is then

$$g_{\rho\rho} = -l^2, \qquad g_{tt} = \left(e^{\rho} - \frac{L + 2\,W\mu}{2l} e^{-\rho}\right)^2,$$

$$g_{\phi\phi} = l^2\left(e^{\rho} + \frac{L - 2\,W\mu}{2l} e^{-\rho}\right)^2 + \frac{4L^2|\mu|^2}{3} - 2\,lW\mu. \tag{41}$$

We observe that there is a horizon, i.e. g_{tt} vanishes, at

$$\rho_+ = \frac{1}{2}\,\ln\left[\frac{L + 2\,W\mu}{2l}\right]. \tag{42}$$

Now, we can introduce the Schwarzschild radial coordinate r [motivated by the definition of the Schwarzschild radial coordinate for the pure $SL(2)$ case],

$$\rho = \ln\left[\frac{r + \sqrt{r^2 - r_+^2}}{2l}\right], \tag{43}$$

where r_+^2 is

$$r_+^2 = 2l(L + 2\,W\mu). \tag{44}$$

In the limit $\mu = 0$, the above equation reduces to the pure $SL(2)$ case, Eq. (14). In the Schwarzschild-like gauge the metric is given by

$$ds^2 = -\frac{l^2}{r^2 - r_+^2} dr^2 + \frac{2(L + 2\,W\mu)}{l r_+^2}(r^2 - r_+^2) dt^2$$
$$+ \left[\left(\frac{rL}{L + 2\,W\mu} + \frac{2\,l\mu W\sqrt{r^2 - r_+^2}}{L + 2\,W\mu}\right)^2 + \frac{4L^2|\mu|^2}{3} - 2\,lW\mu\right] d\phi^2. \tag{45}$$

We also note that $g_{\phi\phi} > 0$, as it is a sum of manifestly positive quantities [W and μ are imaginary quantities with the same sign vide (34)] and there are no closed timelike curves in the ϕ direction.

E. Higher spin quotient cosmologies

Now that we have the metric expressions for the higher spin versions of the Kerr de Sitter universe in Schwarzschild gauge (45) outside the cosmological horizon, one can write the metric for higher spin generalizations of the quotient cosmologies (21) by simply swapping r and t.

$$ds^2 = -\frac{l^2}{t^2 - r_+^2} dt^2 + \frac{2(L + 2W\mu)}{l r_+^2}(t^2 - r_+^2) dr^2 + \left[\left(\frac{Lt + 2\mu W\sqrt{t^2 - r_+^2}}{L + 2W\mu}\right)^2 + \frac{4L^2|\mu|^2}{3} - 2lw\mu\right] d\phi^2. \tag{46}$$

Just as in the case for the $SL(2)$ quotient cosmology, r is now compactified into a circle and this metric cannot be continued inside the horizon r_+. As a result, it contains big bang/big crunch–like singularities at $t = \pm r_+$ when the r circle degenerates to a point, exactly like its $SL(2)$ cousin. It will be interesting to consider the resolution of these singularities along the lines of [28], but we will not pursue this here.

IV. THERMODYNAMICS OF ASYMPTOTICALLY DE SITTER CONNECTIONS

The aim of this section is to derive a consistent thermodynamics for asymptotically dS_3 spin-2 connections in the Chern-Simons language. In a metric (second order) formalism of gravity, more precisely, spin-2 gravity, thermodynamics of spacetimes containing horizons of any kind is provided by the Gibbons-Hawking generalization [40] of the black hole thermodynamics of Bardeen, Carter and Hawking [41]. However, as we shall see, in the Chern-Simons or first order setup, a consistent thermodynamics is obtained extremely efficiently by first mapping de Sitter solutions to Euclidean AdS (EAdS) solutions and then demanding integrability conditions on free energy (equivalently, partition function) of a putative Euclidean CFT located on the future infinity of the asymptotically de Sitter connections (same as the conformal boundary of the analytically continued EAdS solution). Maldacena [12] notes that the conformal patch of dS,[3]

