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Pair creation of rotating black holes

Marco Astorino*

Centro de Estudios Científicos (CECs), Avenida Prat 514 Valdivia, Chile

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An exact and regular solution describing a couple of charged and spinning black holes is generated in an external electromagnetic field, via the Ernst technique, in Einstein-Maxwell gravity. A wormhole instantonic solution interpolating between the two black holes is constructed to discuss, at the semiclassical level, the quantum process of creation rate, in an external magnetic field, of this charged and spinning black hole pair.

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I. INTRODUCTION

In general relativity, few processes are known to allow for the creation of black holes. At the macroscopic classical level it is possible to produce, through the gravitational collapse of stars, a massive black hole whose mass cannot be smaller that the Oppenheimer threshold of about 3 solar masses. At the microscopical level a quantum effect analogous to the Schwinger pair creation in an external field may occur. This effect is the possibility that a spacetime with a source of excess energy will quantum tunnel into a spacetime containing two black holes. Though a well-established theory of quantum gravity is presently not known, some speculations about this Planckian scale effect are studied in the literature [1,2] using the semi-classical Euclidean path integral approach—not only for Einstein-Maxwell gravity but also for the dilatonic coupling [3,4]. Motivations to study this process include, among others, the topological changing process, the black hole information paradox, the counting of black hole microstates, and the microstates interaction [5].

In this framework, several studies were done using the cosmological constant or an external magnetic field as a background, which provides the energy to generate the black hole pair.[1] The fact that the cosmological constant value is fixed and small by observation, while an external magnetic field can be set arbitrarily large—in fact it has recently been measured to be extremely large at the center of some galaxies, including our own [6]—makes this process physically more realistic in the external field setting. In the first case the Plebanski-Demianski solutions are used (see [7,8]), while in the latter case the Ernst solution [9] is needed. The Ernst solution describes two oppositely charged black holes accelerating apart by means of the force supplied by the external magnetic field. Ernst metrics are built with the help of solution generating techniques.

Solution generating techniques—we will focus on the Ernst method [11,12]—are a very powerful tool in general relativity. By exploiting the integrability property of general relativity, they give us a new insight into the theory and are able to generate new and exact solutions, which can be hardly obtained by directly integrating the field equations.

For the pair creation process, as first pointed out by Gibbons in [13] and further analyzed in [1] and [2], the suitable Ernst metric is usually that which describes a couple of accelerating, intrinsically magnetically charged black holes embedded in the external field of the Melvin magnetic universe. The analogy with the Schwinger electron-positron pair creation in an external electric field is apparent, as discussed in Sec. 3. Taking advantage of the electromagnetic duality in four dimensions, a specular treatment can also be done for an electric Reissner-Nordstrom (RN) spacetime in an external electric field. In [14] and [15] it is shown that the pair nucleation rate of the dualized and standard cases are the same; in [16] this electromagnetic equivalence is shown in a general setting. What happens when the Ernst black holes are both electrically and magnetically intrinsically charged at the same time is still unknown, and in this case even a dyonic Ernst-like solution is not known. What one expects is that the black holes acquire rotation because of the Lorentz force interacting between the black hole electrical charge and the external magnetic field, as happens in the case of the nonaccelerating single black hole [17].

The purpose of this work is to explore the possibility of generalizing the Ernst metric to the dyonic case. This can be done by taking advantage of the new form of the C metric offered by [18]; this is more suitable for generating techniques because in this coordinate set, the accelerating space-time can be more easily cast into the Weyl form. This is done in Sec. 2. Then, in Sec. 3, a Euclidean instanton is built to evaluate the pair creation rate.

*marco.astorino@gmail.com

[1] In [10], the possibility of furnishing the energy to produce the black hole pair by a cosmic string in a de Sitter background is also studied.

II. EMBEDDING AN ACCELERATING REISSNER-NORDSTROM BLACK HOLE IN A MAGNETIC UNIVERSE

Consider Einstein gravity coupled to Maxwell electromagnetism. The regularized action for this theory is given by

$$I[g_{\mu\nu},A_\mu] = -\frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\left(\mathrm{R} - \frac{G}{\mu_0}F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{8\pi G}\int_{\partial M} d^3x\sqrt{h}K - \frac{1}{4\pi\mu_0}\int_{\partial M} d^3\sqrt{h}n_\mu A_\nu F^{\mu\nu}, \tag{2.1}$$

where h is the determinant of the induced three-metric and K is the trace of the extrinsic curvature of the boundary. The first boundary term is the standard Gibbons-Hawking regularization [19], while the second is needed for the class of solutions we will discuss, to ensure that the electric charge is fixed on the boundary[2] (as explained in [14]).

