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Rotating AdS black hole stealth solution in $D = 3$ dimensions

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We show that the rotating asymptotically anti-de Sitter black hole solution of new massive gravity in three dimensions can support a static stealth configuration given by a conformally coupled scalar field. By static stealth configuration, we mean a nontrivial time-independent scalar field in which the energy-momentum tensor vanishes identically on the rotating black hole metric solution of new massive gravity. The existence of this configuration is rendered possible because of the presence of a gravitational hair in the black hole metric that prevents the scalar field from being trivial. In the extremality due to the gravitational hair, the stealth scalar field diverges at the horizon, as it occurs for the conformal scalar field of the Bocharova-Bronnikov-Melnikov-Bekenstein solution in four dimensions.

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I. INTRODUCTION

The fundamental tenet of general relativity is the manifestation of the curvature of spacetime produced by the presence of matter. This phenomenon is encoded through the Einstein equations that relate the Einstein tensor or any other gravity tensor $\mathcal{G}_{\mu\nu}$ to the energy-momentum tensor $T_{\mu\nu}$ that arises from the variation of the matter with respect to the metric,

$$\mathcal{G}_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{1}$$

Since the energy-momentum tensor depends explicitly on the metric, both sides of the equations must be solved simultaneously. However, one can ask if, for a fixed geometry solving the vacuum gravity equations, it is possible to find a matter source coupled to this spacetime that does not affect the geometry. Concretely, this problem consists in examining a particular solution of the Einstein equation (1) where both sides of the equation vanish, i.e.,

$$\mathcal{G}_{\mu\nu} = 0 = 8\pi G T_{\mu\nu}. \tag{2}$$

In three dimensions, such gravitationally undetectable solutions called "stealth configurations" have been obtained in [1] for a nonminimally coupled scalar field with the Banados-Teitelboim-Zanelli (BTZ) metric [2]. That is, the gravity action is described by the standard Einstein-Hilbert action with a negative cosmological constant, whereas the matter source is given by

$$S_M = -\int d^3x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{\xi}{2}R\Phi^2 + U(\Phi)\right], \tag{3}$$

where ξ denotes the nonminimal coupling parameter, R the scalar curvature, and $U(\Phi)$ a potential term. As shown in [1], a stealth configuration on the BTZ metric can be obtained for any value of ξ provided that the scalar field is nonstatic (time dependent) and the angular momentum of the BTZ metric is switched off. The same problem has also been considered in higher dimensions for the same stealth matter source but with a flat geometry [3]. Recently, black hole stealth configurations have been obtained with a nonminimal scalar field with a mass term, $U(\Phi) \propto \Phi^2$, and where the gravity side is described by the Einstein-Gauss-Bonnet gravity [4] or its Lovelock generalization [5].

Here, we consider the problem of three-dimensional stealth configurations for which the gravity side of the stealth equations (2) is given by the so-called new massive gravity [6]. This alternative three-dimensional gravity theory has raised a lot of interest in the last five years due to interesting properties. Indeed, this theory presents the advantage of being at the linearized level equivalent to the unitary Fierz-Pauli theory for free massive spin-2 gravitons. There exist examples of black hole solutions for the new massive gravity equations: an asymptotically AdS black hole [7], as well as a solution with a Lifshitz dynamical exponent $z = 3$ [8] and a warped AdS black hole [9]. The rotating version of the AdS black hole solution can easily be obtained by operating an improper boost in the $(t-\varphi)$-plane where t (respectively φ) stands for the time (respectively angular) coordinate and the resulting geometry turns out to be a rotating asymptotically AdS solution of new massive gravity [7]. In the present work, we show that this rotating solution of new massive gravity can support a conformal static stealth configuration given by a nonminimally and conformally coupled scalar field. By conformal stealth configuration, we mean that the solution only exists for $\xi = 1/8$ and for a potential $U \propto \Phi^6$, which

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are the two conditions that ensure that the matter action (3) enjoys the conformal symmetry. We clearly establish that the existence of this stealth configuration is rendered possible because of the presence of a gravitational hair in the black hole metric. Interestingly, we also show that in the extremal case due to the gravitational hair, the stealth scalar field diverges at the horizon as it occurs for the conformal scalar field of the Bocharova-Bronnikov-Melnikov-Bekenstein (BBMB) solution in four dimensions [10,11].

