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Tachyon potentials from a supersymmetric FRW model

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Considering that the effective theory of closed string tachyons can have world sheet supersymmetry, as shown by Vafa, we study a worldline supersymmetric action in a Friedmann–Robertson–Walker background, for which the superpotential originates a tachyon scalar potential. There are such potentials with spontaneously broken supersymmetry at the instability and supersymmetry after tachyon condensation. Furthermore, given a tachyonic potential, the superpotential can be computed by a power series ansatz and has a free parameter that can be chosen such that complex solutions become real.

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I. INTRODUCTION

Many phenomena in physics are related to the transition or decay from an unstable state to another stable one. The corresponding evolution is described frequently by a tachyonic potential, with a "negative" mass term, like in the case of the Higgs potential or in the Landau– Ginzburg theory. String theory has in its lowest mode tachyons. However, the inclusion of supersymmetry is consistent because tachyons can be eliminated by the Gliozzi-Scherk-Olive truncation. Nevertheless, a better knowledge of string theory requires the understanding of the unstable configurations, for which the evolution can be described by the condensation of the tachyonic modes. The complexity of string theory has made this study rather difficult, and it was first performed in the somewhat simpler instance of open strings, resumed by the well-known Sen conjectures [1]. For closed strings the situation is more complicated, in particular because it involves the structure of spacetime, see e.g., [2]. An interesting fact in this case is that closed string tachyons, which are nonsupersymmetric in target space, can have worldsheet supersymmetry [3]. In this sense we address the question of supersymmetric tachyons in the simplified framework of a Friedmann–Robertson–Walker (FRW) background, with "worldline" local supersymmetry, i.e., the time variable is extended to the superspace of supersymmetry.

Supersymmetric quantum cosmology has been studied in various formulations. As usual for uniform spaces, it can be obtained as the "minisuperspace" reduction [4] of four-dimensional supergravity [5]; see also Ref. [6]. The Wheeler–deWitt equation is traced back to the "square root" of the Hamiltonian constraint, i.e., the supersymmetric charge constraints. Additionally to these constraints, there are also the Lorentz transformations of the fermionic fields [5], which strongly restrict the solutions for the wave function of the Universe [7]. An alternative supersymmetrization of these models has been given in Ref. [8], by a worldline one-dimensional superfield approach, where the time variable is extended to a (supersymmetry) superspace. It has been worked out for the Bianchi models [9] and matter has been included as well [8]; see also Ref. [10]. This formulation has the advantage that its one-dimensional supersymmetric structure is much simpler and does not require four- or higher-dimensional supergravity as a starting point. We follow an approach of this type, by means of the covariant formulation of one-dimensional supergravity, given by the so-called "new" Θ variables [13,14], which allows us in a systematic and straightforward way to write local supersymmetric invariant actions. One of the interesting features of the worldline superfield approach is that its fermionic sector has fewer degrees of freedom than the fermionic sector of the minisuperspace of four-dimensional supergravity. On the other side, the Lorentz constraints, which do not occur in the superfield approach, restrict the wave function of the Universe in such a way that it appears to have only two independent components [7], as in the case of the superfield approach [11].

The action we are considering contains two real scalars: one of them is the dilaton, and the other one has a tachyonic potential $V(T)$ [16], coupled to FRW supergravity. We formulate one-dimensional $N = 2$ superspace supergravity following Ref. [14], and the superfield form of the action is taken from Ref. [8]. The final action is obtained after a rescaling and the elimination of the auxiliary fields. For completeness we give also the Hamiltonian formulation, which closes consistently without further complications. As usual in supergravity, the superpotential is related to the scalar potential by a differential equation that is not positive definite. To solve this equation, we consider the case $k = 0$ and make an ansatz of separation of variables. We look for superpotentials corresponding to tachyonic potentials $V(T)$, in particular such that both supersymmetries are spontaneously

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broken at the maximum, and after condensation they are restored. We look also for a general solution of this differential equation by a power series ansatz. With this ansatz, depending on the potential, the superpotential can be complex, with complex values for quantities like the mass. However, there is a parameter that can be chosen in such a way that the solutions are real. In the second section of this work, we give our starting point bosonic action; in the third section, we give the one-dimensional superspace supergravity formulation; in the fourth section, we give the supersymmetric tachyon action; in the fifth section, the Hamiltonian is formulated; in the sixth section, we study the solutions for the superpotential; and in the last section, we sketch conclusions. There are three appendices, the first one on the new Θ-variables formulation; in the second one, there are details of the power series solutions; and in the third one, some computations on one of the solutions are given.

II. CLOSED STRING TACHYON EFFECTIVE ACTION

The closed string tachyon effective action in the bosonic sector is given according to Ref. [16] as

$$S = \frac{1}{2\kappa_D^2}\int \sqrt{-g}e^{-2\phi}[R + 4(\partial\phi)^2 - (\partial T)^2 - 2V(T)]d^D x, \tag{1}$$

where T is the closed string tachyon field, $V(T)$ is the tachyon potential, and ϕ is the dilaton field. This action can be written in the Einstein frame by means of $g_{\mu\nu}^{\rm string} = e^{\phi} g_{\mu\nu}^{\rm Einstein}$, which is more suitable for our cosmological approach. For a four-dimensional FRW metric and in the Einstein frame, the Lagrangian takes the form

$$S = \int \left[-\frac{3\dot{a}^2 a}{\kappa^2 N} + \frac{3Nka}{\kappa^2} + \frac{a^3}{\kappa^2 N}\dot{\phi}^2 + \frac{a^3}{2\kappa^2 N}\dot{T}^2 - \frac{a^3 N e^{2\phi} V(T)}{\kappa^2}\right] dt, \tag{2}$$

where N is the lapse function and a is the scale factor. This Lagrangian is invariant (up to a total derivative) under time reparametrizations of the form $t \to f(t)$. This invariance under time reparametrization is extended to supersymmetry by the introduction of a Grassmann superspace associated to the bosonic time coordinate t (see Tkach *et al.* in Ref. [8]).

As usual, the Hamiltonian of the bosonic theory has the form $H = NH_0$, where N is the lapse function. Then, the associated equation of motion $\partial H/\partial N = 0$ implies the first class constraint

$$H_0 = -\frac{\kappa^2}{12a}\pi_a^2 - \frac{3ka}{\kappa^2} + \frac{\kappa^2}{4a^3}\pi_\phi^2 + \frac{\kappa^2}{2a^3}\pi_T^2 + \frac{a^3 e^{2\phi} V(T)}{\kappa^2} = 0, \tag{3}$$

where π_i are the canonical momenta of the coordinates $i = a, T, \phi$.

III. SUPERSPACE SUPERGRAVITY

Superspace is the natural framework for a geometrical formulation of supersymmetry and supergravity [12]. It extends spacetime by anticommuting Grassmann variables, $x^m \to (x^m, \theta^\mu)$. The field content of the superfields is given by the Grassmann power expansion in the anticommuting variables $\phi(z) = \sum_n 1/n!\theta^{\mu_1}\cdots\theta^{\mu_n}\phi_{\mu_1\cdots\mu_n}(x)$. Supergravity is invariant under local supersymmetry transformations $\xi^m \to \xi^m(x), \xi^\mu \to \xi^\mu(x)$, where ξ^m are spacetime translations and ξ^μ supersymmetry transformation parameters. It can be generalized to superspace diffeomorphisms [13], $z^M \equiv (x^m, \theta^\mu) \to z'^M = z^M + \xi^M(z)$. This generalization actually amounts to introducing additional "superspace" gauge degrees of freedom corresponding to the θ components of the Grassmann expansion of $\xi^M(x,\theta)$. To formulate such a theory, the vierbein and spin connection are generalized to superspace tensors, the vielbein $E_M{}^A(z)$, and the superconnection $\phi_{MA}{}^B$, where $A = (a, \alpha)$ are local Lorentz indices and $M = (m, \mu)$ are superspace world indices [13]. If V_A is a Lorentz supervector, its covariant derivatives are $\mathcal{D}_A V_B = E_A{}^M(\partial_M V_A - \phi_{MB}{}^C V_C)$ and satisfy the graded (anti)-commutators $[\mathcal{D}_A, \mathcal{D}_B]_\pm V_C = -T_{AB}{}^D \mathcal{D}_D V_C - R_{ABC}{}^D V_D$, where $E_A{}^M(z)$ is the inverse vielbein and the torsion and curvature tensors satisfy the graded Bianchi identitites. For the construction of the Lagrangian, in Ref. [8] a somewhat *ad hoc* superspace formulation was proposed. Here we will use the "new" superspace formulation, see Apendix A, that follows from general superspace covariance, by a consistent elimination of the superspace gauge degrees of freedom [13–15], without requiring a gauge fixing. This parametrization corresponds to a field redefinition of the superfield components $\phi_{\mu_1\cdots\mu_n}(x) \to \mathcal{D}_{[\alpha_1}\cdots\mathcal{D}_{\alpha_n]}\phi(z)|_{\theta=0}$, and the superfields are given by $\Phi(x,\Theta) = e^{\Theta^\alpha \mathcal{D}_\alpha}\varphi(z)|_{\theta=0}$, where Θ are anticommuting Lorentz spinor variables. Full manifest covariance can be mantained by keeping the full old-θ dependence, $\Phi(z,\Theta) = e^{\Theta^\alpha \mathcal{D}_\alpha}\phi(z)$ and setting $\theta = 0$ at the end of the computations. In this formulation the supergravity multiplet contains the vierbein, the spin conection, and certain components of the curvature and torsion tensors, constrained by the Bianchi identities. In Appendix A this formulation is reviewed following Ref. [14]. Superfields transform by field dependent transformations (A10), and covariant derivatives can be defined consistently (A14) in such a way that there is a vielbein for which the superdeterminant is an invariant density (A19) and allows us to construct invariant supergravity actions. We use these results for a superspace formulation of one-dimensional supergravity. To set conventions, we first observe that simple one-dimensional supersymmetry has the Grassmannian variables θ and $\bar{\theta} = \theta^\dagger$. The integration properties for these variables are $\int d\theta = 0$, $\int \theta d\theta = 1$. Generic superfields are real and are given by