$$ds^2 = \frac{-d\eta^2 + d\mathbf{x}_d^2}{\eta^2/l^2}, \tag{47}$$

goes over to the Poincare patch of the EAdS, under $l^2 \to -l^2$ and $\eta^2 \to -z^2$,

$$ds^2 = \frac{dz^2 + d\mathbf{x}_d^2}{z^2/l^2}. \tag{48}$$

Then he proposes that for any asymptotic (in time) de Sitter space,

$$\Psi_{\text{Hartle-Hawking}} = Z_{\text{CFT}*}, \tag{49}$$

since for EAdS one has the celebrated AdS-CFT conjecture $Z_{\text{ESUGRA}} = Z_{\text{CFT}}$. Under the identifications, the Euclidean path integral in AdS becomes the Hartle-Hawking wave function of dS. Next we construct a similar map between the *exterior* regions of Kerr de Sitter and Euclidean BTZ black holes and then generalize to the higher spin case where the bulk action would be a first order action instead of a second order (metric) action.

A. Wick rotation from Kerr de Sitter to EBTZ

We simply write down these identifications for the FG gauge,

$$\rho_{\text{dS}} \to \rho_{\text{EAdS}} + i\frac{\pi}{2}, t_{\text{ds}} \to it_{\text{EAdS}}, l_{\text{dS}} \to il_{\text{EAdS}}, \tag{50}$$

$$L_{\text{dS}}, \bar{L}_{\text{dS}} \to -iL_{\text{EAdS}}, -i\bar{L}_{\text{EAdS}}, \quad M_{\text{dS}}, J_{\text{dS}} \to -M_{\text{AdS}}, -J_{\text{AdS}}. \tag{51}$$

Under these identifications, the KdS_3 metric Eq. (12) goes over to

$$ds^2 \to d\tilde{s}^2 = l^2 d\rho^2 + \frac{l}{2}(Ldw^2 + \bar{L}d\bar{w}^2) + \left(l^2 e^{2\rho} + \frac{L\bar{L}}{4}e^{-2\rho}\right) dw d\bar{w} \tag{52}$$

but now with

$$L = \frac{Ml + J}{2}, \qquad \bar{L} = \frac{Ml - J}{2}. \tag{53}$$

This is evidently a Euclidean metric. To determine whether this is the Euclidean BTZ (EBTZ) metric, we write the EBTZ metric expressions directly from Euclideanizing the Lorentzian BTZ,

$$ds^2 = \frac{l}{2}(L^+ dw^{+2} + L^- dw^{-2}) + \left(l^2 e^{2\rho} + \frac{L^+ L^-}{4}e^{-2\rho}\right) dw^+ dw^- + l^2 d\rho^2, \quad w^\pm = \phi \pm \frac{t}{l}, \tag{54}$$

[3]In other words, the upper quadrant in the dS Penrose diagram containing the infinite future at $\eta = 0$ and bounded by the horizon at $\eta = 1$. No light rays from the infinite past can reach this region.

where the "zero modes" L^+, L^- are defined in terms of the mass and the spin by

$$L^+ = \frac{Ml+J}{2}, \qquad L^- = \frac{Ml-J}{2}. \tag{55}$$

Upon replacing $t \to it_E$, we obtain the EBTZ metric,

$$\begin{aligned} ds^2 &= \frac{l}{2}(L^+ dw^2 + L^- d\bar{w}^2) \\ &\quad + \left(l^2 e^{2\rho} + \frac{L^+L^-}{4}e^{-2\rho}\right) dw d\bar{w} + l^2 d\rho^2, \\ w &= \phi + \frac{it_E}{l}, \quad \bar{w} = \phi - \frac{it_E}{l}. \end{aligned} \tag{56}$$

Clearly, this is identical to the wick-rotated KdS_3 metric Eq. (52).