The gravitational and electromagnetic field equations are obtained by extremizing with respect to the metric $g_{\mu\nu}$ and the electromagnetic potential A_μ[3]

$$\mathrm{R}_{\mu\nu} - \frac{\mathrm{R}}{2}g_{\mu\nu} = \frac{2G}{\mu_0}\left(F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right), \tag{2.2}$$

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0. \tag{2.3}$$

A very well-known solution for this theory is given by the dyonic RN spacetime. It represents a static and spherical symmetric black hole equipped with both electric and magnetic intrinsic monopole charges, denoted e and g respectively. A generalization of RN space-time, including an acceleration parameter A, is called a (dyonic) charged C metric. In spherical coordinates this metric is

$$ds^2 = \frac{1}{(1+Ar\cos\theta)^2} \times \left[-Q(r)dt^2 + \frac{dr^2}{Q(r)} + \frac{r^2d\theta^2}{P(\theta)} + r^2P(\theta)\sin^2\theta d\varphi^2\right], \tag{2.4}$$

where

$$Q(r) = (1-A^2r^2)\left(1 - \frac{2m}{r} + \frac{e^2+g^2}{r^2}\right), \tag{2.5}$$

$$P(\theta) = 1 + 2mA\cos\theta + A^2\cos^2\theta(e^2+g^2). \tag{2.6}$$

It is supported by the electromagnetic potential

$$\mathcal{A} = -\frac{e}{r}dt + g\cos\theta d\varphi. \tag{2.7}$$

This is usually interpreted as a couple of twin RN black holes[4] accelerating apart under the force of a string (or a strut), mathematically represented by an axial conical singularity typical of this kind of metric, that will be analyzed after the magnetization process. The metric (2.4)–(2.6) will constitute the "seed" solution of our construction. Apart from the usual RN inner r_- and outer r_+ event horizons, (2.4) has an accelerating horizon r_A located at

$$r_\pm = m \pm \sqrt{m^2-e^2-g^2}, \qquad r_A = \frac{1}{A}. \tag{2.8}$$

In order for the roots of the polynomial $Q(r)$ in (2.5) to be ordered according to the C metric interpretation [18], the physical parameters m, e, g, A must satisfy the following relation:

$$0 \le Ar_- \le Ar_+ \le 1.$$

We recall that in C metrics the azimuthal coordinate range has a hidden parameter C, which can be used, as in [20], to remove one of the characteristic conical singularities: $\phi \in (-C\pi, C\pi]$.

All axisymmetric spacetimes in Einstein-Maxwell gravity, because of the system integrability, have the remarkable property of being generated, in principle, by the group of transformations $SU(2,1)$ (for details see [21]). One element of this group, the Harrison-Elhers transformation, is able to embed a generic spacetime in an external magnetic field [17]. It can be written in this way[5]:

$$\mathcal{E} \to \hat{\mathcal{E}} = \frac{\mathcal{E}}{1 + B\Phi - \frac{B^2}{4}\mathcal{E}}, \qquad \Phi \to \hat{\Phi} = \frac{\Phi + \frac{B}{2}\mathcal{E}}{1 + B\Phi - \frac{B^2}{4}\mathcal{E}}. \tag{2.9}$$

$\mathcal{E}$ and Φ are the Ernst complex gravitational and electromagnetic potentials; for magnetizing purposes, they are defined as

$$\mathcal{E} := f - |\Phi\Phi^*| + ih, \qquad \Phi := A_\varphi + i\tilde{A}_t, \tag{2.10}$$

where

[2]The magnetic charge is automatically fixed by fixing the gauge potential.

[3]Henceforward the Newton constant G and the electromagnetic vacuum permeability μ_0 will be set to 1 for simplicity, without loss of generality.

[4]For the sake of generality we will always consider the dyonic charged case in this paper.

[5]A hat stands for the transformed quantities.

$$\vec{\nabla}\tilde{A}_t := -\frac{f}{\rho}\vec{e}_\phi \times (\vec{\nabla}A_t + \omega\vec{\nabla}A_\phi), \tag{2.11}$$

$$\vec{\nabla}h := -\frac{f^2}{\rho}\vec{e}_\phi \times \vec{\nabla}_\omega - 2\,\mathrm{Im}(\Phi^*\vec{\nabla}\Phi). \tag{2.12}$$

Since we are interested in axisymmetric spacetimes the functions f, ω, γ, A_t, A_ϕ depend only on the coordinates (r, θ). These functions for the seed solution can be obtained by comparing (2.4) with the most general axisymmetric metric, the Weyl-Lewis-Papapetrou metric

$$ds^2 = -f(d\varphi - \omega dt)^2 + f^{-1}[\rho^2 dt^2 - e^{2\gamma}(d\rho^2 + dz^2)]: \tag{2.13}$$

$$f(r,\theta) = -\frac{r^2 P(\theta)\sin^2\theta}{(1+Ar\cos\theta)^2}, \quad \omega(r,\theta) = 0$$
$$\rho(r,\theta) = \frac{r\sin\theta\sqrt{Q(r)P(\theta)}}{(1+Ar\cos\theta)^2},$$
$$z(r,\theta) = \frac{(Ar\cos\theta)[r + m(Ar\cos\theta - 1) - A(e^2+g^2)\cos\theta]}{(1+Ar\cos\theta)^2}. \tag{2.14}$$