$\phi(t,\theta,\bar{\theta}) = A(t) + \theta\psi(t) - \bar{\theta}\bar{\psi}(t) + \theta\bar{\theta}B(t)$, where $A(t)$ and $B(t)$ are real and $\psi^\dagger = \bar{\psi}$. The representation of supersymmetric charges is $Q = \frac{d}{d\theta} - i\bar{\theta}\frac{d}{dt}$ and $\bar{Q} = -\frac{d}{d\bar{\theta}} + i\theta\frac{d}{dt}$, which satisfy $\{Q,\bar{Q}\} = 2i\frac{d}{dt}$; the covariant derivatives are $D = \frac{d}{d\theta} + i\bar{\theta}\frac{d}{dt}$ and $\bar{D} = -\frac{d}{d\bar{\theta}} - i\theta\frac{d}{dt}$. There are also chiral superfields that are complex and satisfy $\bar{D}\phi = 0$ and that in the chiral base have the expansion $\phi(t,\theta) = A(t) + \theta\psi(t)$.

Supergravity in one dimension, as well as gravity, is trivial, and there is no curvature tensor, and in a minimal version, the torsion tensor is the same as in flat superspace; i.e. its only nonvanishing component is $T_{\theta\bar{\theta}}{}^t = 2i$, consistent with the Bianchi identities, as can be easily verified by dimensional reduction from minimal supergravity in four dimensions [13] or by direct computation in one dimension. For the one-dimensional new superspace formulation, unlike Appendix A, we will denote $z^M = (t,\Theta,\bar{\Theta})$, and for simplicity we will omit the tildes. Thus, superfields $\Phi(z) = A(t) + \Theta\chi(t) - \bar{\Theta}\bar{\chi}(t) + \Theta\bar{\Theta}B(t)$ transform as

$$\delta_\xi\Phi(z) = \eta_\xi{}^M(z)\partial_M\Phi(z) \equiv \xi^M(x)\left[\eta^t_M(z)\frac{\partial\Phi(z)}{\partial t} + \eta^\theta_M(z)\frac{\partial\Phi(z)}{\partial\Theta} + \eta^{\bar{\theta}}_M(z)\frac{\partial\Phi(z)}{\partial\bar{\Theta}}\right]. \tag{4}$$

Thus, from Eq. (A12) it turns out that

$$\eta_\xi{}^t = -\xi - ie^{-1}(\Theta\bar{\zeta} + \bar{\Theta}\zeta) + \frac{1}{2}e^{-2}\Theta\bar{\Theta}(\bar{\zeta}\psi - \zeta\bar{\psi}), \tag{5}$$

$$\eta_\xi{}^\theta = -\zeta + \frac{i}{2}e^{-1}(\Theta\bar{\zeta} + \bar{\Theta}\zeta)\psi - \frac{1}{4}e^{-2}\Theta\bar{\Theta}\zeta\psi\bar{\psi}, \tag{6}$$

where the supersymmetry parameters are $\xi^M(x) = (\xi^t, \xi^\theta, \xi^{\bar{\theta}}) \equiv (\xi, \zeta, \bar{\zeta})$ and $\eta_\xi{}^{\bar{\theta}} = (\eta_\xi{}^\theta)^\dagger$. The resulting component transformations are

$$\delta_\xi A = -\xi\dot{A} - \zeta\chi + \bar{\zeta}\bar{\chi}, \tag{7}$$

$$\delta_\xi\chi = -\xi\dot{\chi} - ie^{-1}\bar{\zeta}\dot{A} + \frac{i}{2}e^{-1}\bar{\zeta}(\psi\chi - \bar{\psi}\bar{\chi}) + \bar{\zeta}B, \tag{8}$$

$$\delta_\xi\bar{\chi} = -\xi\dot{\bar{\chi}} + ie^{-1}\zeta\dot{A} - \frac{i}{2}e^{-1}\zeta(\psi\chi - \bar{\psi}\bar{\chi}) + \zeta B, \tag{9}$$

$$\delta_\xi B = -\xi\dot{B} - ie^{-1}(\zeta\dot{\chi} + \bar{\zeta}\dot{\bar{\chi}}) + \frac{1}{2}e^{-2}\zeta\bar{\psi}(\dot{A} + iB) - \frac{1}{2}e^{-2}\bar{\zeta}\psi(\dot{A} - iB) - \frac{1}{12}e^{-2}(\zeta\chi - \bar{\zeta}\bar{\chi})\psi\bar{\psi} \tag{10}$$

as well as one of the $\bar{\chi}$, obtained from Eq. (8) by complex conjugation. Further, the vielbein corresponding to transformations (4), which transforms as $\delta_\xi\nabla_M{}^A = \partial_M\eta^N_\xi\nabla_N{}^A + \eta^N_\xi\partial_N\nabla_M{}^A$, can be obtained from (A17)

$$\nabla_M{}^A = \begin{pmatrix} e + i(\Theta\bar{\Psi} + \bar{\Theta}\psi) & \frac{1}{2}\psi & \frac{1}{2}\bar{\psi} \\ -i\bar{\Theta} & 0 & -1 \\ i\Theta & 1 & 0 \end{pmatrix}, \tag{11}$$

and from its transformation law, we get

$$\delta_\xi e = -\frac{d}{dt}(\xi e) + i(\zeta\bar{\psi} + \bar{\zeta}\psi), \tag{12}$$

$$\delta_\xi\psi = -2\frac{d}{dt}\left(\zeta + \frac{1}{2}\xi\psi\right), \tag{13}$$

which can be verified to be consistent with the usual vielbein transformations [13]. The invariant density is obtained as usual from the superdeterminant $\mathcal{E} = \mathrm{Sdet}(\nabla_M{}^A)$ and transforms as $\delta_\xi\mathcal{E} = (-1)^m\partial_M(\xi^M\mathcal{E})$,

$$\mathcal{E} = -e - \frac{i}{2}(\Theta\bar{\Psi} + \bar{\Theta}\psi). \tag{14}$$

The inverse vielbein can be computed from Eq. (11), and from it we get the covariant derivatives that will be needed for the Lagrangian of the next section:

$$\nabla_\theta\Phi = \chi + ie^{-1}\bar{\Theta}\left[\dot{A} - \frac{1}{2}(\psi\chi - \bar{\psi}\bar{\chi}) - ieB\right] + e^{-1}\Theta\bar{\Theta}\left(-i\dot{\chi} - \frac{1}{2}e^{-1}\bar{\psi}\dot{A} - \frac{1}{4}e^{-1}\psi\bar{\psi}\chi + \frac{i}{2}\bar{\psi}B\right), \tag{15}$$

$$\nabla_{\bar{\theta}}\Phi = \bar{\chi} - ie^{-1}\Theta\left[\dot{A} - \frac{1}{2}(\psi\chi - \bar{\psi}\bar{\chi}) + ieB\right] + e^{-1}\Theta\bar{\Theta}\left(i\dot{\bar{\chi}} - \frac{1}{2}e^{-1}\psi\dot{A} - \frac{1}{4}e^{-1}\psi\bar{\psi}\bar{\chi} - \frac{i}{2}\psi B\right). \tag{16}$$

IV. SUPERSYMMETRY CLOSED TACHYON MODEL

The supersymmetric cosmological model is obtained upon an extension of the time coordinate into a supermultiplet $t \to (t,\Theta,\bar{\Theta})$. Thus, the fields of the model are generalized as superfields, and we write their expansions as

$$\begin{aligned} \mathcal{A}(t,\Theta,\bar{\Theta}) &= a(t) + i\Theta\bar{\lambda}(t) + i\bar{\Theta}\lambda(t) + B(t)\Theta\bar{\Theta}, \\ \mathcal{T}(t,\Theta,\bar{\Theta}) &= T(t) + i\Theta\bar{\eta}(t) + i\bar{\Theta}\eta(t) + G(t)\Theta\bar{\Theta}, \\ \Phi(t,\Theta,\bar{\Theta}) &= \varphi(t) + i\Theta\bar{\chi}(t) + i\bar{\Theta}\chi(t) + F(t)\Theta\bar{\Theta}, \end{aligned} \tag{17}$$

where, $\mathcal{A}$, $\mathcal{T}$, and Φ are the superfields of a, T, and ϕ.

