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Increased infall velocities in galaxy clusters from solitonic collisions?

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Axionlike scalar fields and the Lane-Emden truncation of their periodic potential are analyzed as a model of dark matter halos. The apparent enhancement of infall velocities in merging clusters is intriguing: here it is tentatively explained via an intrinsic inelastic effect during relativistic soliton collisions.

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I. INTRODUCTION

Head-on collisions of massive galaxy clusters, like those occurring in the so-called *bullet cluster* [1] as well as the gigantic merger Abell 520 possibly containing a dark matter (DM) core [2], are a challenge for the cold dark matter (CDM) paradigm, because both the individual galaxies and the hot x-ray emitting interstellar gas are only partially anchored to the DM lumps.

Instead of SUSY-type WIMPs, we are probing axionlike particles [3–5] as dark matter candidates. Observations of astrophysical collisions such as merging clusters provide a valuable test of alternatives to CDM. In particular, we continue our investigation [6] whether or not two dark matter halos, pulling towards one another, can be modeled via soliton-type collisions without invoking [7] a dark energy (DE) mediating "fifth force." The (phase) displacement of two colliding solitons, to some extent, resembles the effects of such a hypothetical force not observed in the laboratory [8]. In cosmology, however, there are arguments [9] to ponder about a quintessencelike scalar field playing the role of DE.

Since solitons are rather stable entities, they behave effectively like colliding particles; i.e., after leaving the interaction region where they may deform due to a temporally inelastic mechanism studied here in a relativistic setting, they ultimately return to their original shapes and velocities. Since the DM distributions in the bullet cluster appear not to be affected during merging, we postulate an axionlike scalar component of galaxies and clusters and analyze its solitary wave behavior, or as in our previous two-dimensional toy model [6], its Lane-Emden (LE) truncation [10],

$$V_{\rm LE}(\phi) = \frac{m^2}{2}\phi^2(1-\chi\phi^4). \tag{1}$$

Recall that in quantum chromodynamics (QCD) axions of inertial mass m are self-interacting via the effective [11,12] periodic potential,

$$V(\phi) = \frac{m^4}{\lambda}\left[1-\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)\right] \simeq V_{\rm LE}(\phi) - \frac{\lambda}{4!}\phi^4 - \cdots. \tag{2}$$

Although globally unbounded due to $\chi = -\lambda^2/(360m^4)$, the LE truncation (1) has the advantage that, in three dimensions, it admits exact spherically symmetric solutions [10] which model quite well [13] DM halos of individual galaxies. More specifically, the LE equation provides us, in three dimensions, with the exact nonsingular [14] radial solution,

$$|\phi(r)| = \chi^{-1/4}\sqrt{\frac{A}{1+A^2r^2}}. \tag{3}$$

This is referred to as a metastable lump, cf. Fig. 12 of Ref. [6], which cannot easily be confounded with domain walls. More than a century ago, it was considered as a crude model for the density of the sun and a bit later for the distribution of stars in globular clusters.

Our previous proposal [15,16] that DM may be composed of a gas of "axion mini clusters" or mini axion stars has been recently adopted [17]. However, numerical simulations of pointlike objects would again run into a "cusp" in the density of the central core, which is in conflict with observations of low-surface brightness galaxies. Moreover, for such Bose-Einstein type condensates, one needs some self-interaction and, in view of the self-similarity of solitons, one would end up with much larger configurations resembling the lump-type halos we will consider here.

II. RELATIVISTIC KINEMATICS OF INELASTIC COLLISIONS

As is well known, the wave operator $\Box := \nabla\cdot\nabla - \partial^2/c^2\partial t^2$ is invariant under the standard Lorentz transformations [18], where c is the velocity of light in vacuum. Thus one can resolve the relativistically invariant semi-classical Klein-Gordon (KG) equation,

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$$\Box\phi = \frac{\partial V(\phi)}{\partial\phi}, \tag{4}$$

for a single scalar field $\phi = \phi(t, \vec{x})$ first in some rest frame. Then, we apply a Lorentz boost

$$\vec{x} \to \gamma(\vec{x} - \vec{u}t), \tag{5}$$

in order to get traveling wave solutions or "moving solitons" needed for studying their collisons. Here,

$$\gamma := 1/\sqrt{1 - \vec{\beta}\bullet\vec{\beta}} = \frac{D + D^{-1}}{2} \tag{6}$$

is the Lorentz factor, $\vec{u}$ the (phase) velocity of the solitary wave, $\vec{\beta} = \vec{u}/c$ the dimensionless relative velocity, and

$$D := \sqrt{\frac{c+u}{c-u}}, \qquad u = c\frac{D^2 - 1}{D^2 + 1}, \tag{7}$$

the relativistic Doppler shift. [19]