The differential operators can be taken as follows[6]:

$$\vec{\nabla}g(r,\theta) \propto \vec{e}_r\sqrt{Q(r)}\partial_r g(r,\theta) + \vec{e}_\theta\sqrt{P(\theta)}\partial_\theta g(r,\theta). \tag{2.15}$$

Then from (2.11) we can obtain the value of $\tilde{A}_t = e\cos\theta$; therefore, the seed Ernst potentials are

$$\Phi = (g + ie)\cos\theta,$$
$$\mathcal{E} = -\frac{r^2 P(\theta)\sin^2\theta}{(1 + Ar\cos\theta)^2} - (g^2 + e^2)\cos^2\theta. \tag{2.16}$$

Now we are able to apply the Harrison transformation (2.9) to get the complex potentials for the magnetized spacetime:

$$\hat{\mathcal{E}} = \frac{-\frac{r^2 P(\theta)\sin^2\theta}{(1+Ar\cos\theta)^2} - (g^2 + e^2)\cos^2\theta}{\Lambda(r,\theta)}, \tag{2.17}$$

$$\hat{\Phi} = \frac{(g - ie)\cos\theta - \frac{B}{2}\left[\frac{r^2 P(\theta)\sin^2\theta}{(1+Ar\cos\theta)^2} + (g^2 + e^2)\cos^2\theta\right]}{\Lambda(r,\theta)}, \tag{2.18}$$

where

$$\Lambda(r,\theta) = 1 - B(g + ie)\cos\theta + \frac{B^2}{4}\left[\frac{r^2 P(\theta)\sin^2\theta}{(1 + Ar\cos\theta)^2} + (g^2 + e^2)\cos^2\theta\right]. \tag{2.19}$$

Finally, we return to the metric notation. From (2.9) it is possible to find how f changes under the Harrison transformation:

$$f(r,\theta) \to \hat{f}(r,\theta) = \frac{f(r,\theta)}{|\Lambda(r,\theta)|^2}. \tag{2.20}$$

From (2.12)we can obtain a relation to get the magnetized $\omega(r,\theta)$:

$$\vec{\nabla}\hat{\omega}(r,\theta) = |\Lambda(r,\theta)|^2\vec{\nabla}\omega - i\vec{e}_\phi \times \frac{\rho}{f}(\Lambda^*\vec{\nabla}\Lambda - \Lambda\vec{\nabla}\Lambda^*). \tag{2.21}$$

Integrating the latter one finds that

$$\hat{\omega}(r,\theta) = \frac{eB^3(1 + 2Ar\cos\theta)Q(r)}{2A^2 r(1 + Ar\cos\theta)^2} + \frac{eB}{2A^2 r}[4A^2 + B^2A^2(e^2 + g^2) - B^2] + \frac{eB^3 m}{A^2 r^2} - \frac{eB^3(e^2 + g^2)}{2A^2 r^3} + \omega_0 \tag{2.22}$$

where ω_0 is an arbitrary constant. From definition (2.10) we have

$$\hat{A}_\phi(r,\theta) = \frac{g\cos\theta - B(e^2 + g^2)\cos^2\theta - \frac{B}{2}(\frac{3gB}{2}\cos\theta - 1)\mathcal{E} - \frac{B^3}{8}\mathcal{E}^2}{|\Lambda(r,\theta)|^2} + k_\phi, \tag{2.23}$$

$$\hat{\tilde{A}}_t(r,\theta) = e\cos\theta\frac{\left\{1 - \frac{B^2}{4}\left[\frac{r^2 P(\theta)\sin^2\theta}{(1+Ar\cos\theta)^2} - (g^2 + e^2)\cos^2\theta\right]\right\}}{|\Lambda(r,\theta)|^2} + \tilde{k}_t. \tag{2.24}$$

Using (2.11) it is possible to obtain the standard electric field component:

$$\hat{A}_t(r,\theta) = -\hat{\omega}(r,\theta)\left[\hat{A}_\phi(r,\theta) + \frac{3}{2B}\right] + \frac{2e}{r} + k_t, \tag{2.25}$$

where k_t, $\tilde{k}_t$, and k_ϕ are generic integration constants. Finally inserting the Harrison transformed quantities ($\hat{f}$ and $\hat{\omega}$, while γ remains unvaried) in (2.13), the

[6]The orthonormal frame is defined by the ordered triad $(\vec{e}_r, \vec{e}_\phi, \vec{e}_\theta)$.

magnetized C-metric solution (2.4), supported by the electromagnetic field (2.23)–(2.25), results in

$$d\hat{s}^2 = \frac{|\Lambda(r,\theta)|^2}{(1+Ar\cos\theta)^2}\left[-Q(r)dt^2 + \frac{dr^2}{Q(r)} + \frac{r^2 d\theta^2}{P(\theta)}\right] + \frac{r^2 P(\theta)\sin^2\theta[d\phi - \omega(r,\theta)dt]^2}{|\Lambda(r,\theta)|^2(1+Ar\cos\theta)^2}. \tag{2.26}$$