The supersymmetric generalization of the action is given by

$$S = S_{R\mathrm{susy}} + S_{M\mathrm{susy}}, \tag{18}$$

where $S_{R\mathrm{susy}}$ is the cosmological supersymmetric generalization of the free FRW model,

$$S_{R\mathrm{susy}} = \int \left(\frac{3\mathcal{E}}{\kappa^2} \mathcal{A} \nabla_{\bar{\theta}} \mathcal{A} \nabla_{\theta} \mathcal{A} - \frac{3\sqrt{k}}{\kappa^2} \mathcal{E} \mathcal{A}^2 \right) d\Theta d\bar{\Theta} dt, \tag{19}$$

and the supersymmetric matter term is

$$S_{M\mathrm{susy}} = \frac{1}{\kappa^2} \int \left[-\mathcal{E}\mathcal{A}^3 \nabla_{\bar{\Theta}} \Phi \nabla_{\Theta} \Phi - \frac{1}{2} \mathcal{E}\mathcal{A}^3 \nabla_{\bar{\Theta}} \mathcal{T} \nabla_{\Theta} \mathcal{T} + \mathcal{E}\mathcal{A}^3 W(\Phi, \mathcal{T}) \right] d\Theta d\bar{\Theta} dt, \tag{20}$$

where $W(\Phi, \mathcal{T})$ is the superpotential. The superpotential expansion can be written as $W(\Phi, \mathcal{T}) = W(\phi, T) + \frac{\partial W}{\partial \varphi}(\Phi - \phi) + \frac{\partial W}{\partial T}(\mathcal{T} - T) + \frac{1}{2}\frac{\partial^2 W}{\partial T^2}(\mathcal{T} - T)^2 + \frac{1}{2}\frac{\partial^2 W}{\partial \phi^2}(\Phi - \phi)^2 + \frac{\partial^2 W}{\partial T \partial \phi}(\mathcal{T} - T)(\Phi - \phi)$. This expansion is finite because the terms $(\mathcal{T} - T)$ and $(\Phi - \phi)$ are purely Grassmannian. Upon integration over the Grassmann parameters, we find the supersymmetric cosmological Lagrangian in the gravity sector,

$$\begin{aligned} L_{FRW\mathrm{susy}} = & -\frac{3a\dot{a}^2}{e\kappa^2} + \frac{3ia}{\kappa^2}(\lambda\dot{\bar{\lambda}} - \dot{\lambda}\bar{\lambda}) + \frac{6e\sqrt{k}\lambda\bar{\lambda}}{\kappa^2} \\ & + \frac{3a\dot{a}}{e\kappa^2}(\psi\lambda - \bar{\psi}\bar{\lambda}) + \frac{3ia\sqrt{k}}{\kappa^2}(\psi\lambda + \bar{\psi}\bar{\lambda}) \\ & - \frac{3aB^2e}{\kappa^2} + \frac{6aBe\sqrt{k}}{\kappa^2} - \frac{3Be\lambda\bar{\lambda}}{\kappa^2} - \frac{3a}{2e\kappa^2}\lambda\bar{\lambda}\psi\bar{\psi}; \end{aligned}$$

the dilaton sector of the matter Lagrangian is given by

$$\begin{aligned} L_{\phi\mathrm{susy}} = & \frac{a^3\dot{\phi}^2}{e\kappa^2} + \frac{a^3\dot{\phi}}{e\kappa^2}(\bar{\psi}\bar{\chi} - \psi\chi) + \frac{ia^3}{\kappa^2}(\dot{\chi}\bar{\chi} - \chi\dot{\bar{\chi}}) \\ & + \frac{3ia^2\dot{\phi}}{\kappa^2}(\lambda\bar{\chi} + \bar{\lambda}\chi) + \frac{a^3eF^2}{\kappa^2} + \frac{3a^2eF}{\kappa^2}(\lambda\bar{\chi} - \bar{\lambda}\chi) \\ & - \frac{3a^2Be\chi\bar{\chi}}{\kappa^2} + \frac{a^3\chi\bar{\chi}\psi\bar{\psi}}{2e\kappa^2} - \frac{6ae\lambda\bar{\lambda}\chi\bar{\chi}}{\kappa^2}, \end{aligned}$$

while for the tachyonic sector, we find

$$\begin{aligned} L_{T\mathrm{susy}} = & \frac{a^3\dot{T}^2}{2e\kappa^2} + \frac{a^3\dot{T}}{2e\kappa^2}(\bar{\psi}\bar{\eta} - \psi\eta) + \frac{ia^3}{2\kappa^2}(\dot{\eta}\bar{\eta} - \eta\dot{\bar{\eta}}) \\ & + \frac{3ia^2\dot{T}}{2\kappa^2}(\lambda\bar{\eta} + \bar{\lambda}\eta) + \frac{a^3eG^2}{2\kappa^2} + \frac{3a^2eG}{2\kappa^2}(\lambda\bar{\eta} - \bar{\lambda}\eta) \\ & - \frac{3a^2Be\eta\bar{\eta}}{2\kappa^2} + \frac{a^3\eta\bar{\eta}\psi\bar{\psi}}{4e\kappa^2} - \frac{3ae\lambda\bar{\lambda}\eta\bar{\eta}}{\kappa^2} \end{aligned}$$

and the superpotential term

$$\begin{aligned} L_W = & -\frac{6aeW\lambda\bar{\lambda}}{\kappa^2} - \frac{3ia^2W}{2\kappa^2}(\bar{\psi}\bar{\lambda} + \psi\lambda) - \frac{3a^2BeW}{\kappa^2} \\ & + \frac{3a^2eW_T}{\kappa^2}(\bar{\lambda}\eta - \lambda\bar{\eta}) - \frac{ia^3W_T}{2\kappa^2}(\bar{\psi}\bar{\eta} + \psi\eta) \\ & - \frac{a^3eGW_T}{\kappa^2} + \frac{3a^2eW_\phi}{\kappa^2}(\bar{\lambda}\chi - \lambda\bar{\chi}) \\ & - \frac{ia^3W_\phi}{2\kappa^2}(\bar{\psi}\bar{\chi} + \psi\chi) - \frac{a^3eFW_\phi}{\kappa^2} - \frac{a^3e\chi\bar{\chi}W_{\phi\phi}}{\kappa^2} \\ & + \frac{a^3eW_{T\phi}}{\kappa^2}(\bar{\chi}\eta - \chi\bar{\eta}) - \frac{a^3e\eta\bar{\eta}W_{TT}}{\kappa^2}. \end{aligned}$$

Thus, the total Lagrangian is

$$\begin{aligned} L = & -\frac{3a\dot{a}^2}{e\kappa^2} + \frac{3a\dot{a}}{e\kappa^2}(\psi\lambda - \bar{\psi}\bar{\lambda}) + \frac{a^3\dot{T}^2}{2e\kappa^2} - \frac{a^3\dot{T}}{2e\kappa^2}(\psi\eta - \bar{\psi}\bar{\eta}) + \frac{3ia^2\dot{T}}{2\kappa^2}(\lambda\bar{\eta} + \bar{\lambda}\eta) + \frac{a^3\dot{\phi}^2}{e\kappa^2} - \frac{a^3\dot{\phi}}{e\kappa^2}(\psi\chi - \bar{\psi}\bar{\chi}) \\ & + \frac{3ia^2\dot{\phi}}{\kappa^2}(\lambda\bar{\chi} + \bar{\lambda}\chi) + \frac{3ia}{\kappa^2}(\lambda\dot{\bar{\lambda}} + \bar{\lambda}\dot{\lambda}) - \frac{ia^3}{2\kappa^2}(\eta\dot{\bar{\eta}} + \bar{\eta}\dot{\eta}) - \frac{ia^3}{\kappa^2}(\chi\dot{\bar{\chi}} + \bar{\chi}\dot{\chi}) + \frac{6e\sqrt{k}\lambda\bar{\lambda}}{\kappa^2} + \frac{3ia\sqrt{k}}{\kappa^2}(\psi\lambda + \bar{\psi}\bar{\lambda}) - \frac{6aeW\lambda\bar{\lambda}}{\kappa^2} \\ & - \frac{3ia^2W}{2\kappa^2}(\psi\lambda + \bar{\psi}\bar{\lambda}) - \frac{ia^3W_T}{2\kappa^2}(\psi\eta + \bar{\psi}\bar{\eta}) + \frac{3a^2eW_T}{\kappa^2}(\bar{\lambda}\eta - \lambda\bar{\eta}) - \frac{ia^3W_\varphi}{2\kappa^2}(\psi\chi + \bar{\psi}\bar{\chi}) + \frac{3a^2eW_\phi}{\kappa^2}(\bar{\lambda}\chi - \lambda\bar{\chi}) \\ & - \frac{a^3eW_{TT}}{\kappa^2}\eta\bar{\eta} + \frac{a^3eW_{T\phi}}{\kappa^2}(\bar{\chi}\eta - \chi\bar{\eta}) - \frac{a^3eW_{\phi\phi}}{\kappa^2}\chi\bar{\chi} - \frac{3a\psi\bar{\psi}\lambda\bar{\lambda}}{2e\kappa^2} + \frac{a^3\psi\bar{\psi}\eta\bar{\eta}}{4e\kappa^2} + \frac{a^3\chi\bar{\chi}\psi\bar{\psi}}{2e\kappa^2} - \frac{3ae\lambda\bar{\lambda}\eta\bar{\eta}}{\kappa^2} - \frac{6ae\lambda\bar{\lambda}\chi\bar{\chi}}{\kappa^2} - \frac{3aB^2e}{\kappa^2} \\ & + \frac{6aBe\sqrt{k}}{\kappa^2} - \frac{3Be\lambda\bar{\lambda}}{\kappa^2} - \frac{3a^2Be\eta\bar{\eta}}{2\kappa^2} - \frac{3a^2Be\chi\bar{\chi}}{\kappa^2} - \frac{3a^2BeW}{\kappa^2} + \frac{a^3eG^2}{2\kappa^2} + \frac{3a^2eG}{2\kappa^2}(\lambda\bar{\eta} - \bar{\lambda}\eta) - \frac{a^3eGW_T}{\kappa^2} + \frac{a^3eF^2}{\kappa^2} \\ & + \frac{3a^2eF}{\kappa^2}(\lambda\bar{\chi} - \bar{\lambda}\chi) - \frac{a^3eFW_\phi}{\kappa^2}, \end{aligned}$$