The plan of this article is as follows: In the next section, we give the stealth field equations (2) where the gravity side is described by the equations of new massive gravity and where the matter source part is given by the variation of (3). We also present the rotating asymptotically AdS solution of new massive gravity as obtained in Ref. [7]. Then, in order to gain clarity, we first derive the stealth configuration in the nonrotating case and present the extremal solution. In the last part of the section, we show that the derivation of the rotating stealth solution is quite similar to the one operated in the nonrotating case. Then, we present the rotating asymptotically AdS black hole stealth configuration as well as its extremal version. Finally, the last section is devoted to the conclusions and future works.

II. FIELD EQUATIONS AND STEALTH CONFIGURATIONS

As said in the Introduction, we consider as a gravity action the so-called new massive gravity action [6] given by

$$S_G = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left[R-2\lambda-\frac{1}{m^2}\left(R_{\mu\nu}R^{\mu\nu}-\frac{3}{8}R^2\right)\right], \tag{4}$$

whose associated field equations read

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\lambda g_{\mu\nu}-\frac{1}{2m^2}K_{\mu\nu}=0, \tag{5}$$

where we have defined

$$K_{\mu\nu}=2\Box R_{\mu\nu}-\frac{1}{2}\nabla_\mu\nabla_\nu R-\frac{1}{2}\Box R g_{\mu\nu}+4R_{\mu\alpha\nu\beta}R^{\alpha\beta}-\frac{3}{2}RR_{\mu\nu}-R_{\alpha\beta}R^{\alpha\beta}g_{\mu\nu}+\frac{3}{8}R^2g_{\mu\nu}.$$

It is easy to see that the field equation (5) admits solutions of constant curvature, that is, $R^{\mu\nu}_{\alpha\beta}=\Lambda\delta^{\mu\nu}_{\alpha\beta}$, with two different radii

$$\Lambda_\pm = 2m(m\pm\sqrt{m^2-\lambda}), \tag{6}$$

and these two radii coincide for

$$m^2=\lambda := -\frac{1}{2l^2}. \tag{7}$$

At the special point (7) where the theory admits a unique maximally symmetric solution, there exists a rotating asymptotically AdS black hole solution that is locally conformally flat [7]. The metric is given by

$$ds^2=-N(r)F(r)dt^2+\frac{dr^2}{F(r)}+r^2(d\varphi+N^\varphi(r)dt)^2, \tag{8}$$

where the structural metric functions N, N^φ, and F read

$$N(r)=\left[1+\frac{bl^2}{4H}\alpha\right]^2, \qquad N^\varphi(r)=-\frac{a}{2r^2}(4GM-bH)$$

$$F(r)=\frac{H^2}{r^2}\left[\frac{H^2}{l^2}+\frac{b}{2}(2-\alpha)H+\frac{b^2l^2}{16}\alpha^2-4GM(1-\alpha)\right],$$

and where the function H is defined by

$$H(r)=\left[r^2-2GMl^2\alpha-\frac{b^2l^4}{16}\alpha^2\right]^{\frac{1}{2}}.$$

In these expressions, the constant α is related to the rotation parameter a through

$$\alpha = 1-(1-a^2/l^2)^{1/2}. \tag{9}$$

Since a ranges from $-l\le a\le l$, the constant $\alpha\in[0,1]$ with $\alpha=0$ corresponds to the nonrotating case. Hence, this solution is described by two constants related to the mass M and the angular momentum $J=Ma$, whereas the constant $b\in\mathbb{R}$, which contributes to the expression of the mass, can be viewed as a sort of gravitational hair since there is no global charge associated with it [12,13]. Nevertheless, this gravitational hair with $b<0$ provides a negative lower bound for the mass, which is useful in order to deal with a physical stealth solution as shown below. We also note that for $b=0$, the solution reduces to the BTZ black hole [2]. The origin of this hair can also be explained as follows. If one looks for the pure quadratic gravity equations $K_{\mu\nu}=0$, a solution is given by [7]