This metric describes a pair of spinning Reissner-Nordstrom dyonically and oppositely charged black holes accelerating away from each other along the axis of a magnetic universe. Remarkably, even though the seed solution was diagonal, (2.26) exhibits rotation due to the appearance of a $\vec{E}\times\vec{B}$ circulating momentum flux in the stress-energy tensor, which serves as a source for a twist potential. This is a typical feature of magnetized black holes when the spacetime possesses an intrinsic charge and an external electromagnetic field of different types—i.e., an electric intrinsic charge and an external magnetic field, or vice versa (see, for instance, [17]). That is because the Ernst potentials are fully complex, not just real or purely imaginary. In fact, the metric (2.26) is the rotating generalization of the one found by Ernst in [9] and studied in [1] and [2]. This latter subcase can be obtained from (2.26) by setting $e = 0$, that is, retaining only the intrinsic magnetic charged black hole. This is why the $e = 0$ case has no rotation.

Due to the accelerating and magnetized asymptotic, it is not known how to compute the angular momentum for these magnetized spacetimes. In the case of no acceleration —for just a single black hole—there have been some recent results, but they disagree.[7] As noted in [22], the gravitational contribution to the angular momentum is exactly compensated by the contrarotation of the external electromagnetic field. Therefore even though the charged black hole in an external magnetic field is rotating, the total angular momentum of the spacetime is null. However, as it is computed in [23], the angular momentum does not vanish.

The spacetime (2.26) is affected by conical singularities (as usually occurs for accelerating metrics) which act as the sources of the acceleration.[8] To study the metric conicity, following [20], a small circle around the half-axis $\theta = 0$ is considered, while keeping the coordinates t and r fixed:

$$\frac{\text{circumference}}{\text{radius}} = \lim_{\theta\to 0}\frac{2\pi C P(\theta)\sin\theta}{\theta|\Lambda(r,\theta)|^2} = \frac{2\pi C[1+2mA+A^2(e^2+g^2)]}{e^2B^2+[1-gB+\frac{B^2}{4}(e^2+g^2)]}. \tag{2.27}$$

To avoid the conical singularity in $\theta = 0$, the parameter C can be fixed such that

$$C = \frac{e^2B^2+[1-gB+\frac{B^2}{4}(e^2+g^2)]}{1+2mA+A^2(e^2+g^2)}. \tag{2.28}$$

Then the coupling between the intrinsic charges and the external magnetic field allows us to regularize the nodal singularity around $\theta = \pi$. In fact, imposing the lack of deficit or excess angle at $\theta = \pi$, as done in (2.27), we obtain a constraint relation between the physical parameters e, g, m, A, and B:

$$\frac{[1+gB+\frac{B^2}{4}(e^2+g^2)]^2+e^2B^2}{[1-gB+\frac{B^2}{4}(e^2+g^2)]^2+e^2B^2}\cdot\frac{1-2mA+A^2(e^2+g^2)}{1+2mA+A^2(e^2+g^2)} = 1. \tag{2.29}$$

This means that the force necessary to accelerate the two black holes is entirely provided by the external magnetic field, without any need for a pulling string.

In the case without the intrinsic electric charge, the nonrelativistic limit of this constraint (i.e., for small acceleration $A \approx 0$) describes the Newtonian force felt by a massive magnetic monopole of intensity g in a uniform magnetic field of strength B. That approximation corresponds, in fact, to the weak magnetic field limit

$$mA \approx -gB. \tag{2.30}$$

The addition of the intrinsic electric charge to the black hole leaves this limit unchanged because eB is a subleading contribution, which is only relevant at higher orders. From a Newtonian perspective, this is related to the fact that the Lorentz force for an electrically charged particle in an external magnetic field is proportional to both the magnetic field and the speed of the particle, which, since in the nonrelativistic limit the speed is small, produces a further factor of damping.

Setting aside the nonrelativistic limit, note how the intrinsic magnetic field plays a prominent role in the angular regularization: when $g = 0$ in (2.29), both conical singularities can be eliminated only in the case of vanishing mass parameter ($m = 0$) or, trivially, in the case of vanishing acceleration ($A = 0$). The role of the intrinsic electromagnetic charges can of course be switched without changing the form of the metric, by an electromagnetic duality rotation, that is, embedding the dyonic black hole in an external electric field.

[7]While [23] refers exactly to the theory we are treating in this paper, i.e., Einstein-Maxwell gravity, Ref. [22] considers a slightly different coupling also involving a scalar dilaton. This could be the reason for the discordance.

[8]Also the seed metric (2.4) has axial deficit/excess angles, which can be quantified in the following computation just by turning off the external magnetic field: $B = 0$.

Therefore, the spacetime (2.26) implemented by the constraint (2.29) is completely regular, since the only remaining singularities—of curvature—are located inside the inner horizon at $r = 0$.