where the subscripts in W denote partial differentiation with respect to ϕ and T, respectively. When we perform the variation of the Lagrangian with respect to the fields B, F and G, as usual the following algebraic constraints are obtained:

$$B = \sqrt{k} - \frac{aW}{2} - \frac{1}{2a}\lambda\bar{\lambda} - \frac{1}{4}a\eta\bar{\eta} - \frac{1}{2}a\chi\bar{\chi},$$
$$G = W_T - \frac{3}{2a}(\lambda\bar{\eta} - \bar{\lambda}\eta),$$
$$F = \frac{W_\phi}{2} - \frac{3}{2a}(\lambda\bar{\chi} - \bar{\lambda}\chi); \tag{21}$$

that is B, F, and G play the role of auxiliary fields, and they can be solved and eliminated from the Lagrangian. When we solve for the auxiliary fields and make the rescalings $\lambda \to \kappa a^{-1/2}\lambda$, $\bar{\lambda} \to \kappa a^{-1/2}\bar{\lambda}$, $\eta \to \kappa a^{-3/2}\eta$, $\bar{\eta} \to \kappa a^{-3/2}\bar{\eta}$, $\chi \to \kappa a^{-3/2}\chi$, $\bar{\chi} \to \kappa a^{-3/2}\bar{\chi}$, we find the Lagrangian

$$\begin{aligned} L = & -\frac{3a\dot{a}^2}{e\kappa^2} + \frac{3\sqrt{a}\dot{a}}{e\kappa}(\psi\lambda - \bar{\psi}\bar{\lambda}) + \frac{3eka}{\kappa^2} + \frac{\dot{T}^2a^3}{2e\kappa^2} - \frac{\sqrt{a^3}\dot{T}}{2e\kappa}(\psi\eta - \bar{\psi}\bar{\eta}) + \frac{3i\dot{T}}{2}(\lambda\bar{\eta} + \bar{\lambda}\eta) + \frac{\dot{\phi}^2a^3}{e\kappa^2} - \frac{\sqrt{a^3}\dot{\phi}}{e\kappa}(\psi\chi - \bar{\psi}\bar{\chi}) \\ & + 3i\dot{\phi}(\lambda\bar{\chi} + \bar{\lambda}\chi) + 3i(\lambda\dot{\bar{\lambda}} + \bar{\lambda}\dot{\lambda}) - \frac{i}{2}(\eta\dot{\bar{\eta}} + \bar{\eta}\dot{\eta}) - i(\chi\dot{\bar{\chi}} + \bar{\chi}\dot{\chi}) + \frac{3e\sqrt{k}\lambda\bar{\lambda}}{a} - \frac{3e\sqrt{k}\eta\bar{\eta}}{2a} - \frac{3e\sqrt{k}\chi\bar{\chi}}{a} + \frac{3i\sqrt{ak}}{\kappa}(\psi\lambda + \bar{\psi}\bar{\lambda}) \\ & + \frac{3eW^2a^3}{4\kappa^2} - \frac{3e\sqrt{k}a^2W}{\kappa^2} - \frac{eW_T^2a^3}{2\kappa^2} - \frac{eW_\phi^2a^3}{4\kappa^2} - \frac{9}{2}eW\lambda\bar{\lambda} + \frac{3}{4}eW\eta\bar{\eta} + \frac{3}{2}eW\chi\bar{\chi} - \frac{3ia^{3/2}W}{2\kappa}(\psi\lambda + \bar{\psi}\bar{\lambda}) \\ & - \frac{i\sqrt{a^3}W_T}{2\kappa}(\psi\eta + \bar{\psi}\bar{\eta}) + \frac{3eW_T}{2}(\bar{\lambda}\eta - \lambda\bar{\eta}) - \frac{i\sqrt{a^3}W_\phi}{2\kappa}(\psi\chi + \bar{\psi}\bar{\chi}) + \frac{3eW_\phi}{2}(\bar{\lambda}\chi - \lambda\bar{\chi}) - eW_{TT}\eta\bar{\eta} + eW_{T\phi}(\bar{\chi}\eta - \chi\bar{\eta}) \\ & - eW_{\phi\phi}\chi\bar{\chi} + \frac{3e\kappa^2}{4a^3}\eta\bar{\eta}\chi\bar{\chi} - \frac{3}{2e}\psi\bar{\psi}\lambda\bar{\lambda} + \frac{1}{2e}\psi\bar{\psi}\chi\bar{\chi} + \frac{1}{4e}\psi\bar{\psi}\eta\bar{\eta}. \end{aligned}$$

Substituting the equations of motion of the auxiliary fields (21) into the supersymetry transformation of the fermions λ, η, and χ from Eq. (8), we get $\delta_\zeta\lambda = \bar{\zeta}(\sqrt{k} - aW/2) + \cdots$, $\delta\eta = \bar{\zeta}W_T + \cdots$, and $\delta\chi = \bar{\zeta}W_\phi + \cdots$. Therefore, if any of the fields on the rhs of these equations has a nonvanishing vacuum expectation value, the corresponding fermion is a Goldstino, and supersymmetry is broken. In the case of λ, the breaking can be due to the cosmological constant or to a nonvanishing W. In fact, if the superpotential has the form $W \sim e^\phi f(T)$, as in the examples in the next section, then $W_\phi = W$; i.e., χ contributes to the Goldstino if $W \neq 0$.

V. HAMILTONIAN ANALYSIS

The canonical momenta are

$$\pi_a = -\frac{6a\dot{a}}{e\kappa^2} - \frac{3\sqrt{a}\bar{\psi}\bar{\lambda}}{e\kappa} + \frac{3\sqrt{a}\psi\lambda}{e\kappa},$$
$$\pi_T = \frac{a^3\dot{T}}{e\kappa^2} + \frac{\sqrt{a^3}\bar{\psi}\bar{\eta}}{2e\kappa} - \frac{\sqrt{a^3}\psi\eta}{2e\kappa} + \frac{3a^{3/2}i\lambda\bar{\eta}}{2\sqrt{a^3}} + \frac{3a^{3/2}i\bar{\lambda}\eta}{2\sqrt{a^3}},$$
$$\pi_\phi = \frac{2a^3\dot{\phi}}{e\kappa^2} + \frac{\sqrt{a^3}\bar{\psi}\bar{\chi}}{e\kappa} - \frac{\sqrt{a^3}\psi\chi}{e\kappa} + \frac{3a^{3/2}i\lambda\bar{\chi}}{\sqrt{a^3}} + \frac{3a^{3/2}i\bar{\lambda}\chi}{\sqrt{a^3}},$$
$$\pi_\lambda = -3i\bar{\lambda}, \pi_{\bar{\lambda}} = -3i\lambda,$$
$$\pi_\eta = \frac{i}{2}\bar{\eta}, \pi_{\bar{\eta}} = \frac{i}{2}\eta,$$
$$\pi_\chi = i\bar{\chi}, \pi_{\bar{\chi}} = i\chi.$$

As usual, we can see the appearance of the fermionic constraints

$$\Omega_\lambda = \pi_\lambda + 3i\bar{\lambda}, \quad \Omega_{\bar{\lambda}} = \pi_{\bar{\lambda}} + 3i\lambda,$$
$$\Omega_\eta = \pi_\eta - \frac{i}{2}\bar{\eta}, \quad \Omega_{\bar{\eta}} = \pi_{\bar{\eta}} - \frac{i}{2}\eta,$$
$$\Omega_\chi = \pi_\chi - i\bar{\chi}, \quad \Omega_{\bar{\chi}} = \pi_{\bar{\chi}} - i\chi. \tag{22}$$

According to the Dirac formalism, the previous constraints are second class, and the dynamics of the system is obtained when we impose the set of constraints (22) and introduce the Dirac brackets, and we obtain

$$\{a, \pi_a\}_D = 1,\ \{\varphi, \pi_\varphi\}_D = 1,\ \{T, \pi_T\}_D = 1,$$
$$\{\lambda, \bar{\lambda}\}_D = -\frac{1}{6i},\ \{\chi, \bar{\chi}\}_D = -\frac{i}{2},\ \{\eta, \bar{\eta}\}_D = -i. \tag{23}$$