$$ds^2=-(br-4GM)dt^2+\frac{dr^2}{(br-4GM)}+r^2d\varphi^2,$$

and hence the hair already appears. Adding the Einstein piece with a negative cosmological constant, the metric solution acquires the usual cosmological term to become $F(r)=r^2/l^2+br-4GM$.

We now investigate whether the black hole spacetime geometry (8) may accommodate a stealth configuration given by a nonminimally coupled scalar field. To be more precise, we are interested in finding a static nontrivial scalar field $\Phi=\Phi(r)$ and eventually a potential term such that the following equations,

$$G_{\mu\nu} + \lambda g_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} = 0 = T_{\mu\nu}, \tag{10a}$$

$$\Box \Phi = \xi R \Phi + \frac{dU}{d\Phi}, \tag{10b}$$

are satisfied on the rotating black hole background given by (8) at the special point (7). Here, $T_{\mu\nu}$ is the stress tensor associated with the variation of the matter action (3) and is given by

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma \Phi \partial^\sigma \Phi + U(\Phi)\right) + \xi(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu + G_{\mu\nu})\Phi^2. \tag{11}$$

We already know that the gravity part of the stealth equations (10a) is satisfied on the background (8) at the point (7), and so it only remains to solve the equations $T_{\mu\nu} = 0$. Indeed, the nonlinear Klein-Gordon–type equation (10b) is automatically satisfied as a consequence of the conservation of the energy-momentum tensor.

In order to gain clarity, we first present the details of the computations in the nonrotating case $a = 0$ (or equivalently $\alpha = 0$), and then we switch to the rotating case. For a vanishing rotation parameter $a = 0$, the following combination $T_t^t - T_\varphi^\varphi = 0$ yields a first-order differential equation for the scalar field in which the solution is given by

$$\Phi_\pm(r) = \frac{C}{\sqrt{\pm(-br + 8GM)}},$$

where C is an integration constant. Injecting this expression into the combination $T_r^r - T_\phi^\phi = 0$, one obtains the following constraint,

$$\frac{1}{4}\frac{(-r^2 - brl^2 + 4GMl^2)}{(-br + 8GM)^3 l^2} C^2 b^2 (8\xi - 1) = 0,$$

which is solved for $C = 0$, or $b = 0$, or $\xi = 1/8$. However, the first two options imply that the scalar field becomes constant, and hence, in order to satisfy the constraint with a nontrivial scalar field, the nonminimal coupling parameter must take its three-dimensional conformal value $\xi = 1/8$. Finally, the remaining independent equation given by the combination $T_t^t - T_r^r - T_\varphi^\varphi = 0$ allows us to express the potential term U as a local expression of the scalar field as

$$U(\Phi) = \eta \Phi^6, \tag{12}$$

where η is a constant. It is interesting to note that this form of the potential together with the coupling $\xi = 1/8$ are precisely those that ensure that the matter action (3) is conformally invariant. We then conclude that the static solutions of the stealth equations (10) on the nonrotating background (8) with $\alpha = 0$ require a conformal scalar field source and are given by

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\varphi^2,$$
$$F(r) = \frac{r^2}{l^2} + br - 4GM,$$
$$\Phi(r) = \left[\frac{16G^2M}{2\eta l^2}\left(M + \frac{b^2 l^2}{16G}\right)\right]^{\frac{1}{4}} \frac{1}{\sqrt{(-br + 8GM)}}. \tag{13}$$