At spatial infinity, that is, for $\theta \to \pi$, $r \to A^{-1}$, the solution is not asymptotically Melvin, as the $e = 0$ case is. This is a typical feature of spinning magnetized black holes [24,25] . To visualize it, consider that the value of the electric field is not converging to zero, as it is in the Melvin universe.

In the case of null acceleration the metric describes a single spinning RN black hole embedded in an external magnetic field [17].[9]

III. INSTANTONIC PAIR CREATION

In order to interpret the solution as a black hole pair creation and evaluate its nucleation rate (as discussed in [2,3] and [4]), the (y, x) coordinates are always preferred to the spherical coordinates. They are related by the following transformation:

$$T = At, \qquad y = -\frac{1}{Ar}, \qquad x = \cos\theta.$$

In this set of coordinates the dyonic magnetized C-metric solution (2.26) becomes

$$ds^2 = \frac{|\Lambda(y,x)|^2}{A^2(x-y)^2}\left[G(y)dT^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)}\right] + \frac{G(x)[d\phi - \omega(y,x)dT]^2}{|\Lambda(y,x)|^2 A^2(x-y)^2}, \tag{3.1}$$

where

$$G(\xi) = (1-\xi^2)(1 + r_- A\xi)(1 + r_+ A\xi), \tag{3.2}$$

$$\Lambda(y,x) = 1 - Bx(g - ie) + \frac{B^2}{4}\left[\frac{G(x)}{A^2(x-y)^2} + (e^2+g^2)x^2\right], \tag{3.3}$$

$$\begin{aligned}
\omega(y,x) &= \frac{B^3 e(y-2x)}{2A^2(x-y)^2}G(y) - \frac{Bey}{2A^2}[4A^2 - B^2 + A^2B^2(e^2+g^2)] + \frac{B^3 emy^2}{A} + \frac{B^3 ey^3}{2}e(e^2+g^2) + \omega_0,\\
A_\varphi(y,x) &= \frac{\{gx - \frac{B}{2}[\frac{G(x)}{A^2(x-y)^2} + (e^2+g^2)x^2]\}\{1 - gxB + \frac{B^2}{4}[\frac{G(x)}{A^2(x-y)^2} + (e^2+g^2)x^2]\} - Be^2g^2}{|\Lambda(y,x)|^2} + k_\phi,\\
A_T(y,x) &= -\omega(y,x)\left[A_\phi(y,x) + \frac{3}{2B}\right] - 2ey + k_T.
\end{aligned} \tag{3.4}$$

In this new set of coordinates the nonaccelerating limit is not as explicit as it is in (2.26); also, the geometrical interpretation is clearer in spherical coordinates. On the other hand, it is clear that (3.1)–(3.4) are the generalization of the metric considered in [1] and [2], which can be obtained by vanishing the electric charge e. There is only a subtle difference in the parametrization of the polynomial $G(\xi) = -r_- r_+ A^2 \prod_{i=1}^4 (\xi - \xi_i)$, according to the insight of [18]; therefore, the roots do not always coincide. This means that the location of the horizons and range of the coordinates may differ. The angular coordinates are (x, ϕ), and in order for the metric to have a Lorentz signature we require $\xi_3 \le x \le \xi_4$ so that the sign of $G(x)$ is positive. Because of the conformal factor $1/(x-y)^2$ in the metric, the spatial (and conformal) infinity is reached by fixing t and letting both y and x approach ξ_3. The inner, event, and accelerating horizons are located at $y = \xi_1$, $y = \xi_2$, and $y = \xi_3$, respectively. The $x = \xi_3$ axis points towards spatial infinity, and the $x = \xi_4$ axis points towards the other black hole. Usually the constant k_ϕ is fixed in order to confine the Dirac string of the magnetic field to the axis $x = \xi_4$. This can be accomplished by fixing k_ϕ so that $A_\phi(x = \xi_3) = 0$.

Similarly, for the case of $e = 0$ [2], to ensure that the metric is free from conical singularities, we impose the following on both poles $x = \xi_3, \xi_4$:

$$G'(\xi_3)|\Lambda(\xi_4)|^2 = -G'(\xi_4)|\Lambda(\xi_3)|^2 \tag{3.5}$$

and

$$\Delta\phi_E = \frac{4\pi|\Lambda(\xi_3)|^2}{G'(\xi_3)}, \tag{3.6}$$

which are precisely equivalent to the constraints (2.28) and (2.29) we have previously obtained in spherical coordinates. Note that $\Lambda(\xi_i) \coloneqq \Lambda(x = \xi_i)$ are just constants.

The black hole pair production probability $|\Psi|^2$ is described[10]—according to the no-boundary Hartle-Hawking proposal—by the functional integral over all possible manifold topologies, metrics, and electromagnetic potentials interpolating between two boundary hypersurfaces Σ_1, Σ_2,

[9] For a notation similar to the one used here, see also Appendix A in [26], fixing $s = 0$.