Using the standard definition for the Hamiltonian and imposing the constraints (22), we can write the Hamiltonian of the theory as

$$H = NH_0 + \frac{1}{2}\psi S - \frac{1}{2}\bar{\psi}\bar{S}, \tag{24}$$

where

$$
\begin{aligned}
H_0 = &-\frac{\kappa^2\pi_a^2}{12a} + \frac{\kappa^2\pi_T^2}{2a^3} - \frac{3i\kappa^2\pi_T}{2a^3}(\lambda\bar{\eta} + \bar{\lambda}\eta) + \frac{\kappa^2\pi_\phi^2}{4a^3} \\
&- \frac{3i\kappa^2\pi_\phi}{2a^3}(\lambda\bar{\chi} + \bar{\lambda}\chi) - \frac{3a^3}{4\kappa^2}W^2 + \frac{3\sqrt{k}a^2}{\kappa^2}W - \frac{3ka}{\kappa^2} \\
&+ \frac{a^3}{2\kappa^2}W_T^2 + \frac{a^3}{4\kappa^2}W_\phi^2 + \frac{9}{2}W\lambda\bar{\lambda} - \frac{3}{4}W\eta\bar{\eta} - \frac{3}{2}W\chi\bar{\chi} \\
&+ \frac{3}{2}W_T(\lambda\bar{\eta} - \bar{\lambda}\eta) + \frac{3}{2}W_\phi(\lambda\bar{\chi} - \bar{\lambda}\chi) + W_{TT}\eta\bar{\eta} \\
&+ W_{T\phi}(\chi\bar{\eta} - \bar{\chi}\eta) + W_{\phi\phi}\chi\bar{\chi} - \frac{3\sqrt{k}}{a}\lambda\bar{\lambda} + \frac{3\sqrt{k}}{2a}\eta\bar{\eta} \\
&+ \frac{3\sqrt{k}}{a}\chi\bar{\chi} - \frac{9\kappa^2}{2a^3}\lambda\bar{\lambda}\chi\bar{\chi} - \frac{9\kappa^2}{4a^3}\lambda\bar{\lambda}\eta\bar{\eta} - \frac{3\kappa^2}{4a^3}\eta\bar{\eta}\chi\bar{\chi},
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
S = &\frac{\kappa\pi_a}{\sqrt{a}}\lambda + \frac{\kappa\pi_T}{\sqrt{a^3}}\eta + \frac{\kappa\pi_\phi}{\sqrt{a^3}}\chi - \frac{6i\sqrt{ak}}{\kappa}\lambda + \frac{3i\sqrt{a^3}}{\kappa}W\lambda \\
&+ \frac{i\sqrt{a^3}}{\kappa}W_T\eta + \frac{i\sqrt{a^3}}{\kappa}W_\phi\chi + \frac{3i\kappa}{2a^{3/2}}\lambda\eta\bar{\eta} + \frac{3i\kappa}{a^{3/2}}\lambda\chi\bar{\chi},
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
\bar{S} = &\frac{\kappa\pi_a}{\sqrt{a}}\bar{\lambda} + \frac{\kappa\pi_T}{\sqrt{a^3}}\bar{\eta} + \frac{\kappa\pi_\phi}{\sqrt{a^3}}\bar{\chi} + \frac{6i\sqrt{ak}}{\kappa}\bar{\lambda} - \frac{3i\sqrt{a^3}}{\kappa}W\bar{\lambda} \\
&- \frac{i\sqrt{a^3}}{\kappa}W_T\bar{\eta} - \frac{i\sqrt{a^3}}{\kappa}W_\phi\bar{\chi} - \frac{3i\kappa}{2a^{3/2}}\bar{\lambda}\eta\bar{\eta} - \frac{3i\kappa}{a^{3/2}}\bar{\lambda}\chi\bar{\chi}
\end{aligned} \tag{27}
$$

satisfy the Dirac algebra $\{S, \bar{S}\}_D = 2H_0$, $\{H_0, S\}_D = \{H_0, \bar{S}\}_D = 0$.

VI. SUPERPOTENTIAL SOLUTIONS

From H_0, Eq. (25), we identify the scalar potential

$$U(a, \phi, T) = -\frac{3k}{a^2} + e^{2\phi}V(T), \tag{28}$$

which is related to the superpotential $W(\phi, T)$ by

$$U = -\frac{3W^2}{4} + \frac{3\sqrt{k}}{a}W - \frac{3k}{a^2} + \frac{1}{4}\left(\frac{\partial W}{\partial\varphi}\right)^2 + \frac{1}{2}\left(\frac{\partial W}{\partial T}\right)^2. \tag{29}$$

The form of the scalar potential (28) of a FRW geometry is consistent with $k = 0$; hence, we restrict ourselves to this geometry, and we get the equation

$$e^{2\phi}V(T) = -\frac{3W^2}{4} + \frac{1}{4}\left(\frac{\partial W}{\partial\varphi}\right)^2 + \frac{1}{2}\left(\frac{\partial W}{\partial T}\right)^2. \tag{30}$$

This suggests to us a separation of variables of the form $W(\varphi, T) = \frac{1}{\sqrt{2}}e^\phi f(T)$. With this ansatz we obtain the relation between the tachyon potential and the tachyonic component of the superpotential,

$$(f')^2 - f^2 = V(T), \tag{31}$$

where the prime denotes differentiation with respect to T. To find solutions to this equation, we must fix the function $V(T)$. For example, if $V(T) = 0$, the solution is $f(T) = e^T$. Further, for

$$V(T) = \frac{m^2}{2}\left(-T^2 + \frac{1}{4}T^4\right), \tag{32}$$

which according to the analysis of Zwiebach *et al.* produces a big crunch scenario as the final state of the Universe [16], there is an imaginary solution $f(T) = imT^2/(2\sqrt{2})$, for which the superpotential is

$$W(T) = \frac{i}{4}me^\phi T^2, \tag{33}$$

which generates complex fermion masses. However, as we show in Appendix B, there is also a real solution given by an infinite power series, which can be written as (B5)

$$
\begin{aligned}
W(T) = \frac{1}{\sqrt{2}}e^\phi\Bigg\{&e^T - \frac{1}{12}m^2T^3\Bigg[1 - \frac{1}{2}T \\
&+ \frac{1}{20}\left(1 + \frac{3}{2}m^2\right)T^2 + \mathcal{O}(T^3)\Bigg]\Bigg\}.
\end{aligned} \tag{34}
$$

Other proposals are potentials of the form $V(T) = \exp(\nu T)$ [16]; they are known to prevent the tachyon from reaching infinity in certain cases; with $\nu \geq 2$ there is no initial (positive) tachyon velocity for which the tachyon can reach $T = \infty$. For this potential we find $f(T) = \pm 2(\nu^2 - 4)^{-1/2}\exp(\nu T/2)$, and the superpotential is in this case

$$W(\phi, T) = \pm\frac{2}{\sqrt{\nu^2 - 4}}\exp\left(\phi + \frac{\nu}{2}T\right), \quad \nu \neq 2. \tag{35}$$

Another interesting proposal is, for instance,

$$W(\phi, T) = e^\phi f(T) = \frac{ie^\phi[(T - \tau)(\tau + T) + 2]}{2\tau^2}. \tag{36}$$

As in the case of Eq. (33) this superpotential generates complex fermion masses; however, it can be made to be real in complete analogy with Eq. (34). The tachyon potential corresponding to Eq. (36) is

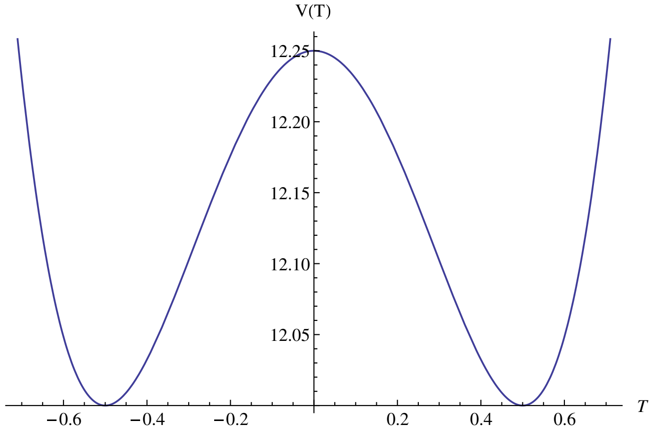

FIG. 1 (color online). Tachyon potential V(T) from Eq. (37) for $\tau = 1/2$.