It is evident that the existence of this stealth configuration is indicative of the presence of the gravitational hair b since in the limit $b = 0$ the scalar field becomes constant. This result is not in contradiction with the one in Ref. [1] where the authors showed that in the BTZ case, $b = 0$, the scalar field must be nonstatic. In fact, the results obtained in Ref. [1] together with the solution (13) clearly establish the correlation between the presence of the gravitational hair b and the possibility of having a nontrivial static stealth scalar field. The scalar field diverges at $r_d = 8GM/b$; however, for $b < 0$ and $M \geq -b^2 l^2/(16G)$, which are the lower bounds allowed for the mass M [12,13], this divergence is always hidden by the horizon r_+ and the scalar field is real outside of the horizon. This is not true for $b > 0$, where $r_d > r_+$, but in some sense, this situation is similar to the one of the BBMB solution where the scalar field diverges at the horizon [10,11].

In the extremal case, that is, for

$$M = -b^2 l^2/(16G), \tag{14}$$

the derivation of the stealth solution along the same line as before implies again that $\xi = 1/8$, but the potential must be zero. The resulting extremal stealth solution reads

$$ds^2 = -\left(\frac{r}{l} + \frac{bl}{2}\right)^2 dt^2 + \frac{dr^2}{(\frac{r}{l} + \frac{bl}{2})^2} + r^2 d\varphi^2, \tag{15a}$$

$$\Phi(r) = \frac{A}{\sqrt{(2r + bl^2)}}, \tag{15b}$$

where now the scalar field depends on an arbitrary constant A. Various comments can be made concerning this extremal solution. First, this latter can be obtained from the nonextremal one (13) by taking the limit $M \to -b^2 l^2/(16G)$ but at the same time $\eta \to 0$ such that $(M + b^2 l^2/(16G)) = \mathcal{O}(\eta)$. In doing so, the scalar field depends on an arbitrary constant. The occurrence of this arbitrary constant can be easily explained since, in the absence of the potential term, the energy-momentum tensor has a scaling symmetry $\Phi \to \Omega\Phi$, where Ω is an arbitrary constant. Hence, the presence of the arbitrary constant A is just a consequence of this symmetry. We also stress that the scalar field diverges at the horizon $r_+ = -\frac{bl^2}{2}$ as it occurs for the BBMB solution in four dimensions. This is intriguing in the sense that the BBMB solution shares some features with this extremal stealth configuration. Indeed, the BBMB solution is a solution of a static conformal scalar field in four dimensions without potential and whose metric is also extremal (the extremal Reissner-Nordstrom spacetime). The divergence of the BBMB scalar field at the horizon makes its physical interpretation and the problem of its stability a subject of debate [14,15]. A way of circumventing this problem is to introduce a cosmological constant, in which the effect is to precisely push this singularity behind the horizon, as has been done in Ref. [16]. Because of these two examples (the BBMB solution and the extremal stealth configuration), one is tempted to associate this pathology with the extremal character of the metric together with the conformal symmetry of the source.

Let us now consider the rotating case, $a \neq 0$, for which the different steps to obtain the stealth solution turn out to be analogous to those operating in the nonrotating case. Indeed, as before, the combination $T_t^t - T_\varphi^\varphi = 0$ permits us to obtain the expression of the scalar field, while injecting this form into the combination $T_r^r - T_\varphi^\varphi = 0$ yields a rather complicated constraint. This latter is satisfied and yields a nontrivial solution only in the case $\xi = 1/8$. Finally, the combination $T_t^t - T_r^r - T_\varphi^\varphi = 0$ allows us to express the potential term U, and one obtains again the conformal potential (12) while the remaining independent Einstein equation $T_\varphi^t = 0$, being proportional to the combination $T_t^t - T_\varphi^\varphi = 0$, is also satisfied. We end with the following rotating asymptotically anti-de Sitter stealth configuration given by the metric (8) together with the scalar field,

$$\Phi(r) = \left(\frac{256M^2G^2 + 16MGb^2l^2(\alpha+1) + b^4l^4\alpha}{2\eta l^2}\right)^{\frac{1}{4}} \frac{1}{\sqrt{(-b\sqrt{16r^2 - 32\alpha MGl^2 - b^2l^4\alpha^2} + b^2l^2\alpha + 32MG)}}, \tag{16}$$