[10] Up to a normalization factor.

$$\Psi_{12} = \int \mathcal{D}[\mathcal{M}]\mathcal{D}[g]\mathcal{D}[\mathcal{A}] \exp(-iI[\mathcal{M}, g, \mathcal{A}]). \quad (3.7)$$

The measure $\mathcal{D}[\mathcal{M}]\mathcal{D}[g]\mathcal{D}[\mathcal{A}]$ on the functional space is not well defined and, even if properly defined, it would be computationally impractical to handle. Fortunately, in analogy with the flat case, we can make use of a semiclassical simplifying assumption which relies on the existence of an instanton. An instanton is a Euclidean regular solution which interpolates between the initial (1) and final (2) states of a classically forbidden transition. It is a saddle point for the Euclidean path integral that describes the pair nucleation probability. The transition probability amplitude is well approximated, at the lowest order in the Planck length, by

$$\Psi_{12} \approx e^{-I_e}. \quad (3.8)$$

Therefore, to obtain the pair creation rate between the two black holes described by the C metric and their magnetic background, one has to build the instanton from (3.1). I_e is a real action evaluated on a Riemannian solution of the Einstein-Maxwell equations (2.2), which does not necessarily have to be real. For a rotating solution such as ours, we can choose to consider either a real or a complex instanton. It is argued in [8,15,27] that for this stationary pair production a complex instanton is more physical, because to enforce reality one must impose some imaginary charge parameters. This means, in that case, that the Euclidean and Lorentzian solutions do not properly match because the positions of the horizons are different, or worse, that some horizons may disappear because the number of real roots may change. In addition, neither electromagnetic charge nor angular momentum would be conserved in the pair production. However, the more problematic point is that the extrinsic curvature, the induced metric, and the induced electromagnetic field will not match on the spatial hypersurface joining the Euclidean and the Lorentzian solutions. The introduction of extra thin wall matter would be necessary to fix this issue. For these reasons, we will consider the possibility of having a complex instanton, provided that the action evaluated on this solution is real; thus, the creation probability remains real as well.

A standard way to generate the instanton (as in [2–4]) is to Euclideanize the Ernst solution (3.1) by setting $\tau = iT$ and then fixing the Euclidean period to regularize the conical singularity in the (y, τ) section. In [8] (see also [15]) it is shown that this is equivalent to requiring regularity to the extrinsic curvature K_{ij}, the induced metric h_{ij}, and the induced electromagnetic field (E_i, B_i) on the gluing spacelike hypersurfaces Σ_τ, defined by constant τ. In this $3+1$ foliation, the Euclidean spacetime takes the usual form

$$ds^2 = N^2 d\tau^2 + h_{ij}(dx^i + iV^i d\tau)(dx^j + iV^j d\tau), \quad (3.9)$$

where N and V^j are the lapse function and the shift vectors, which can depend only on (x, y) coordinates to respect axisymmetry. The prescription for the supporting electromagnetic field is given by

$$F_{tj} = i\tilde{F}_{tj}, \qquad F_{jt} = i\tilde{F}_{jt}, \qquad F_{jk} = \tilde{F}_{jk}. \quad (3.10)$$

Since the metric is complex, its signature is not clearly defined. Therefore we can adopt the meaning of the Euclidean from [8], if at any point x_0^α there exists a complex spatial-coordinate transformation $x^j = \tilde{x}^j - iV^j(x_0^\alpha)\tau$, that, absorbing the shift vector V^j, puts the metric in Euclidean diagonal form

$$ds^2|_{x=x_0} = N^2 d\tau^2 + h_{ij} d\tilde{x}^i d\tilde{x}^j. \quad (3.11)$$

We are interested in the lukewarm solution, which is defined as having the event and accelerating horizons not degenerate and at thermal equilibrium. Therefore we impose the surface gravity and the temperature are the same on both horizons $y = \xi_2$ and $y = \xi_3$. This can be implemented by further constraining the structure constants of the black hole to avoid the conical singularity on the (y, τ) section. This is done in two steps: first, fixing the period of the Euclidean time to be

$$\Delta\tau = \frac{4\pi}{G'(\xi_3)} \quad (3.12)$$

on the accelerating horizon $y = \xi_3$ and then requiring that this value coincides with the one of the event horizon in $y = \xi_3$, that is,

$$G'(\xi_2) = -G'(\xi_3). \quad (3.13)$$

These conditions on $G(\xi)$ are formally identical to the electrically neutral case (i.e., nonrotating) studied in [2]. The basic difference is that the $G(\xi)$ differs with respect to the $e = 0$ case, mainly in the horizon positions. In addition we have one more parameter (related to the intrinsic electric charge) to accomplish the regularity constraints (3.5),(3.6), (3.12), and (3.13). For this reason, in the pair creation process a more general black hole can be produced with respect to the static case [2].