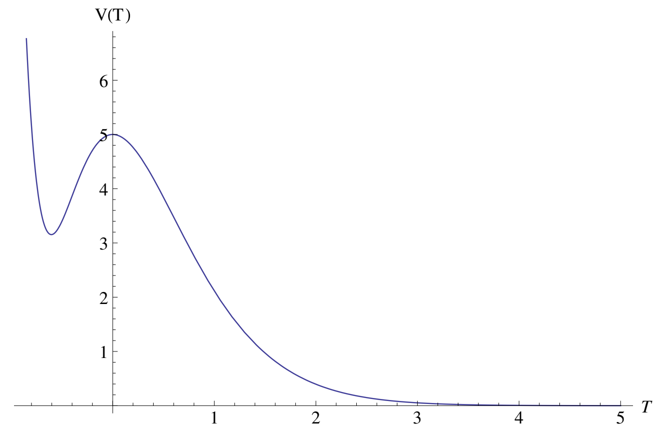

FIG. 2 (color online). Tachyon potential V(T) produced by $f(T)$ in Eq. (C1) for $a = 10$, $n = 3$, and $V_0 = 5$.

$$V(T) = \frac{1}{4\tau^4}[(\tau^2 - 2)^2 + T^4 - 2\tau^2 T^2]. \tag{37}$$

This potential, shown in Fig. 1, has a maximum at $T = 0$, with $V(0) = 1/4 - 1/\tau^2 + 1/\tau^4$, and minima at $T = \pm\tau$, with $V(\tau) = V(-\tau) = -1/\tau^2 + 1/\tau^4$; it also holds that $V(0) - V(\pm\tau) = 1/4$. Thus, if we let $\tau \to \infty$, the potential difference will remain the same, as in the case of Sen's conjectures. If we compute the superpotential corresponding to Eq. (37) following Appendix B, it can be shown that $W_T(\phi, 0) = e^{\phi}\sqrt{4(1-\tau^2) + \tau^4(A^2 - 1)}/\tau^2$, where $A = f(0)$. Further, at the minimum, i.e., at $T = \tau$, we make the power expansion around this point, and we get $W_T(\phi, \tau) = e^{\phi}\sqrt{4(1-\tau^2) + \tau^4(B^2 - 2)}/\tau^2$, where $B = f(\tau)$, which in the limit $\tau \to \infty$ gives $W_T(\phi) = e^{\phi}\sqrt{B^2 - 2}$, which can be set to zero if in this limit $B = \sqrt{2}$.

An interesting question regards potentials with suitable supersymmetric properties, of the type of Sen conjectures. For instance, the potential

$$V(T) = \exp(-nT)[\alpha_0 + \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3 + \alpha_4 T^4]. \tag{38}$$

In this case we have $f(T) = (a + bT + cT^2)\exp(-nT/2)$, from which we obtain

$$W(T, \phi) = (a + bT + cT^2)\exp(\phi - nT/2); \tag{39}$$

the explicit coefficients α_i depend on the free parameters a, n, and V_0. In fact we demand the presence of a maximum for $T = 0$, and this provides us with two equations that can be solved for b and c in Eq. (39), and with the condition $V(0) < 0$, details of the calculations are given in Appendix C; see Fig. 2. At the maximum of this potential, $W_T(0, \phi) \neq 0$ and $W_\phi(0, \phi) \neq 0$; hence, supersymmetry is broken. Further, after condensation supersymmetry is restored because $W_T \to 0$ and $W_\phi \to 0$, when $T \to \infty$.

VII. CONCLUSIONS

We have studied a worldline supersymmetric theory in a FRW background, with a closed string tachyon. We have constructed the action in the formalism of the new Θ variables in one dimension, which allows us to systematically construct supergravity actions. For given tachyonic potentials, we considered the solutions of the differential equation of the superpotential. We show that there are solutions with broken supersymmetry at the unstable, tachyonic, configuration, and supersymmetric at the stable minimum. Furthermore, the superpotentials can have simple forms but correspond to complex fermionic masses. These superpotentials can be obtained as well by a power series ansatz, for which the general solution depends on a real parameter that can be chosen such that the complex solutions can be mapped to real solutions. Potentials like these have been considered in cosmological models, e.g. in Ref. [16,19] where inflationary and big crunch scenarios are given, and it would be interesting to consider supersymmetric versions, in particular their quantization [20].

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APPENDIX A: SUPERSPACE SUPERGRAVITY

In this appendix we shortly review the new superspace formulation of supergravity following Ref. [14]. Superfields are defined as

$$\phi(z) \to \Phi(z) = e^{\Theta^\alpha \mathcal{D}_\alpha}\phi(z)|_{\theta=0}, \tag{A1}$$

where Θ are anticommuting Lorentz $[SL(2,C)]$ covariant spinorial variables. To ensure full covariance for these new superfields, the whole "old" superspace can be kept, setting $\theta = 0$ at the end of the computations, i.e., $\Phi(z,\Theta) = e^{\Theta^\alpha \mathcal{D}_\alpha}\phi(z)$.

The preceding redefinition of superfields is complemented by the usual redefinition of local supersymmetry transformations in such a way that Lorentz covariance is kept. The way is to add a local Lorentz transformation to the local superspace translations as follows [17]:

$$\delta_\xi \phi_A(z) = -\xi^B E_B{}^M(\partial_M \phi_A - \phi_{MA}{}^B \phi_B) = -\xi^B \mathcal{D}_B \phi_A(z); \tag{A2}$$

hence, the new superfields, for which the components are Lorentz covariant, transform as $\delta_\xi \Phi(z,\Theta) = -\xi^A \mathcal{D}_A \Phi(z,\Theta)$, i.e.,

$$\delta_\xi \Phi_A(z,\Theta) = -\xi^B \mathcal{D}_B \phi_A - \Theta^\beta \xi^B \mathcal{D}_B \mathcal{D}_\beta \phi_A - \frac{1}{2}\Theta^{\beta_1}\Theta^{\beta_2}\xi^B \mathcal{D}_B \mathcal{D}_{\beta_1}\mathcal{D}_{\beta_2}\phi_A + \cdots \tag{A3}$$

The computation of this expression is done taking into account the fact that the multiple covariant derivatives arising from the exponential in Eq. (A1) appear as fully antisymmetrized products. Thus, when a further derivative is applied on this product, the result must be antisymmetrized, e.g.,

$$\mathcal{D}_\alpha \mathcal{D}_\beta \phi_A = \frac{1}{2}\{\mathcal{D}_\alpha, \mathcal{D}_\beta\}\phi_A + \frac{1}{2}[\mathcal{D}_\alpha, \mathcal{D}_\beta]\phi_A = -T_{\alpha\beta}{}^C \mathcal{D}_C \phi_A - R_{\alpha\beta A}{}^B \phi_B + \mathcal{D}_{[\alpha}\mathcal{D}_{\beta]}\phi_A, \tag{A4}$$

where the last term is precisely the second-order term of $\Phi(z,\Theta)$. Following these lines it can be shown that Eq. (A3) can be cast into the form

$$\delta_\xi \Phi_A(z,\Theta) = \eta_\xi{}^\alpha(z,\Theta)\frac{\partial}{\partial\Theta^\alpha}\Phi_A(z,\Theta) + \eta_\xi{}^a(z,\Theta)\mathcal{D}_a \Phi_A(z,\Theta) + \eta_{\xi A}{}^B(z,\Theta)\Phi_B(z,\Theta), \tag{A5}$$

where the coefficients $\eta_\xi{}^A(z,\Theta)$ and $\eta_{\xi A}{}^B(z,\Theta)$ depend on components of the curvature and torsion tensors and their covariant derivatives.

To have a geometric formulation in the new superspace, following the Wess–Zumino gauge [18], which eliminates the gauge degrees of freedom introduced by the generalization of local supersymmetry to superspace diffeomorphisms, a new vielbein is introduced. Let us consider a vector field, $V_m = E_m{}^a V_a + E_m{}^\alpha V_\alpha$; this relation can be inverted to

$$V_a = E^{(-1)m}{}_a(V_m - E_m{}^\alpha V_\alpha) = \tilde{E}_a^{\tilde{M}} V_{\tilde{M}}, \tag{A6}$$

where the indices $\tilde{M} \equiv (m,\alpha)$ contain a spacetime world index and a spinorial local index, i.e., $\tilde{E}_a^{\tilde{M}} = (\tilde{E}_a^m, \tilde{E}_a^\alpha) \equiv (E^{(-1)m}{}_a, -E^{(-1)m}{}_a E_m{}^\alpha)$. With this definition and $\tilde{E}_\alpha^{\tilde{M}} = \delta_\alpha^{\tilde{M}}$, an inverse vielbein $\tilde{E}_A^{\tilde{M}}$ can be defined. The corresponding vielbein is then

$$\tilde{E}_{\tilde{M}}{}^B = \begin{pmatrix} E_m{}^b & E_m{}^\beta \\ 0 & \delta_\alpha{}^\beta \end{pmatrix},$$

i.e., $\tilde{E}_A^{\tilde{M}}\tilde{E}_{\tilde{M}}{}^B = \delta_A^B$ and $\tilde{E}_{\tilde{M}}{}^A \tilde{E}_A^{\tilde{N}} = \delta_{\tilde{M}}^{\tilde{N}}$. Even if this vielbein seems to correspond to the Wess–Zumino gauge, full covariance can be kept by considering certain components of the torsion and curvature as independent degrees of freedom of the supergravity multiplet. If we define in this basis covariant derivatives as usual by $\mathcal{D}_A V_{\tilde{M}} = (-1)^{(m+b)a}\tilde{E}_{\tilde{M}}{}^B \mathcal{D}_A V_B$ and $\mathcal{D}_A V^{\tilde{M}} = \mathcal{D}_A V^B \tilde{E}_B^{\tilde{M}}$, then

$$[\mathcal{D}_{\tilde{M}}, \mathcal{D}_{\tilde{N}}]_\pm V_A = -T_{\tilde{M}\tilde{N}}{}^{\tilde{P}}\mathcal{D}_{\tilde{P}} V_A - R_{\tilde{M}\tilde{N}A}{}^B V_B, \tag{A7}$$

which supplemented by the corresponding Bianchi identities contains all the information of supergravity. As Eq. (A7) does not contain derivatives of the old θ variables, the different levels in the θ expansion decouple, and the limit $\theta^\mu = 0$ does not require gauge fixing. Actually, supergravity transformations can be written as $\delta_\xi \Phi(z,\Theta) = -\tilde{\xi}^{\tilde{M}}\tilde{\mathcal{D}}_{\tilde{M}}\Phi(z,\Theta)$, and following the same lines as for Eq. (A5), we get

$$\delta_\xi \Phi(z,\Theta) = \left[\tilde{\eta}_\xi{}^\alpha(z,\Theta)\frac{\partial}{\partial\Theta^\alpha} + \tilde{\eta}_\xi{}^m(z,\Theta)\mathcal{D}_m\right]\Phi(z,\Theta). \tag{A8}$$