1. *Schwarzschild gauge wick rotation*

In Schwarzschild-like coordinates, the Kerr-dS_3 metric 9, using the identifications 51, becomes

$$\begin{aligned} d\tilde{s}^2 &= N^2 dt^2 + N^{-2} dr^2 + r^2(iN^\phi dt + d\phi)^2, \\ N^2 &= -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \qquad N^\phi = \frac{J}{2r^2}. \end{aligned} \tag{57}$$

The Lorentzian exterior BTZ metric Eq. (54) reads

$$\begin{aligned} ds^2 &= -N^2 dt^2 + N^{-2} dr^2 + r^2(N^\phi dt + d\phi)^2, \\ N^2 &= -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \qquad N^\phi = \frac{J}{2r^2}, \end{aligned} \tag{58}$$

which, upon Euclideanizing, i.e. $t \to it_E$,

$$\begin{aligned} ds^2 &= N_E^2 dt_E^2 + dr^2/N_E^2 + r^2(iN_E^\phi dt_E + d\phi)^2, \\ N_E^2 &= -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \qquad N_E^\phi = \frac{J}{2r^2}. \end{aligned} \tag{59}$$

One can write a metric expression in terms of outer and inner horizons for the BTZ along the lines of 12,

$$\begin{aligned} ds^2 &= -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2 l^2} dt^2 \\ &\quad + \frac{r^2 l^2}{(r^2-r_+^2)(r^2-r_-^2)} dr^2 + r^2\left(d\phi + \frac{r_+ r_-}{r^2}\frac{dt}{l}\right)^2, \\ & r > r_+ \end{aligned} \tag{60}$$

with

$$r_\pm^2 = \frac{Ml^2}{2}\left(1 \pm \sqrt{1 - \left(\frac{J}{Ml}\right)^2}\right). \tag{61}$$

So the identifications are

$$r_+ \to r_+, \quad (r_-)_{KdS_3} \to -i(r_-)_{BTZ}. \tag{62}$$

For KdS_3 note that in terms of L, $\bar{L}$,

$$r_+ = \frac{\sqrt{Ll} + \sqrt{\bar{L}l}}{\sqrt{2}}, \qquad r_- = \frac{\sqrt{Ll} + \sqrt{\bar{L}l}}{\sqrt{2}} \tag{63}$$

$$r_+^2 + r_-^2 = 2\sqrt{L\bar{L}l^2}. \tag{64}$$

So, the temperature inverse of KdS_3 in terms of L, $\bar{L}$ is

$$\frac{\beta}{2\pi} = \frac{l^2}{2}\left(\frac{1}{\sqrt{2\,\mathrm{Ll}}} + \frac{1}{\sqrt{2\bar{L}l}}\right). \tag{65}$$

For nonrotating KdS_3, $\tau = \beta/2\pi$, and we have

$$\frac{L}{l} = \frac{l^2}{2\tau^2}. \tag{66}$$

Again, the metric (59) is exactly that of the Wick-rotated dS metric in Schwarzschild coordinates, Eq. (57).

B. The dS-AdS wick rotation at work: Equivalence of thermodynamics in the metric formulation

In order to further solidify our heuristic identifications, we show that under these identifications the Gibbons-Hawking thermodynamics [40], including the temperature and entropy of the Kerr-dS_3 solution, maps onto those of the wick-rotated EBTZ solutions.

(1) The entropies for either geometry are the same since the entropy of either cosmological or black hole horizons in the Gibbons-Hawking framework is given by

$$S = \frac{1}{4G}(\text{Horizon Area}) = 2(2\pi r_+) = 4\pi r_+. \tag{67}$$

This is borne out by our heuristic identifications, since $r_+ \to r_+$.