From (3.13) one obtains

$$(\xi_4 - \xi_3)(\xi_3 - \xi_1) = (\xi_4 - \xi_2)(\xi_2 - \xi_1), \quad (3.14)$$

which, in the nondegenerate case $\xi_3 \neq \xi_2$, can be further simplified to

$$\xi_4 - \xi_3 = \xi_2 - \xi_1. \quad (3.15)$$

In terms of the physical parameters Eq. (3.15) means

$$A(e^2+g^2)=\sqrt{m^2-e^2-g^2}. \tag{3.16}$$

This condition further restricts the regularity constraint (2.29), and hence also the pair production process.

The resulting instanton has topology $S^2\times S^2-\{pt\}$, where the removed point is $y=x=\xi_3$. In the literature this instanton is interpreted as representing the creation, in an external magnetic field, of a pair of oppositely charged black holes which subsequently uniformly accelerate away from each other [2,4]. The two black holes are connected by a wormhole throat containing the event horizon, which is located at a finite proper distance from the wormhole mouth.

To compute the black hole pair creation rate, we need to evaluate the action (2.1) on the instanton we have just built and compare this with the value of the action on the background. Using the trace of the equation of motion (2.2) and using the Stokes theorem for the Maxwell term F^2, the action (2.1) can be recast, on shell, as a boundary term

$$I=-\frac{1}{8\pi G}\int_{\partial\mathcal{M}}d^3x\sqrt{h}[F^{\mu\nu}n_\mu A_\nu+\nabla_\mu n^\mu], \tag{3.17}$$

where n^μ is the normalized vector orthogonal to the boundary surface $y=x=\xi_3$. We explicitly evaluate the action at $y=x-\epsilon$ and then take the limit $\epsilon\to 0$; ϵ acts as a regularization. The non-null components of the unit outward pointing normal to the surface $y=x-\epsilon$ are

$$\begin{aligned} n^y&=-\frac{A(x-y)G(y)}{|\Lambda(y,x)|\sqrt{G(x)-G(y)}},\\ n^x&=-\frac{A(x-y)G(x)}{|\Lambda(y,x)|\sqrt{G(x)-G(y)}}, \end{aligned} \tag{3.18}$$

while the instantonic induced three-metric on the $y=x-\epsilon$ hypersurface is

$$d\hat{s}^2=\frac{|\Lambda(y,x)|^2}{A^2(x-y)^2}\left[-G(y)d\tau^2+\frac{G(x)-G(y)}{G(x)G(y)}dx^2\right] +\frac{G(x)[d\phi+i\omega(y,x)d\tau]^2}{|\Lambda(y,x)|^2A^2(x-y)^2}\bigg|_{y=x-\epsilon}. \tag{3.19}$$

According to the no-boundary proposal, the creation rate of the dyonic RN black hole pair with respect to the background (bkgr) is given by

$$\Gamma_{\frac{\text{dyon RN}}{\text{bkgr}}}\propto\frac{|\Psi_{\text{dyon RN}}|^2}{|\Psi_{\text{bkgr}}|^2}\propto \mathrm{e}^{-2(I_{\text{dyon RN}}-I_{\text{bkgr}})}. \tag{3.20}$$

Unfortunately in the case of rotating Ernst metrics, the behavior at infinity is not clear (see [25] for recent developments) so in our case evaluating the background contribution is problematic. At most we might speculate that the naive regularization carried out in [3], consisting in eliding the divergent term with the background contribution, also works in the rotating case.[11]

The analogy with the Schwinger electron-positron production (of charge $\pm\hat{e}$) in an external electric field ($\hat{E}$) is manifest when the intrinsic black hole electric charge e is null; therefore, the RN black hole couple is not spinning (the Ernst potentials and the instanton are real). In that case the boundary action (3.17), according to [14], reduces to the following [2,3]:

$$I=-\frac{1}{8\pi}\int_{\partial\mathcal{M}}d^3x\sqrt{h}e^{-\delta}\nabla_\mu(e^\delta n^\mu), \tag{3.21}$$

where $e^{-\delta}=\Lambda\frac{(y-\xi_1)}{(x-\xi_1)}$. Then, performing the trivial integrations over τ and ϕ, the action evaluated on the instanton becomes

$$I=-\frac{1}{8\pi}\Delta\tau\Delta\phi_E\int_{\xi_3}^{\xi_3+\epsilon}dx\frac{\sqrt{h}}{\sqrt{g}}e^{-\delta}\partial_\mu(e^\delta\sqrt{g}n^\mu)|_{y=x-\epsilon}. \tag{3.22}$$

Expanding in powers of ϵ and integrating (3.22) we obtain[12]

$$I=I_0+\frac{\Lambda^2(\xi_3)}{A^2G'(\xi_3)}\frac{\pi}{\xi_3-\xi_1}. \tag{3.23}$$