An inverse Lorentz boost is later denoted by

$$\begin{aligned}\zeta &:= \gamma(\tilde{x} + u\tilde{t}) \\ &= \frac{D^{-1} + D}{2}\left(\tilde{x} - \frac{1 - D^2}{1 + D^2}c\tilde{t}\right) \\ &= \xi D + \eta D^{-1}.\end{aligned} \tag{8}$$

In the last equation we have made use of the fact that, in two dimensions, solutions of the KG equation can conveniently be written in terms of light-cone coordinates

$$\xi := \frac{1}{2}(\tilde{x} + c\tilde{t}), \qquad \eta := \frac{1}{2}(\tilde{x} - c\tilde{t}). \tag{9}$$

After dividing by the Compton wave length $\lambda_{\mathrm{Compton}} = h/mc$ of a particle, i.e., $\tilde{x} := x/\lambda_{\mathrm{Compton}}$ and $\tilde{t} := t/\lambda_{\mathrm{Compton}}$, these coordinates become dimensionless.

Let us now consider inelastic collisions of two particles of mass m_1 and m_2, respectively, moving with velocities $\vec{u}_1$ and $\vec{u}_2$, as measured from a positive oriented frame. From the conservation of the relativistic energy $E = \gamma mc^2$ and relativistic momenta $\vec{p} = \gamma m\vec{u}$ in two dimensions, there results [20] the collinear velocity

$$U = \frac{m_1\gamma_1 u_1 + m_2\gamma_2 u_2}{m_1\gamma_1 + m_2\gamma_2} \tag{10}$$

of the (newly generated) composite particle of mass

$$M = \frac{m_1\gamma_1 + m_2\gamma_2}{\gamma(U)}. \tag{11}$$

Here the Lorentz factor (6) applies for different velocities u_1, u_2, and U.

For particles of initially equal masses $m_1 = m_2$, Eq. (10) degenerates to

$$\mathbb{B} := \frac{U}{c} = \frac{\gamma_1\beta_1 + \gamma_2\beta_2}{\gamma_1 + \gamma_2} \equiv \frac{1 + \beta_1\beta_2 - \gamma_1^{-1}\gamma_2^{-1}}{\beta_1 + \beta_2}. \tag{12}$$

This dimensionless common velocity, coinciding with Eq. (3.11) of Ref. [21], also arises in the multisoliton solutions. At times, $u_1 = -|\vec{u}_1|$ is adopted in the center of the collision frame. Thus, we can surmise that in some soliton collisions an intermediate state [22] occurs which is temporarily inelastic. Indirectly, this may affect all nonlinear waves [23] due to Doppler broadening of the shape of interacting solitons.

Quite generally, not only inelastic collisions of point masses but also the peaks of multisolitons are suffering from some "sticky" or inelastic interaction during merging. A long way after the collision, however, the only remnant from this intermediate state is a (phase-) shift,

$$\delta_i := \gamma_i^{-1}\ \ln|\mathbb{B}|, \tag{13}$$

of the centers of the asymptotic solitons. This leads to an "overtaking process" as has been noted before [21,24,25], without revealing, to our knowledge, its elementary relativistic origin.

For light pulses, there occurs a repulsion phenomenon by which the intensity in the central collision region is decreased during the overlap of the two solitons: this leads to a decrease of the effective refractive index and, hence, an ejection of the peaks from the central region. Again, a related phase shift is observed after the collision, cf., Ref. [26].

Here we will analyze this intriguing effect in the case of the Lane-Emden truncation.

III. SOLITON COLLISIONS

When several solitons collide, one expects that they recuperate their initial shapes and velocities after some time has passed. Remnants from crossing the scattering region may be the concomitant phase shifts or displacements of the centers of the individual solitary waves due the nonlinear, partially inelastic interaction.

For constructing multisolitons, the well-established Bäcklund transformation (BT), cf., Refs. [25,27,28], is employed, which may also bridge between different types of nonlinear equations.

Let us depart from the sine-Gordon (sG) equation [29] which in dimensionless light-cone coordinates (9) acquires the form

$$\theta_{\xi\eta} = \sin\theta. \tag{14}$$

In a moving frame, it has the exact kink solution,

$$\theta = 4C \arctan [\exp \gamma(\tilde{x} - u\tilde{t})], \tag{15}$$

for $C = 1$ and antikink for $C = -1$. Since its spatial derivative,

$$\theta_{\tilde{x}} = 2\gamma C \mathrm{sech}[\gamma(\tilde{x} - u\tilde{t})], \tag{16}$$

becomes localized and square integrable, its absolute value will facilitate a subsequent comparison with the scattering behavior of solitons or lumps regarded as Bose-Einstein condensates [30,31] of DM.