where α is defined in (9). It is evident that in the vanishing rotation limit $a \to 0$ (or equivalently $\alpha \to 0$), this solution reduces to (13). Moreover, as before, we emphasize again that this is the presence of the gravitational hair b that prevents the scalar field from being trivial. In the rotating case, as discussed in [12], there are two notions of extremality to be considered. The "standard one" is reached for a particular relation between the angular momentum and the mass $J^2 = M^2l^2$, and in our case, this is translated to the condition $\alpha = 1$ (9); there is an extremality due to the presence of the gravitational hair as it already occurs in the nonrotating case (14). In the former situation, the limit $\alpha = 1$ in the expression (16) does not present any pathology, and the resulting scalar field is well defined at the horizon. However, for the extremal version due to the gravitational hair (14), one is forced to rederive the solution from the beginning, and as in the nonrotating case, one ends with a coupling $\xi = 1/8$ as well as a vanishing potential $U = 0$ while the scalar field is given by

$$\Phi(r) = \frac{A}{\sqrt{\pm(-b\sqrt{16r^2 + b^2l^4\alpha(2-\alpha)} + l^2b^2(\alpha-2))}},$$

where A is an arbitrary constant. In contrast with the standard extremal case, the scalar field diverges at the horizon, and hence, in some sense, the extremality due to the gravitational hair seems to be stronger than the standard one, as was also observed in [12]. We finally note that this extremal configuration can be obtained by taking the limits $M \to -b^2l^2/(16G)$ and $\eta \to 0$ such that $(M + b^2l^2/(16G)) = \mathcal{O}(\eta)$ in (16).

III. COMMENTS AND CONCLUSIONS

Here, we have shown that the rotating asymptotically anti-de Sitter black hole solution of new massive gravity in three dimensions can support a nontrivial static stealth configuration given by a nonminimally and conformally coupled scalar field. We have clearly established that this is the presence of the gravitational hair b that prevents the scalar field from being trivial. The extremal version of this stealth configuration presents the same pathology (namely the divergence of the scalar field at the horizon) as the BBMB solution in four dimensions.

There are many issues related to the present work that will be interesting to explore, but we would like to emphasize the thermodynamics issue. Indeed, since we have obtained a black hole solution, it is natural to wonder about the thermodynamics. However, in order to compute the mass, the temperature, and the entropy of this solution, we are faced with the following problem. In fact, one may note that the stealth equations (10) can be viewed as a particular solution of the field equations arising from the variation of the action

$$S = S_G + S_M, \tag{17}$$

where S_G is the new massive gravity action (4) and S_M is the source action (3). For simplicity, let us consider the nonrotating case. The temperature is given as in the free source case [12,13] by

$$T = \frac{1}{\pi l}\sqrt{GM + \frac{b^2 l^2}{16}}, \tag{18}$$

while the entropy $\mathcal{S}$ computed with the help of the Wald formula [17] yields

$$\mathcal{S} = \frac{2\pi l}{\sqrt{G}}\sqrt{M + \frac{b^2 l^2}{16G}} - \frac{\pi}{8}\sqrt{\frac{2GM}{\eta}}. \tag{19}$$

However, a simple computation shows that the product $Td\mathcal{S}$ is not a total derivative, and hence, we are faced with the problem that the first law is not satisfied unless there is some additional charge to be considered. It will be interesting to further explore the thermodynamic issue of the stealth solutions found here.

Other questions can be asked related to this work: for example, what is the precise role of the gravitational hair in the emergence of such a configuration? Also, we have been interested in looking only for a static stealth configuration that can be supported by the rotating solution of new massive gravity. From Ref. [1], we learn that in the BTZ case, the stealth scalar field configuration must be nonstatic and the angular momentum must be zero. We may ask whether there exists a nonstatic stealth configuration in the case of the rotating solution of new massive gravity. As an open question, it will be interesting to explore the physical meaning of this stealth configuration in the dual Conformal Field Theory.

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