The first factor I_0 is divergent on the boundary, when $\epsilon\to 0$; but, by evaluating the pair production rate relative to the Euclideanized magnetic universe, this term is compensated by the background contribution. In fact, this can be checked by evaluating, up to the order $O(\epsilon)$, the action on the Melvin background, setting $r_\pm=0$ on the instanton metric. Finally, the production rate of a pair of nonspinning, magnetically charged RN black holes, in an external magnetic field background, with respect to the Melvin background is given by

$$\Gamma_{\frac{\text{RN}}{\text{Melvin}}}\propto\exp\left[\frac{-2\pi\Lambda^2(\xi_3)}{A^2G'(\xi_3)(\xi_3-\xi_1)}\right]. \tag{3.24}$$

In terms of the value of the magnetic field at infinity (which coincides with the Melvin background magnetic field) $\hat{B}=\frac{B}{2}\frac{G'(\xi_3)}{\sqrt{\Lambda(\xi_4)}}$, and the intrinsic physical magnetic charge

[11]Even though the result of [3] is eventually correct, some subtleties in this asymptotic regularization process have to be carefully considered, as analyzed in [2].

[12]Note that there seems to be a factor $1/2$ discrepancy in I_{Ernst} between Refs. [2] and [14], from the same authors, even though they claim to obtain the same result. The final result of Γ is obtained exactly by changing the definition of the pair creation rate, from [2] to [14], by a compensating factor of 2.

$$\hat{q} \coloneqq \frac{1}{4\pi}\int_\Sigma F_{\mu\nu}dx^\mu \wedge dx^\nu = \frac{1}{4\pi}\int_0^{\Delta\varphi_E} d\phi \int_{\xi_3}^{\xi_4} dx\partial_x A_\phi = g\frac{\xi_4-\xi_3}{G'(\xi_3)}\frac{\Lambda^{3/2}(\xi_3)}{\Lambda^{1/2}(\xi_4)}, \tag{3.25}$$

where Σ is any two-sphere surrounding the black hole horizon, Eq. (3.24) can be rewritten as

$$\Gamma_{\frac{\mathrm{RN}}{\mathrm{Melvin}}} \propto \exp\left[-4\pi\hat{q}^2\frac{(1-\hat{B}\,\hat{q})^2}{1-(1-\hat{q}\,\hat{B})^4}\right]. \tag{3.26}$$

Expanding (3.26) for small $\hat{q}\,\hat{B}$, we obtain a similar behavior with respect to the leading term of the Schwinger pair production $\pi m^2/\hat{e}\,\hat{E}$:

$$\Gamma_{\frac{\mathrm{RN}}{\mathrm{Melvin}}} \approx \exp\left[-\hat{q}^2\left(\frac{\pi}{\hat{B}\,\hat{q}}-\frac{\pi}{2}+\dots\right)\right]. \tag{3.27}$$

Therefore, even though the new C-metric parametrization introduced in [18] is not completely physically equivalent to the older one,[13] the pair creation rate remains the same as [1–3], at least in the nonrotating case.

IV. COMMENTS AND CONCLUSIONS

In this paper, by means of Ernst's solution generating technique, we generate a generalization of the Ernst metric describing a couple of accelerating intrinsically electrically and magnetically charged black holes in the presence of an external electromagnetic field. The main novelty, with respect to the only intrinsically magnetically charged case, consists in the fact that the presence of the electric charge embedded in an external magnetic field makes the Reissner-Nordstrom black hole pair rotate, due to the Lorentz force. Then, because of the presence of the external magnetic field, it is possible to regularize the conical singularity typical of these accelerating solutions. Therefore there is no need for a cosmic string or strut to provide the acceleration; rather, the acceleration is furnished by the external magnetic field.

The relevance of this result is that this is a completely regular, analytic, rotating, two black hole exact solution. To our knowledge, it represents the first example of this kind in the theory of pure Einstein-Maxwell general relativity.

From this metric an instantonic solution is built. It interpolates between the two classical states— the black hole pair, and its magnetic background. As a saddle point for the Euclidean path integral, it is used to describe, at the semiclassical level, the quantum nucleation probability between the two forbidden classical states. This is analogous to the Schwinger electron-positron pair creation in an external electric field. The instanton considered here is of a more general type compared to the usual one studied in the literature because it also includes the electric charge.

A better understanding of the asymptotic behavior may be useful to clarify the charges of the solution considered here and also to evaluate the contribution of the rotation on the pair creation rate.

It would also be interesting to extend this analysis starting, as a seed, with a more general black hole pair that includes rotation from the beginning, that is, an accelerating Kerr-Newman metric. It would, at the same time, generalize and unify both the Ernst family of solutions describing black holes embedded in an external (electro)magnetic field, and the Plebanski-Demianski family, which describes accelerating metrics. By including six physical parameters—mass, rotation, acceleration, external electromagnetic fields, and intrinsic electric and magnetic charges—it would represent the most generic physical black hole metric for electrovacuum general relativity.[14] Works in this direction are in progress.

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[13] In case of rotation, this coordinate system makes the accelerating Kerr black hole free from torsion singularities that generate closed timelike curves [28].

[14] We mean Maxwell electromagnetism coupled with pure general relativity without cosmological constant, because in presence of cosmological constant some symmetries, such as the Harrison transformation, are broken [29].

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