Due to Bianchi's permutability theorem of BTs, there results the "nonlinear superposition" principle

$$\tan [(\theta_3 - \theta_0)/4] = \mathbb{B} \tan [(\theta_1 - \theta_2)/4], \tag{17}$$

which allows us to construct algebraically multikink solutions of the sG equation *a la* Perring and Skyrme [32,33].

Our relativistic KG equation is not only Lorentz but also CPT invariant, where $C: \theta \to -\theta$ is the topological charge conjugation for a real scalar and $P: \vec{x} \to -\vec{x}$ and $T: t \to -t$ are space and time reflections, respectively. This will allow us to distinguish solitons from antisolitons.

Here we focus on the collision of two kinks (instead of a collision of a kink and its CP-odd antikink, as in Ref. [6]) and obtain from the trivial seed solution $\theta_0 = 0$ the exact solution,

$$\theta_{\mathrm{kk}} = 4 \arctan [K(\zeta_1, \zeta_2)], \tag{18}$$

where the kinetic factor

$$\begin{aligned} K(\zeta_1, \zeta_2) :&= \mathbb{B} \frac{\exp(\zeta_1) + C \exp(\zeta_2)}{\exp(\zeta_1 + \zeta_2) - C} \\ &= \frac{\exp(\zeta_1 + \gamma_1\delta_1) + C \exp(\zeta_2 + \gamma_2\delta_2)}{\exp(\zeta_1 + \zeta_2) - C} \\ &\simeq \exp(\zeta_1 + \gamma_1\delta_1) + C \exp(\zeta_2 + \gamma_2\delta_2) \end{aligned} \tag{19}$$

depends on the initial velocities. This is also known as Hirota's formula [34].

At large separations from the interaction region, cf. Fig. 1, the solution (18) clearly decouples asymptotically into a (noninteracting) kink-kink or kink-antikink pair [24] distinguished by the sign $C = \pm 1$ of the topological charge.

A. Approximate Bäcklund transformation via mapping

For the Lane-Emden equation of interest here, an exact auto-Bäcklund transformation has not yet been found. Instead a generalized transformation will serve as a guide in constructing a multilump solution. Consider the Lane-Emden potential (1) as a truncation of (2), then the corresponding nonlinear KG equation in light-cone coordinates simplifies to

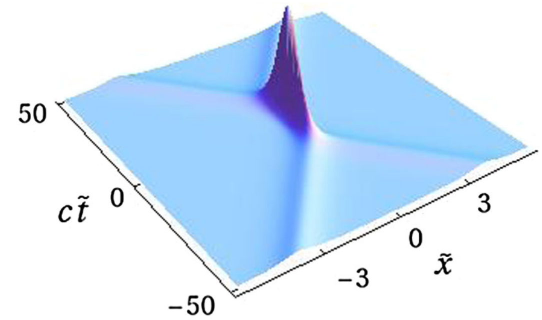

FIG. 1. (color online). Kink-kink collision monitored via the absolute value of its spatial derivative.

$$\phi_{\xi\eta} = \frac{\partial V}{\partial \phi} = \phi(1 - 3\chi\phi^4). \tag{20}$$

Integration leads us to the explicit exact solution

$$\phi = \frac{1}{\sqrt[4]{\chi}} \sqrt{\mathrm{sech}(2\zeta)}, \tag{21}$$

where an integration constant has been suppressed.

Due to the identity $\sin(2 \arctan y) = \mathrm{sech}(\ln y)$ for (hyperbolic) trigonometric functions, the Lane-Emden solution (21) is related to the kink (15) of the sG equation via the nonlinear mapping,

$$\varphi = \varphi(\theta) = \frac{1}{\sqrt[4]{\chi}} [\sin (\theta/2)]^{1/2} = \frac{1}{\sqrt[4]{\chi}} \sqrt{\mathrm{sech}(2\zeta)}. \tag{22}$$

This suggests that we generate a two-soliton solution,

$$\begin{aligned} \phi(\theta_{\mathrm{kk}}) \sqrt[4]{\chi} &= (\sin \{2 \arctan [K(\zeta_1, \zeta_2)]\})^{1/2} \\ &= (\mathrm{sech}\{\ln [K(\zeta_1, \zeta_2)]\})^{1/2}, \end{aligned} \tag{23}$$

of the LE equation via the same mapping, using again identities for trigonometric (hyperbolic) functions. Thus far, a more precise approximation is not yet available.

In the resulting spacetime diagram, the scattering of two solitons behaves as expected: after crossing the collision region, the individual solitons regain their original velocities, their trajectories are asymptotically the same as the initial ones, and the phase shift or displacement of the centers of the lumps is completely determined by Eq. (13). For solitons, some temporary "bouncing" of the center of a lump in the collision region occurs, cf. Fig. 2, whereas an antisoliton may "tunnel" through a solition during merging, cf. Ref. [6].