(2) The temperature of KdS_3 is given by Gibbons-Hawking thermodynamics by the conical singularity trick,

$$T_{KdS_3} = \frac{r_+^2 + r_-^2}{2\pi l^2 r_+}. \tag{68}$$

Using the identification Eq. (62) and the additional identification $T_{\rm ds} \to -T_{\rm AdS}$,[4] this temperature continues to the Hawking temperature of the corresponding BTZ black hole,

$$T_{\rm BTZ} = \frac{r_+^2 - r_-^2}{2\pi l^2 r_+}. \tag{72}$$

(3) The chemical potential conjugate to the angular momentum is

$$\Omega_{{\rm KdS}_3} = -T\frac{\partial S}{\partial J} = -\frac{r_-}{r_+ l}. \tag{73}$$

Again, under the identifications, we obtain the expected behavior $\Omega_{\rm dS} \to \Omega_{\rm AdS}$ since $J_{\rm dS} \to -J_{\rm AdS}$.[5] We note that $\Omega_{\rm BTZ} = \frac{r_-}{r_+ l}$. Parenthetically, we note that when we move to Euclidean BTZ, we need to define $J_{\rm EAdS} = -iJ_{\rm AdS}$ and consequently the new conjugate $\Omega_{\rm EAdS} = i\Omega_{\rm AdS}$, so that the respective identifications are $J_{\rm dS} \to -iJ_{\rm EAdS}$ and $\Omega_{{\rm KdS}_3} \to -i\Omega_{\rm EBTZ}$.

Since under the identifications, one can successfully map any dS thermodynamic quantities like entropy, internal energy, angular charges and their respective conjugates to AdS quantities, the laws of thermodynamics will continue as well. When higher spin charges are added, we will demand a similar statement to hold with higher spin charges and chemical potential added to the thermodynamical relations.

C. Thermodynamics in the Chern-Simons formulation

So far everything we discussed was in the $SL(2,C)$ sector of the theory with just metric or spin-2 fields turned on, but we extend this analogy to the full $SL(3,C)$ sector, i.e. when both metric and spin-3 fields are present. In that case though we do not know the generalization of the Gibbons-Hawking thermodynamics [41]. However, taking dS/CFT as a principle, we can propose that the thermodynamics of a dS connection is identical to that of a suitably continued Euclidean AdS connection, i.e. a higher spin AdS black hole [29]. The thermodynamics of $SL(3,C)$-valued Euclidean $\rm AdS_3$ connections for higher spin black holes (connected to BTZ, i.e. the so-called "BTZ" branch) has been shown to be dictated by the integrability conditions of the free energy of a dual CFT [29]. These conditions can be cast in a gauge-invariant form by the *holonomy* conditions [29]. Under the correct identifications of charges and potentials, the integrability or holonomy conditions of a dS connection should continue to those of an AdS connection. Turning this fact around, we expect the charges we obtain as functions of the potentials μ and T on solving the integrability conditions on the dS side (34), to reproduce the respective solutions of the AdS integrability conditions [42] upon making the dS-to-AdS identifications. For AdS, the solution to the holonomy conditions is

$$\frac{L^+}{l} = \frac{l^2}{2\tau^2} + \frac{10\alpha^2 l^4}{3\ \tau^6} + \cdots, \qquad \frac{W^+}{l} = -\frac{4\alpha l^4}{3\ \tau^5} + \cdots. \tag{75}$$

We have the identifications for the spin-3 charges when going from dS to AdS,

$$\alpha_{\rm AdS} = \frac{\alpha_{\rm dS}}{2}, \qquad W^+_{\rm AdS} = -2iW_{\rm dS}, \tag{76}$$

or

$$\mu_{\rm AdS} = -\frac{\mu_{\rm dS}}{2}, \qquad W^+_{\rm AdS} = -2iW_{\rm dS}. \tag{77}$$

1. The action and Free energy in the CS theory

To compute $Z_{\rm CFT}$ from the bulk gauge theory, we make use of the saddle-point approximation,

$$Z_{\rm CFT} = Z_{(E){\rm SUGRA}} = e^{I_E^{\text{On-shell}}} \tag{78}$$

where I_E is the Euclidean bulk action, defined in terms of the original action I by

$$I_E[F(\mathbf{x},t)] = iI[F(\mathbf{x},it_E)]. \tag{79}$$

The Chern-Simons action without any supplementary boundary terms,

$$I_{\rm CS} = \frac{k}{4\pi\epsilon_R}\int {\rm Tr}\left(AdA + \frac{2}{3}A^3\right), \quad \epsilon_R = 4, \tag{80}$$