Further, the covariant derivative $\mathcal{D}_m$ on the rhs of this expression acts on the components of the superfield $\Phi(z,\Theta)$ as in Eq. (A3), which can be written as

$$\mathcal{D}_m \Phi(z,\Theta) = \partial_m \phi + \Theta^\beta(\partial_m - \omega^\gamma_{m\beta})\mathcal{D}_\gamma \phi - \frac{2}{2}\Theta^{\beta_1}\Theta^{\beta_2}(\partial_m - \omega^\gamma_{m\beta_1})\mathcal{D}_\gamma \mathcal{D}_{\beta_2}\phi + \cdots = \partial_m \Phi(z,\Theta) + \Theta^\beta \omega^\gamma_{m\beta}\partial_\gamma \Phi(z,\Theta). \tag{A9}$$

Therefore, including a Lorentz index, Eq. (A8) can be written as

$$\delta_\xi \Phi_A(z,\Theta) = \hat{\eta}_\xi{}^{\tilde{M}}(z,\Theta)\partial_{\tilde{M}}\Phi_A(z,\Theta) + \hat{\eta}_{\xi A}{}^B(z,\Theta)\Phi_B(z,\Theta), \tag{A10}$$

where $\hat{\eta}^\alpha_\xi = \tilde{\eta}^\alpha_\xi - \tilde{\eta}^m_\xi \Theta^\beta \phi^\alpha_{m\beta}$, $\hat{\eta}_\xi{}^m = \tilde{\eta}_\xi{}^m$ and $\hat{\eta}_{\xi A}{}^B$ is Lie algebra valued. Further, $\tilde{\eta}_\xi{}^{\tilde{M}} = \tilde{\xi}^{\tilde{N}}\tilde{\eta}_{\tilde{N}}{}^{\tilde{M}}$ and $\tilde{\eta}_m{}^{\tilde{N}} = \delta_m{}^{\tilde{N}}$,

$\tilde{\eta}_{mD}{}^{B} = 0$ and $\tilde{\eta}'^{\tilde{N}}_{\alpha} = \tilde{\eta}^{\tilde{N}}_{\alpha} - \delta^{\tilde{N}}_{\alpha}$ can be obtained from the recursion relation

$$\left(1 + \Theta^{\beta}\frac{\partial}{\partial\Theta^{\beta}}\right)\tilde{\eta}'^{\tilde{N}}_{\alpha} = \Theta^{\beta}\mathcal{D}_{\beta}\tilde{\eta}^{\tilde{N}}_{\alpha} + \Theta^{\alpha_1}(-\Theta^{\alpha_2}R_{\alpha_2\alpha\alpha_1}{}^{\tilde{N}} + T_{\alpha\alpha_1}{}^{\tilde{L}}\tilde{\eta}^{\tilde{N}}_{\tilde{L}}) + \tilde{\eta}^{m}_{\alpha}\Theta^{\alpha_1}(-\Theta^{\alpha_2}R_{\alpha_2 m\alpha_1}{}^{\tilde{N}} + T_{m\alpha_1}{}^{\tilde{L}}\tilde{\eta}^{\tilde{N}}_{\tilde{L}}) - \tilde{\eta}_{\alpha}{}'^{\gamma}\tilde{\eta}_{\gamma}{}'^{\tilde{N}} \tag{A11}$$

and a similar one for $\tilde{\eta}_{\xi A}{}^{B}$. It turns out that

$$\tilde{\eta}_{\xi}{}^{\tilde{M}}(z,\Theta) = -\tilde{\xi}^{\tilde{M}} + \Theta^{\gamma}\left(\frac{1}{2}\tilde{\xi}^{\beta}T_{\gamma\beta}{}^{\tilde{M}} + \tilde{\xi}^{n}\phi_{n\gamma}{}^{\tilde{M}}\right) - \frac{1}{2}\Theta^{\gamma}\Theta^{\delta}\left(-\frac{2}{3}\tilde{\xi}^{\beta}R_{\delta\beta\gamma}{}^{\tilde{M}} + \frac{1}{3}\mathcal{D}_{\delta}T_{\gamma\beta}{}^{\tilde{M}} + T_{\delta\beta}{}^{\tilde{n}}\phi_{n\gamma}{}^{\tilde{M}} - \frac{1}{3}T_{\delta\beta}{}^{\tilde{n}}T_{n\gamma}{}^{\tilde{M}} + \frac{1}{6}T_{\delta\beta}{}^{\tilde{\epsilon}}T_{\epsilon\gamma}{}^{\tilde{M}}\right) + \cdots \tag{A12}$$

Consistently with these ideas, covariant derivatives can be defined as

$$\nabla_{A} = e^{\Theta^{\alpha}\mathcal{D}_{\alpha}}\mathcal{D}_{A}e^{-\Theta^{\alpha}\mathcal{D}_{\alpha}}. \tag{A13}$$

Following a similar reasoning as that which lead to Eq. (A10), it can be shown that

$$\nabla_{A}\Phi_{B} = \nabla_{A}{}^{\tilde{M}}\partial_{\tilde{M}}\Phi_{B} + \nabla_{AB}{}^{C}\Phi_{C}, \tag{A14}$$

where $\nabla_{A}{}^{\tilde{M}}$ is the inverse vielbein of the new superspace, and if we write it as $\nabla_{A}{}^{\tilde{M}} = \tilde{E}_{A}{}^{\tilde{N}}\nabla^{\tilde{M}}_{\tilde{N}}$, it can be obtained from the recursion relations

$$\left(\delta_{\tilde{M}}{}^{\gamma}\delta_{\gamma}{}^{\tilde{L}} + \delta_{\tilde{M}}{}^{\tilde{L}}\Theta^{\beta}\frac{\partial}{\partial\Theta^{\beta}}\right)\nabla'_{\tilde{L}}{}^{\tilde{N}} = -T_{\tilde{M}}{}^{\tilde{L}}\nabla_{\tilde{L}}{}^{\tilde{N}} - \nabla'_{\tilde{M}}{}^{\gamma}\nabla'_{\gamma}{}^{\tilde{N}} - (-1)^{m}\Theta^{\alpha_1}\nabla_{\tilde{M}\alpha_1}{}^{\gamma}\nabla_{\gamma}{}^{\tilde{N}} \tag{A15}$$

and a similar one for $\nabla_{AB}{}^{C}$. The vielbein $\nabla_{\tilde{M}}{}^{A}$, i.e., $\nabla_{\tilde{M}}{}^{A}\nabla_{A}{}^{\tilde{N}} = \delta^{\tilde{N}}_{\tilde{M}}$, transforms as

$$\delta_{\xi}\nabla_{\tilde{M}}{}^{A} = \partial_{\tilde{M}}\hat{\eta}^{\tilde{N}}_{\xi}\nabla_{\tilde{N}}{}^{A} + \hat{\eta}^{\tilde{N}}_{\xi}\partial_{\tilde{N}}\nabla_{\tilde{M}}{}^{A} - \nabla_{\tilde{M}}{}^{B}\hat{\eta}_{\xi B}{}^{A} \tag{A16}$$

and to second order is given by

$$\nabla_{m}{}^{B} = E_{m}{}^{B} + \Theta^{\gamma}(T_{\gamma m}{}^{B} + \phi_{m\gamma}{}^{B}) + \frac{1}{2}\Theta^{\gamma}\Theta^{\delta}(-R_{m\delta\gamma}{}^{B} + \mathcal{D}_{\delta}T_{\gamma m}{}^{B} + T_{m\delta}{}^{A}T_{A\gamma}{}^{B} - \phi_{m\delta}{}^{\beta}T_{\gamma\beta}{}^{B}) + \cdots$$

$$\nabla_{\alpha}{}^{B} = \delta_{\alpha}{}^{B} + \frac{1}{2}\Theta^{\gamma}T_{\gamma\alpha}{}^{B} + \frac{1}{6}\Theta^{\gamma}\Theta^{\delta}(-R_{\delta\alpha\gamma}{}^{B} + 2\mathcal{D}_{\delta}T_{\gamma\alpha}{}^{B} + T_{\delta\alpha}{}^{D}T_{D\gamma}{}^{B}) + \cdots \tag{A17}$$

As in ordinary supergravity, the superdeterminant of the vielbein is an invariant density

$$\mathcal{E} = \mathrm{Sdet}(\nabla_{\tilde{M}}{}^{A}) \equiv \det(\nabla_{m}{}^{a} - \nabla_{m}{}^{\beta}\nabla^{(-1)}{}_{\beta}{}^{\gamma}\nabla_{\gamma}{}^{a})/\det(\nabla_{\alpha}{}^{\beta}), \quad \text{A18}$$

which transforms as

$$\delta_{\xi}\mathcal{E} = (-1)^{m}\partial_{\tilde{M}}(\eta^{\tilde{M}}_{\xi}\mathcal{E}), \tag{A19}$$

and the superspace integral of the product of the invariant density with any Lorentz invariant superfield will be by construction invariant under supergravity transformations.