Thus interacting solitons behave more like "extended particles," where due to the inelastic effects discussed above, a continuous interchange of inertia (invariant mass) and interaction energy occurs. In Ref. [24], this non-Newtonian behavior is referred to as a local version of Mach's principle, inasmuch as the total inertia of a lump depends also on the relative motion of other solitons in its vicinity.

Although Eq. (23) is not an exact solution to the nonlinear KG equation (20), the same relativistic factor occurs in the LE truncation.

IV. DISCUSSION

In astrophysical scenarios, the Compton wave length of DM particles is of the order of the size of a galaxy, i.e., $\lambda_{\mathrm{Compton}} = h/mc \sim 10$ kpc, but above the Hubble scale of 10^{-31} eV. This naive estimate would lead to an ultralight mass of $m \simeq 10^{-26}$ eV, which is 20 orders of magnitude below the usual mass range $m_a \simeq \mu$eV of invisible axions. Another possibility is that, in a first stage, μeV axions collapse to mini-machos [16], which then form the halo via large DM clouds.

A mysterious central "dark core" [2] in the merging cluster Abell 520 presents another challenge to standard CDM (cf., however, Ref. [35]). In our tentative soliton-type model of DM, however, such a "sticky" behavior of DM can be associated with the temporarily inelastic scattering of two or more LE lumps close to the origin of Fig. 2. On the contrary, in the case of the bullet cluster, the DM lumps appear already well separated due to their higher mutual "infall" velocities, cf. Ref. [36].

The estimated collision velocities for the Bullet cluster and Abell 520 are 4700 km/s and 1066 km/s, respectively. The Musket ball clusters appear much older [37] and slower.

Cluster	Velocity U/c	Shift 2δ
Bullet	0.016	−8.3
Abell 520	0.0033	−11.4

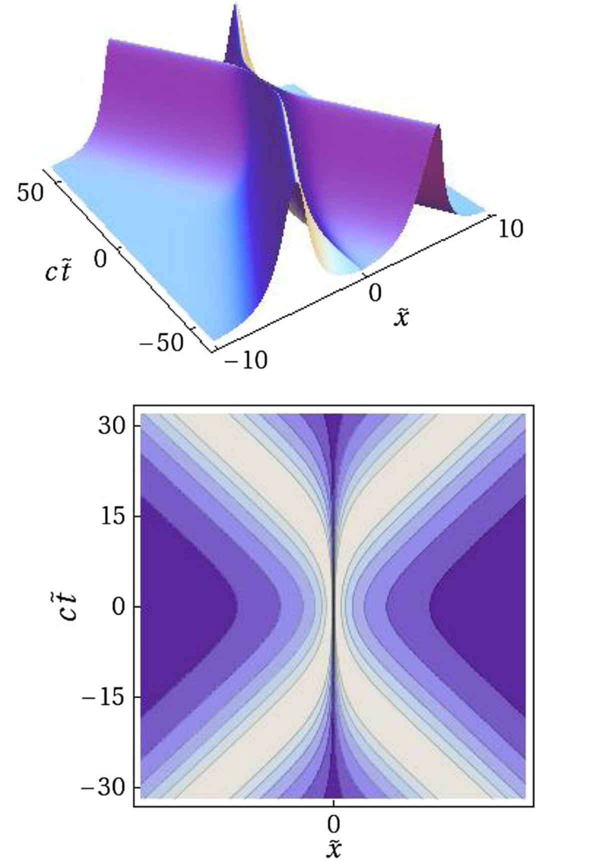

FIG. 2. (color online). Collision of two lump-type solitons in the Lane-Emden ϕ^6 model. The contour plot below amplifies the scattering region of our spacetime diagram.

Thus, the intrinsic mechanism [6,25] of temporarily "sticky" solitons during the non-Newtonian scattering process could, to some extent, explain the enhanced infall velocity of the bullet cluster, taking into consideration that the dimensionless displacement 2δ has to be multiplied by the Compton wave length $\lambda_{\mathrm{Compton}}$ of axionlike particles.

In the case of the galaxy cluster CL0024 + 17, one suspects that it collided with another cluster about a billion years ago, leaving a well-separated DM ring [38] resembling the angular momentum toruses [31,39] found in the three-dimensional LE equation.

As is well known [40], the LE truncation provides only metastable lumps, whose decay time $\tau \simeq \lambda_{\mathrm{Compton}}/c$ is proportional to the Compton wave length. Since this is shorter than the collision time as well as the age of the Universe, such lumps need to be stabilized via their self-generated gravity [41–43], similarly as in the case of (colliding) boson stars.

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