[4]This temperature sign flip is a direct result of the flip in the sign of the mass parameter or "internal energy" M in identification 51. The conjugacy relation

$$T^{-1} = \frac{\partial S}{\partial M} \tag{69}$$

shows us that one needs to perform

$$T_{\rm dS} \to -T_{\rm AdS} \tag{70}$$

in consonance with

$$M_{\rm dS} \to -M_{\rm AdS}. \tag{71}$$

[5]Since going from dS to AdS implies the replacements $S \to S$, $J \to -J$ and $\beta \to -\beta$, we must have $\Omega \to \Omega$ in order to reproduce the correct thermodynamic relation,

$$\frac{\partial S}{\partial J} = -\Omega\beta. \tag{74}$$

is the right action for the $SL(2)$ sector. On shell this becomes [43]

$$I_{\rm CS}[A] = -\frac{k}{4\pi\epsilon_R}\int dtd\phi {\rm Tr}(A_t A_\phi). \tag{81}$$

For the $SL(3)$ sector one needs to add new boundary terms as formulated in [35,36]. But it is easy to see that a similar map, as we are presenting below, will also hold for the boundary terms, so in the following, we will illustrate it only for the bulk terms. Using (5.1) of [29],

$$I^{\rm EAdS}_{\text{On-shell}} = -\frac{2\ \beta L}{l} + \frac{16\beta\mu^2 L^2}{3l^2}. \tag{82}$$

The higher spin de Sitter on-shell action turns out to be[6]

$$\tilde{I}^{\rm dS}_{\text{On-shell}} = -\left(\frac{2\ \beta L}{l} + \frac{4}{3}\frac{\beta\mu^2 L^2}{l^2}\right). \tag{85}$$

Again, using the identifications, we see that the dS on-shell action reproduces the EAdS on-shell action (82),

$$\tilde{I}^{\rm dS}_{\text{On-shell}} = I^{\rm EAdS}_{\text{On-shell}}. \tag{86}$$

Thus, we have demonstrated that the higher spin generalizations of Kerr de Sitter universes are related to (higher spin) AdS black holes just as they were in the pure gravity (spin-2) case in the metric formulation. However, this on-shell action is not yet equal to a $-\beta\Phi$, where Φ is the grand "higher spin" canonical potential $\Phi = E - TS - \mu W$. But it is possible to add boundary or supplementary terms and change the action $\tilde{I}^{\rm dS}_{\text{On-shell}}$ to a new action $I_{\text{On-shell}}$ such that

$$-I_{\text{On-shell}} = \beta\Phi. \tag{87}$$

Such a procedure was conducted in the anti–de Sitter case in [42] [see their Sec. (2.2)]. For our de Sitter case, the necessary extra terms can be obtained from their expressions by the AdS-dS identifications (51) and (77), in exact analogy with our computation here for the bulk terms. The match between our entropy and the higher spin AdS_3 black hole entropy [44] is a natural consequence, and we have explicitly checked this. This concludes our discussion about the connection between the thermodynamics of the Kerr-dS_3 solution and that of higher spin black holes in AdS_3.

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[6]To get this on-shell action, we perform integration over the t circle $(0, i\beta)$ and over the ϕ circle $(0, 2\pi)$,

$$\tilde{I}^{\rm dS}_{\text{On-shell}} = i\left(-\frac{k}{4\pi\epsilon_R}\right)\int_0^{-\beta}(idt)\int_0^{2\pi} d\phi[-i{\rm Tr}(A_t A_\phi - \bar{A}_t\bar{A}_\phi)], \tag{83}$$

with

$$k = -2\ il. \tag{84}$$

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