Therefore, local supersymmetry can be formulated in the new superspace in a geometrical way, with the only difference that now the transformation parameters are field dependent, depending on components of the torsion and curvature and their covariant derivatives, subject to the Bianchi identities. This formulation is manifestly covariant in the framework of the highly redundant superspace (z,Θ). However, as in the transformations, (A7), (A10), (A16) and (A19), there are no derivatives of the old θ^{μ} variables, they can be set to zero without loss of generality.

APPENDIX B: POWER SERIES ANSATZ

Equation (31) can be solved by a power series ansatz. Let us set $V(T) = \sum_{l\geq 0} v_l T^l$ and $f(T) = \sum_{l\geq 0} f_l T^l$; then

$$V(T) = \sum_{l\geq 0}\left[\sum_{m=0}^{l+2} m(l-m+2)f_m f_{l-m+2}T^l - \sum_{m=0}^{l} f_m f_{l-m}\right]T^l; \tag{B1}$$

that is

$$v_l = 2(l+1)f_1 f_{l+1} + \sum_{m=2}^{l} m(l-m+2)f_m f_{l-m+2} - \sum_{m=0}^{l} f_m f_{l-m}, \tag{B2}$$

which can be solved as follows. If $f_1 = \pm\sqrt{v_0 + f_0^2}$ does not vanish, then for $l > 1$, f_{l+1} can be obtained in terms of v_l and f_l,

$$\begin{aligned} f_2 &= \tfrac{1}{4f_1}(2f_0f_1 + v_1),\\ f_3 &= \tfrac{1}{6f_1}(f_1^2 + 2f_0f_2 - 4f_2^2 + v_2),\\ f_4 &= \tfrac{1}{6f_1}(2f_1f_2 + 2f_0f_3 - 12f_2f_3 + v_3).\\ &\vdots \end{aligned} \tag{B3}$$

This solution depends on the free parameter f_0 and in general is singular in f_1. For example, in the case of the exponential potential, $V(T) = e^{2\kappa T}$, it can be verified that Eq. (B3) coincides with $f(T) = \frac{1}{\sqrt{\kappa^2-1}}e^{\kappa T}$, with $f_0^2 = 1/(\kappa^2 - 1)$. Further, in the singular case when $f_1 = 0$, which corresponds to $f_0 = \pm\sqrt{-v_0}$, we see from the first equation of Eq. (B3) that there are solutions only if $v_1 = 0$. In this case we get

$$\begin{aligned} f_2 &= \tfrac{1}{4}\left(f_0 \pm \sqrt{f_0^2 + 4v_2}\right),\\ f_3 &= -\tfrac{v_3}{2(f_0-6f_2)},\\ f_4 &= \tfrac{1}{2(f_0-8f_2)}(-f_2^2 + 9f_3^2 - v_4).\\ &\vdots \end{aligned} \tag{B4}$$

The square roots in these solutions can lead to imaginary terms, similarly to the "imaginary mass" of the tachyon. Such problems can be avoided if the integration constant f_0 is suitably chosen, as can be seen for the potential $V(T) = v_2T^2 + v_4T^4$. In this case $f_1 = f_0$ and $f_0(2f_2 - f_0) = 0$, and if we choose $f_0 = 0$, then $f_1 = 0$, $f_2 = \pm\frac{1}{2}\sqrt{v_2}$, $f_3 = \frac{1}{12f_2}v_3$, $f_4 = \frac{1}{16f_2}(f_2^2 - 9f_3^2 + v_4)$, $f_5 = \frac{1}{20f_2}(f_2f_3 - 24f_3f_4 + v_5)$, etc. This is the situation of Eq. (32), where $v_2 < 0$ and $v_4 = -v_2/4$; hence, f_2 becomes imaginary, and $f_l = 0$ for $l > 3$, as in Eq. (33). However, if we keep $f_0 \neq 0$, then $f_1 = f_0$, and from Eq. (B3) we get another solution that, setting $v_2 = -m^2/2$, is given by an infinite series:

$$\begin{aligned} f(T) = f_0\Bigg[&1 + T + \frac{1}{2}T^2 + \frac{1}{3!}\left(1 - \frac{m^2}{2f_0^2}\right)T^3\\ &+ \frac{1}{4!}\left(1 + \frac{m^2}{f_0^2}\right)T^4 + \frac{1}{5!}\left(1 - \frac{m^2}{2f_0^2} - \frac{3m^4}{4f_0^4}\right)T^5 + \cdots\Bigg]. \end{aligned} \tag{B5}$$

APPENDIX C: POTENTIAL (38)

We are interested in potentials fulfilling the type of requirements of Sen's conjectures. We start from a superpotential of the form $f(T) = \exp[-nT/2(a + bT + cT^2)]$; i.e., it rolls down to zero when $T \to \infty$. By means of Eq. (31), we compute the corresponding scalar potential, which has the form $V(T) = \exp(-nT)[\alpha_0 + \alpha_1T + \alpha_2T^2 + \alpha_3T^3 + \alpha_4T^4]$. Imposing the conditions that $V(0) = V_0 > 0$ and $V'(0) = 0$, we get

$$f_\pm(T) = e^{-\frac{1}{2}(nx)}\left[a + \frac{1}{2}\left(an \pm 2\sqrt{a^2 + V_0}\right)x + \frac{an^2\sqrt{a^2+V_0} + 4a\sqrt{a^2+V_0} \pm 4n(a^2+V_0)}{8\sqrt{a^2+V_0}}x^2\right]. \tag{C1}$$

If we choose $f_-(T)$ (it would be similar for f_+), we get for the parameters of $V(T)$:

$$\begin{aligned} \alpha_0 &= V_0,\\ \alpha_1 &= nV_0,\\ \alpha_2 &= \begin{cases} \frac{n^2V_0^2}{a^2+V_0} + \frac{1}{8}an^3\sqrt{a^2+V_0} + \frac{2a^2n^2V_0}{a^2+V_0} + \frac{5}{2}an\sqrt{a^2+V_0}\\ -\frac{anV_0}{\sqrt{a^2+V_0}} - \frac{7a^2n^2}{4} - a^2 + \frac{a^4n^2}{a^2+V_0} - \frac{a^3n}{\sqrt{a^2+V_0}} - \frac{5n^2V_0}{4} - V_0, \end{cases}\\ \alpha_3 &= \begin{cases} -nV_0 + \frac{n^3V_0}{4} - \frac{n^3V_0^2}{2(a^2+V_0)} - \frac{1}{16}an^4\sqrt{a^2+V_0} + \frac{an^4V_0}{8\sqrt{a^2+V_0}} - \frac{a^2n^3V_0}{a^2+V_0}\\ +\frac{3an^2V_0}{2\sqrt{a^2+V_0}} + a\sqrt{a^2+V_0} - 2a^2n - \frac{a^4n^3}{2(a^2+V_0)} + \frac{a^3n^4}{8\sqrt{a^2+V_0}} + \frac{3a^3n^2}{2\sqrt{a^2+V_0}}, \end{cases}\\ \alpha_4 &= \begin{cases} \frac{n^4V_0^2}{16(a^2+V_0)} - \frac{n^2V_0^2}{4(a^2+V_0)} - \frac{an^5V_0}{32\sqrt{a^2+V_0}} + \frac{a^2n^4V_0}{8(a^2+V_0)} - \frac{a^2n^2V_0}{2(a^2+V_0)} + \frac{anV_0}{2\sqrt{a^2+V_0}} + \frac{a^2n^6}{256}\\ +\frac{a^2n^4}{64} - \frac{a^2n^2}{16} - \frac{a^2}{4} + \frac{a^4n^4}{16(a^2+V_0)} - \frac{a^4n^2}{4(a^2+V_0)} - \frac{a^3n^5}{32\sqrt{a^2+V_0}} + \frac{a^3n}{2\sqrt{a^2+V_0}}. \end{cases} \end{aligned} \tag{C2}$$

Now we look for a potential of the form of Fig. 2, so we must have $n > 0$. We require also that $V''(0) < 0$, and in addition, for convenience, we set $\alpha_i > 0$, resulting in the following constraints for the parameters a, n, and V_0:

$$n > 2,\ a > 0,\ \frac{a^2n^6 - 12a^2n^4 + 48a^2n^2 - 64a^2}{36n^4 + 96n^2 + 64} < V_0 < \frac{a^2n^4 - 8a^2n^2 + 16a^2}{16n^2}.$$

Within this rank are located the potentials with profiles like the one in Fig. 2.

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